Strictly Localizable Field Theories with Rising Trajectories*

I. T. Grodsky
Department of Physics, The Cleveland State University, Cleveland, Ohio 44115
(Received 3 April 1970)

We investigate the use of the vehicle of perturbative strictly localizable field theory in examining generalized dual-resonance models, with a view towards finding a class of examples rich enough to provide a Born-like input to an order-by-order Reggeized viable unitarization scheme. Although we manage to construct a field-theoretic Lagrangian capable of reproducing the original dual-resonance model as a tree approximation, it is found that this approach is often misleading, and the pitfalls are explicated. We conclude with a brief polemic designed to indicate that it is yet not out of the question to aspire to the hope of obtaining a disease-free tree approximation, and thereby making a step towards a realistic unitary theory of hadronic interaction.

I. INTRODUCTION

Since Veneziano[1] first rediscovered the statistical beta function and indicated its remarkable structure when viewed as a meromorphic approximation to some scattering amplitudes, there has been strong interest in investigating the general properties of classes of various models giving rise to amplitudes with similar properties. A proposal has recently been put forth which generalizes Veneziano's original ansatz for four-point processes to any number of external lines. This set of prescriptions has come to be known as the dual-resonance models.[2] While the known dual-resonance models possess many wonderful properties, in particular, universal Regge asymptotics or a true bootstrap, they are afflicted by two distressing pathologies. First, factorization implies an exponential increase in the number of states at a given mass as the mass increases, and second, there exists an infinite number of ghosts or imaginary coupling constants.[3]

There are basically two different attitudes one can take when contemplating the properties of these models. One can assume that the models represent a narrow-resonance approximation to the scattering matrix, but *not* in the sense of a Feynman-like tree approximation. Then, the negative residues may be considered to represent repulsive channels, and one can hope that these trajectories go away when the proper method of unitarization is discovered.[4] On the other hand, one can attempt to demand that the models do represent a sort of super-Born approximation and try to unitarize them in a Feynman-like fashion.[5] Because of the ambiguity inherent in the first approach as to which way to go, it is the latter attitude which will be adopted in this work.

Initial attempts to unitarize these models have shown quite pleasing formal properties,[6] but unfortunately none of the higher-order diagrams exists and the divergences are rather horrible—probably of order $\exp(\infty)$. It would appear that these bad divergences arise more from the level structure than from the occurrence of ghosts.[5]

If one is going to pursue the program of attempting to unitarize via Feynman-like diagrams, it is only natural to ask if an effective Lagrangian field-theory approach might shed light on the structure and pathologies of the possible models. There are many questions needing answers. For example, what is the nature of field theories which give rise to Veneziano-like amplitudes? More importantly, we must know whether the entire set of possible dual-resonance models contain any members whose pathologies (level structure and ghosts) are relieved or at least diminished. Indeed, if such example classes obtain, we would be at the first step to a truly self-consistent theory of hadronic interactions.

In this paper, we investigate and partially answer some of these questions. In Sec. II we examine several examples of strictly localizable field theories capable of giving rise to Veneziano-like amplitudes. Section III consists of an analysis of the structure of these theories and the difficulties involved in their interpretation. Finally, in Sec. IV, we discuss the general problem of exhausting the properties of all possible models, and end with a few speculations as to the direction this line of research should take.

II. SOME MODEL FIELD THEORIES

Interest in the properties of field theories which contain an infinite number of spinning particles is certainly not new. During the last several decades it has become experimentally evident that the narrow resonances apparently continue upwards in energy indefinitely. The evolution of sum rules from current algebra and superconvergence (or finite-energy sum-rule) assump-

* Work supported in part by the U. S. Office of Naval Research.
[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).
[2] K. Bardakci and H. Ruegg, Phys. Letters **28B**, 342 (1968); M. A. Virasoro, Phys. Rev. Letters **22**, 37 (1969); H. M. Chan, Phys. Letters **28B**, 425 (1969); H. M. Chan and Tsou S. Tsun, *ibid.* **28B**, 485 (1969); C. Goebel and B. Sakita, Phys. Rev. Letters **22**, 257 (1969); K. Bardakci and H. Ruegg, Phys. Rev. **181**, 1884 (1969); S. Mandelstam, *ibid.* **184**, 1625 (1969); K. Bardakci and S. Mandelstam, *ibid.* **184**, 1640 (1969); D. J. Gross, CERN Report No. TH 1048, 1969 (unpublished).
[3] S. Fubini and G. Veneziano, Nuovo Cimento **64A**, 811 (1969).
[4] This attitude has been advanced by S. Mandelstam (see Ref. 2).
[5] K. Kikkawa, B. Sakita, and M. A. Virasoro, Phys. Rev. **184**, 1701 (1969); K. Bardakci, M. B. Halpern, and J. A. Shapiro, *ibid.* **185**, 1910 (1969).
[6] J. C. Polkinghorne, Phys. Rev. **186**, 1670 (1969).

tions generated a strong interest in infinite-component field theories with a view towards obtaining nontrivial form factors and saturating the sum rules.[7]

In general, the theories under scrutiny used representations of the homogeneous Lorentz group containing at least one infinite-dimensional irreducible representation, as these gave elegant nonderivative couplings (in terms of the so-called "big" fields) between an infinity of states, and rather exciting form factors. Regrettably, all of these theories possessed serious diseases connected with locality and the spectral condition, and it had been found that these difficulties were a universal property of theories of this type.[8]

Because of this state of affairs it is natural to turn to theories containing an infinite tower of the finite-dimensional representations of $SL(2,c)$. By doing so we are giving up hope of obtaining nontrivial saturations of current algebras in the canonical fashion, but rather focus our attention upon the purely hadronic properties of the states. This matter will be further discussed in Sec. III.

If the coupling constants decrease in a reasonably slow manner as the masses and spins increase, it is highly unlikely that the perturbative n-point functions will be tempered distributions; that is, we will not in general be dealing with Wightman field theories.[9] However, Jaffe[10] has been able to show that the physical requirements of locality do not demand that the n-point functions be tempered distributions, but rather imply a weaker bound which considerably enlarges the possibilities. Further, Abarbanel[11] has recently demonstrated by example that strictly localizable field theories (as the Jaffe species are called) are rich enough to be made to yield a Veneziano-like amplitude in the Born approximation. His example will be briefly examined at the end of this section.

We now turn to the construction of some model strictly localizable field theories.[12]

A. Four-Dimensional Oscillator

We choose this case because of its applicability to our next example, the original dual-resonance model.

[7] See, for example, C. Fronsdal, Phys. Rev. **171**, 1881 (1968), and references therein.

[8] I. T. Grodsky and R. F. Streater, Phys. Rev. Letters **20**, 695 (1968). This result has recently been extended to include strictly localizable field theories: A. I. Oksak and I. T. Todorov, Phys. Rev. D **1**, 3511 (1970). We remark here, however, that C. Fronsdal [Phys. Rev. **182**, 1514 (1969)] has shown that these fields can be considered as solutions to the Bethe-Salpeter equation in the crossing-symmetry-violating ladder approximation. This sheds new light on the proper physical interpretation and use of these wave functions.

[9] R. F. Streater and A. S. Wightmann, *PCT, Spin and Statistics, and All That* (Benjamin, New York, 1964).

[10] A. M. Jaffe, Phys. Rev. **158**, 1454 (1967).

[11] H. D. I. Abarbanel, Princeton University report, 1969 (unpublished).

[12] In this work the phrase "field theory" simply means a set of bare couplings, masses, spins, and internal quantum numbers, along with the usual perturbative combinatorial rules.

Consider the group $SU(3,2)$ and its compactified companion $[SU(4,1)]$. The generators can be written

$$C_{\alpha\beta}=D_\alpha{}^+D_\beta+D_\beta{}^+D_\alpha \tag{2.1}$$

for $SU(4,1)$, and

$$c_{\alpha\beta}=d_\alpha{}^+d_\beta+d_\beta{}^+d_\alpha \tag{2.2}$$

for $SU(3,2)$. Here,

$$\alpha,\beta=(1,2,3,0,5)\,, \tag{2.3}$$

$$(D_\alpha{}^+,D_\beta)=\mathcal{G}_{\alpha\beta}\,, \tag{2.4}$$

$$(d_\alpha{}^+,d_\beta)=g_{\alpha\beta}\,, \tag{2.5}$$

all others commuting, and

$$\begin{aligned}&\mathcal{G}_{\alpha\beta}=\delta_{\alpha\beta}\,,\ \alpha=1,2,3,0;\quad \mathcal{G}_{5\beta}=-\delta_{5\beta}\,,\\ &g_{\alpha\beta}=\delta_{\alpha\beta}\,,\ \alpha=1,2,3;\quad g_{\alpha\beta}=-\delta_{\alpha\beta}\,,\ \alpha=0,5\,.\end{aligned} \tag{2.6}$$

Since we wish to obtain a representation of $SU(3,2)$ which contains a tower of finite-dimensional representations of $SL(2,c)$, we perform a sort of inverse Weyl trick. We fix an infinite-dimensional unitary representation of $SU(4,1)$ by setting

$$C_{\alpha\alpha}=D_\alpha{}^+D_\alpha=0 \tag{2.7}$$

and

$$(D_\alpha{}^+)^\dagger=D_\alpha\,. \tag{2.8}$$

A basis for this representation of $SU(4,1)$ also labels a basis for an infinite-dimensional nonunitary representation of $SU(3,2)$, for which

$$c_{\alpha\alpha}=d_\alpha{}^+d_\alpha=0\,, \tag{2.9}$$

$$(d_\alpha{}^+)^\dagger=d_\alpha\,,\quad \alpha=1,2,3,5 \tag{2.10}$$

and

$$(d_0{}^+)^\dagger=-d_0\,. \tag{2.11}$$

Now the boost generators in the $SL(2,c)$ subgroup of $SU(3,2)$ will be anti-Hermitian, ensuring that this representation contains only a tower of finite representations of $SL(2,c)$, $D^{(j_1/2,\,j_2/2)}$. Let

$$|n\rangle=|n_1,n_2,n_3,n_0\rangle \tag{2.12}$$

be a basis for this representation R of $SU(3,2)$;

$$d_{(\alpha)}{}^+d_{(\alpha)}|n\rangle=n_\alpha|n\rangle\,,\quad \alpha=1,2,3 \tag{2.13}$$

$$d_0{}^+d_0|n\rangle=-n_0|n\rangle \tag{2.14}$$

(since $d_0{}^+d_0$ is negative semidefinite), and the n_μ are the positive integers.

In order to form multilinear Lorentz invariants, we shall need a metric operator G with the properties

$$(G,d_\alpha)_-=(G,d_\alpha{}^+)_-=0\,,\quad \alpha=1,2,3,5 \tag{2.15}$$

$$[G,d_0]_+=[G,d_0{}^+]_+=0\,, \tag{2.16}$$

$$G=G^\dagger\,, \tag{2.17}$$

and

$$\langle 0|G|0\rangle=1\,. \tag{2.18}$$

It is clear that G can only be a function of $d_0{}^+d_0$,

$$G=\sum c_n(d_0{}^+d_0)^n. \tag{2.19}$$

Since

$$(d_0{}^+d_0)^n d_0=d_0(d_0{}^+d_0-1)^n\,, \tag{2.20}$$

we have

$$G(d_0{}^+d_0)d_0=d_0G(d_0{}^+d_0-1)\,. \tag{2.21}$$

Thus $G(x)$ satisfies the functional equations

$$G(x)=-G(x-1)\,,\quad G(0)=1\,. \tag{2.22}$$

The most general solution is therefore

$$G(d_0{}^+d_0)=\cos(\pi d_0{}^+d_0)\,. \tag{2.23}$$

All other forms are operator-equivalent to Eq. (2.23), in view of the eigenspectrum of $d_0{}^+d_0$.

Let $\Psi_n(x_\mu)$ be an infinite-component field over this representation,

$$\Psi_n(x_\mu)=\langle n|\Psi(x_\mu)\rangle\,, \tag{2.24}$$

and consider the following free Lagrangian density in momentum space:

$$L_F(p_\mu)=\Psi_n(p_\mu)G_{nn'}[p_\mu p^\mu-m_0{}^2 -m_1{}^2(d_5{}^+d_5)_{n'n''}]\Psi_{n''}(p_\mu)\,, \tag{2.25}$$

where $m_0{}^2$, $m_1{}^2>0$. Here we are using index notation,

$$G_{nn'}=\langle n|G|n'\rangle\,, \tag{2.26}$$

etc. The wave equation is simply

$$[p_\mu p^\mu-m_0{}^2-m_1{}^2(d_5{}^+d_5)_{nn'}]U_{n'}(\mathbf{p}Nn_j)=0\,. \tag{2.27}$$

Since $d_5{}^+d_5=-d_\mu{}^+d_\mu$, we label the wave functions with

$$N=n_1+n_2+n_3+n_0 \tag{2.28}$$

and

$$\{n_j\}=(n_1,n_2,n_3)\,. \tag{2.29}$$

The U's are, of course, just the matrix elements of the helicity boosts from the rest frame,

$$U_n(\mathbf{p}Nn_j)=\langle n|B(\mathbf{p},N,n_j)|Nn_j\rangle. \tag{2.30}$$

The mass formula is linear and rising:

$$M^2=m_0{}^2+m_1{}^2N\,, \tag{2.31}$$

and, of course, only timelike solutions obtain. The degeneracy is just that of a four-dimensional harmonic oscillator, i.e., an infinite sum of Lorentz poles spaced by one unit, so that the degeneracy of the daughter Regge trajectories increases linearly as we go down in spin.

The free fields are given by

$$\Psi_n(x_\mu)=\sum_{N,n_j}c^N\int\frac{d^3p}{\epsilon_N}\{e^{-ip_N\cdot x}U_n(\mathbf{p}Nn_j)a^\dagger(\mathbf{p}Nn_j) +e^{ip_N\cdot x}U_n{}^*(\mathbf{p}Nn_j)a(\mathbf{p}Nn_j)\}\,, \tag{2.32}$$

with

$$\epsilon_N=+(\mathbf{p}^2+m_0{}^2+m_1{}^2N)^{1/2}\,, \tag{2.33}$$

and the $\{a^\dagger,a\}$ are the usual canonical creation and annihilation operators. The $\{c^N\}$ are growth dampeners, always chosen so that the commutation relations of the fields satisfy Jaffe's strict locality conditions.[13]

One way to turn on interaction would be in terms of the matrix elements $\langle n_3|G|n_1n_2\rangle$, but since we are dealing here with towers of finite representations of $SL(2,c)$, this would couple a given state only to a finite set of other ones. Thus, unlike the infinite-dimensional irreducible-representation-based field theories, we must employ derivative couplings.

In our representation we have a natural four-vector operator which raises and lowers the $\{n_\mu\}$,

$$c_{\mu5}=d_\mu{}^+d_5+d_5{}^+d_\mu\,. \tag{2.34}$$

Accordingly, we define the trilinear matrix elements

$$\Theta_{n_3n_2n_1}=\langle n_3|G\left[\sum_{r=0}^{\infty}g_r{}^{123}\left(c_{\mu5}\frac{\partial}{\partial x_\mu}\right)^r\right]|n_1n_2\rangle\,, \tag{2.35}$$

where by $g_r{}^{123}$ we mean that these numbers can depend upon the Lorentz-invariant masses, spins, and degeneracies of the states 1, 2, and 3; and we postulate the following interaction Lagrangian:

$$L_I(x_\mu)=\Theta_{n_3n_2n_1}\Psi_{n_3}(x_\mu)(G\Psi)_{n_2}(x_\mu)(G\Psi)_{n_1}(x_\mu)+\text{H.c.} \tag{2.36}$$

Observe that for any finite n_1, n_2, and n_3, only finite powers of a finite number of derivatives survive the inner product in Eq. (2.35), so for finite-mass trilinear couplings, the interaction is manifestly local. Our only constraint on the $\{g_r{}^{123}\}$ is that the interaction Lagrangian again satisfy the Jaffe conditions. We note that the actual "bare" couplings are products of the $\{g_r{}^{123}\}$ and the growth dampeners $\{c^N\}$.

It is a straightforward although tedious exercise to show that the rich freedom of choosing the constants $\{g_r{}^{123}\}$ and $\{c^N\}$ allows us to invent interactions within the Jaffe bounds such as to obtain *any* of the generalized Veneziano functions,[13] and, in particular, the beta function as the Born approximation for the four-point amplitude where the external lines are the spinless lowest rungs of the tower defined by the wave equation above.[14]

B. Original Dual-Resonance Model

Here it is our goal to make contact with the original dual-resonance model[15] in which the satellite degeneracy increases exponentially, so we need another infinite degree of freedom. To this end, consider an infinite tensor product of independent representations iso-

[13] See Sec. IV.

[14] Those interested in seeing the mechanics involved in such a construction should consult Ref. 11.

[15] By this phrase we mean the spinless quark model considered by all authors in Ref. 2 except the last.

morphic to the one examined in the previous example,

$$T=\prod_{i=1}^{\infty} R_i\,, \tag{2.37}$$

i.e., a "Fock" space of the nonunitary $SU(3,2)$ representations R, and let a basis be denoted by $|n_1,n_2,\ldots,n_j,\ldots\rangle$.

We define an infinite-infinite-component field (∞^2 field) over T by

$$\Psi_{\Pi\{n\}}(x_\mu)=\Psi_{n_1n_2\cdots n_j\cdots}(x_\mu)$$
$$=\langle n_1,n_2,\ldots,n_j,\ldots|\Psi(x_\mu)\rangle. \tag{2.38}$$

Next we adjoin a single integer generator $l^\dagger l$ by introducing another degree of freedom (one-dimensional harmonic oscillator):

$$(l^\dagger,l)=1\,,\quad (l^\dagger)^\dagger=l\,,\quad (l,c_{\alpha\beta}{}^i)=0\,. \tag{2.39}$$

Consider the free Lagrangian

$$L_F(p_\mu)=\Psi_{\Pi\{n\}}(p_\mu)G_{\Pi\{n,n'\}}$$
$$\times\{p_\mu p^\mu-m_0{}^2-m_1{}^2[l^\dagger l+\sum_{i=1}^{\infty} i(d_5{}^{\dagger i}d_5{}^i)_{n'{}^i n''{}^i}]\}$$
$$\times\Psi_{\Pi\{n''\}}(p_\mu)\,, \tag{2.40}$$

where

$$G_{\Pi\{n,n'\}}=G_{n_1n_1'}G_{n_2n_2'}\cdots G_{n_jn_j'}\cdots\,, \tag{2.41}$$

etc., and we have suppressed the indices for the operator $l^\dagger l$. The solutions to the wave equation have been designed to yield exactly the mass spectra and level structure of the original dual-resonance model,[3]

$$[p_\mu p^\mu-m_0{}^2-m_1{}^2(l^\dagger l+\sum_i d_5{}^{+i}d_5{}^i)]U(\mathbf{p}N\delta)=0\,, \tag{2.42}$$

where all indices have been suppressed, and δ indicates the degeneracies (including spin and helicity). For fixed N, the number of tensor products contributing to $U(\mathbf{p}N\delta)$ cannot exceed N, and the degeneracy at fixed N is given by the number of distinct solutions to

$$\epsilon+\sum i\epsilon_i=N\,, \tag{2.43}$$

where ϵ, ϵ_i, and N are integers greater than 0, and the ϵ_i represent four-dimensional oscillator energy levels,

$$\epsilon_i=(n_0+n_1+n_2+n_3)_i\,, \tag{2.44}$$

with the accompanying degeneracies. Again, from covariance the U's are just the matrix elements of the helicity boosts in the Fock space T:

$$U_{\Pi\{n\}}(\mathbf{p}N\delta)=\langle\prod^{N}\{n\}|B_{n1}(\mathbf{p}N\delta)B_{n2}(\mathbf{p}N\delta)\cdots$$
$$B_N(\mathbf{p}N\delta)|N\delta\rangle\,. \tag{2.45}$$

The free fields are given by

$$\Psi_{\Pi\{n\}}(x_\mu)=\sum_{N,\delta} c^{N,\delta}\int\frac{d^3p}{\epsilon_N}$$
$$\times e^{-ip_Nx}U_{\Pi\{n\}}(\mathbf{p}N\delta)a^\dagger(\mathbf{p}N\delta)+\text{H.c.}\,, \tag{2.46}$$

with a notation made evident from the example set forth in Sec. II A.

To turn on interaction, we simply take the tensor product of the interaction introduced in the example of Sec. II A,

$$\Xi_{\Pi\{n_3n_2n_1\}}=\prod_{i,j,k}\Theta_{n_3{}^in_2{}^jn_1{}^k}\,, \tag{2.47}$$

where we now have all of the constants $\{g_r(i,j,k,1,2,3)\}$ to play with. The interaction Lagrangian is now

$$L_I(x_\mu)=\Xi_{\Pi\{n_3n_2n_1\}}\Psi_{\Pi\{n_3\}}(x_\mu)(G\Psi)_{\Pi\{n_2\}}(x_\mu)$$
$$\times(G\Psi)_{\Pi\{n_1\}}(x_\mu)+\text{H.c.} \tag{2.48}$$

With the huge freedom at our disposal in choosing the constants $\{g_r(i,j,k,1,2,3)\}$ and $\{c^{N,\delta}\}$, we can easily reconstruct the original dual-resonance model as the tree approximation to this theory. Troubles involving ghosts (remember $G=\cos\pi d_0{}^+d_0$ has eigenvalues ±1, so it is an indefinite metric) and a level structure have been clarified.

C. Abarbanel Model

Here we briefly indicate the result of an example considered by Abarbanel.[11] Seeking simplicity, he took a single tower of the tensor $(D^{(N/2,N/2)})$ representations of $SL(2,c)$, and created the fields

$$\Psi_{lm}(x_\mu)=\sum_{NS\lambda} c^N\int\frac{d^3p}{\epsilon_N}$$
$$\times[e^{-ip_Nx}U_{lm}(\mathbf{p}\lambda NS)a^\dagger(\mathbf{p}\lambda(NS))+\text{H.c.}]\,, \tag{2.49}$$

where N and s are integers

$$0\leqslant s\leqslant N\,, \tag{2.50}$$

the U's are matrix elements of the boosts in the $D^{(N/2,N/2)}$ representation, and the masses are taken to be linear in N. Here we have exactly one "bare" creation operator for each mass and spin, and a spectrum resembling a three-dimensional oscillator. Using these fields and derivative trilinear interactions satisfying the Jaffe bounds, he has been able to obtain the beta function for the Born approximation to the lowest-rung four-point amplitude.

III. DISCUSSION

Let us first make the following observation: A glance at the first example in Sec. II will reveal that the free Lagrangian is quite capable of yielding a representation of current algebras at infinite momentum in the canonical way. Unhappily, this current will merely couple a finite set of states to any given one, and the form factors will possess only kinematic singularities in the finite t plane. Thus, models of this type can only give trivial representations of current algebras, as can be expected from the no-go theorems.[8] Even when we take the derivative hadronic couplings into account, this will only add difficulties like those connected with Schwinger terms, unless we can learn how to sum infinite sets of hadronic graphs. If we take the narrow-width approximation seriously for the hadronic interactions, it is clear that the canonical method of introducing the weaker interactions does not represent a viable physical approach to the connection between the various interactions in this picture.[16]

The most striking result of Sec. II would appear to be the fact that several (indeed, certainly an infinity of) distinct theories are quite capable of yielding the same Veneziano-like Born approximation to the lowest-rung four-point amplitude. The only real difference between the theories considered above and typically Weinberg-like perturbation theories is the existence of an infinite number of fields, with distinct mass and spin, accumulating at infinity. But of course this difference profoundly changes the nature and physical interpretation of the theories. Summing an infinite set of poles opens a Pandora's box of possibilities. As we have known for some time, we can completely change the value of the residues in the Born approximation, so that the bare couplings have only remote connections with the actual couplings, since another contribution arises from summing the crossed-channel poles.

We can now ask: What about the connection between the bare fields and the number of physical states, even in the Born approximation? Consider, for simplicity, the Abarbanel model.[11]

Some time ago, Bronzan and Jones[17] obtained an interesting result. They showed that if one assumes (a) Poincaré covariance, (b) analyticity at s, t, $u=0$, (c) universal leading Regge asymptotics in at least two four-point, spinless, equal- and unequal-mass channels, and (d) only one state for each mass and spin, it follows that the ratios of the daughter residues are such as to generate exactly a single Lorentz pole (Gegenbauer function) at $u=0$. In the theories considered above, the tree approximations are such as to have parallel trajectories irrespective of the values of s, t, and u. Analyticity at the origin is easily arranged.[18] Finally, the Poincaré covariance of the theories is unquestionable. Since a Gegenbauer function is not meromorphic, but rather has a cut, and since Abarbanel's example gives a beta function with only dynamical poles (an infinite sum of Lorentz poles), he must be violating at least one of the Bronzan-Jones assumptions (γ) or (δ).

Given the freedom in the choice of the rest of the constants besides $\{c^N\}$ and $\{g_r^{k00}\}$, and using a straightforward induction from the Bronzan-Jones result, the conclusion is evident: In general, not only is there no direct connections between the bare couplings and pole residues in the tree approximations to these theories; there is also no direction connection between the number of bare field operators and the degeneracies of the physical states in the tree approximation. At least one of the two following situations usually obtains: The Regge asymptotics are not even so universal as to guarantee that *any* two spinless unequal-mass channels will share the same leading trajectory—a situation which is highly unlikely—or, much more probably, the connection between the creation and annihilation operators and the physical states is already lost in the Born approximation.

Our second example was designed to show that it *is* possible to generate a field theory which can faithfully connect the bare fields to the physical states in the tree approximation. However, it is now evident that this by no means must occur in the general case, and indeed, the field theories do not indicate when this will or will not occur. The lesson is apt: The use of strictly localizable field theories designed to utilize the infinite-pole-series trick for Reggeization can be misleading. The physical content in the tree approximation is evidently almost as obscure as in the direct approach of amplitude construction. The effective-Lagrangian attack to this problem may have utility, but it needs alteration in order to illuminate the physical particle structure. What we need is a better language.

IV. GENERAL PROBLEM

The basic question facing us is that of analyzing the entire set of dual-resonance-like models in order to see if it contains a class of examples less pathological than the ones already known.[2] The fruits of such a discovery are by now self-evident, and if the class is void, at least we will know what directions not to take. Here, rather distant from actually solving this problem, we make some general observations.

Any prescription obtainable from an effective Lagrangian of the type considered above will yield a Born approximation to the spinless four-point amplitude which can be written (considering only linear tra-

[16] The point here is that these effective Lagrangians—usable as "perturbative" approximations to the amplitudes—are not meant to imply weak couplings, but rather to emphasize the strong resonant (or narrow-reduced-width) nature of many amplitudes. Since the weaker interactions actually see the "cloud of virtual states" surrounding a point hadronic field, one should be very aware that they do not see our effective fields as simple entities. It is likely that the approach of the irreducible infinite-component field theory comes much closer to representing the physical hadronic structure as seen by the weaker currents (see Ref. 8).

[17] J. B. Bronzan and C. E. Jones, Phys. Rev. Letters **21**, 564 (1968).

[18] I suspect that Jaff's strict locality constraints guarantee this property. In fact, we make the following conjecture: The looser Jaffe constraints cannot change the local structure of amplitudes in the finite complex planes, but only conditions in infinity.

jectories and, for simplicity, only the s and t channels)

$$A^B(s,t)=a(s,t)+a(t,s)\,, \tag{4.1}$$

$$a(s,t)=\sum_{n=0}^{\infty}\frac{f_n(s,t)}{t-an-b}\,, \tag{4.2}$$

where $f_n(s,t)$ is a polynomial of order not higher than n,

$$f_n(s,t)=\sum_{i,j=0}^{n} a^{ij}s^it^j. \tag{4.3}$$

Our constraints upon the couplings are constructed to retain the features of dual-resonance models. We insist that the $f_n(s,t)$ are such as to do no more in Eq. (4.2) than generate simple poles in t at $t=an+b$. Further, appealing to Chew's maximal-hadronic-strength hypothesis, we demand that these poles in t actually occur (a weak form of that nebulous concept called "duality"), for otherwise the couplings would decrease ridiculously rapidly. For apparent reasons, we denote these as Wightman-like field theories.

A double Mellin representation of all $a(s,t)$ satisfying these constraints is given by

$$a(s,t)=\int_0^1 du_1du_2u_1{}^{-\alpha_s}u_2{}^{-\alpha_t}G(u_1,u_2)\,,$$
$$\alpha_s=as+b,\quad \alpha_t=at+b. \tag{4.4}$$

Here, $G(u_1,u_2)$ is a distribution of the type

$$G(u,u)=\sum_{r=0}^{\infty} g_r(u_1)\frac{d^r}{du_1{}^r}\delta(u_1+u_2-1)\,, \tag{4.5}$$

where the $\{g_r(u_1)\}$ are any functions analytic in the closed interval $[0,1]$, and the series converges uniformly in this interval in the sense of distribution.[19] One can readily check that all such functions have Regge asymptotic behavior on all rays except those too close to the positive real s axis.

Gross has recently investigated a class of models which contain a series of satellite trajectories,[20] and has found that in general this only increases the degeneracies of the levels, as conjectured earlier by Mandelstam.[2] However, a cursory glance at our representation, Eq. (4.4), will show that the class of four-point amplitudes is rich enough to allow for any finite number of residues to be *completely* arbitrary.[21,22] Further, Khuri[23] has been able to show that it is possible to find classes of such four-point amplitudes in which all ghosts are eliminated. Indeed, given the interpretive problems inherent in a strictly localizable field theory, one can envisage using towers of the $D^{(0,j/2)}$ and $D^{(j/2,0)}$ representations of $SL(2,c)$ in such a way as to make all ghosts manifestly vanish from the effective Lagrangians. It is not out of the question to hope that such an approach, coupled with some additional coupling constraints, would parallel Khuri's class.

Given this vast freedom in constructing spinless four-point amplitudes of Veneziano type, it is natural to begin an attack on the general n-point tree functions. Unfortunately, in the light of at least my initial attempts, it appears that the techniques used by Fubini and Veneziano[3] and by Gross[2] are not efficacious for our generalized class.[24]

The conclusion for now is that the answer to the general problem is as yet unknown; what we need is a language intermediate between effective Lagrangians and amplitude construction capable of illuminating the physical content (degeneracies and couplings) of the possible models.

ACKNOWLEDGMENTS

I would like to thank members of the Battelle Institute and Professor V. Bargmann for their kind hospitality at the Rencontres on Group Theory in Mathematics and Physics, 1969, where this work was initiated. I would also like to thank H. Leutwyler and W. Rühl for many useful discussions.

[19] Regions of convergence are not indicated. The expressions are assumed valid where they converge and are defined elsewhere via analytic continuation.

[20] See the last entry under Ref. 2.

[21] Including the u channel in no way vitiates this remark.

[22] What this indicates concerning phenomenological fitting will be left to the reader.

[23] N. N. Khyri, Phys. Rev. **185**, 1876 (1969).

[24] Effectively, our series of derivatives of δ functions, when generalized to n-point functions, interferes with the factorization properties exploited by the authors referred to in the text. But this is encouraging, as it is this very property that leads to such large degeneracy.