Intermediate Vector Mesons, Unitary Symmetry, and the CP Violation*

YUKIO TOMOZAWA
Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan 48104
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We present a model of the weak-interaction Lagrangian based on SU_3 symmetry and the assumed existence of intermediate vector mesons (W mesons). The requirement that the effective Lagrangian resulting from contraction of the W mesons should induce no $|\Delta S|=2$ transition in the lowest order leads us to assume complex values for some of the coupling constants, which implies that the model is intrinsically CP violating. The effects of the CP violation will be found in processes of W-meson production or those involving higher orders in the Fermi coupling constant G_F. As far as the K^0-$\bar{K}^0$ system is concerned, the prediction of the model is similar to that of a superweak theory, although in this model the observed CP violation is due to a combined effect of the higher-order weak interactions and the other CP-conserving but SU_3-symmetry-breaking interactions (such as the electromagnetic *and* the medium-strong interactions). Some other possibilities for the cause of CP violation are also discussed, in which case the prediction is not necessarily the same as that of a superweak theory. The Cabibbo angle is defined as the ratio between the coupling constants of the SU_3-symmetric and symmetry-breaking hadronic terms of the weak interactions.

I. INTRODUCTION

THE simplest assumption on which we may derive the observed vectorial nature of the weak-interaction currents[1] and universality of the coupling constants in leptonic processes is the existence of the intermediate W mesons that couple with the hadronic as well as the leptonic currents. After a decade of various investigations in this direction,[2-5] we have yet to understand the meaning of the Cabibbo angle,[6,7] the isospin selection rules[8] of decay processes, and the observed CP violation in the K^0-$\bar{K}^0$ system.[9] In this paper we present a model which sheds some light on these problems.

One of the distinctive characteristics of the strong and electromagnetic interactions is the SU_3-symmetry property. These interactions consist of the SU_3-invariant term and symmetry-breaking terms that transform as octet representations of the group. The isospin selection rules of weak decay processes indicate that the weak interactions also obey SU_3 symmetry and transform in a simple manner. Therefore, we will assume that the currents and the W mesons should transform according to the SU_3 group. A natural choice is the octet or nonet. (We will not consider the possibility of triplet W mesons as has been discussed in the literature,[3] since it corresponds to the U_3 group.)

In Sec. II, the model which introduces the CP-violating phase in a natural way is presented. Various possibilities for explaining the observed CP violation in the K^0-$\bar{K}^0$ system are discussed in Sec. III.

II. MODEL

The interaction Lagrangian which was suggested in Sec. I may be expressed as follows:

$$-L_W=g_1\,\mathrm{Tr}(JW)+g_2[(JW)_3{}^2e^{i\chi}+(WJ)_2{}^3e^{-i\chi}]+g_3[(JW)_2{}^3e^{-i\varphi}+(WJ)_3{}^2e^{i\varphi}]+g_l(lW_2{}^1+l^\dagger W_1{}^2)\,, \quad (1)$$

where $(J_\lambda)_\beta{}^\alpha$ ($\alpha,\beta=1,2,3$) is a nonet (or an octet) of hadronic currents defined by

$$(J_\lambda)_\beta{}^\alpha=(V_\lambda)_\beta{}^\alpha+(A_\lambda)_\beta{}^\alpha\,, \quad (2)$$

$(V_\lambda)_\beta{}^\alpha$ and $(A_\lambda)_\beta{}^\alpha$ being the vector and axial-vector currents, and l_λ is the charged leptonic current:

$$l_\lambda=\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e+\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu\,. \quad (3)$$

The nonet W mesons $(W_\lambda)_\beta{}^\alpha$ are described by the 3×3 SU_3 matrix

$$W=\hat{W}+(1/\sqrt{3})W_\eta{}'\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}, \quad (4)$$

where

$$\hat{W}=\begin{bmatrix}W_{\pi}{}^0/\sqrt{2}+W_\eta/\sqrt{6} & W_{\pi^-} & W_{K^-}\\ W_{\pi^+} & -W_{\pi}{}^0/\sqrt{2}+W_\eta/\sqrt{6} & W_{\bar{K}^0}\\ W_{K^+} & W_{K^0} & -(2/\sqrt{6})W_\eta\end{bmatrix} \quad (5)$$

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[6] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

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[8] See, e.g., T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965); **16**, 471 (1966); **17**, 513 (1967).

[9] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turley, Phys. Rev. Letters **13**, 138 (1964).

stands for a traceless octet matrix. The suffixes which denote pseudoscalar mesons in Eqs. (4) and (5) indicate the relevant quantum numbers. A similar structure is assumed for the hadronic currents. The space-time indices will be omitted whenever no confusion arises.

In Eq. (1), the first term is SU_3 symmetric, while the rest transform as octet members (assuming that the leptonic current is an SU_3 singlet). All the coupling constants g_i ($i=1, 2, 3$) and g_l are real, and the complex phases in the second and third term are inserted for a reason that will be given later. Note that complex phases which may be attached to the leptonic terms have no physical consequences, since they could be absorbed in the definition of the leptonic fields.

We first consider the case where the W mesons form a nonet. (The currents may belong either to a nonet or an octet.) The free Lagrangian of the massive W mesons is assumed to be SU_3 symmetric, and the effective Lagrangian $L_{\rm eff}$ for weak processes with low momentum transfer will be derived by contracting the W mesons in Eq. (1). In doing so, we require the following phenomenological selection rules to be satisfied.

(I) There are no $|\Delta S|=2$ transitions in lowest order.

(II) The universality of the leptonic interactions in the sense of Gell-Mann, Lévy, and Cabibbo.[6,7]

The effective Lagrangian in lowest order, then, reads

$$\begin{aligned}-M_W{}^2L_{\rm eff}{}^{(2)}=&\tfrac{1}{2}g_1{}^2\,{\rm Tr}(JJ)+g_2{}^2(JJ)_2{}^2+g_3{}^2(JJ)_3{}^3\\&+g_l{}^2l^\dagger l+2g_2g_3J_2{}^2J_3{}^3\cos(\varphi-\chi)\\&+[\tfrac{1}{2}(g_2{}^2e^{2i\chi}+g_3{}^2e^{2i\varphi})J_3{}^2J_3{}^2\\&+g_1(g_2e^{i\chi}+g_3e^{i\varphi})(JJ)_3{}^2\\&+g_ll(g_1J_2{}^1+g_2e^{i\varphi}J_3{}^1)+{\rm H.c.}],\end{aligned}\tag{6}$$

where M_W is the mass of the W meson.

In carrying out the contraction, all the mesons $W_\beta{}^\alpha$ can be handled as if they are independent particles, because of the assumption of nonet symmetry. This will be the case neither for the octet W meson nor if there is a mass splitting among the W-meson multiplets.

As can be immediately observed, Eq. (6) contains a term $J_3{}^2J_3{}^2$ that induces the $|\Delta S|=2$ transition and therefore violates the requirement (I). As a result, we have to demand that its coefficient be zero; thus we obtain

$$g_2=g_3\tag{7}$$

and

$$e^{2i\chi}+e^{2i\varphi}=0\,,\tag{8a}$$

i.e.,

$$\chi-\varphi=\pm\tfrac{1}{2}\pi\,({\rm mod}\;2\pi)\,.\tag{8b}$$

If we use the definition

$$g_1=g\cos\theta\,,\quad g_2=g_3=g\sin\theta\,,\tag{9}$$

the requirement (II) leads to the condition

$$g_l=g.\tag{10}$$

Therefore, using the relations (7)–(10), we rewrite the interaction and the effective Lagrangian as

$$\begin{aligned}-L_W=g\{&{\rm Tr}(JW)\cos\theta+[((JW)_3{}^2e^{\pm\pi i/2}+(WJ)_3{}^2)\\&\times\sin\theta\,e^{i\varphi}+lW_2{}^1+{\rm H.c.}]\}\end{aligned}\tag{1'}$$

and

$$\begin{aligned}-L_{\rm eff}{}^{(2)}=(G_F/\sqrt{2})\{&\tfrac{1}{2}\,{\rm Tr}(JJ)(1+\sin^2\theta)\\&-(JJ)_1{}^1\sin^2\theta+l^\dagger l\\&+[\sqrt{2}(JJ)_3{}^2\sin\theta\cos\theta e^{i\varphi\pm\pi i/4}\\&+l(J_2{}^1\cos\theta+J_3{}^1\sin\theta\,e^{i\varphi})+{\rm H.c.}]\}\,,\end{aligned}\tag{6'}$$

where

$$G_F/\sqrt{2}=g^2/M_W{}^2$$

denotes the Fermi coupling constant ($G_F=10^{-5}M_N{}^{-2}$, M_N being the nucleon mass).

If the W mesons had belonged to an octet representation, a similar consideration would have led to the Lagrangian (see the Appendix)

$$\begin{aligned}-L_W=g\{&{\rm Tr}(J\hat{W})\cos\theta\\&+[((J\hat{W})_3{}^2e^{\pm i\pi/3}+(\hat{W}J)_3{}^2)\sin\theta\,e^{i\varphi}\\&+l\hat{W}_2{}^1+{\rm H.c.}]\}\end{aligned}\tag{11}$$

and

$$\begin{aligned}-L_{\rm eff}{}^{(2)}=(G_F/\sqrt{2})\{&\tfrac{1}{2}[{\rm Tr}(JJ)-\tfrac{1}{3}({\rm Tr}J)^2]\\&+[\tfrac{2}{3}({\rm Tr}J)^2-J_1{}^1\,{\rm Tr}J]\sin^2\theta+l^\dagger l\\&+[\sqrt{3}((JJ)_3{}^2-\tfrac{1}{3}J_3{}^2\,{\rm Tr}J)\sin\theta\cos\theta\,e^{i\varphi\pm\pi i/6}\\&+l(J_2{}^1\cos\theta+J_3{}^1\sin\theta\,e^{i\varphi})+{\rm H.c.}]\}\,,\end{aligned}\tag{12}$$

where we may set ${\rm Tr}J=0$ if the currents are assumed to form an octet.[10]

The Lagrangians for the two cases, i.e., Eqs. (1′) and (6′) for the nonet and Eqs. (11) and (12) for the octet W mesons, differ slightly from each other, but have many features in common. By inspecting their forms we see the following.

(1) All observed isospin selection rules in weak decay processes ($\Delta I=\frac{1}{2}$ rule for strangeness-changing decay and $\Delta I=1$ rule for strangeness-conserving decay), as well as the $\Delta S=\Delta Q$ rule for semileptonic decay, are satisfied. We assume that the $\Delta I=\frac{3}{2}$ transition, occuring, for example, in the process $K^+\to\pi^++\pi^0$, is due to the electromagnetic corrections to our Lagrangians.[11]

(2) The strangeness-changing hadronic terms in the $L_{\rm eff}{}^{(2)}$'s (members of an octet in both cases) have different coefficients, namely, $\sqrt{2}$ for the nonet and $\sqrt{3}$ for the octet W mesons. If a reliable method of estimating the effects of the strong-interaction corrections to the weak-interaction vertex could be established, then we would be able to determine[12] the number of W mesons in the basic Lagrangian.

[10] If the nonet current is decomposed into an octet $\hat{J}$ and a singlet J_0, $(J)_\beta{}^\alpha=(\hat{J})_\beta{}^\alpha+(1/\sqrt{3})J_0\delta_{\alpha\beta}$, we can use the following identities in order to simplify the terms in Eq. (12): ${\rm Tr}(JJ)-\frac{1}{3}({\rm Tr}J)^2={\rm Tr}(\hat{J}\hat{J})$, $\frac{2}{3}({\rm Tr}J)^2-J_1{}^1({\rm Tr}J)=-\sqrt{3}J_0\hat{J}_1{}^1+(J_0)^2$, and $(JJ)_3{}^2-\frac{1}{3}J_3{}^2({\rm Tr}J)=(\hat{J}\hat{J})_3{}^2+(1/\sqrt{3})J_0J_3{}^2$.

[11] See, e.g., S. Okubo, R. E. Marshak, and V. S. Mathur, Phys. Rev. Letters 19, 407 (1967).

(3) The Cabibbo angle θ is defined by the ratio between the coupling constants of the SU_3-symmetric and symmetry-breaking hadronic term in the interaction Lagrangian.

(4) There is no **27** component in the $L_{\text{eff}}^{(2)}$, and the symmetry-breaking hadronic terms with $\Delta S=0$ in $L_{\text{eff}}^{(2)}$ that transform as octets are multiplied by a factor $\sin^2\theta \approx \frac{1}{16}$. The latter feature, common to the Cabibbo Lagrangian,[7] is consistent with the recent analysis of the observed circular polarization of nuclear γ rays,[13] in contradistinction to the theory based on triplet W mesons.[3,13]

(5) No CP violation can be observed in the lowest order of G_F for processes in which the W meson is not produced. This is because the complex phases in $L_{\text{eff}}^{(2)}$ appear as an over-all phase factor for each process.[14]

(6) CP violation will be detected in processes with real production of the $W_3{}^2$ and $W_2{}^3$ mesons in the lowest order of

$$g=(5\sqrt{2})^{1/2}(M_W/M_N)\times 10^{-3}. \tag{13}$$

In order to see this, we point out that the $W_3{}^2=W_{\bar{K}^0}$ meson is coupled to the current

$$g[J_2{}^3\cos\theta+(J_2{}^2e^{i\rho}+J_3{}^3)\sin\theta\, e^{i\varphi}], \tag{14}$$

where

$$\begin{aligned}\rho&=\pm\tfrac{1}{2}\pi \quad \text{for the nonet } W \text{ mesons}\\ &=\pm\tfrac{1}{3}\pi \quad \text{for the octet } W \text{ mesons,}\end{aligned} \tag{15}$$

and hence the second term, which has matrix elements for the $\Delta S=\Delta I=0$ transition, contains the observable CP-violating phase ρ.

(7) For the mass of the W meson ≈ 10 GeV, we obtain $g\approx 2.8\times 10^{-2}$. It would not be impossible to observe the W meson in the near future if this is the case. A typical lifetime for the process

$$W_{\bar{K}^0}\to K^-+\pi^+,\ \pi^-+\pi^+,\ \text{or } K^-+K^+,\ \text{etc.,} \tag{16}$$

would then be of the order of

$$\tau\approx(g^2M_W)^{-1}\approx 10^{-22}\ \text{sec}. \tag{17}$$

A possible way to detect the neutral W meson would be, e.g., through the reaction

$$p+p\to p+p+W_{\bar{K}^0} \searrow K^-+\pi^+\ \text{etc.,} \tag{18}$$

i.e., apparently strangeness-nonconserving processes. For the charged W meson, weak production by neutrinos is also possible, as has been discussed extensively in the literature.[8]

(8) In order to observe CP or T violation in W_{K^0}-meson production, we have to look into the $\Delta S=0$, $\Delta I=0$ decays, as can be seen from Eq. (14). An example of such a decay is given by

$$p+p\to p+\Sigma^++W_{K^0} \searrow p+\bar{p}\ \text{or}\ \Sigma^++\bar{\Sigma}^+. \tag{19}$$

If we neglect the phase shift due to final-state interactions, the measurement of transverse polarization of the final particle, viz.,

$$\boldsymbol{\sigma}\cdot\langle \mathbf{J}_{||}\rangle\times\mathbf{p}, \tag{20}$$

would be evidence against time-reversal invariance. Here $\boldsymbol{\sigma}$ and $\mathbf{p}$ denote the spin operator and the momentum of one of the final baryons in the rest system of the W meson, and $\langle\mathbf{J}_{||}\rangle$ represents the average polarization of the W meson in the direction of its momentum. The actual situation may be more complicated, because of the mixing of all neutral W mesons.

(9) CP violation will be observed to order $G_F{}^2$ for $|\Delta S|=1$ transitions and to order $G_F{}^3$ for transitions with $|\Delta S|=0$ or 2. This conclusion follows from the fact that leptonic term in $L_{\text{eff}}^{(2)}$ induces a term with $\Delta S=1$ in $(L_{\text{eff}}^{(2)})^2$ such as

$$\tfrac{1}{2}G_F{}^2(l^\dagger J_1{}^2)(lJ_3{}^1)\sin\theta\cos\theta\, e^{i\varphi}, \tag{21}$$

while other terms with $\Delta S=1$ in $L_{\text{eff}}^{(2)}$ or $(L_{\text{eff}}^{(2)})^2$ have the common phase $e^{i\rho/2+i\varphi}$, so that the phase difference $\frac{1}{2}\rho$ induces a CP violation. The statement concerning $\Delta S=0$ or 2 transitions can be derived as a result of interference between the aforementioned two kinds of terms with mismatched phases, in the higher-order contribution.

(10) As corollaries: (a) The electric dipole moment will be of the order of $G_F{}^3$. (b) There is no CP violation to order $G_F{}^2$ for the K^0-$\bar{K}^0$ mixture, according to the Lagrangians (1′) and (6′) or (11) and (12). It appears in the order $G_F{}^3$.

III. CP VIOLATION IN K^0-$\bar{K}^0$ SYSTEM

We notice that the Lagrangians (6′) or (12) do not give CP violation in any order of G_F if the leptonic interactions are neglected. This is because the CP-violating phase $\frac{1}{2}\rho$ can be removed by choosing the arbitrary phase $\varphi=-\frac{1}{2}\rho$ or by defining the phase of the SU_3 bases in a suitable way [see item (5) of the discussion in Sec. II]. Moreover, any interactions which do not involve the W mesons, such as the SU_3 symmetry-breaking medium-strong interactions (MSI), will not change this conclusion.[15]

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[12] Y. Tomozawa (unpublished); I. Kimel (unpublished).

[13] V. M. Lobashov *et al.*, Zh. Eksperim. i Teor. Fiz. Pis'ma Redaktsiyu **5**, 73 (1967) [Soviet Physics—JETP Letters **5**, 59 (1967)]; B. H. J. McKellar, Phys. Rev. Letters **20**, 1542 (1968); **21**, 1822 (1968), and the references in these articles.

[14] The model proposed by L. Wolfenstein [Phys. Letters **15**, 196 (1965)] has a structure similar to that of the effective Lagrangian in the present article. The author is indebted to Professor R. R. Lewis for drawing his attention to this reference.

[15] In earlier papers [Y. Tomozawa, Phys. Letters **30B**, 543 (1969); in *Proceedings of the Coral Gables Conference on Fundamental Interactions at High Energies University of Miami, 1970*, edited by A. Perlmutter *et al.* (Gordon and Breach, New York, 1970), p. 319], the possibility that the SU_3 symmetry-breaking MSI, along with Lagrangians (1′) or (11), might cause the CP-violating effect has been considered. The statement made there about the MSI is incorrect.

In the following, we will consider various possibilities for the explanation of the observed CP violation in the K^0-$\bar{K}^0$ system, which is of the order of $10^{-3}\,G_F{}^2$ in the mass matrix.

A. Electromagnetic Interactions and MSI

The electromagnetic interaction induces a mass shift among members of the W mesons, which, in turn, may modify the effective Lagrangian (6′) or (12). In particular, the mass splitting between the $W_2{}^2$ and $W_3{}^3$ for the nonet W mesons, and between the W_η and W_{π^0} for the octet, requires an additional term in $L_{\rm eff}{}^{(2)}$,

$$-\delta L_{\rm eff}{}^{(2)}=\frac{G}{\sqrt{2}}\frac{\Delta M^2}{M_W{}^2}J_3{}^2J_3{}^2\sin^2\theta\, e^{2i\varphi}, \tag{22}$$

where

$$\Delta M^2=\tfrac{1}{2}(M_{W_2{}^2}{}^2-M_{W_3{}^3}{}^2)\quad\text{for the nonet } W \text{ mesons} \tag{23}$$

or

$$\Delta M^2=\tfrac{1}{4}(W_\eta{}^2-W_{\pi^0}{}^2)\quad\text{for the octet } W \text{ mesons.} \tag{24}$$

The simplest diagram producing a mass splitting of ΔM^2 is given in Fig. 1. Note that since the electromagnetic interaction transforms as an SU_3 tensor $T_1{}^1$, the SU_3-breaking MSI must be incorporated in order to create a mass difference between the $W_2{}^2$ and $W_3{}^3$ mesons. For the octet W-meson model, too, the inclusion of the MSI, e.g., through the magnetic moments of the hadrons, is considered favorable to obtain a significant contribution to the mass splitting of the form of $\Delta M^2\propto M_W{}^2$. To be more definite, we may estimate the mass splitting as

$$\Delta M^2\approx r\alpha g^4 M_W{}^2 \tag{25}$$

from dimensional considerations, where r stands for the SU_3-breaking effect of order, say, $\frac{1}{5}$, and $\alpha=e^2/4\pi$, the fine-structure constant. We may have an extra factor such as $(1/\pi)1/(4\pi)^2$ in Eq. (25). We have neglected a possible cutoff dependence of ΔM^2, since there is an indication that the cutoff due to the strong interactions may be small.

With these preparations, we now compare the term in Eq. (22) with that of the $\Delta S=2$ transition in $(L_{\rm eff}{}^{(2)})^2$. They have a definite relative phase $e^{i\rho}$ which can cause CP violation through the mass matrix.[16] The model is, therefore, equivalent to a superweak theory[17] as far as the K^0-$\bar{K}^0$ system is concerned, and the CP-violation parameter ϵ is given by

$$\epsilon=-\frac{\mathrm{Im}M_{12}}{\frac{1}{2}\gamma_S-i\Delta M_{LS}}\approx-\frac{\sqrt{2}\,\mathrm{Im}M_{12}}{\gamma_S}e^{\pi i/4}, \tag{26}$$

where M_{12} represents the off-diagonal element of the mass matrix, and γ_S and ΔM_{LS} are the decay rate of the short-lived neutral kaon and the mass difference of the two neutral kaons, respectively.

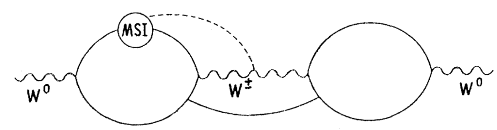

FIG. 1. Diagram which gives the mass splitting of the $W_2{}^2$ and $W_3{}^3$ mesons. The solid, wavy, and dashed lines represent hadrons, W mesons, and photons, respectively.

A crude estimate of $\mathrm{Im}M_{12}$ will be made by taking the vacuum intermediate state,

$$\begin{aligned}\mathrm{Im}M_{12}&=\langle K^0|(G_F/\sqrt{2})(\Delta M^2/M_W{}^2)(J_\mu)_3{}^2(J_\mu)_3{}^2\sin^2\theta\sin\rho|\bar{K}^0\rangle\\&=(G_F/\sqrt{2})r\alpha g^4\sin^2\theta\sin\rho\langle K^0|(J_\mu)_3{}^2|0\rangle\langle 0|(J_\mu)_3{}^2|\bar{K}^0\rangle\\&=-(G_F{}^3/\sqrt{2})M_W{}^4r\alpha\sin^2\theta\sin\rho\tfrac{1}{2}(f_K{}^2M_K).\end{aligned} \tag{27}$$

Here f_K denotes the $K\to\mu\nu$ decay amplitude ($f_K=1.28f_\pi=1.2M_\pi$). From Eqs. (26) and (27) and the experimental data,[18] $\gamma_S=0.81\times10^{-14}M_N$, we obtain

$$\epsilon\approx4.6\times10^{-8}(M_W/M_N)^4\,e^{\pi i/4}. \tag{28}$$

Identifying ϵ with the observed parameter η_{+-},

$$|\epsilon|=|\eta_{+-}|=|\eta_{00}|=2\times10^{-3}, \tag{29}$$

gives an estimate of the W-meson mass M_W:

$$M_W\approx14M_N. \tag{30}$$

Needless to say, this is merely an order-of-magnitude estimate. If, for example, a factor $(1/\pi)(1/4\pi)^2$ had been inserted in Eq. (25), we would have obtained

$$M_W\approx66M_N.$$

In summary:

(11) The incorporation of the electromagnetic interaction, the SU_3-breaking MSI and CP-violating weak interaction (1′) or (11) leads to a model equivalent to a superweak theory. The observed CP-violating parameter ϵ can be obtained by the mass value of the W mesons in the range 10–100 GeV.

B. Effect of Lepton Loop

The contraction of the lepton loop in the expression (21) introduces an additional term in the effective Lagrangian

$$-\delta_l L_{\rm eff}=\frac{G_F}{\sqrt{2}}\frac{G_F\Lambda_l{}^2}{4\pi}J_1{}^2J_3{}^1\sin\theta\cos\theta\,e^{i\varphi}, \tag{31}$$

where Λ_l is the cutoff associated with the lepton-loop integration. Lacking a damping effect as in the strong interaction, the appearance of the cutoff Λ_l in Eq. (31) cannot be avoided. The CP violation which results from the interference between the term (31) and the $\Delta S=1$

[16] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).
[17] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).
[18] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **42**, 87 (1970).

term in $L_{\text{eff}}^{(2)}$ will be significant if

$$\frac{G_F\Lambda_l^2}{4\pi}=\frac{10^{-5}}{4\pi}\left(\frac{\Lambda_l}{M_N}\right)^2\approx|\epsilon|\,, \tag{32}$$

i.e.,

$$\Lambda_l/M_N\approx 50\,. \tag{33}$$

If this is the case, the model predicts a violation of CP which is different from a superweak theory; since the $\Delta I=\frac{3}{2}$ transition amplitude allowed by Eq. (31) can contribute to the decay matrix $\text{Im}\Gamma_{12}$, the CP-violating parameter

$$\epsilon=\frac{i\,\text{Im}\Gamma_{12}-\text{Im}M_{12}}{\frac{1}{2}\gamma_S-i\Delta M_{LS}} \tag{34}$$

will have a phase different from that required by a superweak theory. The relation (29) is also violated.

This possibility as an explanation of the CP problem was discussed by Wolfenstein[14] some years ago. As for the upper bound for the lepton-loop cutoff Λ_l on the basis of various experimental data, we refer to a work of Ioffe,[19] although some of his arguments should be revised because of the discovery of the two neutrinos since then.

If the effects so far considered in this section are not sufficient to explain the observed CP violation, we must seek a new type of interaction which promotes such an effect. The following section contains an example.

C. Neutral Leptonic Current l_λ^0

Assume that the neutral leptonic current exists in L_W as

$$-L_W^{\text{NL}}=(\sqrt{r_C})g(l^0W_2^2)\,, \tag{35}$$

where $\sqrt{r_C}$ stands for the coupling strength relative to that of the charged leptonic term. By contracting the W mesons, we obtain an additional term in $L_{\text{eff}}^{(2)}$:

$$-L_{\text{eff}}^{\text{NL}}=(\sqrt{r_C})(G_F/\sqrt{2})l^0\{J_2^2\cos\theta+(J_3^2e^{i\varphi}+J_2^3e^{-i\varphi})\sin\theta\}\,. \tag{36}$$

Hence, the $\Delta S=2$ terms in $(L_{\text{eff}}^{(2)})^2$ are

$$\tfrac{1}{2}G_F^2[s(JJ)_3^2(JJ)_3^2e^{i\varphi}\cos^2\theta+r_C(l^0J_3^2)(l^0J_3^2)]\sin^2\theta\,e^{2i\varphi}\,, \tag{37}$$

where

$$\begin{aligned} s&=2 \quad \text{for the nonet } W \text{ mesons}\\ &=3 \quad \text{for the octet } W \text{ mesons.}\end{aligned} \tag{38}$$

Equation (37) creates a CP-violating effect through the off-diagonal elements of the mass matrix as well as the decay matrix. Therefore, the model gives the relation (29) with the CP-violating parameter

$$|\epsilon|\approx r_C(\Lambda_l/\Lambda_S)\,, \tag{39}$$

where Λ_S is the strong-interaction cutoff parameter. The phase of ϵ is determined by the relation (34), which does not lead to any clear-cut prediction, in contrast to the case presented in Sec. III A.

As for the magnitude r_C, the present experimental data on various neutral leptonic currents[18] give the following upper limits:

$$\begin{aligned}(r_C)_{\nu\bar\nu}&=\frac{R(K^\pm\to\pi^\pm\nu\bar\nu)}{R(K^\pm\to\pi^0e^\pm\nu)}<\frac{10^{-4}}{0.049}=2\times10^{-3}\,,\\ (r_C)_{e\bar e}&=\frac{R(K^\pm\to\pi^\pm e^+e^-)}{R(K^\pm\to\pi^0e^\pm\nu)}<\frac{0.4\times10^{-6}}{0.049}\\ &=0.8\times10^{-5}\,,\\ (r_C)_{\mu\bar\mu}&=\frac{R(K^\pm\to\pi^\pm\mu^+\mu^-)}{R(K^\pm\to\pi^0e^\pm\nu)}<\frac{2.4\times10^{-6}}{0.049}=4.9\times10^{-5}\,.\end{aligned} \tag{40}$$

While the possibility that a neutral leptonic current composed of $(\bar ee)$ or $(\bar\mu\mu)$ is responsible for the observed CP violation seems to be excluded, the possibility of a $(\bar\nu\nu)$ current has yet to be explored. However, the present experimental upper bound for the latter is just on the verge of producing the right size for the observed CP-violating effect[20] only if $\Lambda_l/\Lambda_S\approx 1$.

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APPENDIX: EFFECTIVE LAGRANGIAN FOR OCTET W MESONS

For the octet W mesons, we have to use the following rule for contraction of the diagonal elements of the octet matrix W:

$$\langle\hat W_1^1\hat W_1^1\rangle=\langle\hat W_2^2\hat W_2^2\rangle=\langle\hat W_3^3\hat W_3^3\rangle=\tfrac{2}{3}(1/M_W^2)$$

and

$$\langle\hat W_1^1\hat W_2^2\rangle=\langle\hat W_2^2\hat W_3^3\rangle=\langle\hat W_3^3\hat W_1^1\rangle=-\tfrac{1}{3}(1/M_W^2)\,. \tag{A1}$$

Starting with the interaction Lagrangian (1) in the text (except that W is replaced by $\hat W$), the contraction of the W mesons leads to

$$\begin{aligned}-M_W^2L_{\text{eff}}^{(2)}=&\tfrac{1}{2}g_1^2[\text{Tr}(JJ)-\tfrac{1}{3}(\text{Tr}J)_3^2]\\ &+g_2^2[(JJ)_2^2-\tfrac{1}{3}J_3^2J_2^3]\\ &+g_3^2[(JJ)_3^3-\tfrac{1}{3}J_2^3J_3^2]\\ &+2g_2g_3\cos(\varphi-\chi)(J_2^2J_3^3-\tfrac{1}{3}J_3^2J_2^3)\\ &+g_l^2l^\dagger l+\{\tfrac{1}{3}(g_2^2e^{2i\chi}+g_3^2e^{2i\chi}\\ &-g_2g_3e^{i(\varphi+\chi)})J_3^2J_3^2\\ &+g_1(g_2e^{i\chi}+g_3e^{i\varphi})[(JJ)_3^2-\tfrac{1}{3}J_3^2\,\text{Tr}J]\\ &+g_ll(g_1J_2^1+g_2J_3^1e^{i\varphi})+\text{H.c.}\}\,.\end{aligned} \tag{A2}$$

[19] B. L. Ioffe, Zh. Eksperim. i Teor. Fiz. **42**, 1411 (1962) [Soviet Phys. JETP **15**, 978 (1962)]; Zh. Eksperim. i Teor. Fiz. **38**, 1608 (1960) [Soviet Phys. JETP **11**, 1158 (1960)].

[20] For a possible link between a neutral leptonic current and the CP violation in a different context, see M. L. Good, L. Michel, and E. deRafael, Ref. 3; R. J. Oakes, Phys. Rev. Letters **20**, 1539 (1968).

Hence, from the requirement (I), we obtain the condition

$$g_2^2e^{2i\chi}+g_3^2e^{2i\varphi}-g_2g_3e^{i(\varphi+\chi)}=0\,, \qquad \text{(A3)}$$

which is equivalent to

$$g_2=g_3$$

and

$$\chi-\varphi=\pm\tfrac{1}{3}\pi \quad (\text{mod } 2\pi)\,, \qquad \text{(A4)}$$

while the requirement (II) leads to the same condition as before, Eq. (10). The corresponding Lagrangians are then given by Eqs. (11) and (12) in the text.

PHYSICAL REVIEW D VOLUME 2, NUMBER 8 15 OCTOBER 1970

Bootstrap Conditions from Unitarity at High Energy*

JEROME FINKELSTEIN†
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305
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The unitarity relation for the high-energy scattering of two particles is investigated under the assumption that the production amplitudes which enter the unitarity sum are given by a single Regge-pole exchange; the further assumption of multi-Regge exchange is not made. It is seen that the unitarity sum can be expressed as an integral involving the particle-Reggeon scattering amplitude (which may be defined in terms of the six-point function), and by representing this amplitude by Regge exchange, one obtains an equation which leads to self-consistency conditions on the Regge parameters. Solutions to this equation have, in addition to Regge poles, Regge cuts which interfere destructively with the poles. Comparisons are made with related models, in particular, the multi-Regge model. It is suggested that, while the Regge cuts calculated by the multi-Regge method are model dependent, the bootstrap condition on the Regge pole is more general.

I. INTRODUCTION

ONE of the more interesting applications of the multi-Regge model for production processes has been the derivation of equations from which the high-energy limit of elastic two-body processes may be studied. Chew, Goldberger, and Low[1] found that, by using the multi-Regge expression for the production matrix element as input into the unitarity sum for the absorptive part of the two-body process, they were able to obtain an integral equation for the two-body amplitude from which it followed that this amplitude would exhibit Regge behavior at high energy. They also pointed out that a bootstrap condition resulted from the requirement of consistency between the Regge behavior assumed in the input and that found in the output.[2]

In this paper we attempt to assess the implications for two-particle scattering of a model for production processes which may be somewhat more general than is the multi-Regge model. Nevertheless, the assumptions which we shall make do closely parallel some of those made in the multi-Regge calculation, and in fact we shall be able to use our model as a laboratory in which to test the generality of many of the results of the multi-Regge bootstrap.

In brief, our approach, which shall be more fully explained in Sec. II, is as follows: Our starting point is the assumption that production processes at high energy may be described in terms of a single Regge exchange, as is depicted in Fig. 1. By this assumption we do not mean to exclude the possibility that the vertex functions on the right-hand side of Fig. 1 may themselves Reggeize, as would be the case in the multi-Regge model; we merely require that there be Regge behavior where we have indicated. Our next step is to use this single-Regge expression for the production amplitudes as input in the unitarity sum for the absorptive part of the two-body elastic amplitude. We shall argue that in summing over the multiplicity of particles in the intermediate state, we must keep the number of particles emerging from one of the vertices fixed so as to avoid double counting. We

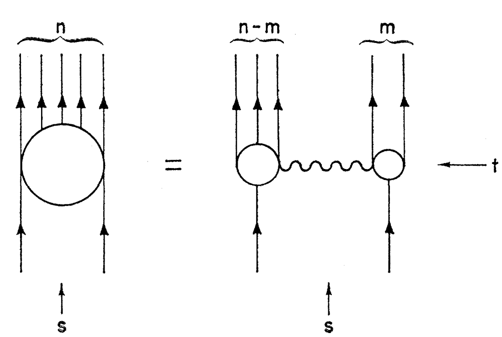

FIG. 1. Approximation of a production amplitude by single-Regge-pole exchange. Straight lines are particles, the wavy line is a Reggeon.

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† Alfred P. Sloan Foundation Fellow. Address 1970–71: CERN, Geneva, Switzerland.

[1] G. F. Chew, M. L. Goldberger, and F. Low, Phys. Rev. Letters **22**, 208 (1969).

[2] See also G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968).