It is interesting to note that even when we increase our kernel strength by a factor of 5 to get the output Pomeranchuk pole at 1.0, the average multiplicity is only 1.04 ln$s$ [see (4.3e)]. Our model seems to indicate that the average multiplicity for N-N scattering is closer to 1 ln$s$ than to 2 ln$s$ (see Ref. 15).

Note added in proof. The Michigan experiment of Ref. 15 has now been completed. They obtained an average multiplicity per inelastic collision to be 1.14 ln$s$. We wish to thank Dr. Donald E. Lyon for informing us of this result before publication.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Geoffrey F. Chew for suggesting this problem and for his continued guidance and encouragement. I would also like to thank Dr. G. Marchesini, Dr. Dale R. Snider, and Peter D. Ting for numerous discussions and suggestions.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Multichannel Model of the $K^*(890)$

DAVID C. CAREY*
Department of Physics, The City College of the City University of New York, New York, New York 10031
(Received 13 February 1970)

The inelastic channels $K^*\pi$ and K are included in a three-channel model of the $I=\frac{1}{2}$ p-wave $K\pi$ amplitude. Feynman diagrams for vector-meson exchange are used as input to multichannel N/D equations, which are solved to obtain the scattering amplitude. Coupling constants which cannot be measured experimentally or calculated using $SU(3)$ are obtained by employing $SU(6)_W$ in the rest system of one of the particles at a given vertex. The $K^*\pi$ channel, neglected in previous calculations, is observed to influence the amplitude strongly, but the width of the $K^*(890)$ is calculated to be 210 MeV or about four times the experimental value.

I. INTRODUCTION

COMPARATIVELY little work has been done on the effect of inelastic channels on the parameters of resonances in the K_π system. Those channels which lie lowest and should therefore be considered first are $K^*\pi$ and $K\eta$ with threshold energies of 1030 and 1042 MeV, respectively. The $K\eta$ channel has been included in a two-channel model of the p-wave $K\pi$ system in an article by Fulco, Shaw, and Wong.[1] Aside from this latter and an article by Gupta, Saxena, and Mathur[2] which presents the results of a single-channel calculation of s-, p-, and d-wave $K\pi$ phase shifts, the literature on the $K\pi$ interaction seems relatively sparse.

By contrast, multichannel models of the $\pi\pi$ interaction have been studied in great detail. Two-channel models of the ρ meson which have been studied include a treatment of the $\pi\pi$-$\pi\omega$ system by Zemach and Zachariasen,[3] and of the $\pi\pi$-$K\bar{K}$ system by Balázs.[4] A complete comparison of the several possibilities $\pi\pi$-$\pi\omega$, $\pi\pi$-$K\bar{K}$, and $\pi\pi$-$\pi\omega$-$K\bar{K}$ was also performed by Fulco, Shaw, and Wong.

There are, however, difficulties in a calculation of the p-wave $K\pi$ interaction using $K^*\pi$ as an inelastic channel which are not encountered in an analysis in which $K\eta$ is the only inelastic channel or in any of the multichannel $\pi\pi$ calculations mentioned. In the latter cases, all coupling constants may be determined either directly from experimental measurements of decay rates or indirectly by $SU(3)$. In a model of the $K\pi$ interaction which includes $K^*\pi$ as an inelastic channel, one encounters coupling constants which must be evaluated either by appeal to higher symmetries or by making assumptions about ϕ-ω mixing in the vector-meson octet. We have chosen the former approach and evaluate such coupling constants by using $SU(6)_W$ in the rest system of one of the particles at a given vertex. The model discussed in this paper is a three-channel one with $K^*\pi$ and $K\eta$ taken as inelastic channels.

The single-particle exchange contributions are calculated as Feynman diagrams and are used as input into a multichannel N/D equation. The diagrams used and their partial-wave analysis are discussed in Sec. II. In Sec. III we describe in greater detail the N/D formalism used and the exact method of solution employed. The results of the calculation are summarized in Sec. IV.

II. INPUT DIAGRAMS

We use as input terms the set of diagrams shown in Fig. 1. These consist of all permissible t- or u-channel exchanges of pseudoscalar or vector mesons. Since we

* Present address: National Accelerator Laboratory, Batavia, Ill. 60510.
[1] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).
[2] K. C. Gupta, R. P. Saxena, and V. S. Mathur, Phys. Rev. **141**, 1479 (1966).
[3] C. Zemach and F. Zachariasen, Phys. Rev. **128**, 849 (1962).
[4] L. A. P. Balázs, Phys. Rev. **137**, B168 (1965).

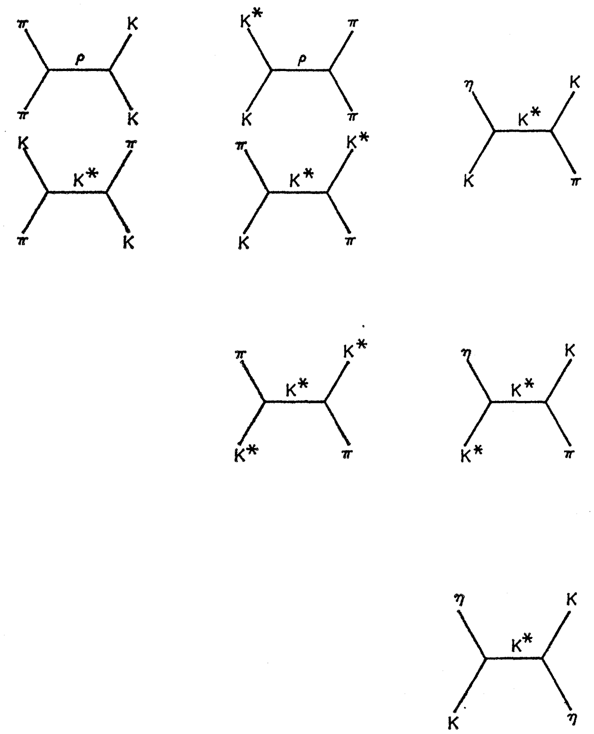

FIG. 1. Input diagrams. Channel 1 is $K\pi$, channel 2 is $K^*\pi$, and channel 3 is $K\eta$.

are considering only P-P or P-V states in the direct channel, we appeal to crossing symmetry by ruling out diagrams which contain such states in the t or u channel. The only allowed pseudoscalar-meson ex-

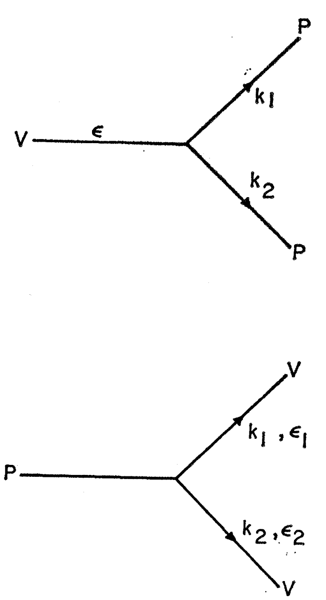

FIG. 2. Vertex diagrams used. The upper vertex is for $V\rightarrow PP$ and the lower for $P\rightarrow VV$. The k's represent the momenta of the particles and the ϵ's the polarization vectors.

changes are energetically possible in the s channel and their effect is therefore taken to be included via the requirement of unitarity.

The form of the vertex functions shown in Fig. 2 for $V\rightarrow PP$ is taken to be

$$g_{VPP}\epsilon_\mu(\lambda)(k_1-k_2)^\mu$$

and for $P\rightarrow VV$ is taken to be

$$i(f_{PVV}/m_P)\epsilon^{\mu\lambda\nu\sigma}k_{1\mu}k_{2\lambda}\epsilon_{1\nu}\epsilon_{2\sigma}.$$

The k's represent the momenta of either the pseudoscalar or vector mesons, and the ϵ's with a single numerical subscript are the vector-meson polarization vectors. The ϵ with superscripts is the completely antisymmetric tensor and m_P is the pseudoscalar-meson mass. If we consider the rest frame of the pseudoscalar meson, then the latter vertex can be considered a collinear process and the coupling constant can be evaluated via $SU(6)_W$.[5] However, in order to conserve energy and momentum, we must take one of the vector mesons slightly off the mass shell. We assume that the coupling constant is independent of the masses of the particles and can then evaluate all relevant coupling constants as

$$\begin{aligned}2g_{\rho K\bar{K}}=(2\sqrt{\tfrac{2}{3}})g_{K^*K\pi}=g_{\pi K^*\bar{K}^*}&=-(\sqrt{\tfrac{2}{3}})g_{KK^*\rho}\\&=-\sqrt{3}g_{\eta K^*\bar{K}^*}=2(\sqrt{\tfrac{2}{3}})g_{K^*K\eta}=g_{\rho\pi\pi}.\end{aligned}$$

Following Sakurai,[6] we take $g_{\rho\pi\pi}/4\pi$ to be equal to 2.4, which corresponds to a ρ width of approximately 125 MeV.

In order to evaluate the diagrams involving vector mesons in initial or final states, we must project the amplitude onto parity-conserving helicity states. For the negative-parity P-wave state in which the $K^*(890)$ appears, the only possible parity-conserving helicity amplitude is $(1/\sqrt{2})[\epsilon(+)-\epsilon(-)]$ where the sign represents the helicity state of the vector meson. We project the PV channels in our amplitudes onto such helicity states and remove the kinematic singularities to arrive at amplitudes suitable for input into the N/D equations. Kinematic singularities for processes of the form $PV\leftrightarrow PP$ will be proportional to $W=\sqrt{s}$ and those for $PV\leftrightarrow PV$ will be proportional to s. Partial-wave projections of the amplitudes will yield further kinematic singularities of the form k^{2l+1}. If we remove all kinematic singularities and use isospin crossing matrices[7] to project onto $I=\frac{1}{2}$, $J=1$ P-wave states, we arrive at

$$\begin{aligned}B(K\pi\leftrightarrow K\pi)=&\frac{g_{\rho K\bar{K}}g_{\rho\pi\pi}}{4\pi}F_1(s,\mu_K,\mu_\pi,\mu_K,\mu_\pi,m_\rho)\\&-\frac{1}{3}\frac{g_{K^*K\pi}{}^2}{4\pi}F_1(s,\mu_k,\mu_\pi,\mu_\pi,\mu_K,m_{K^*}),\end{aligned}$$

[5] H. J. Lipkin and S. Meshkov, Phys. Rev. **143**, 1269 (1966).
[6] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966).
[7] D. E. Neville, Phys. Rev. **160**, 1375 (1967).

$$B(K\pi\leftrightarrow K^*\pi)=\frac{1}{2\pi\sqrt{3}}\frac{1}{\mu_K}g_{\rho\pi\pi}g_{K^*K\rho}F_2(s,\mu_K,\mu_\pi,\mu_\pi,m_{K^*},m_\rho)+\frac{1}{4\pi\sqrt{3}}\frac{1}{\mu_K}g_{\pi K^*\bar{K}^*}g_{K^*K\pi}\times F_2(s,\mu_K,\mu_\pi,\mu_\pi,m_{K^*},m_{K^*}),$$

$$B(K\pi\leftrightarrow K\eta)=\frac{g_{K^*K\eta}g_{K^*K\pi}}{4\pi}F_1(s,\mu_K,\mu_\pi,\mu_\eta,\mu_K,m_{K^*}),$$

$$B(K^*\pi\leftrightarrow K^*\pi)=\frac{1}{64\pi k^2 s}\frac{g_{\pi K^*\bar{K}^*}{}^2}{\mu_\pi{}^2}F_3(s,m_{K^*},\mu_\pi,m_{K^*}),$$

$$B(K^*\pi\leftrightarrow K\eta)=\frac{1}{4\pi}\frac{1}{\mu_\eta}g_{K^*K\pi}g_{\eta K^*\bar{K}^*}\times F_2(s,\mu_K,\mu_\eta,\mu_\pi,m_{K^*},m_{K^*}),$$

$$B(K\eta\leftrightarrow K\eta)=\frac{g_{K^*K\eta}g_{K^*K\pi}}{4\pi}F_1(s,\mu_K,\mu_\eta,\mu_\eta,\mu_K,m_{K^*}),$$

where

$$F_1(s,\mu_1,\mu_2,\mu_3,\mu_4,m)=\frac{1}{4k^2k'^2}\left(2s+m^2-\sum_i\mu_i{}^2+\frac{(\mu_1{}^2-\mu_3{}^2)(\mu_2{}^2-\mu_4{}^2)}{m^2}\right)\times\left(\frac{1}{2}x\ln\frac{x+1}{x-1}-1\right)$$

with

$$x=\left(s-\sum_i\mu_i{}^2+\frac{(\mu_1{}^2-\mu_2{}^2)(\mu_3{}^2-\mu_4{}^2)}{s}+2m^2\right)\Big/4kk',$$

$$F_2(s,\mu_1,\mu_2,\mu_3,m_1,m)=\frac{1}{4kk'}\left(\frac{1}{2}(1-x^2)\ln\frac{x+1}{x-1}+x\right),$$

with

$$x=\left[s-(\mu_1{}^2+\mu_2{}^2+\mu_3{}^2+m_1{}^2)+\frac{(\mu_1{}^2-\mu_2{}^2)(\mu_3{}^2-m_1{}^2)}{s}+2m^2\right]\Big/4kk',$$

$$F_3(s,m_1,m_2,m)=\frac{1}{8k^2s}\left(\frac{1}{2}b_0\ln\frac{x+1}{x-1}+b_1-\frac{1}{3}k^2\right),$$

with

$$b_0=a_0+b_1x,$$
$$b_1=a_1+a_2x+a_3x^2,$$
$$a_0=k^2+m_2{}^2,$$
$$a_1=2m_1{}^2+2m_2{}^2-2s+3k^2,$$
$$a_2=2m_1{}^2+m_2{}^2+3k^2,$$
$$a_3=k^2,$$
$$x=(s-2k^2-2m_1{}^2-2m_2{}^2+m^2)/2k^2.$$

III. METHOD OF SOLUTION

The requirement of unitarity for the relativistically invariant amplitudes with the kinematic singularities removed and the proper threshold behavior reads

$$\mathrm{Im}(T_{ij}{}^{-1})=-\delta_{ij}\theta(s-s_i)\theta(\Lambda^2-s)\rho_i(s),$$

where s_i is the threshold for the ith channel and Λ is the cutoff used in solving the equations. The phase-space factor is given by

$$\rho_i=k^3/\sqrt{s}\quad\text{for } i=1,3$$
$$=k^3\sqrt{s}\quad\text{for } i=2.$$

If we employ the Uretsky form[8] of the N/D equations, we require that

$$T_{ij}=(ND^{-1})_{ij},$$

where N is given by the integral equation

$$N_{ij}(s)=B_{ij}(s)+\frac{1}{\pi}\sum_{i=1}^{3}\int_{s_i}^{\Lambda}ds'\,K_{ik}(s,s')N_{kj}(s'),$$

and the kernel K is calculated using the Born terms as

$$K_{ij}(s)=\left(\frac{1}{s'-s}\right)\left[B_{ij}(s')-\left(\frac{s-s_0}{s'-s_0}\right)B_{ij}(s)\right]\rho_i(s').$$

The denominator term D is given in terms of N via

$$D_{ij}(s)=\delta_{ij}-\left(\frac{s-s_0}{\pi}\right)\int_{s_i'}^{\Lambda}ds'\frac{\rho_i(s')N_{ij}(s')}{(s'-s_0)(s'-s)}.$$

The actual numerical calculation of N was done using the matrix inversion method of Fulco, Shaw, and Wong.[1] The N function was then integrated to give the real part of D. The imaginary part of D is just the phase-space factor multiplied by N. The total cross section was then calculated from the imaginary part of the invariant amplitude and the cutoff adjusted so that the the peak corresponded to the mass of the $K^*(890)$.

The appendix to the Fulco, Shaw, and Wong paper is quite complete, but a few additional remarks might be in order. The expressions for the Born terms given above are not entirely appropriate for use in the integral equations over the entire energy range considered. Below the upper threshold for an inelastic diagram and close to a threshold for any diagram, it is convenient to expand the kinematic singularity-free Born terms in a power series in the variable labeled above as x. The singularities then appear more explicitly and analytic continuation of the terms below threshold is possible. Also, the grid employed to solve the integral equations is invariably much too coarse to show the fine structure of the resonance. The actual values of the N and ReD were obtained by Lagrange interpolation on the values at nearby grid points. These functions vary fairly smoothly near resonance points while the phase-space

[8] J. L. Uretsky, Phys. Rev. **123**, 1459 (1961).

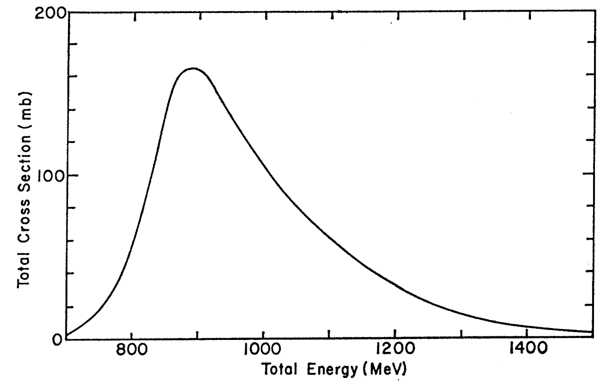

FIG. 3. Total cross section for $I=\frac{1}{2}$ p-wave $K\pi$ scattering versus center-of-mass energy.

factor is calculated explicitly. The amplitude, which shows more rapid variation, is obtained by taking the appropriate quotient.

IV. RESULTS

A graph of the total cross section as a function of energy is presented in Fig. 3. A cutoff of $3.87m_\pi$ gives a peak at 890 MeV. The general shape of the curve is similar to previous results. The cutoff is noticeably lower, but the width is still on the same order as previous calculations using N/D. We derive a value of about 210 MeV which is comparable to the results of Fulco, Shaw, and Wong and about four times the experimental value. One is led to doubt whether the inclusion of further channels will significantly improve the situation and that the defect is inherent in N/D.

The large change in the cutoff from previous results conclusively demonstrates the strong influence of the $K^*\pi$ channel on the $K\pi$ amplitude, and future multichannel models should include it. This conclusion and the simple method for calculating the coupling constants for inclusion of such channels seem to be the major results of this work.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor Ngee-Pong Chang, Professor Michael Arons, and Professor Gordon Kane for helpful discussions and encouragement.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Forward Proton Compton Scattering and Continuous Dispersion Sum Rules*

YU-CHIEN LIU AND IAN J. MCGEE
Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada
(Received 6 March 1970)

The new invariant amplitudes of Bardeen and Tung for nucleon Compton scattering, which are free of both kinematic singularities and zeros, are examined. The forward scattering amplitude, and the continuous-dispersion sum rules derived therefrom, are obtained. Using the data of a recent calculation by Damashek and Gilman, tests of these sum rules are shown to be quite satisfactory, indicating the validity of the dispersion relation, the good parametrization of the forward proton Compton scattering amplitude, and the presence of a $J=0$ fixed pole within the accuracy of present experiment.

I. INTRODUCTION

HISTORICALLY, dispersion relations as applied to particle physics were first derived and analyzed by Gell-Mann, Goldberger, and Thirring[1] in 1954. Owing to kinematical complexity and experimental unfeasibility, a full-scale analysis[2] of nucleon Compton scattering was not available until a recent effort in the accurate measurement of the unpolarized total photoabsorption cross section.[3] This permitted a calculation of the real part[3] of the spin-averaged forward amplitude from threshold to 20 GeV, although the comparison of such a calculation with experiment has yet to be done.[4]

The form of the invariant amplitudes for (nucleon) Compton scattering was first investigated over ten years ago. In 1958, Prange[5] wrote down six invariant amplitudes based on the principles of Lorentz, gauge, parity, and charge-conjugation invariance. They were

* Work supported in part by the National Research Council of Canada.
[1] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954).
[2] See the references quoted in R. Köberle, Phys. Rev. **166**, 1558 (1968). Also P. S. Baranov, L. V. Fil'kov, and G. A. Sokol, Fortschr. Physik **16**, 595 (1968); G. C. Fox and D. Z. Freedman, Phys. Rev. **182**, 1628 (1969).
[3] M. Damashek and F. J. Gilman, Phys. Rev. D **1**, 1319 (1970).
[4] S. J. Brodsky, A. C. Hearn, and R. G. Parsons, Phys. Rev. **187**, 1899 (1969).
[5] R. E. Prange, Phys. Rev. **110**, 240 (1958). Actually he dealt with electron Compton scattering.