Spin-Two Mesons, the Stress Tensor, and a Field-Source Identity. I*

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(Received 15 May 1969; revised manuscript received 7 January 1970)

It is suggested that the hypothesis that the matrix elements of the stress tensor are dominated by the neutral isoscalar spin-two mesons may be expressed by a field-source identity. This states that, to the lowest order in the gravitational constant κ, the traceless part of the complete stress-energy tensor is proportional to a linear combination of the renormalized field operators of the neutral, isoscalar spin-two mesons. It is shown how this identity may be realized in a Lagrangian field theory, in a manner analogous to the work of Kroll, Lee, and Zumino for the vector-meson dominance of the electromagnetic current. The conditions imposed by Lorentz covariance are discussed. The field-source identity determines the parts of the singular terms in the stress-tensor commutation relations which are the most singular in the coupling strength.

I. INTRODUCTION

THE hypothesis of a universal conserved vector current and the vector-meson dominance of the electromagnetic current has been very fruitful in the theory of the electromagnetic and weak interactions of the hadrons.[1,2] This idea was expressed by Kroll, Lee, and Zumino[3] in terms of a Lagrangian field theory; they exhibited a Lagrangian field theory in which the total electromagnetic current operator for the hadrons was, to a good approximation, identical with a linear combination of the renormalized field operators for the neutral vector mesons.

Since the discovery of spin-two mesons, it has been of interest to examine the parallel hypothesis that the matrix elements of the traceless part of the stress tensor are dominated by spin-two (tensor) mesons.[4] An additional hypothesis that led to interesting results was that the matrix elements of the trace of the energy-momentum tensor were dominated by scalar mesons.[5]

In this paper, we show how the idea of tensor-meson dominance of the matrix elements of the stress tensor can be expressed in terms of a Lagrangian field theory with a "field-source identity." Our work is carried out in a manner similar to that of Kroll, Lee, and Zumino.[3]

Just as the response of a system to an external electromagnetic field measures the electromagnetic current for the system, the response to an external gravitational field can be used for measuring the stress tensor of the system.[6] We therefore introduce the coupling of the gravitational field with the hadrons.

As is well known, a theory of the gravitational field is essentially nonlinear and must be described by a transcendental Lagrangian density, in order that it give the gravitational field equation with the stress tensor as source.[7] However, the small magnitude of the gravitational constant κ makes it meaningful to consider an expansion in κ. In this paper we shall make the approximation of treating the gravitational interaction to the lowest order in κ. To this order in κ, the free (linearized) gravitational field will be treated as a zero-mass spin-two field. For convenience, we shall often refer to this as the "graviton" field.

We now suggest that tensor-meson dominance may be expressed by the hypothesis, that to the lowest order in the gravitational constant κ, the irreducible spin-two part $\tilde{\Theta}_{\mu\nu}$ of the complete stress-tensor operator of the hadrons is identical to a linear combination of the irreducible spin-two parts $\tilde{U}_{\mu\nu}$ of the (renormalized) field operators of the neutral isoscalar spin-two mesons,[8] and that similarly, the trace of the (divergenceless) stress-tensor operator is a linear combination of the traces of the spin-two meson field operators. The complete

* Work supported in part by the U. S. Atomic Energy Commission (Report No. NYO-2262TA-204).
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[2] J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960).
[3] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).
[4] This idea was perhaps first exploited in detail by H. Pagels, University of North Carolina report (unpublished). Related ideas have been discussed by R. Delbourgo, A. Salam, and J. Strathdee, ICTP Report No. IC/66/15, Trieste (unpublished); and by W. Królikowski, Phys. Letters **24B**, 305 (1967). These authors discuss results obtained from a postulated relation between the matrix elements of the stress tensor and those of the 2^+ meson fields. In connection with the idea of the universality of the coupling to the Pomeranchuk pole, the matrix elements of the stress tensor were considered by P. G. O. Freund, *ibid.* **2**, 136 (1962).
[5] Recently, P. G. O. Freund and Y. Nambu [Phys. Rev. **174**, 1741 (1968)] have shown how the coupling of a scalar field to the trace of the stress tensor can be formulated in a Lagrangian theory with a nonpolynomial interaction Lagrangian.
[6] See, e.g., J. Schwinger, Phys. Rev. **130**, 406 (1962). The stress tensor $\Theta_{\mu\nu}$ can be defined in terms of the response of the system to an external gravitational field through the variational equation $\delta_g W=\frac{1}{2}\int d^4x(-g)^{1/2}\Theta_{\mu\nu}\delta g_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor and $g=\det g_{\mu\nu}$. W is the action integral and the variation $\delta_g W$ is induced by an infinitesimal coordinate transformation. Thus the stress tensor $\Theta_{\mu\nu}$ is proportional to the functional derivative of the action integral with respect to the metric: $\Theta_{\mu\nu}=2(-g)^{-1/2}\delta W/\delta g_{\mu\nu}$.
[7] See, for instance, S. N. Gupta, Rev. Mod. Phys. **29**, 334 (1957); Phys. Rev. **96**, 1682 (1954); P. G. O. Freund and Y. Nambu, *ibid.* **174**, 1741 (1968).
[8] We use the term "irreducible" in the group-theoretic sense. By the irreducible spin-two part of a symmetric tensor operator we mean the traceless, divergenceless part of this operator. Note that a symmetric tensor operator which has a nonzero trace and divergence is the sum of an irreducible spin-two operator, a part involving an irreducible (divergenceless) spin-one operator, and a part involving two scalar operators. For a symmetric, divergenceless operator such as the stress tensor, the spin-one part is absent, and the two scalar operators reduce to one.

stress-tensor operator $\Theta_{\mu\nu}$ is a linear combination of the divergenceless operators $(U_{\mu\nu}-\eta_{\mu\nu}Uvv)$ for the neutral spin-two mesons fields. These relations will be referred to collectively as the "field-source identity."

In this paper, we show how the tensor-meson dominance hypothesis can be formulated as a field-source identity in a Lagrangian theory; in this theory, the linearized gravitational equation (in an arbitrary gauge) is of the form

$$\mathfrak{D}_\lambda(\Gamma^\lambda{}_{\mu\nu})=-\kappa[\Theta_{\mu\nu}+\Theta_{\mu\nu}{}^{(l)}], \tag{1.1}$$

where

$$\mathfrak{D}_\lambda(\Gamma^\lambda{}_{\mu\nu})\equiv 2\partial_\lambda[\Gamma^\lambda{}_{\mu\nu}-\tfrac{1}{2}\delta^\lambda{}_\mu\Gamma^\alpha{}_{\nu\alpha} -\tfrac{1}{2}\delta^\lambda{}_\nu\Gamma^\alpha{}_{\mu\alpha}-\tfrac{1}{2}\eta_{\mu\nu}(\Gamma^\lambda{}_{\alpha\alpha}-\Gamma_\alpha{}^{\lambda\alpha})], \tag{1.2}$$

and $\Theta_{\mu\nu}$ is found to be of the form

$$\Theta_{\mu\nu}=\beta_f[U_{\mu\nu}{}^{(f)}-\eta_{\mu\nu}u^{(f)}]+\beta_{f'}[U_{\mu\nu}{}^{(f')}-\eta_{\mu\nu}u^{(f')}], \tag{1.3}$$

with $u\equiv U_\sigma{}^\sigma$.

Here, $\Gamma^\lambda{}_{\mu\nu}$ is the affinity (for the gravitational field), $\Theta_{\mu\nu}{}^{(l)}$ is the leptonic contribution to the stress tensor, and $\Theta_{\mu\nu}$ gives the hadronic contribution. $U_{\mu\nu}{}^{(f)}$ and $U_{\mu\nu}{}^{(f')}$ are the field operators for the neutral spin-two mesons f and f'.[9] $\eta_{\mu\nu}$ is the pseudo-Euclidean (Minkowski) metric.

Here we do not explore the consequences of the additional hypothesis that the matrix elements of the trace of the stress tensor may be dominated by scalar mesons. We hope to discuss elsewhere a theory with both tensor-meson dominance and scalar-meson dominance.

In Sec. II, we outline the description we use for the massive neutral spin-two meson fields and for the gravitational field. In Sec. III we formulate the condition on the (gauge-invariant) interaction Lagrangian that would lead to a field-source identity; this is done without examining the detailed structure of the strong-interaction Lagrangian. We briefly discuss the relations between the matrix elements of the stress tensor $\Theta_{\mu\nu}$ and the source tensors $J_{\mu\nu}$ for the neutral tensor mesons, that follow from the field-source identity. In Sec. IV we discuss the question of the renormalization of a single neutral tensor meson, considering only its strong interactions. We also briefly discuss the propagator of the unmixed tensor meson. In Sec. V we discuss the constraints imposed by Lorentz covariance on a theory with a field-source identity. In particular, we point out that the field-source identity determines the parts of the singular terms in the stress-tensor equal-time commutation relations which are the most singular in the coupling strength g. It further requires that some of the singular terms be q numbers.

In subsequent papers we shall discuss applications and further questions relating to the tensor-meson dominance hypothesis.

[9] We recall that the observed tensor mesons are the $A_2(1300)$, $K_T(1420)$, $f(1260)$, and $f'(1515)$, where the numbers give the approximate masses in MeV.

II. NEUTRAL MASSIVE SPIN-TWO MESON FIELD AND LINEARIZED GRAVITATIONAL FIELD

In this section we briefly outline the description we shall use of the massive spin-two meson field and the massless "graviton" field, i.e., the gravitational field in the linear approximation. The derivation of the field-source identity can be carried out in a class of theories of the spin-two meson interacting with the gravitational field.

In choosing a description of the free spin-two meson field, we shall impose the following requirements.

(a) The energy of the field should be positive definite.

(b) The canonical variables and the commutation relations should follow in a simple way from the Lagrangian. This will be true if the Lagrangian density is of the first order in the time derivative of the field variables; the action principle then allows a simple determination of a set of canonical variables.[10]

(c) It is convenient and elegant to use a theory of the massive spin-two meson field which goes over to a simple gauge-invariant theory of the linearized gravitational field in the limit of zero mass.

For this, we shall describe the gravitational field by the affinity and the metric tensor, and use the linearized form of the Palatini Lagrangian density.[11,12] For the massive spin-two meson field, we shall use a similar Lagrangian density, with a mass term added.[13] In this paper we shall treat the gravitational integraction to the lowest order in the gravitational constant κ.

The part of the Lagrangian density describing a neutral massive spin-two meson, interacting strongly with the hadrons, will be taken to be the following[14]:

$$\mathfrak{L}=\mathfrak{L}_0+\mathcal{G}(U^{\mu\nu}), \tag{2.1a}$$

$$\mathfrak{L}_0=(U^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}u)(2\partial_\rho\Pi^\rho{}_{\mu\nu}-\partial_\mu\Pi_\nu-\partial_\nu\Pi_\mu) +2(\eta^{\mu\nu}\Pi^\beta{}_{\alpha\mu}\Pi^\alpha{}_{\beta\nu}-\Pi_\alpha\tilde{\Pi}_\alpha)-\tfrac{1}{2}m^2(U^{\mu\nu}U_{\mu\nu}-u^2), \tag{2.1b}$$

where $\mathcal{G}(U^{\mu\nu})$ describes the strong interaction of the spin-two mesons.[15] Here

$$U_{\mu\nu}=U_{\nu\mu},\quad \Pi^\lambda{}_{\mu\nu}=\Pi^\lambda{}_{\nu\mu}, \tag{2.2}$$

and we have used the notation

$$u=U_\sigma{}^\sigma,\quad \Pi_\mu=\Pi^\alpha{}_{\alpha\mu},\quad \tilde{\Pi}^\mu=\Pi^\mu{}_{\alpha\alpha}. \tag{2.3}$$

[10] J. Schwinger, Phys. Rev. **82**, 914 (1951); **91**, 713 (1953).

[11] E. Schrödinger, *Space-Time Structure* (Cambridge U. P., Cambridge, England, 1963), p. 107.

[12] R. Arnowitt and S. Deser, Phys. Rev. **113**, 745 (1959).

[13] Such a Lagrangian theory of the massive spin-two meson field has been studied by S. J. Chang, Phys. Rev. **148**, 1259 (1966), and by S. Deser, J. Trubatch, and S. Trubatch, Can. J. Phys. **44**, 1715 (1968).

[14] The free Lagrangian $\mathfrak{L}_0$ leads to a positive-definite energy (although not a positive-definite energy density), as shown by S. J. Chang (see Ref. 13).

[15] If $g_TJ_{\mu\nu}\equiv\delta\mathcal{G}/\delta U^{\mu\nu}$ were independent of $U_{\mu\nu}$, then $\mathcal{G}$ would be just $g_TJ_{\mu\nu}U^{\mu\nu}$.

In (2.1), the terms arising from the gravitational interaction of the spin-two mesons have been omitted, so that (2.1) is correct to the zeroth order in the gravitational constant κ. $\eta^{\mu\nu}$ is the pseudo-Euclidean (Minkowski) metric $(1, -1, -1, -1)$.

The field equations obtained from (2.1) are the following:

$$\partial_\rho \Pi^\rho{}_{\mu\nu} - \tfrac{1}{2}(\partial_\mu \Pi_\nu + \partial_\nu \Pi_\mu) - \tfrac{1}{2}\eta_{\mu\nu}\partial_\lambda(\tilde{\Pi}_\lambda - \Pi_\lambda) = \tfrac{1}{2}m^2(U_{\mu\nu} - \eta_{\mu\nu}u) - \tfrac{1}{2}g_T J_{\mu\nu}, \quad (2.4)$$

$$(\Pi^\nu{}_\lambda{}^\mu - \tfrac{1}{2}\delta_\lambda{}^\mu \tilde{\Pi}^\nu) + (\Pi^\mu{}_\lambda{}^\nu - \tfrac{1}{2}\delta_\lambda{}^\nu \tilde{\Pi}^\nu) - \eta^{\mu\nu}\Pi_\lambda = \partial_\lambda W^{\mu\nu} - \tfrac{1}{2}\delta_\lambda{}^\mu(\partial_\sigma W^{\nu\sigma}) - \tfrac{1}{2}\delta_\lambda{}^\nu(\partial_\sigma W^{\mu\sigma}), \quad (2.5)$$

where

$$g_T J_{\mu\nu} = \delta \mathcal{G}(U^{\mu\nu})/\delta U^{\mu\nu} \quad (2.6a)$$

and

$$W^{\mu\nu} \equiv U^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}u. \quad (2.6b)$$

We shall assume that the tensor current $J^{\mu\nu}$ describing the hadronic interaction of the spin-two mesons, to zeroth order in the gravitational constant κ, is divergenceless to this order:

$$\partial_\mu J^{\mu\nu} = 0. \quad (2.7)$$

Equations (2.4)–(2.7) lead to the following equations:

$$\partial^\mu(U_{\mu\nu} - \eta_{\mu\nu}u) = 0, \quad (2.8)$$

$$\Pi^\lambda{}_{\mu\nu} = \tfrac{1}{2}(\partial_\mu U_{\lambda\nu} + \partial_\nu U_{\nu\mu} - \partial_\lambda U_{\mu\nu}), \quad (2.9)$$

$$u = (-g_T/3m^2)j, \quad j \equiv J_\sigma{}^\sigma, \quad (2.10)$$

$$(\Box^2 + m^2)U_{\mu\nu} - (\partial_\mu\partial_\nu + m^2\eta_{\mu\nu})u = g_T J_{\mu\nu}. \quad (2.11)$$

For the linearized free gravitational field, we shall take the linear approximation to the Palatini Lagrangian density[12]:

$$\mathcal{L}_{\text{grav}}{}^{(0)} = (h^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}h)\{2\partial_\rho \Gamma^0{}_{\mu\nu} - \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu\} + 2(\eta^{\mu\nu}\Gamma^\beta{}_{\alpha\mu}\Gamma^\alpha{}_{\beta\nu} - \Gamma_\alpha \tilde{\Gamma}_\alpha), \quad (2.12)$$

where $h^{\mu\nu}$ gives the deviation, to order κ, of the metric tensor $g^{\mu\nu}$ in the gravitational equations from a flat (pseudo-Euclidean) metric $\eta^{\mu\nu}$:

$$g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}, \quad (2.13)$$

$h = h_\alpha{}^\alpha$, and $\Gamma^\lambda{}_{\mu\nu}$ is the affinity. Γ_μ and $\tilde{\Gamma}_\mu$ are defined in terms of $\Gamma^\lambda{}_{\mu\nu}$ in a manner analogous to (2.3). κ is the gravitational constant. (2.12) gives the following free-field equations:

$$\partial_\alpha \Gamma^\alpha{}_{\mu\nu} - \tfrac{1}{2}\partial_\mu\partial_\nu h - \tfrac{1}{2}\eta_{\mu\nu}(\partial^\lambda\partial^\sigma h_{\lambda\sigma} - \Box^2 h) = 0, \quad (2.14)$$

$$\Gamma^\lambda{}_{\mu\nu} = \tfrac{1}{2}(\partial_\mu h_{\lambda\nu} - \partial_\nu h_{\lambda\mu} - \partial_\lambda h_{\mu\nu}). \quad (2.15)$$

The action integral and the field equations are invariant under the gauge transformation

$$h_{\mu\nu} \to h_{\mu\nu} + (\partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu), \quad (2.16)$$

$$\Gamma^\alpha{}_{\mu\nu} \to \Gamma^\alpha{}_{\mu\nu} + \partial_\mu\partial_\nu \Lambda_\alpha. \quad (2.17)$$

In terms of $h_{\mu\nu}$, the free-field equation may be written as

$$\partial_\mu\partial^\lambda h_{\lambda\nu} + \partial_\nu\partial^\lambda h_{\lambda\mu} - \partial_\mu\partial_\nu h - \Box^2 h_{\mu\nu} - \eta_{\mu\nu}(\partial^\lambda\partial^\sigma h_{\lambda\sigma} - \Box^2 h) = 0. \quad (2.18)$$

So far the gauge has been left arbitrary. In the Hilbert gauge,[16] defined by

$$\partial^\sigma(h_{\nu\sigma} - \tfrac{1}{2}\eta_{\nu\sigma}h) = 0, \quad (2.19)$$

the free-field equation takes the form

$$\Box^2(h_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}h) = 0, \quad (2.20)$$

which gives

$$\Box^2 h_{\mu\nu} = 0. \quad (2.21)$$

In this paper, however, we shall write all relations in a gauge-independent form.

III. INTERACTING FIELDS

For simplicity we first consider one neutral spin-two meson interacting with the gravitational field. The Lagrangian for the system will be written as

$$\mathcal{L} = \mathcal{L}_{\text{grav}} + \mathcal{L}_T + \mathcal{L}_{\text{int}}, \quad (3.1)$$

where $\mathcal{L}_T$, the free Lagrangian density for the spin-two meson field, is given by (2.1b), and $\mathcal{L}_{\text{grav}}$, the free Lagrangian density for the linearized gravitational field, is given by (2.12). The interaction Lagrangian density $\mathcal{L}_{\text{int}}$ will be written as

$$\mathcal{L}_{\text{int}} = r\kappa h_{\mu\nu}J^{\mu\nu} + \mathcal{L}_{\text{int}}', \quad (3.2)$$

where r is a parameter to be fixed later, and $J^{\mu\nu}$ is the source tensor describing the strong interaction of the neutral spin-two meson and is given by (2.6a). The part $\mathcal{L}_{\text{int}}'$ of the interaction Lagrangian density does not involve $h_{\mu\nu}$, but involves derivatives of $h_{\mu\nu}$; it is to be determined by the requirement that $\mathcal{L}_{\text{int}}$ be gauge invariant and that it should lead to a field-source identity expressing the source of the gravitational field as a linear function of the spin-two field operator $U_{\mu\nu}$.

These requirements do not specify $\mathcal{L}_{\text{int}}$ uniquely; they define a class of theories of the neutral tensor meson interacting with the gravitational field.

For simplicity, we shall first express the field source identity in terms of the tensor field $U_{\mu\nu}$ and its trace, rather than in terms of the irreducible part of $U_{\mu\nu}$. We denote the source of the gravitational field by $\Theta_{\mu\nu}$, and identify it with the total stress-tensor density, including that of the gravitational field as well as that of the matter fields.[17] Thus $\Theta_{\mu\nu}$ has zero divergence:

$$\partial_\mu \Theta_{\mu\nu} = 0. \quad (3.3)$$

[16] See, e.g., W. Thirring, Ann. Phys. (N.Y.) 16, 96 (1961).

[17] Note, however, that here we are considering $\Theta_{\mu\nu}$ only to the zeroth order in κ; to this order, the (divergenceless) stress tensor does not involve the terms arising from the gravitational field.

We shall here seek a field-source identity of the simplest form, such that $\Theta_{\mu\nu}$ is a linear function of $U_{\mu\nu}$ alone and does not depend on its derivatives. From (3.3) and (2.8) it then follows that we must seek a relation expressing $\Theta_{\mu\nu}$ as a multiple of $U_{\mu\nu}-\eta_{\mu\nu}U_\sigma{}^\sigma$.

It is clear that $\mathfrak{L}_{\rm int}'$ must be nonzero, in order that such a relation follow from the Lagrangian (3.1). The equations of motion for the linearized gravitational field $h_{\mu\nu}$ with interaction, that follow from (3.1), are (2.15) and the following:

$$\partial_\alpha\Gamma^\alpha{}_{\mu\nu}-\tfrac{1}{2}\partial_\mu\partial_\nu h-\tfrac{1}{2}\eta_{\mu\nu}[\partial^\lambda\partial^\sigma h_{\lambda\sigma}-\Box^2 h]=-\tfrac{1}{2}\kappa\Theta_{\mu\nu}, \quad (3.4)$$

where

$$\kappa\theta_{\mu\nu}=\frac{\delta\mathfrak{L}_{\rm int}}{\delta h_{\mu\nu}}=\frac{\partial\mathfrak{L}_{\rm int}}{\partial h_{\mu\nu}}-\partial_\lambda\left[\frac{\partial\mathfrak{L}_{\rm int}}{\partial h_{\mu\nu,\lambda}}\right], \quad (3.5)$$

if we assume that, to lowest order in κ, we can obtain the desired identity with an interaction Lagrangian density that involves only the first time derivative of $h_{\mu\nu}$.

We thus look for $\mathfrak{L}_{\rm int}$ satisfying

$$\frac{\delta\mathfrak{L}_{\rm int}}{\delta h_{\mu\nu}}\equiv\frac{\partial\mathfrak{L}_{\rm int}}{\partial h_{\mu\nu}}-\partial_\lambda\left(\frac{\partial\mathfrak{L}_{\rm int}}{\partial h_{\mu\nu,\lambda}}\right)=\bar\beta\kappa(U_{\mu\nu}-\eta_{\mu\nu}U_\sigma{}^\sigma), \quad (3.6)$$

where $\bar\beta$ is a constant to be determined.

From the equation of motion (2.4) for $U_{\mu\nu}$, we may write

$$[U_{\mu\nu}-\eta_{\mu\nu}U_\sigma{}^\sigma]=(g_T/m^2)J_{\mu\nu}+m^{-2}\mathfrak{D}(\Pi^\lambda{}_{\mu\nu}), \quad (3.7)$$

where

$$\mathfrak{D}(\Pi^\lambda{}_{\mu\nu})=2[\partial_\lambda\Pi^\lambda{}_{\mu\nu}-\tfrac{1}{2}(\partial_\mu\Pi_\nu+\partial_\nu\Pi_\mu)-\tfrac{1}{2}\eta_{\mu\nu}\partial_\mu(\tilde\Pi_\lambda-\Pi_\lambda)], \quad (3.8)$$

as on the left-hand side of (2.4).

From (3.6), (3.2), and (3.7), it now follows that[18]

$$rJ^{\mu\nu}-\kappa^{-1}\partial_\lambda\left[\frac{\partial\mathfrak{L}_{\rm int}'}{\partial h_{\mu\nu,\lambda}}\right]=\frac{\bar\beta g_T}{m^2}J^{\mu\nu}+\frac{\bar\beta}{m^2}\mathfrak{D}(\Pi^\lambda{}_{\mu\nu}). \quad (3.9)$$

We look for a simple solution, such that

$$\bar\beta=m^2r/g_T \quad (3.10)$$

and

$$\partial_\lambda\left(\frac{\partial\mathfrak{L}_{\rm int}'}{\partial h_{\mu\nu,\lambda}}\right)=-(\kappa\bar\beta/m^2)\mathfrak{D}(\Pi^\lambda{}_{\mu\nu}). \quad (3.11)$$

This gives

$$\partial\mathfrak{L}_{\rm int}'/\partial h^{\mu\nu,\lambda}=(-2\kappa\bar\beta/m^2)[\eta_{\lambda\rho}\Pi^\rho{}_{\mu\nu}-\tfrac{1}{2}(\eta_{\mu\lambda}\Pi_\nu+\eta_{\nu\lambda}\Pi_\mu)-\tfrac{1}{2}\eta_{\mu\nu}(\tilde\Pi_\lambda-\Pi_\lambda)]+\mathcal{g}_{\mu\nu\lambda}, \quad (3.12)$$

where $\mathcal{g}_{\mu\nu\lambda}$ is a function of $h_{\mu\nu,\lambda}$ with the properties

$$\mathcal{g}_{\mu\nu\lambda}=\mathcal{g}_{\nu\mu\lambda}, \quad \partial^\lambda\mathcal{g}_{\mu\nu\lambda}=0. \quad (3.13)$$

As $J^{\mu\nu}$ is divergenceless, $\mathfrak{L}_{\rm int}'$ by itself must be gauge invariant in the usual sense, that is, it must give a gauge-invariant contribution to the action integral. Equation (3.12) does not lead in a simple manner to a gauge-invariant expression for $\mathfrak{L}_{\rm int}'$. We therefore use the field equation (2.9) for $U_{\nu\mu}$ and $\Pi^\lambda{}_{\mu\nu}$, and obtain from Eq. (3.12) the following:

$$\partial\mathfrak{L}_{\rm int}'/\partial h_{\mu\nu,\lambda}=(\kappa\bar\beta/m^2)[U^{\mu\nu,\lambda}-U^{\lambda\mu,\nu}-U^{\lambda\nu,\mu}+\tfrac{1}{2}\eta^{\lambda\nu}U_\alpha{}^{\alpha,\mu}+\tfrac{1}{2}\eta^{\lambda\mu}U_\alpha{}^{\alpha,\nu}+\eta^{\mu\nu}(U_\alpha{}^{\lambda,\alpha}-U_\alpha{}^{\alpha,\lambda})]+\mathcal{g}^{\mu\nu\lambda}. \quad (3.14)$$

A simple gauge-invariant solution for $\mathfrak{L}_{\rm int}'$ is

$$\mathfrak{L}_{\rm int}'=(r\kappa/g_T)(h_{\mu\nu,\rho}U^{\mu\nu,\rho}-2h_{\mu\nu,\rho}U^{\rho\mu,\nu}+h_{\mu\sigma}{}^{,\sigma}U_\rho{}^{\rho,\mu}+h_\rho{}^\rho{}_{,\tau}U^{\tau\sigma}{}_{,\sigma}-h_\mu{}^{\mu,\rho}U_\nu{}^\nu{}_{,\rho}), \quad (3.15)$$

to the first order in κ. This is, of course, not unique; for instance, any term of the form $h_{\mu\nu,\lambda}\mathcal{g}^{\mu\nu\lambda}$ may be added, where $\mathcal{g}^{\mu\nu\lambda}$ has the properties (3.13)

With $\mathfrak{L}_{\rm int}$ given by (3.2) and (3.15), we thus obtain

$$\Theta_{\mu\nu}=(rm^2/g_T)(U_{\mu\nu}-\eta_{\mu\nu}U_\sigma{}^\sigma). \quad (3.16)$$

This may be checked directly using (3.5), which gives, to the lowest order in κ,

$$\Theta_{\mu\nu}=r\{J_{\mu\nu}-g_T{}^{-1}[\Box^2U_{\mu\nu}-\partial^\rho\partial_\nu U_{\mu\rho}-\partial^\rho\partial_\mu U_{\nu\rho}+\partial_\mu\partial_\nu U_\rho{}^\rho+\eta_{\mu\nu}(\partial^\rho\partial^\sigma U_{\rho\sigma}-\Box^2U_\rho{}^\rho)]\}. \quad (3.17)$$

On using Eqs. (2.8) and (2.11) for the massive spin-two meson field, this gives (3.16).

Equation (3.16) implies the relations

$$\Theta_{\mu\nu}{}^{(t)}=(rm^2/g_T)U_{kj}{}^{(t)}, \quad (3.18)$$

$$\Theta_\sigma{}^\sigma=-r(3m^2/g_T)U_\tau{}^\tau=rJ_\sigma{}^\sigma, \quad (3.19)$$

where the transverse part $U_{kj}{}^{(t)}$ of U_{kj},

$$U_{kj}{}^{(t)}=(U_{kj}-\tfrac{1}{3}\delta_{kj}U_{mm}), \quad k,j,m=1,2,3 \quad (3.20)$$

gives a set of independent dynamical variables for the spin-two meson field, and $U_\tau{}^\tau$ is the scalar part of the field $U_{\mu\nu}$. Similarly, we have defined

$$\Theta_{kj}{}^{(t)}=\Theta_{kj}-\tfrac{1}{3}\delta_{kj}\Theta_{mm}. \quad (3.21)$$

Note that for the free spin-two field $U_{\mu\nu}$, the scalar part $U_\tau{}^\tau$ is a redundant variable, while for the interacting part, it is related to the trace of the source tensor $J_{\mu\nu}$. In the present work, we do not relate $U_\tau{}^\tau$ to a physical scalar field.

The above discussion applies to a system in which there is just one neutral spin-two meson coupled to the gravitational field, with the internal quantum numbers of the vacuum. The normalization of $J_{\mu\nu}(x)$ may be chosen by fixing the scale of the coupling constant g_T such that the constant r in (3.2) or (3.19) is unity.[19]

[18] Note that, to the lowest order in κ, the terms in $\partial\mathfrak{L}/\partial h_{\mu\nu}$ that would arise from the dependence of $J_{\mu\nu}$ on the graviton field $h_{\lambda\rho}$ (that is, the terms arising from $\delta J_{\mu\nu}/\delta h_{\lambda\rho}$) do not contribute.

[19] C. A. Orzalesi, J. Sucher, and C. H. Woo [Phys. Rev. Letters **21**, 1550 (1968)] have suggested that for a local Hermitian operator $G_{\mu\nu}(x)$ obeying certain conditions, one must have $\int d^3x\, G_{\mu 0}(x)=cP_\mu$, where c is a constant and P_μ is the momentum operator of the system. If $J_{\mu\nu}(x)$ is a local operator obeying these conditions, then our normalization would correspond to choosing $\int d^3x\, J_{\mu\nu}(x)=P_0$, or $\int d^3x[J_{00}{}^{(f')}(x)\cos\theta_T+J_{00}{}^{(f)}(x)\sin\theta_T]=P_0$, for a theory with two spin-two mesons which can mix.

We then obtain

$$\Theta_{\mu\nu}=(m^2/g_T)(U_{\mu\nu}-\eta_{\mu\nu}U_\sigma{}^\sigma), \tag{3.22}$$

which is the field-source identity for the tensor field $U_{\mu\nu}$. It is the analog of the field-current identity for the vector field. Here we have derived the field-source identity without reference to the detailed structure of $J_{\mu\nu}$ or $\Theta_{\mu\nu}$, using the definition of $\Theta_{\mu\nu}$ in terms of the response of the system to an external gravitational field and using the assumptions of a divergenceless source tensor $J_{\mu\nu}$ for the massive spin-two meson field and a gauge-invariant interaction Lagrangian density $\mathcal{L}_{\rm int}$ satisfying the requirement (3.6).

We stress that the field-source identity (3.22) is a relation involving the interacting spin-two field $U^{\mu\nu}$; it can hold only when there is a nonzero coupling between the spin-two field and other matter fields (or a self-interaction of the spin-two field with itself). One cannot postulate a similar relation between the stress tensor and the free spin-two field, as this would be inconsistent with Poincaré invariance. This may be seen from the equal-time commutation relations (ETCR). Thus, Lorentz invariance requires the ETCR

$$i[\Theta_{00}(x),\Theta_{00}(y)]_{x_0=y_0}=[\Theta_{0l}(x)\partial_l+\Theta_{0l}(y)\partial_l]\delta(x-y), \tag{3.23}$$

while the canonical ETCR of the *free* spin-two fields leads to the ETCR

$$i[U_{00}(x)-\eta_{00}u(x),U_{00}(y)-\eta_{00}u(y)]_{x_0=y_0}=0. \tag{3.24}$$

That one cannot postulate an identity between $\Theta_{\mu\nu}$ and a combination of the free spin-two field operators is also evident from the way the identity was derived; it was necessary to have an additional term $\mathcal{L}_{\rm int}'$ in the Lagrangian, proportional to g_T^{-1}, which gave rise to the factor g_T^{-1} in the field source identity. The identity cannot be derived when $g_T=0$. The field-source identity thus expresses a property of a particular kind of interacting spin-two field theory.

The nature of the interaction $J_{\mu\nu}$ is constrained by the requirement of Lorentz covariance. The explicit nature of these constraints will be briefly discussed in Sec. V of the paper. These constraints must be satisfied in order that a theory with a field-source identity be Lorentz covariant.

So far we have not considered the internal symmetry properties of $U_{\mu\nu}(x)$, $J_{\mu\nu}(x)$, and $\Theta_{\mu\nu}(x)$. The discussion given above can be directly extended to take these into account and to include the interaction of more than one neutral spin-two meson with the gravitational field.

For simplicity we consider here a theory in which $\theta_{\mu\nu}$ is a singlet under SU_3; it then must be proportional to a linear combination of spin-two meson field operators which transforms as a singlet under SU_3.[20,21] We write equations of motion analogous to (2.4) and (2.5) for the f and f' mesons, with source terms $g_TJ_{\mu\nu}{}^{(f)}$ and $g_TJ_{\mu\nu}{}^{(f')}$, which we assume to be divergenceless. We take into account $f-f'$ mixing by taking the interaction Lagrangian to be

$$\mathcal{L}_{\rm int}=r[\kappa h_{\mu\nu}(J_{\mu\nu}{}^{(f')}\cos\theta_T+J_{\mu\nu}{}^{(f)}\sin\theta_T)]+\mathcal{L}_{\rm int}', \tag{3.25}$$

where $\mathcal{L}_{\rm int}'$ is obtained from (3.15) by replacing $U_{\mu\nu}$ by $U_{\mu\nu}{}^{(f')}\cos\theta_T+U_{\mu\nu}{}^{(f)}\sin\theta_T$. This combination transforms as a singlet under SU_3. Again we may normalize $J_{\mu\nu}(x)$ by choosing $r=1$.[19]

We now obtain the equation of motion for $h_{\mu\nu}$ to be (3.4), where $\Theta_{\mu\nu}$ is determined, using the equations of motion for $U_{\mu\nu}{}^{(f)}$ and $U_{\mu\nu}{}^{(f')}$, to be

$$\Theta_{\mu\nu}=(g_T)^{-1}\{m_{f'}{}^2[U_{\mu\nu}{}^{(f')}-\eta_{\mu\nu}U_\sigma{}^{\sigma(f')}]\cos\theta_T+m_f{}^2[U_{\mu\nu}{}^{(f)}-\eta_{\mu\nu}U_\sigma{}^{\sigma(f)}]\sin\theta_T\}. \tag{3.26}$$

The relation (3.26) is the field-source identity; it expresses the stress tensor as a linear combination of the renormalized field operators of the f and f' mesons.

With this identity, the matrix element of $\Theta_{\mu\nu}$ between two physical states A and B is related to the matrix elements of $J_{\mu\nu}{}^{(f)}$ and $J_{\mu\nu}{}^{(f')}$ between the same states. This relation is, in general, more complicated than the corresponding one for the vector mesons and the electromagnetic current, because the field operators $U_{\mu\nu}{}^{(f)}$ and $U_{\mu\nu}{}^{(f')}$ are, in general, neither traceless or divergenceless. This gives rise to additional terms, depending on the trace of $U_{\mu\nu}{}^{(f')}$ and $U_{\mu\nu}{}^{(f)}$, like those occurring on the left-hand side of (2.11).

When the source tensors $J_{\mu\nu}{}^{(f)}$ and $J_{\mu\nu}{}^{(f')}$ have zero trace as well as zero divergence, then $u(f)=U_\sigma{}^{\sigma(f)}=0$, $u_{(f')}=0$, and the equations of motion give

$$(\Box^2+m^2)U_{\mu\nu}=g_TJ_{\mu\nu} \tag{3.27}$$

for the f and the f'. The field-source identity now takes the simple form

$$\Theta_{\mu\nu}=(m_{f'}{}^2/g_T)(\cos\theta_T)U_{\mu\nu}{}^{(f')}+(m_f{}^2/g_T)(\sin\theta_T)U_{\mu\nu}{}^{(f)}. \tag{3.28}$$

In this case we have the following relation between the matrix elements of $\Theta_{\mu\nu}$ and $J_{\mu\nu}$:

$$\langle B|\Theta_{\mu\nu}(x)|A\rangle=[m_{f'}{}^2/(m_{f'}{}^2-q^2)]\cos\theta_T\langle B|J_{\mu\nu}{}^{(f')}(x)|A\rangle+[m_f{}^2/(m_f{}^2-q^2)]\sin\theta_T\langle B|J_{\mu\nu}{}^{(f)}(x)|A\rangle, \tag{3.29}$$

where $q=(p_B-p_A)$ is the four-momentum transfer between the states A and B. Expanding the matrix elements of $\Theta_{\mu\nu}$, $J_{\mu\nu}{}^{(f)}$, and $J_{\mu\nu}{}^{(f')}$ in terms of a set of covariants, we can obtain relations analogous to (3.29) for the corresponding invariant form factors. For instance, when A and B are spin-zero mesons of equal

[20] In general, in the presence of SU_3 and SU_2 symmetry breaking, the stress tensor can have parts that transform like the hypercharge component of an octet or like the third component of isospin. We shall discuss this elsewhere, while discussing applications of the field-source identity.

[21] It is also of interest to consider the results obtained for an octet or a nonet generalization of the stress tensor, such as that suggested by H. Pagels (Ref. 4).

mass, we write

$$\langle B|\Theta_{\mu\nu}(0)|A\rangle = [(q_\mu q_\nu - q^2\eta_{\mu\nu}) - (3q^2/P^2)P_\mu P_\nu]\Theta(q^2), \quad (3.30)$$

where $P=\frac{1}{2}(p_A+p_B)$, and obtain

$$\Theta(q^2) = \frac{m_{f'}^2}{m_{f'}^2-q^2}\cos\theta_T\, \mathfrak{F}_{(f')}(q^2) + \frac{m_f^2}{m_f^2-q^2}\sin\theta_T\, \mathfrak{F}_{(f)}(q^2), \quad (3.31)$$

where $\mathfrak{F}_{(f)}(q^2)$ and $\mathfrak{F}_{(f')}(q^2)$ are related to the matrix elements of $J_{\mu\nu}^{(f)}(x)$ and $J_{\mu\nu}^{(f')}(x)$ in a manner analogous to (3.30).

When $J_{\mu\nu}^{(f)}$ and $J_{\mu\nu}^{(f')}$ are divergenceless but not traceless, we have the equation

$$(\Box^2+m^2)[U_{\mu\nu}^{(f)} - \eta_{\mu\nu}u^{(f)}] = g_T[J_{\mu\nu}^{(f)} - (1/3m_f^2)(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box^2)j^{(f)}], \quad (3.32)$$

and a similar equation for the f'. Here $j^{(f)}=J_\sigma{}^{\sigma(f)}$. These give the relation

$$\langle B|\Theta_{\mu\nu}(x)|A\rangle = \frac{m_{f'}^2\cos\theta_T}{m_{f'}^2-q^2}\langle B|\hat{J}_{\mu\nu}^{(f')}(x)|A\rangle + \frac{m_f^2\sin\theta_T}{m_f^2-q^2}\langle B|\hat{J}_{\mu\nu}^{(f)}(x)|A\rangle, \quad (3.33)$$

where

$$\hat{J}_{\mu\nu}(x) = J_{\mu\nu}(x) - (1/3m^2)(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box^2)j = \bar{J}_{\mu\nu}(x) - \eta_{\mu\nu}\bar{J}_\sigma{}^\sigma(x) \quad (3.34)$$

and

$$\bar{J}_{\mu\nu}(x) = (\Box^2+m^2)U_{\mu\nu}(x). \quad (3.35)$$

Note that $\partial_\mu\hat{J}^{\mu\nu}(x)=0$. As an example, we again consider the matrix element between spin-zero (pseudoscalar) mesons A and B. We write the decomposition

$$\langle B|\Theta_{\mu\nu}(0)|A\rangle = R_{\mu\nu}\Theta_1(q^2) + S_{\mu\nu}\Theta_2(q^2), \quad (3.36)$$

where

$$R_{\mu\nu} = (q_\mu q_\nu - \eta_{\mu\nu}q^2), \quad (3.37)$$

$$S_{\mu\nu} = \frac{P_\mu P_\nu}{p^2} + \frac{(P\cdot q)}{q^2p^2}[(P\cdot q)\eta_{\mu\nu} - (P_\mu q_\nu + q_\mu P_\nu)]. \quad (3.38)$$

Note that $q^\mu R_{\mu\nu}=0$ and $q^\mu S_{\mu\nu}=0$.

For the vertex function coupling a tensor meson f to the pseudoscalar mesons A and B, we similarly define the form factors $F_i^{(f)}(q^2)$ by

$$\langle B|\hat{J}_{\mu\nu}^{(f)}(0)|A\rangle = R_{\mu\nu}F_1^{(f)}(q^2) + S_{\mu\nu}F_2^{(f)}(q^2) \quad (3.39)$$

$$\to [q_\mu q_\nu F_1^{(f)}(q^2) + (\eta_{\mu\nu} - 3P_\nu P_\mu/P^2)F_2^{(f)}(q^2)] \quad (3.40)$$

when $m_A=m_B$. A similar decomposition is written for $\langle B|\hat{J}_{\mu\nu}^{(f')}(0)|A\rangle$. $F_i^{(f)}(q^2)$ and $F_i^{(f')}(q^2)$ are assumed to be regular at $q^2=m_f^2$ and $q^2=m_{f'}^2$, respectively. Using (3.39), we obtain relations of the form (3.31) between the "energy-momentum structure form factors"[22] $\Theta_i(q^2)$ and the form factors $F_i^{(f)}(q^2)$ and $F_i^{(f')}(q^2)$ for the vertex functions coupling the tensor mesons f and f' to the particles A and B.

The complication in equations such as (2.11), (3.26), and (3.22) arises because in order to write local equations for fields with spin higher than 1, it is necessary to introduce auxiliary variables, which become redundant for the free field, but are, in general, no longer so when an interaction is introduced.

The relations involving the irreducible parts of $U_{\mu\nu}$, $J_{\mu\nu}$, and $\Theta_{\mu\nu}$ are simpler. These irreducible parts (which are the traceless, divergenceless parts of these tensors) are the following:

$$\tilde{U}_{\mu\nu} = U_{\mu\nu} - \partial_\nu\partial_\mu(\Box^2)^{-1}U_\sigma{}^\sigma = U_{\mu\nu} + (g_T/3m^2)\partial_\mu\partial_\nu(\Box^2)^{-1}J_\sigma{}^\sigma, \quad (3.41)$$

$$\tilde{J}_{\mu\nu} = J_{\mu\nu} - \tfrac{1}{3}[\eta_{\mu\nu} - \partial_\mu\partial_\nu(\Box^2)^{-1}]J_\sigma{}^\sigma, \quad (3.42)$$

$$\tilde{\Theta}_{\mu\nu} = \Theta_{\mu\nu} - \tfrac{1}{3}[\eta_{\mu\nu} - \partial_\mu\partial_\nu(\Box^2)^{-1}]\Theta_\sigma{}^\sigma. \quad (3.43)$$

The operation with the inverse d'Alembertian operator is defined as an integration over the appropriate Green's function.[23] In terms of these new variables (which are nonlocal in the way they are defined here), the field equation (2.11) gives just

$$(\Box^2+m^2)\tilde{U}_{\mu\nu} = g_T\tilde{J}_{\mu\nu}, \quad (3.44)$$

and we obtain the following relation between the irreducible spin-two parts of $\Theta_{\mu\nu}$, $U_{\mu\nu}^{(f)}$, and $U_{\mu\nu}^{(f')}$:

$$\tilde{\Theta}_{\mu\nu} = \left(\frac{m_{f'}^2}{g_T}\cos\theta_T\right)\tilde{U}_{\mu\nu}^{(f')} + \left(\frac{m_f^2}{g_T}\sin\theta_T\right)\tilde{U}_{\mu\nu}^{(f)}. \quad (3.45)$$

Equation (3.45) and the relation

$$\Theta_\sigma{}^\sigma = \left(\frac{m_{f'}^2}{g_T}\cos\Theta_T\right)U_\sigma{}^{\sigma(f')} + \left(\frac{m_f^2}{g_T}\sin\Theta_T\right)U_\sigma{}^{\sigma(f)} \quad (3.46)$$

are equivalent to the relation (3.26).

The relation between the matrix elements of $\tilde{\Theta}_{\mu\nu}$ and $\tilde{J}_{\mu\nu}^{(f)}$, $\tilde{J}_{\mu\nu}^{(f')}$ is of the simple form (3.33).

IV. RENORMALIZED SOURCE OPERATORS FOR NEUTRAL TENSOR MESON; TENSOR MESON PROPAGATOR

We first discuss the question of the renormalization of the neutral tensor meson, considering only its strong interactions. We assume that renormalization constants can be defined for the neutral tensor meson[24]; we make

[22] H. Pagels, Phys. Rev. **144**, 1250 (1966).

[23] Thus $(\Box_x{}^2)^{-1}F(x) = \int dx'\,D(x-x')F(x')$, where $D(x)$ is defined by $\Box_x{}^2D(x)=\delta(x)$ and appropriate boundary conditions.

[24] For instance, we can define renormalization constants for the neutral tensor meson in particular models like the chain model (sum of bubble diagrams).

no reference to perturbation theory or the electromagnetic or gravitational interactions. For simplicity, we consider here a problem with a one neutral tensor meson, so that there is no mixing. This can be extended directly to take into account f-f'.

The Lagrangian density for the spin-two meson and its strong interaction will be taken to be of the form (2.1), but in terms of the unrenormalized fields $U_{\mu\nu}^{(0)}$ and $\Pi^{\lambda}{}_{\mu\nu}{}^{(0)}$, and the unrenormalized source $J_{\mu\nu}^{(0)}$, with the (unrenormalized) coupling g_0. The unrenormalized mass will be denoted by m_0. We shall assume that

$$\partial^{\mu}J_{\mu\nu}^{(0)}=0. \tag{4.1}$$

The field equations for the unrenormalized fields are again of the form (2.4) and (2.5), or (2.8)–(2.11).[25]

We shall normalize $J_{\mu\nu}^{(0)}$ in the same manner as $J_{\mu\nu}$ was normalized in the previous section. Introducing a wave-function renormalization constant Z, we define the renormalized field operator $U_{\mu\nu}$ by

$$U_{\mu\nu}=Z^{-1/2}U_{\mu\nu}^{(0)}. \tag{4.2}$$

The Lagrangian density may be written in terms of the renormalized field operators as follows:

$$\begin{aligned}\mathfrak{L}=&\,\mathfrak{L}_0(U^{\mu\nu})\\&+(Z-1)\{(U^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}u)[2\partial_\rho\Pi^{\rho}{}_{\mu\nu}-\partial_\mu\Pi_\nu-\partial_\nu\Pi_\nu]\\&+2(\eta^{\mu\nu}\Pi^{\beta}{}_{\alpha\mu}\Pi^{\alpha}{}_{\beta\nu}-\Pi_\alpha\tilde{\Pi}_\alpha)\}\\&-\tfrac{1}{2}(Zm_0{}^2-m^2)(U^{\mu\nu}U_{\mu\nu}-u^2)+g_0Z^{1/2}U^{\mu\nu}J_{\mu\nu}^{(0)}.\end{aligned} \tag{4.3}$$

The renormalized field obeys Eq. (2.4), where

$$\begin{aligned}J_{\mu\nu}=&\,g^{-1}g_0(m/m_0)^2Z^{-1/2}J_{\mu\nu}^{(0)}-2g^{-1}(1-m^2/m_0{}^2)\\&\times[\partial_\rho\Pi^{\rho}{}_{\mu\nu}-\tfrac{1}{2}(\partial_\mu\Pi_\nu+\partial_\nu\Pi_\mu)-\tfrac{1}{2}\eta_{\mu\nu}\partial_\lambda(\tilde{\Pi}_\lambda-\Pi_\lambda)].\end{aligned} \tag{4.4}$$

As we have normalized both the unrenormalized and the renormalized source operators, we obtain the relations

$$Z^{1/2}g/g_0=m^2/m_0{}^2, \tag{4.5}$$

$$\begin{aligned}U_{\mu\nu}&=\frac{g}{g_0}\frac{m_0{}^2}{m^2}U_{\mu\nu}^{(0)},\\J_{\mu\nu}&=J_{\mu\nu}^{(0)}-\frac{1}{g}\left(1-\frac{m^2}{m_0{}^2}\right)\mathfrak{D}(\Pi^{\lambda}{}_{\mu\nu}).\end{aligned} \tag{4.6}$$

In (4.6), $\mathfrak{D}(\Pi^{\lambda}{}_{\mu\nu})$ denotes twice the expression within square brackets in (4.4). The relation between $\Pi^{\lambda}{}_{\mu\nu}{}^{(0)}$ and $\Pi^{\lambda}{}_{\mu\nu}$ is the same as that between $U_{\mu\nu}^{(0)}$ and $U_{\mu\nu}$.

This shows how the theory for the neutral tensor meson may be written in a form similar to that given by Kroll, Lee, and Zumino[3] for the neutral vector meson. We here get the result

$$J_{\mu\nu}^{(0)}=\frac{m^2}{g}(U_{\mu\nu}-\eta_{\mu\nu}u)-g^{-1}\frac{m^2}{m_0{}^2}\mathfrak{D}(\Pi^{\lambda}{}_{\mu\nu}). \tag{4.7}$$

[25] Here, we have assumed for simplicity that $J_{\mu\nu}^{(0)}$ does not depend on $U_{\mu\nu}^{(0)}$.

If the bare mass m_0 of the tensor meson is infinite, then the unrenormalized source operator becomes proportional to the renormalized operator $(U_{\mu\nu}-\eta_{\mu\nu}u)$:

$$J_{\mu\nu}^{(0)}=(m^2/g)(U_{\mu\nu}-\eta_{\mu\nu}u). \tag{4.8}$$

If we assume the interaction Lagrangian density given by (3.2) and (3.15), obtaining the field-source identity (3.26), then we obtain the relation

$$\Theta_{\mu\nu}=J_{\mu\nu}^{(0)}+g^{-1}(m^2/m_0{}^2)\mathfrak{D}(\Pi^{\lambda}{}_{\mu\nu}). \tag{4.9}$$

We finally remark that, as with vector-meson dominance, one may try an alternative hypothesis to express tensor-meson dominance, namely, that the stress tensor is proportional to the unrenormalized source operator $J_{\mu\nu}^{(0)}$.[26] With our normalization, this gives

$$\Theta_{\mu\nu}=J_{\mu\nu}^{(0)}, \tag{4.10}$$

which leads to

$$\Theta_{\mu\nu}=\frac{m^2}{g_T}(U_{\mu\nu}-\eta_{\mu\nu}u)-g^{-1}\frac{m^2}{m_0{}^2}\mathfrak{D}(\Pi^{\lambda}{}_{\mu\nu}). \tag{4.11}$$

The two alternatives coincide when the bare mass m_0 is finite. If m_0 is finite, then the two alternatives lead to different relations between the energy-momentum structure form factors and the vertex functions for the tensor mesons. Thus, instead of (3.31), we would obtain the following relation, analogous to that for the vector mesons[3]:

$$\begin{aligned}\Theta(q^2)=&\,\frac{m_{f'}{}^2\cos\theta_T}{m_{f'}{}^2-q^2}\left(1-\frac{q^2}{(m_{f'}{}^2)_0}\right)\mathfrak{F}_{(f')}(q^2)\\&+\frac{m_f{}^2\sin\theta_T}{m_f{}^2-q^2}\left(1-\frac{q^2}{(m_f{}^2)_0}\right)\mathfrak{F}_{(f)}(q^2),\end{aligned} \tag{4.12}$$

with the assumption (4.10), so that in principle, the two different hypotheses for tensor-meson dominance, which lead to (4.9) and (4.10), may be distinguished experimentally.

We finally briefly discuss the propagator for the tensor meson. The canonical equal-time commutation relations lead to the relation

$$[U_{pq}{}^T(x,t),\pi_{kl}(y,t)]=-iZ^{-1}\delta_{pq,kl}{}^T\delta(x-y), \tag{4.13}$$

where

$$U_{pq}{}^T=U_{pq}-\tfrac{1}{3}\eta_{pq}U_l{}^l, \tag{4.14a}$$

$$\begin{aligned}\pi_{kl}=&-2\Pi^0{}_{kl}+2(\nabla^2-\tfrac{3}{2}m^2)^{-1}\\&\times(\partial_k\partial_l-\tfrac{1}{2}m^2\eta_{kl})\Pi^0{}_{mm},\end{aligned} \tag{4.14b}$$

and

$$\delta_{pq,kl}{}^T=\tfrac{1}{2}(\eta_{pk}\eta_{ql}+\eta_{qk}\eta_{pl}-\tfrac{2}{3}\eta_{pq}\eta_{kl}). \tag{4.15}$$

For the vacuum-expectation value of the commutator $[U^{\mu\nu}(x),U^{\lambda\sigma}(y)]$, we may write the spectral represen-

[26] This would be analogous to the hypothesis made in Refs. 1 and 2 for vector-meson dominance. For a discussion of this for the vector current, see Ref. 3.

tation, in general, as

$$\langle 0|[U^{\mu\nu}(x),U^{\lambda\sigma}(y)]|0\rangle$$
$$=\int_0^\infty ds\{\mathfrak{D}^{\mu\nu,\lambda\sigma}(\partial,s)\rho_2(s)+\mathfrak{V}^{\mu\nu,\lambda\sigma}(\partial,s)\rho_1(s)$$
$$+\tfrac{2}{3}d^{\mu\nu}d^{\lambda\sigma}\rho_{0a}(s)+(1/24)l^{\mu\nu}l^{\lambda\sigma}\rho_{0b}(s)$$
$$+\tfrac{1}{12}(l^{\mu\nu}d^{\lambda\sigma}+d^{\mu\nu}l^{\lambda\sigma})\rho_{0c}(s)\}\Delta(x-y,s)\,, \quad (4.16)$$

where

$$l^{\mu\nu}=\left(g^{\mu\nu}+4\frac{\partial^\mu\partial^\nu}{s}\right),\quad d^{\mu\nu}=\left(g^{\mu\nu}+\frac{\partial^\mu\partial^\nu}{s}\right),$$

$$\mathfrak{V}^{\mu\nu,\lambda\sigma}(\partial,s)=\tfrac{1}{2}(\partial^\mu\partial^\sigma d^{\nu\lambda}+\partial^\nu\partial^\sigma d^{\mu\lambda}$$
$$+\partial^\mu\partial^\lambda d^{\nu\sigma}+\partial^\nu\partial^\lambda d^{\mu\sigma})\,. \quad (4.17)$$

When $\partial_\mu J^{\mu\nu}=0$, we have the relations

$$\rho_1=0,\quad \rho_{0a}=\tfrac{1}{16}\rho_{0b}=-\tfrac{1}{8}\rho_{0c}=\rho_0\,, \quad (4.18)$$

say. Equation (4.13) now leads to the spectral-function sum rules[27]

$$\int_{4m_\pi^2}^\infty ds\,\rho_2(s)=Z^{-1}, \quad (4.19a)$$

$$\int_{4m_\pi^2}^\infty ds\frac{\rho_2(s)}{s}=\frac{CZ^{-1}}{m_0^2}=m_0^2\int_{4m_\pi^2}^\infty\frac{ds}{s^2}[\rho_2(s)+9\rho_0(s)]\,, \quad (4.19b)$$

where C is a positive constant, which depends, in general, on the interaction. For the free field, $C=1$.

The lower limit of the spectral integrals is $4m_\pi^2$ when we consider the spectral functions for the neutral isoscalar tensor meson. From (4.19a) and (4.19b), using the positive-definiteness of $\rho_2(s)$ and $\rho_0(s)$, we obtain

$$m_0^2>4m_\pi^2. \quad (4.20)$$

For a stable neutral tensor meson, we thus obtain $m_0^2\geq m^2$.

For the renormalized propagator in momentum space, we make the decomposition

$$\mathcal{P}^{\mu\nu,\lambda\sigma}(p)=\int\frac{ds}{s-p^2-i\epsilon}\left[\mathfrak{D}^{\mu\nu,\lambda\sigma}(p,s)\rho_2(s)+\frac{6p^\mu p^\nu p^\lambda p^\sigma}{s^2}\rho_0(s)\right] \quad (4.21)$$

$$\equiv\mathcal{P}_2^{\mu\nu,\lambda\sigma}(p)+\mathcal{P}_0^{\mu\nu,\lambda\sigma}(p)\,, \quad (4.22)$$

in an obvious notation. Making the further decomposition

$$\mathcal{P}_2^{\mu\nu,\lambda\sigma}(p)=\mathfrak{D}^{\mu\nu,\lambda\sigma}(p)F_2(p^2)+\cdots\,, \quad (4.23)$$

analogous to the decomposition in (4.16), we define the square of the renormalized mass m^2 as the value of p^2, where

$$\mathrm{Re}[F_2(m^2)]^{-1}=0\,. \quad (4.24)$$

Proceeding as for the vector meson,[3] we may obtain the relations

$$F_2(p^2)\approx Z_1Z^{-1}/(m^2-p^2-i\gamma m)\,, \quad (4.25)$$

with

$$ZZ_1^{-1}=\frac{d}{dp^2}[\mathrm{Re}(F_2)^{-1}]\bigg|_{p^2=m^2},\quad \gamma\approx\frac{Z_1}{\pi mZ\rho_2(m^2)}. \quad (4.26)$$

The possible choices of Z and the resulting normalizations of the matrix element $\langle T|U_{\mu\nu}(0)|0\rangle$ between the vacuum and a one-particle state may be discussed as for the vector meson; we shall not give the details here.

This discussion may be extended to the problem with f-f' mixing.

V. CONSTRAINTS IMPOSED BY LORENTZ COVARIANCE

The invariance of a field theory under the Poincaré group imposes constraints on the theory, which may be expressed in the form of the equal-time commutation relations (ETCR) among the components of the stress tensor.[28,29] The general form of the stress-tensor ETCR consistent with Poincaré invariance and locality has been discussed by Boulware and Deser,[28] who give the following form:

$$i[\Theta_{00}(x),\Theta_{00}(y)]_{x_0=y_0}=[\Theta_{0k}(x)+\Theta_{0k}(y)]\partial_k(x-y)+\omega_{00,00}(x,y)\,, \quad (5.1)$$

$$i[\Theta_{00}(x),\Theta_{0k}(y)]_{x_0=y_0}=[\Theta_{kl}(x)+\Theta_{00}(y)\eta_{kl}]\partial_l\delta(x-y)+\omega_{00,0k}(x,y)\,, \quad (5.2)$$

$$i[\Theta_{00}(x),\Theta_{kl}(y)]_{x_0=y_0}=[-\partial_0\Theta_{kl}(x)+\Theta_{0k}(y)\partial_l+\Theta_{0l}(y)\partial_k]+\delta(x-y)+\omega_{00,kl}(x,y)\,, \quad (5.3)$$

$$i[\Theta_{0k}(x),\Theta_{0l}(y)]_{x_0=y_0}=[\Theta_{0l}(x)\partial_k+\Theta_{0k}(x')\partial_l]\delta(x-y)+\omega_{0k,0l}(x,y)\,, \quad (5.4)$$

$$i[\Theta_{0k}(x),\Theta_{mn}(y)]_{x_0=y_0}=[\Theta_{mn}(x)\eta_{kl}-\Theta_{ml}(x)\eta_{nk}-\Theta_{nl}(y)\eta_{mk}]\partial_l\delta(x-y)+\omega_{0k,mn}(x,y)\,. \quad (5.5)$$

Here, the operators $\omega_{00,00}$, etc., are subject to the constraints discussed in Ref. 28.

To find the implications of the Lorentz-covariance conditions (5.1)–(5.5) on a theory with a field-source

[27] The spectral-function sum rules for spin-two fields have been discussed by the author in an earlier paper [see K. Raman, Nuovo Cimento **55A**, 650 (1968); and Erratum (to be published)], where these are given also for the more general possibility where the source tensor $J_{\mu\nu}$ for the spin-two field has nonzero divergence and trace.

[28] P. A. M. Dirac, Rev. Mod. Phys. **34**, 1 (1962); J. Schwinger, Phys. Rev. **127**, 324 (1962); **130**, 406 (1963); **130**, 800 (1963).

[29] D. G. Boulware and S. Deser, J. Math. Phys. **8**, 1468 (1967).

identity, we substitute the field-source identity into the ETCR's (5.1)–(5.5) and reexpress the resulting equations in terms of the dynamical variables U_{kl}^T and π_{kl}. We then obtain the following results.

(1) The field-source identity determines the parts of the singular terms in the stress-tensor ETCR which are c numbers and which are the most singular in the coupling constant g.[30] These are the following[31]:

$$\omega_{00,0k}{}^s(x,y) = -(2/3g^2)\nabla^2\partial_k\delta(x-y)\,, \tag{5.6}$$

$$\begin{aligned}\omega_{0k,mn}{}^s(x,y) &= (m^2/g^2)\left[\tfrac{1}{2}(\eta_{km}\partial_n+\eta_{kn}\partial_m-\tfrac{2}{3}\eta_{nm}\partial_k)\delta(x-y)\right.\\ &\qquad\left.-(2/3m^2)\partial_m\partial_n\partial_k\delta(x-y)\right].\end{aligned} \tag{5.7}$$

Here we have used the notation

$$\omega_{00,0k} = g^{-2}(g^2\omega_{00,0k}{}^s) + g^{-1}\omega_{00,0k} + (\text{terms regular in } g)\,. \tag{5.8}$$

Note that (5.7) and (5.8) lead to the relation

$$\omega_{0k,mm}{}^s(x,y) = \omega_{00,0k}{}^s(x,y)\,. \tag{5.9}$$

This is in agreement with the result noted by Boulware and Deser,[28] that in the vacuum-expectation values $\langle\omega_{00,0k}\rangle_0$ and $\langle\omega_{0k,mn}\rangle_0$, the parts proportional to the third derivative of $\delta(x-y)$ are the same.

(2) The field-source identity and the stress-tensor ETCR impose explicit constraints on the nature of the field dependence of the source density $J_{\mu\nu}(x)$ in the field equations for the neutral spin-two mesons.[32]

Details of these are given elsewhere.[33] Here we shall merely quote two important qualitative results following from these constraints:

(3) When we require the Jacobi identity to hold for triple commutators involving a component of $J_{\mu\nu}$ and the canonical variables U_{kl}^T and π_{kl}, then the constraints following from the field-source identity and the stress-tensor ETCR require that some of the singular terms in the stress-tensor ETCR must be q numbers.

(4) Also, as a consequence of some of these constraints, the interaction term $gJ_{\mu\nu}(x)$ for the spin-two field must be nonlinear in the coupling strength g.

These results indicate in what way a field-source identity restricts a Lorentz-invariant theory.

VI. CONCLUSIONS

In this paper, we have suggested that the hypothesis of tensor-meson dominance of the stress tensor may be expressed by the requirement that the irreducible spin-two part of the complete stress-tensor operator of the hadrons is identical to a linear combination of the irreducible spin-two parts of the renormalized field operators for the neutral isoscalar spin-two mesons, and that the trace of the stress tensor is, similarly, a linear combination of the traces of these spin-two meson field operators.

We have shown how this hypothesis may be realized in a class of Lagrangian field theories of neutral tensor mesons with a gauge-invariant coupling to the gravitational field. We have here treated the gravitational interaction to lowest order; this has enabled us to proceed in a manner analogous to that used by Kroll, Lee, and Zumino for the electromagnetic current.

We have also briefly discussed here the constraints imposed by Lorentz covariance on a theory with a field-source identity. These are in the form of conditions on the source $J_{\mu\nu}$ describing the strong interactions of the spin-two mesons. We have not exhibited an explicit model satisfying these constraints. However, we have not found any inconsistency among these conditions, and we take this to support the existence of nontrivial theories with a field-source identity.

Some of the questions that arise in this work are (a) the explicit construction of theories with a field-source identity, (b) the extension of the results obtained here to all orders of the gravitational interaction, and (c) the formulation of a theory with tensor-meson dominance of the traceless parts of the stress tensor and scalar-meson dominance of the trace.

These and other questions relating to the tensor-meson dominance hypothesis will be discussed in subsequent work.

ACKNOWLEDGMENT

The author is obliged to Professor J. A. Cronin for a discussion.

[30] This is analogous to the result that the field-current identities for the vector and axial-vector currents determine the singular terms in the commutation relations for these currents. See T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[31] When we write a spectral decomposition of the vacuum-expectation value $\langle 0|[\Theta_{\mu\nu}(x),\Theta_{\lambda\sigma}(y)]|0\rangle$, then the results (5.6) and (5.7) lead to sum rules for the spectral functions of the stress-tensor two-point function. These will be discussed elsewhere.

[32] For a discussion of noncommutation and field-dependence requirements on (divergenceless) sources of spin-two fields, see K. Raman, J. Math. Phys. (to be published).

[33] K. Raman, Brown University report, 1969 (unpublished).