Some Predictions of the Amati-Bertocchi-Fubini-Stanghellini-Tonin Multiperipheral Model*

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The predictions of the Amati-Bertocchi-Fubini-Stanghellini-Tonin multiperipheral model based on pion pole dominance are calculated quantitatively and compared with experimental data, by using the Veneziano representation for the π-π amplitude, with a cutoff at the mass of the g resonance for the input kernel, and neglecting off-shell effects. We find $\alpha_P(0)=0.30$, $\alpha_\rho(0)=0.16$, an average multiplicity growing as 0.74 ln$s$, an elasticity factor of 0.69, an average pion-pair transverse momentum equal to 0.43 GeV/c, and an average invariant mass squared equal to 0.84 GeV^2. The only serious discrepancies with experiment are the trajectory heights, which correspond to a kernel strength $2\frac{1}{2}$–5 times too weak. The inclusion of the high-subenergy contribution and the inclusion of off-shell effects by several methods of off-shell continuation are considered, but none is found to be satisfactory. The importance of interference terms, due to different ways of arranging the final-state particles along the multiperipheral chain, is discussed.

I. INTRODUCTION

IN the last two years there has been interest in the multi-Regge model (MRM) of production processes (see Fig. 1) and the corresponding multi-Regge bootstrap by way of unitarity.[1] However, the multi-Regge amplitude can only be expected to approximate the physical production amplitude when all subenergies are large ($\gg$1 GeV), whereas experimentally the important subenergies are of the order of or less than 1 GeV.[2] To justify the MRM then requires the unreasonable assumption that duality holds even at such small energies. Furthermore, in order to simplify their equations, some authors[1] have made the kinematic approximation that

$$s \propto s_1 s_2 \cdots s_n. \tag{1.1}$$

This kinematic approximation is good only if all the s_i's are large compared with the masses and momentum transfers involved.

For these reasons, there has recently been renewed interest[3] in another model of production processes: the multiperipheral model with pion pole dominance first proposed by Amati, Bertocchi, Fubini, Stanghellini, and Tonin (ABFST) in 1962.[4] This model is shown in Fig. 2. The model assumes that all final-state particles (with the possible exception of the first and last links) are pions, since experimentally at energies from tens of GeV to thousands of GeV the large majority of the produced particles are observed to be pions.[5] The model furthermore assumes that all exchanged particles are pions. A possible justification for this last assumption is that the one-pion-exchange (OPE) model works well when one or two pions are produced[6]; it is therefore plausible that increasing the number of final pions just increases the number of exchanged pions. Another way of saying this is that two-body and quasi-two-body processes are peripheral, i.e., the amplitude is large only when the momentum transfer is small; it is therefore plausible that general production processes are multiperipheral, i.e., the amplitude is large only when all the momentum transfers are small. Because of the small pion mass, this implies the dominance of pion exchange.

The ABFST multiperipheral model predicts[4] some features of high-energy scattering which are in qualitative agreement with experiment. The model predicts

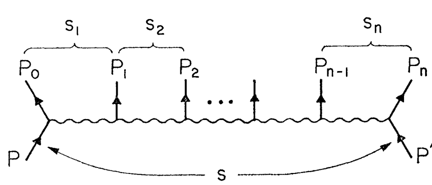

FIG. 1. Diagram for multi-Regge amplitude. Wavy lines correspond to Regge poles; $s\equiv(P+P')^2$; $s_1\equiv(P_0+P_1)^2$, $\ldots$, $s_n\equiv(P_{n-1}+P_n)^2$.

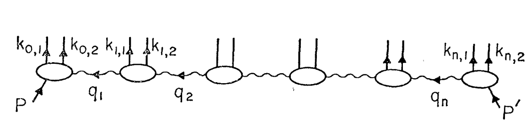

FIG. 2. Diagram for ABFST multiperipheral amplitude. Solid lines and wavy lines correspond to on-mass-shell and off-mass-shell pions, respectively; $t_i=q_i^2$, for $i=1, 2, \ldots, n$.

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[1] G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968); G. F. Chew, M. L. Goldberger, and F. E. Low, Phys. Rev. Letters **22**, 208 (1969); I. G. Halliday, Nuovo Cimento **60A**, 177 (1969); I. G. Halliday and L. M. Saunders, *ibid.* **60A**, 494 (1969); L. Caneschi and A. Pignotti, Phys. Rev. **180**, 1525 (1969); **184**, 1915 (1969); G. F. Chew and W. R. Frazer, *ibid.* **181**, 1914 (1969); J. S. Ball and G. Marchesini, *ibid.* **188**, 2209 (1969); S. Pinsky and W. I. Weisberger (unpublished).

[2] See, e.g., S.-J. Chang and R. Rajaraman, Phys. Rev. **183**, 1517 (1969); Aachen-Berlin-Bonn-CERN-Warsaw Collaboration, Nucl. Phys. **B8**, 471 (1968).

[3] G. F. Chew, T. W. Rogers, and D. R. Snider, UCRL Report No. UCRL-19457, 1970 (unpublished); J. S. Ball and G. Marchesini, Phys. Rev. **188**, 2508 (1969).

[4] L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento **25**, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, *ibid.* **26**, 896 (1962). This model will hereafter be called the ABFST multiperipheral model.

[5] See, e.g., article by P. V. Ramana Murthy, in ANL Report No. ANL/HEP 6909, 1968 (unpublished).

[6] See, e.g., G. Wolf, Phys. Rev. **182**, 1538 (1969).

Regge asymptotic behavior, average multiplicity growing as $\ln s$, a constant elasticity (ratio of the average laboratory energy of the primary outgoing particle to that of the incident particle),[7] and a transverse-momentum distribution of secondary particles which is independent of the energies of the incident and secondary particles. However, before we can seriously accept this model, we would like to have a quantitative comparison of the predictions of the model with experimental data.

This paper carries out such a quantitative comparison. In Sec. II we briefly rederive the relevant equations in AFS's paper,[4] with the generalization of AFS's equations to include off-shell pions in the input kernel. We also discuss the approximations that are used in deriving these equations. In Sec. III we describe the π-π elastic cross sections used in our calculations. In Sec. IV we describe the predictions of the model and compare them with experimental data. We also show that these predictions are almost independent of the detailed shape of the input π-π elastic cross sections. In Sec. V we discuss some possible modifications of the model by the relaxation of some of the approximations used in Sec. II. We end with a conclusion in Sec. VI.

II. DERIVATION OF RELEVANT EQUATIONS

A. Integral Equation for Absorptive Part

Consider the process where $(n+1)$ pairs of pions are produced. The kinematic variables are shown in Fig. 2. We first consider the incident particles to be pions with zero isospin; we will later consider the general case of physical isospin and particles other than pions. Our model assumes that the production amplitude is given by the factorized form

$$\begin{aligned}T_n(P,P';k_{0,1},k_{0,2},k_{1,1},k_{1,2},\ldots,k_{n,1},k_{n,2})\\ =[(q_1{}^2-\mu^2)(q_2{}^2-\mu^2)\cdots(q_n{}^2-\mu^2)]^{-1}\\ \times[T^R(P,q_1;k_{0,1},k_{0,2})T^R(q_1,q_2;k_{1,1},k_{1,2})\cdots\\ T^R(q_{n-1},q_n;k_{n-1,1},k_{n-1,2})T^R(q_n,P';k_{n,1},k_{n,2})]\,,\end{aligned}\tag{2.1}$$

where the superscript R denotes elastic scattering, and $\mu=m_\pi$. The optical theorem can then be used to relate elastic scattering to production processes. This is illustrated in Fig. 3. Following ABFST,[4] we can derive a recursion relation for the $(2n+2)$-particle contribution to the off-shell absorptive part of the forward elastic amplitude,

$$A_n(s,u)=\frac{1}{8\pi^4}\int ds_0\int\int ds'du'\times\frac{A^R(s_0,-u',-u)Q(s,u;s',u';s_0)A_{n-1}(s',u')}{(u'+\mu^2)^2}.\tag{2.2}$$

[7] AFS calls this the inelasticity, but our notation agrees with that of most authors.

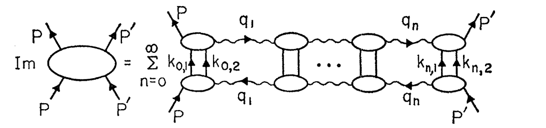

FIG. 3. Unitarity relates the production amplitudes to the forward absorptive part of the elastic amplitude.

Here

$$\begin{aligned}Q(s,u;s',u';s_0)\\ =[\pi/2(S^2+4\mu^2u)^{1/2}]\Theta(uS'^2+u'S^2-\mu^2S_0{}^2\\ -SS'S_0+4\mu^2uu')\Theta(s^{1/2}-s'^{1/2}-s_0{}^{1/2})\,,\end{aligned}\tag{2.3}$$

where

$$\begin{aligned}S&=s+u-\mu^2\,,\\ S'&=s'+u'-\mu^2\,,\\ S_0&=s_0+u+u'\,.\end{aligned}\tag{2.4}$$

Therefore, $Q(s,u;s',u';s_0)$ determines the limits of integration in (2.2). In (2.2), $A^R(s_0,-u',-u)$ is the off-shell continuation of $A^R(s_0,\mu^2,\mu^2)$, where

$$A^R(s_0,\mu^2,\mu^2)=2|\mathbf{q}^{\mathrm{c.m.}}|s_0{}^{1/2}\sigma_{\mathrm{el}}(s_0,\mu^2,\mu^2)\,,\tag{2.5}$$

with $|\mathbf{q}^{\mathrm{c.m.}}|$ being the center-of-mass momentum. There is no unique or *a priori* correct continuation of (2.5) to obtain $A^R(s_0,-u',-u)$. In Sec. V we will consider several methods of continuing off shell. Summing (2.2), we get

$$A(s,u)=A^R(s,u)+\frac{1}{8\pi^4}\int ds_0\int\int ds'du'\times\frac{A^R(s_0,-u',-u)Q(s,u;s',u';s_0)A(s',u')}{(u'+\mu^2)^2}.\tag{2.6}$$

The physical forward absorptive part of the elastic amplitude is given by $A(s,u)$ continued to $u=-\mu^2$. The integral equation (2.6) is schematically illustrated in Fig. 4.

If we take the high-energy limit and assume $s\gg\mu^2$, u, s_0, then $Q(s,u;s',u';s_0)$ gives the following limits of integration for (2.6):

$$A(s,u)=A^R(s,u)+\frac{1}{16\pi^3}\int_{4\mu^2}ds_0\int_0^s\frac{ds'}{s}\int_{(s'/s)[u+s_0(1-s'/s)^{-1}]}^{\infty}du'\times\frac{A^R(s_0,-u',-u)A(s',u')}{(u'+\mu^2)^2}.\tag{2.7}$$

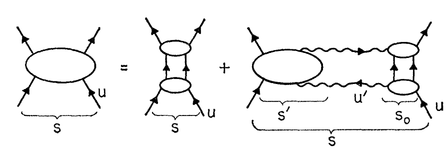

FIG. 4. Schematic representation of Eq. (2.6).

Note that there is no upper limit on s_0 except that $s_0 \ll s$. The model therefore appears to depend on where we cut off the s_0 integration. However, we know experimentally that for general production processes, the invariant mass of two adjacent pions in a multiperipheral chain (arranged, for example, according to longitudinal momentum) is bounded. This means there must be a $(s_0)_{\max}$ such that the contribution from $s_0 > (s_0)_{\max}$ is negligible. For asymptotic s, we can therefore let the upper integration limit for s_0 be either $(s_0)_{\max}$ or ∞. AFS conjectured that $(s_0)_{\max}$ is a few GeV^2. Of course, we can check the validity of this conjecture only *a posteriori*. This point will be further discussed in Secs. IV and V.

The relatively large multiplicity corresponding to this small invariant mass allows us to neglect $A^R(s,u)$ in the high-energy limit compared with $A(s,u)$, since the former is just the elastic contribution to the absorptive part, whereas the latter is the total absorptive part. Therefore,

$$A(s,u) \approx \frac{1}{16\pi^3}\int_{4\mu^2} ds_0 \int_0^s \frac{ds'}{s} \int_{(s'/s)[u+s_0(1-s'/s)^{-1}]}^{\infty} du' \times \frac{A^R(s_0,-u',-u)A(s',u')}{(u'+\mu^2)^2}. \quad (2.8)$$

Equation (2.8) is an integral equation in two variables. This equation has the special property that it can be reduced to an integral equation of the Fredholm type in one variable by assuming a solution of the form

$$A(s,u) = s^{\alpha}\phi_\alpha(u). \quad (2.9)$$

Equations (2.8) and (2.9) imply

$$\phi_\alpha(u) = \frac{1}{16\pi^3(\alpha+1)(2u)^{\alpha+1}}\int_0^\infty du' \times \phi_\alpha(u')\int_{4\mu^2} ds_0\, A^R(s_0,-u',-u) \times \frac{\{s_0+u+u'-[(s_0+u+u')^2-4uu']^{1/2}\}^{\alpha+1}}{(u'+\mu^2)^2}. \quad (2.10)$$

From (2.9) we see that the model predicts Regge asymptotic behavior, with α being the position of the Regge pole. Equation (2.10) has a solution only for certain values of α, and a knowledge of $A^R(s_0,-u',-u)$ allows us to calculate these values. For asymptotic s, we are interested in the nearest value, corresponding to the leading Regge pole.

B. Average Multiplicity

Since $A_n(s,-\mu^2)$ is proportional to the contribution to the total cross section from the production process where $(n+1)$ pairs of pions are produced, it is also proportional to the probability of producing $(n+1)$ pairs of pions. This means the average number of produced pions is

$$\langle n\rangle = \sum_{n=0}^{\infty} 2(n+1)A_n(s,-\mu^2) \Big/ \sum_{n=0}^{\infty} A_n(s,-\mu^2). \quad (2.11)$$

If we represent the variation of the strength of the π-π elastic cross section by an over-all coupling constant g, where $g=1$ corresponds to the actual coupling strength, one can easily show[4]

$$\langle n\rangle = 2\left[g\frac{\partial\alpha}{\partial g}\right]_{g=1}\ln s + 2\left[g\frac{\partial}{\partial g}(\ln\phi_\alpha{}^{\text{phys}})\right]_{g=1}. \quad (2.12)$$

Thus the model predicts that average multiplicity grows as $\ln s$ as $s\to\infty$.

C. Elasticity, Transverse-Momentum Distribution, and Average Subenergy

To derive the elasticity, we single out the primary link (one closest to the incident particle) in the multiperipheral chain. This is shown in Fig. 5. If we let $\bar{E}_{\text{pion pair}}$ be the average laboratory energy of the primary outgoing pair of pions and E_{inc} be the laboratory energy of the incident particle, then one can show[4]

$$\bar{E}_{\text{pion pair}}/E_{\text{inc}} = \frac{1}{16\pi^3\phi_\alpha{}^{\text{phys}}}\int_0^\infty du' \frac{\phi_\alpha(u')}{(u'+\mu^2)^2}\int_{4\mu^2} ds_0 \times A^R(s_0,-u',\mu^2)\int_0^{x_\mu} dx(1-x)x^{\alpha}, \quad (2.13)$$

where

$$x_\mu = \frac{[(s_0-\mu^2+u')^2+4\mu^2u']^{1/2}-(s_0-\mu^2+u')}{2\mu^2}.$$

If we assume this pion pair is the decay product of a pure resonance, e.g., the ρ, then on the average each pion will carry away half of the laboratory energy of the resonance. The elasticity is therefore $\bar{E}_{\text{one pion}}/E_{\text{inc}} = \frac{1}{2}\bar{E}_{\text{pion pair}}/E_{\text{inc}}$. However, if we cannot specify which of these two pions corresponds to the incoming pion (this is in general the case in cosmic-ray experiments), then the fast outgoing pion on the average will carry away three-fourths of the laboratory energy of the resonance. The elasticity is therefore

$$\bar{E}_{\text{one pion}}/E_{\text{inc}} = \tfrac{3}{4}\bar{E}_{\text{pion pair}}/E_{\text{inc}}. \quad (2.14)$$

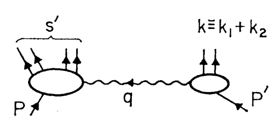

FIG. 5. Diagram used in calculating elasticity; $s_0 \equiv k^2$.

We see that the model predicts an elasticity which is independent of s. This last effect, due to the fact that either decay particle may be considered to be the fast outgoing particle, does not apply when one of the decay particles is much more massive than the other, as in the decay of $\Delta(1236)$ into N-π.

To calculate the transverse-momentum distribution and the average invariant mass squared of the secondary pion pairs, one has to consider diagrams of the form of Fig. 6. Calling $\mathbf{k}_T$ the total transverse momentum of a pion pair in the over-all c.m. frame (or the laboratory frame) and neglecting the pions near the ends of the multiperipheral chain (so that we can neglect all masses and momentum transfers relative to s' and s''), one can show[4] that the transverse-momentum distribution and the invariant-mass-squared distribution are given by the expression $F d\mathbf{k}_T{}^2 dk^2$, where[8]

$$F=(k^2+\mathbf{k}_T{}^2)^{1+\alpha}\int\int du'du''\times\frac{A^R(k^2,-u',-u'')\phi_\alpha(u')\phi_\alpha(u'')}{(u'+\mu^2)^2(u''+\mu^2)^2}\int\int dxdy\times x^\alpha y^\alpha T[u'-(k^2+\mathbf{k}_T{}^2)x(1+y);\ u''-(k^2+\mathbf{k}_T{}^2)y(1+x);\mathbf{k}_T{}^2],\quad(2.15)$$

and

$$T(a,b,c)=\frac{\Theta(-a^2-b^2-c^2+2ab+2ac+2bc)}{(-a^2-b^2-c^2+2ab+2ac+2bc)^{1/2}},\quad x=s'/s_1,\quad y=s''/s_2,\quad(2.16)$$

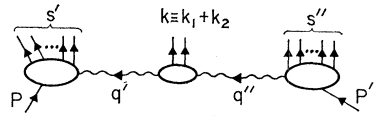

FIG. 6. Diagram used in calculating transverse-momentum distribution.

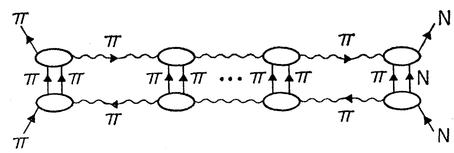

FIG. 7. Diagram for π-N scattering.

with $s_1=(P+k)^2$, $s_2=(P'+k)^2$. In (2.15), the upper limits of integration for x and y depend on the energy of the secondary, k_0, and on the total energy $s^{1/2}$. But as noted by AFS,[4] when $k_0 \ll s^{1/2}$, then these upper limits are outside the support of the Θ function. For $k_0 \ll s^{1/2}$, F is therefore independent of k_0 and $s^{1/2}$, i.e.,

$$F=F(k^2,\mathbf{k}_T{}^2).\quad(2.17)$$

Thus the model correctly predicts that the transverse-momentum distribution is independent of both the incident and secondary energies.

D. Extension to N-N Scattering

First consider π-N scattering. The general multiperipheral diagram is shown in Fig. 7. Proceeding in essentially the same manner as in the π-π case, one can show that the total absorptive part is given by

$$A_{\pi N}(s,u)=A_{\pi N}{}^R(s,u)+\frac{1}{8\pi^4}\int_{(\mu+m)^2}ds_0\int\int ds'du'\frac{A_{\pi N}{}^R(s_0,-u',-u)A_{\pi\pi}(s',u')Q(s,u;s',u';s_0)}{(u'+\mu^2)^2},\quad(2.18)$$

where $m=m_N$ and $A_{\pi N}{}^{\mathrm{phys}}(s)$ is given by $A_{\pi N}(s,-m^2)$. This equation is schematically represented in Fig. 8. Making approximations similar to those in the π-π case, we have

$$A_{\pi N}(s,u)=s^\alpha\phi_\alpha{}^{(\pi N)}(u).\quad(2.19)$$

Here

$$\phi_\alpha{}^{(\pi N)}(u)=\frac{1}{16\pi^3(\alpha+1)(2u)^{\alpha+1}}\int_0^\infty du'\,\phi_\alpha{}^{(\pi\pi)}(u')\int_{(\mu+m)^2}ds_0\times\frac{A_{\pi N}{}^R(s_0,-u',-u)\{s_0+u+u'-[(s_0+u+u')^2-4uu']^{1/2}\}^{\alpha+1}}{(u'+\mu^2)^2},\quad(2.20)$$

where α is determined by the integral equation for π-π scattering, i.e., Eq. (2.10).

We can easily generalize this to N-N scattering. The general multiperipheral diagram is shown in Fig. 9. The total absorptive part is given by

$$A_{NN}(s,u)=A_{NN}{}^R(s,u)+\frac{1}{8\pi^4}\int_{(\mu+m)^2}ds_0\int\int ds'du'\frac{A_{\pi N}{}^R(s_0,-u',-u)A_{\pi N}(s',u')Q(s,u;s',u';s_0)}{(u'+\mu^2)^2},\quad(2.21)$$

where $A_{NN}{}^{\mathrm{phys}}(s)$ is given by $A_{NN}(s,-m^2)$. This equation is schematically represented in Fig. 10. Making approx-

[8] AFS left out a factor of $x^\alpha y^\alpha$ in their expression for F.

imations as before, we get

$$A_{NN}(s,u)=s^{\alpha}\phi_{\alpha}{}^{(NN)}(u)\,. \tag{2.22}$$

Here

$$\phi_{\alpha}{}^{(NN)}(u)=\frac{1}{16\pi^3(\alpha+1)(2u)^{\alpha+1}}\int_0^{\infty}du'\,\phi_{\alpha}{}^{(\pi N)}(u')\int_{(\mu+m)^2}ds_0 \times\frac{A_{\pi N}{}^R(s_0,-u',-u)\{s_0+u+u'-[(s_0+u+u')^2-4uu']^{1/2}\}^{\alpha+1}}{(u'+\mu^2)^2}\,, \tag{2.23}$$

where α is again determined by (2.10) for π-π scattering. Similarly, the ratio of the laboratory energy of the outgoing N-π pair to the laboratory energy of the incident N can be easily shown to be

$$\bar{E}_{N\text{-}\pi\text{ pair}}/E_{\text{inc}} =\frac{1}{16\pi^3\phi_{\alpha}{}^{(NN),\text{phys}}}\int_0^{\infty}du'\frac{\phi_{\alpha}{}^{(\pi N)}(u')}{(u'+\mu^2)^2}\int_{(\mu+m)^2}ds_0 \times A_{\pi N}{}^R(s_0,-u',m^2)\int_0^{x_m}dx(1-x)x^{\alpha}, \tag{2.24}$$

where

$$x_m=\frac{[(s_0-m^2+u')^2+4m^2u']^{1/2}-(s_0-m^2+u')}{2m^2}\,.$$

If we now assume that the N-π pair is the decay product of $\Delta(1236)$, then on the average N carries away 78% of the laboratory energy of $\Delta(1236)$. The elasticity is therefore

$$\bar{E}_N/E_{\text{inc}}=0.78\bar{E}_{N\text{-}\pi\text{ pair}}/E_{\text{inc}}\,. \tag{2.25}$$

Note that if we had used a more massive N-π resonance, then N would carry away even a smaller part of the resonance's energy. We can also easily show that for N-N scattering, the average number of produced pions is

$$\langle n\rangle_{N\text{-}N}=2+2\left[g\frac{\partial\alpha}{\partial g}\right]_{g=1}\ln s +2\left[g\frac{\partial}{\partial g}(\ln\phi_{\alpha}{}^{(NN),\text{phys}})\right]_{g=1}. \tag{2.26}$$

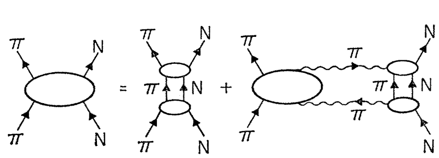

FIG. 8. Schematic representation of Eq. (2.18).

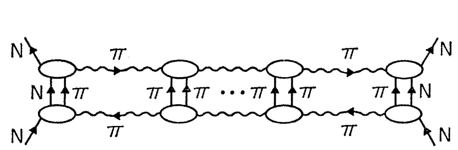

FIG. 9. Diagram for N-N scattering.

The transverse-momentum and invariant-mass-squared distributions are given by (2.15) with the substitution of $\phi_{\alpha}{}^{(\pi N)}(u')\phi_{\alpha}{}^{(\pi N)}(u'')$ for $\phi_{\alpha}(u')\phi_{\alpha}(u'')$.

E. Extension to Include Isospin

The generalization to include isospin is trivial once we realize that a definite isospin in the t channel is carried through the multiperipheral chain because of isospin conservation. See Fig. 11. If we work with an amplitude of definite I_t, inclusion of isospin then just introduces the parameter I_t into our equations. Using a subscript and a superscript to denote isospin in the t and s channel, respectively, we get, in place of (2.9) and (2.10),

$$A_I(s,u)=s^{\alpha_I}\phi_{\alpha_I}(u)\,, \tag{2.27}$$

where

$$\phi_{\alpha_I}(u)=\frac{1}{16\pi^3(\alpha_I+1)(2u)^{\alpha_I+1}}\int_0^{\infty}du' \times\phi_{\alpha_I}(u')\int_{4\mu^2}ds_0\,A_I{}^R(s_0,-u',-u) \times\frac{\{s_0+u+u'-[(s_0+u+u')^2-4uu']^{1/2}\}^{\alpha_I+1}}{(u'+\mu^2)^2} \tag{2.28}$$

and

$$A_I{}^R(s_0,-u',-u)=\sum_T C_I{}^T A^{RT}(s_0,-u',-u)\,, \tag{2.29}$$

with $C_I{}^T$ being the appropriate isospin crossing matrix. To obtain $A(s,u)$ for any physical process, one has to

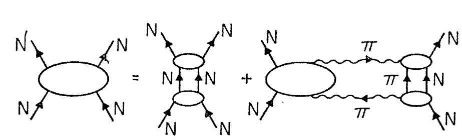

FIG. 10. Schematic representation of Eq. (2.21).

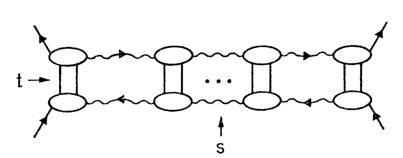

FIG. 11. A definite isospin in the t channel is carried through the multiperipheral chain because of isospin conservation.

take only the appropriate linear combination of $A_I(s,u)$. For later applications, we here list the π-π and π-N isospin crossing matrices:

$$(C_I{}^T)_{\pi\pi}=(C^T{}_I)_{\pi\pi}=\begin{bmatrix}\frac{1}{3} & 1 & 5/3\\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6}\\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6}\end{bmatrix}, \qquad (2.30\text{a})$$

$$(C_I{}^T)_{\pi N}=\begin{pmatrix}\frac{1}{3}(6)^{1/2} & \frac{2}{3}(6)^{1/2}\\ \frac{2}{3} & -\frac{2}{3}\end{pmatrix}, \qquad (2.30\text{b})$$

$$(C^T{}_I)_{\pi N}=\begin{pmatrix}1/(6)^{1/2} & 1\\ 1/(6)^{1/2} & -\frac{1}{2}\end{pmatrix}. \qquad (2.30\text{c})$$

Because the amplitude is proportional to s^{α_I}, in the high-energy limit, $\langle n\rangle$, $\bar{E}_{N \text{ or } \pi}/E_{\text{inc}}$, $\langle|\mathbf{k}_T|\rangle$, and $\langle k^2\rangle$ can be calculated to a good approximation by considering just the $I_t=0$ case.

III. π-π ELASTIC CROSS SECTIONS

From the discussion in Sec. II, we see that the basic equation to solve is (2.28). To accomplish this, we need a knowledge of $A_I{}^R(s_0,-u',-u)$, where, by (2.5) and (2.29),

$$A_I{}^R(s_0,\mu^2,\mu^2)=2|\mathbf{q}^{\text{c.m.}}(\mu^2,\mu^2)|s_0{}^{1/2}\sum_T C_I{}^T\sigma_{\text{el}}{}^T(s_0,\mu^2,\mu^2). \qquad (3.1)$$

As discussed in Sec. II, there is no unique or *a priori* correct way of continuing $A_I{}^R$ off the mass shell; in Sec. V we will consider several methods of continuation. We now discuss what to use for $A_I{}^R(s_0,\mu^2,\mu^2)$.

At present, there is no π-π scattering experiment; so there are no direct π-π data. Our present experimental knowledge about π-π scattering comes from studying $\pi N\rightarrow\pi\pi N$ reactions using an OPE model. However, we need to know $\sigma_{\text{el}}{}^T(s_0)$ for all three T's. At present, we do not have sufficient data to extract this information reliably. Besides, there is no unique way of using an OPE model to extract π-π cross sections. In this paper, we use the Veneziano model for π-π scattering[9] to give us $A_I{}^R(s_0,\mu^2,\mu^2)$ in the resonance region.

At low energy, the π-π elastic cross section is approximately equal to the π-π total cross section (the ρ and f resonances decay totally or almost totally into 2π) which by the optical theorem is equal to the absorptive part of the forward elastic amplitude. We represent this elastic amplitude by the π-π Veneziano formula. Therefore, $A^R(s_0,\mu^2,\mu^2)$ is just a sum of δ functions. In our calculations, we cut off the s_0 integration at the mass of the g resonance and include the ρ, f, and g contributions (and the contributions of their daughters). In Sec. V we will discuss the implications of the s_0 cutoff. The inclusion of the g contribution is somewhat questionable, since its dominant decay mode may be 4π. We include it as a compensation for the higher s_0 contribution, which we have neglected. In any case, including or excluding the g does not change any essential features of the model, but only changed somewhat the numbers obtained.

Following Shapiro,[9] if we use a single Veneziano term, normalized to $\Gamma_{\rho\pi\pi}=125$ MeV and $m_\rho=764$ MeV, and require the correct $I=L=0$ π-π phase shifts, then we get the following representation for the π-π amplitude with s,t, and $\bar{u}$ being the usual Mandelstam invariants (we put a bar over u so as not to confuse this $\bar{u}$ with the previous u):

$$T_s{}^0=\tfrac{3}{2}F(s,t)+\tfrac{3}{2}F(s,\bar{u})-\tfrac{1}{2}F(t,\bar{u}), \qquad (3.2\text{a})$$

$$T_s{}^1=F(s,t)-F(s,\bar{u}), \qquad (3.2\text{b})$$

$$T_s{}^2=F(t,\bar{u}), \qquad (3.2\text{c})$$

where

$$F(x,y)=\beta\frac{\Gamma(1-\alpha(x))\Gamma(1-\alpha(y))}{\Gamma(1-\alpha(x)-\alpha(y))}, \qquad (3.3)$$

where $\alpha(x)=a+bx$, with

$$a=0.48, \qquad (3.4\text{a})$$

$$b=0.90\ \text{GeV}^{-2}, \qquad (3.4\text{b})$$

$$\beta=-1.22\quad (\text{Ref. }10). \qquad (3.4\text{c})$$

Equations (2.29), (2.30a), and (3.2)–(3.4) together imply[11]

$$A_0{}^R(s_0,\mu^2,\mu^2)=16\pi(-\pi\beta/b)\times[0.71\delta(s_0-m_\rho{}^2)+1.13\delta(s_0-m_f{}^2)+1.27\delta(s_0-m_g{}^2)], \qquad (3.5\text{a})$$

$$A_1{}^R(s_0,\mu^2,\mu^2)=16\pi(-\pi\beta/b)\times[0.48\delta(s_0-m_\rho{}^2)+0.71\delta(s_0-m_f{}^2)+0.88\delta(s_0-m_g{}^2)], \qquad (3.5\text{b})$$

$$A_2{}^R(s_0,\mu^2,\mu^2)=16\pi(-\pi\beta/b)\times[0.03\delta(s_0-m_\rho{}^2)-0.11\delta(s_0-m_f{}^2)+0.09\delta(s_0-m_g{}^2)]. \qquad (3.5\text{c})$$

IV. PREDICTIONS OF MODEL AND COMPARISON WITH EXPERIMENT

When ABFST first proposed this multiperipheral model, they conjectured that the important contribution to the kernel in (2.28) comes from the low-energy (the resonance region) π-π elastic cross section and that the dependence of the latter on the mass of the virtual pions is negligible. In this section, we calculate the predictions of this model under these two approximations. In (2.28), we put a cutoff for the s_0 integration at $m_g{}^2$ and replace $A_I{}^R(s_0,-u',-u)$ by $A_I{}^R(s_0,\mu^2,\mu^2)$.

[9] J. A. Shapiro, Phys. Rev. **179**, 1345 (1969).

[10] Shapiro (Ref. 9) normalized to $\Gamma_\rho=112$ MeV; he found $\beta=-1.09$.

[11] Note that there is a difference of (16π) in the normalization used by AFS in Ref. 4 and Shapiro in Ref. 9: $A^{\text{AFS}}(s,t)\equiv(16\pi)\times A^{\text{Shapiro}}(s,t)$.

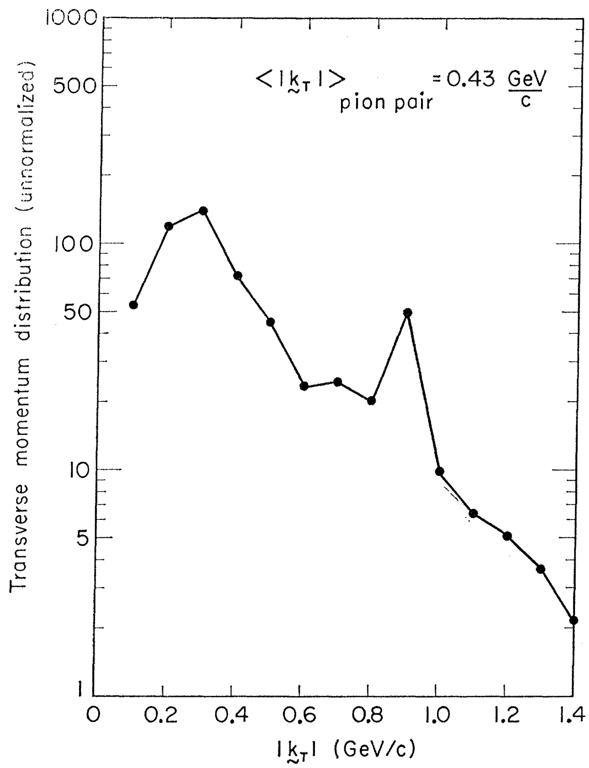

FIG. 12. Transverse-momentum distribution of pion pairs.

In Sec. V, we will consider several modifications of the model by the relaxation of these approximations, but we will find that none of these is satisfactory.

Our calculations show the Regge trajectory intercepts to be

$$\alpha_{I=0}=0.30\,, \tag{4.1a}$$

$$\alpha_{I=1}=0.16\,, \tag{4.1b}$$

the average multiplicity to be[12]

$$\langle n\rangle \approx 0.74 \ln s\,, \tag{4.1c}$$

the elasticity to be

$$\bar{E}_N/E_{\rm inc}=0.69\,, \tag{4.1d}$$

the average transverse momentum of the produced pion pairs to be

$$\langle |\mathbf{k}_T|\rangle_{\text{pion pair}}=0.43 \text{ GeV}/c\,, \tag{4.1e}$$

and the average invariant mass squared for each pair of outgoing pions to be

$$\langle k^2\rangle = 0.84 \text{ GeV}^2. \tag{4.1f}$$

The first three numbers given are the same for all scattering processes. The last three numbers are for N-N scattering[13]; these corresponding numbers for π-π scattering[14] are $\bar{E}_\pi/E_{\rm inc}=0.64$, $\langle |\mathbf{k}_T|\rangle_{\text{pion pair}}=0.39$ GeV/c, and $\langle k^2\rangle=0.81$ GeV^2. The pion transverse-momentum distribution for N-N scattering is plotted in Fig. 12. We do not have an explanation for the peak at 0.9 GeV/c. The corresponding experimental numbers are

$$\alpha_{I=0}\approx 1.0\,, \tag{4.2a}$$

$$\alpha_{I=1}\approx 0.5\,, \tag{4.2b}$$

$$\langle n\rangle \approx 1 \ln s - 2 \ln s \quad \text{(Ref. 15)}, \tag{4.2c}$$

$$\bar{E}_N/E_{\rm inc}\approx 0.5\text{–}0.6 \quad \text{(Ref. 16)}, \tag{4.2d}$$

$$\langle |\mathbf{k}_T|\rangle_{\text{one pion}}\approx 0.30\text{–}0.35 \text{ GeV}/c \quad \text{(Ref. 17)}, \tag{4.2e}$$

$$\langle k^2\rangle \approx 0.7\text{–}0.8 \text{ GeV}^2 \quad \text{(Ref. 18)}, \tag{4.2f}$$

Comparing these two sets of numbers, we see that the model's predictions for elasticity, average transverse momentum, average invariant mass, and possibly average multiplicity are close to the experimental values, but the predictions for the Regge-trajectory intercepts are too small. To generate the experimental P and ρ intercepts, the kernel of our integral equation (2.28) must be multiplied by a factor of 5 and 2.5, respectively. If we increase the kernel strength by a

[12] Using (2.12) and (2.26), we have also calculated the constant term in the expression for $\langle n\rangle$. We found $\langle n\rangle_{\pi-\pi}=3.7+0.74 \ln s$ and $\langle n\rangle_{N-N}=1.7+0.74 \ln s$. However, these constant terms are fairly sensitive to the value of α. Note that to calculate the constant term, we have to solve the inhomogeneous integral Eq. (2.7), since there is an arbitrary normalization in the solution of the homogeneous integral equation.

[13] For N-N scattering, we need to know the π-N elastic cross sections, as is evident from (2.23) and (2.20). We use those given in G. Källén, *Elementary Particle Physics* (Addison-Wesley, Reading, Mass., 1964).

[14] For π-π scattering, we also found $\alpha_{I=2}=-0.80$. In calculating this number, we have included only the ρ contribution, since in this channel, the Veneziano formula gives a negative contribution from the f resonance [see (3.5c)], and since cross sections should be intrinsically positive.

[15] Experimentally, this number is not precisely known. From experimental production cross sections at laboratory energies, Chew and Pignotti (see Ref. 1) conclude that $\langle n\rangle \approx 1.1 \ln s$. On the other hand, cosmic-ray data give $\langle n\rangle \approx 2 \ln s$ for the average number of produced pions. See P. V. Ramana Murthy, Ref. 5. However, cosmic-ray data are most likely overestimating the average multiplicity, because of the use of heavy nuclei as targets. In a preliminary analysis of their new cosmic-ray data, L. W. Jones *et al.* found that in the 100–400-GeV energy range the average multiplicity with a liquid-hydrogen target is about 70% of that with a carbon target. See L. W. Jones *et al.*, University of Michigan report, 1969 (unpublished).

[16] P. V. Ramana Murthy, Ref. 5; F. Turkot, in *Proceedings of the Topical Conference on High Energy Collisions* (CERN, Geneva, 1968).

[17] G. Cocconi, Nuovo Cimento **57A**, 837 (1968); P. V. Ramana Murthy, Ref. 5; Aachen-Berlin-Bonn-CERN-Cracow-Heidelberg-Warsaw Collaboration, Nucl. Phys. **B13**, 571 (1969).

[18] Experimentally, this number is not precisely known, but experiments at laboratory energies with low multiplicity indicate that the invariant mass is slightly above m_ρ. See, e.g., Aachen-Berlin-Bonn-CERN-Warsaw Collaboration, Ref. 17; I. R. Kenyon, Nucl. Phys. **B13**, 255 (1969).

TABLE I. Variation of $\alpha_{I=0}$ with u_c' at three different values of $(s_0)_{\max}$.

$(s_0)_{\max}$ (GeV^2)	$m_g^2=2.7$			12.7			52.7		
u_c' (GeV^2/c^2)	∞	2	5	∞	2	5	∞	2	5
$\alpha_{I=0}$	0.30	0.275	0.28	0.40	0.34	0.38	0.55	0.43	0.48
% of total contribution	100	94	95	100	84	95	100	71	83

factor of 5, then we get

$$\alpha_{I=0}=1.00\,, \tag{4.3a}$$

$$\alpha_{I=1}=0.78\,, \tag{4.3b}$$

$$\langle n\rangle=1.04\ \ln s\,, \tag{4.3c}$$

$$\bar{E}_N/E_{\rm inc}=0.62\,, \tag{4.3d}$$

$$\langle|\mathbf{k}_T|\rangle_{\text{pion pair}}=0.48\ \text{GeV}/c\,, \tag{4.3e}$$

$$\langle k^2\rangle=0.76\ \text{GeV}^2. \tag{4.3f}$$

We conclude that with the present approximations, the kernel in our model is of insufficient strength.

Although we use the Veneziano formula for $A_I^R(s_0,\mu^2,\mu^2)$ to get the above results, these results are actually almost independent of the detailed shape of $A_I^R(s_0,\mu^2,\mu^2)$. Instead of evaluating (3.5a) with δ functions, we have substituted Breit-Wigner forms, normalizing to the constants in (3.5a), and got almost the same answers.

V. POSSIBLE MODIFICATIONS OF THE MODEL

A. Inclusion of High-s_0 Contribution

The most obvious way of modifying our calculations is to raise the s_0 cutoff $(s_0)_{\max}$ in (2.28), for this will surely increase our kernel strength. This was the approach of Ball and Marchesini.[3] However, increasing $(s_0)_{\max}$ increases the average subenergy of each pair of outgoing pions and so decreases the average multiplicity. But our multiplicity is already smaller than the experimental value. In their calculations, Ball and Marchesini can get $\alpha_{I=0}=0.95$, but at the expense of getting $\langle n\rangle\approx 0.2\ \ln s$. Furthermore, increasing $(s_0)_{\max}$ increases the average momentum transfers. One way to see this is to use a u' cutoff u_c' in (2.28), and observe the change in α_I as u_c' is varied. If we let $\sigma_{I_t=0}(s_0)=(5\text{ mb})/\ln s_0$ for $s_0>m_g^2$, then we obtain the result shown in Table I.

The last row represents the ratio (in percent) of the contribution with a cutoff in u' to that without a cutoff in u'. These numbers are obtained by varying the kernel strength to get the appropriate α's. This increase in average momentum transfers undermines the whole basis of the ABFST multiperipheral model—small momentum transfers and consequently the dominance of pion exchange. We therefore conclude that the inclusion of the large subenergy contribution is not the solution to our problem. If there is a modification which can fix up this model, it must damp out the large s_0 contribution.[19] This also means that our model will be independent of $(s_0)_{\max}$ as long as $(s_0)_{\max}$ is above the resonance region.

B. Off-Shell Continuation

As there is no unique or *a priori* correct off-shell continuation of $A^R(s_0,\mu^2,\mu^2)$, we consider several methods of continuation.

Since the Mandelstam invariants satisfy the usual constraints

$$s+t+\bar{u}=\sum_{i=1}^{4} m_i^2\,, \tag{5.1}$$

the Veneziano representation (3.2) has a natural off-shell continuation by requiring $\bar{u}$ at fixed s ($t=0$ in our case of forward scattering) to always satisfy (5.1) when the m_i^2 are taken off-shell. Assuming that the Veneziano coupling constant β does not vary as we go off-shell, instead of (3.5) we now have

$$\begin{aligned}A_0^R(s_0,-u',-u)&\\ =16\pi(-\pi\beta/b)\{&[0.48-0.45\bar{u}]\delta(s_0-m_\rho^2)\\ &+[0.72-0.40\bar{u}^2-0.88\bar{u}]\delta(s_0-m_f^2)\\ &+[0.87-0.25\bar{u}^3-0.9\bar{u}^2-1.2\bar{u}]\delta(s_0-m_g^2)\}\,,\end{aligned} \tag{5.2a}$$

$$\begin{aligned}A_1^R(s_0,-u',-u)&\\ =16\pi(-\pi\beta/b)\{&0.48\delta(s_0-m_\rho^2)+0.71\delta(s_0-m_f^2)\\ &+0.88\delta(s_0-m_g^2)\}\,,\end{aligned} \tag{5.2b}$$

$$\begin{aligned}A_2^R(s_0,-u',-u)&\\ =16\pi(-\pi\beta/b)\{&[0.48+0.90\bar{u}]\delta(s_0-m_\rho^2)\\ &+[0.81\bar{u}^2+1.76\bar{u}+0.71]\delta(s_0-m_f^2)\\ &+\tfrac{1}{2}[\bar{u}^3+3.6\bar{u}^2+5.0\bar{u}+1.8]\delta(s_0-m_g^2)\}\,,\end{aligned} \tag{5.2c}$$

where

$$\bar{u}=-2(u+u')-s_0\,. \tag{5.3}$$

From (5.2a) and (5.3), we see that this continuation increases A_0^R. However, as is evident from (5.2a) and (5.3), the average momentum transfers are also drastically increased and so the basis of a multiperipheral model with pion pole dominance is undermined. Furthermore, this continuation leaves A_1^R unchanged, because $A_1^R=\text{Im}T_1^t=\text{Im}F(s,t=0)$, which is independent of $\bar{u}$, and it causes A_2^R to be negative when u or $u'\to\infty$, as is evident from (5.2c) and (5.3). This off-shell continuation is therefore unsatisfactory.

The difficulty of increasing the average momentum transfers is again encountered if in (3.1) we use the

[19] This point is discussed in detail by Chew, Rogers, and Snider in Ref. 3.

off-shell partial cross sections obtained from the Born approximation (meaning lowest-order perturbative diagram) and related to the on-shell partial cross sections by[20]

$$\sigma^l(s_0,u',u)=[q(u',u)/q]^{2l-1}\sigma^l(s_0)\,, \tag{5.4}$$

where

$$q=[\tfrac{1}{4}(s_0-4\mu^2)]^{1/2}, \tag{5.5a}$$

and

$$q(u',u)=\{[s_0^2+2s_0(u+u')+(u'-u)^2]/4s_0\}^{1/2}. \tag{5.5b}$$

Equation (5.4) implies that the partial cross sections blow up as u or $u'\to\infty$ for $l\geq 1$. This same difficulty of increasing momentum transfers also arises if in (3.1) we continue $|\mathbf{q}^{\text{c.m.}}|$ off-shell.

Wolf[6] has successfully fitted single and double pion production for $|t|<1$ GeV$^2/c^2$ and laboratory momentum between 1.6 and 20 GeV$/c$ by using an OPE model with the Benecke-Dürr off-shell continuation of partial cross sections[21]:

$$\sigma^l(s_0,u)=\frac{u_l(q_uR)}{u_l(qR)}\frac{q}{q_u}\sigma^l(s_0)\,, \tag{5.6}$$

where q_u is given by (5.5b) with $u'=-\mu^2$, R is a parameter (one for each l) determined by fitting data, and

$$u_l(x)=(2x^2)^{-1}Q_l(1+1/2x^2)\,, \tag{5.7}$$

where $Q_l(z)$ are Legendre functions of the second kind. Using the values of R obtained by Wolf, we find $\sigma^l(s_0,u)/\sigma^l(s_0)<1$ for $l=0,1,2,3$ and for almost all relevant values of s_0 and u. Although in OPE only one pion is off-shell, we believe taking two pions off-shell by replacing q_u by $q(u',u)$ will probably make the off-shell cross sections even smaller.

Another method of continuation is the Lovelace-Wagner unitarized π-π Veneziano formula.[22] For some values of s_0 and u this method gives off-shell partial cross sections that are larger than the on-shell partial cross sections, but for the important values of s_0 and u, its off-shell partial cross sections are smaller than the on-shell partial cross sections. Therefore, this continuation again decreases the kernel strength.

Thus, the methods of off-shell continuation that we have studied either decrease the kernel strength or drastically increase the momentum transfers.

C. Inclusion of Interference Terms

In a general production process in which many particles are produced, there are many ways of arranging the final particles along the multiperipheral chain with all these arrangements corresponding to the same physical process. The total amplitude is then actually a superposition of amplitudes. Thus in calculating production cross sections or n-particle contributions to the unitarity sum, interference terms appear. These interference terms correspond to nonplanar diagrams, as shown in Fig. 13. In all our discussion so far, we have neglected such interference terms, assuming that when the momentum transfers are small for one arrangement, they must be large for all other arrangements. It may turn out that this assumption is a poor one. Including interference terms may increase the kernel strength, but at present no one has calculated how large a contribution will come from such terms.

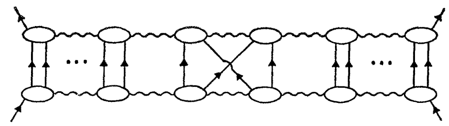

FIG. 13. Nonplanar diagrams corresponding to interference terms.

VI. CONCLUSIONS

The predictions of the ABFST multiperipheral model for elasticity, average transverse momentum, average invariant mass, and possibly average multiplicity are in the neighborhood of the experimental values. The model's predictions of trajectory heights are, however, much too small, corresponding to a kernel strength which is too weak by a factor of $2\frac{1}{2}$–5.

What is the explanation for the inadequate strength of our kernel? We have seen that the explanation is not due to our neglect of the high-subenergy contribution, since the inclusion of the latter decreases the average multiplicity and increases the average momentum transfers. A possible explanation lies in the off-shell continuation of the π-π cross sections, but we have seen that it is not easy for an off-shell continuation to increase the kernel strength without simultaneously increasing the average momentum transfers. The inclusion of K's, η's, and other particles, as well as π's, will definitely add to our kernel strength. Since experimentally the large majority of produced particles in high-energy collisions are π's, this added strength is probably not significant. Furthermore, the use of unmodified propagators for more massive exchanged particles is probably an overestimate of their effect, since these more massive poles are farther from the physical region.

A promising possibility is the inclusion of interference terms, but at present we have not calculated the sign and the strength of such terms. Another possibility is that the physical π-π cross sections may be larger than those given by our Veneziano prescription; this is especially important if it turns out that there are a strong threshold effect and/or a strong, low-mass s-wave resonance. On this point we shall have to wait until we have more information on π-π scattering.

[20] See, e.g., H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967), pp. 279 ff.
[21] J. Benecke and H. P. Dürr, Nuovo Cimento **56A**, 269 (1968).
[22] C. Lovelace, in Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 1969 (unpublished); F. Wagner, Nuovo Cimento **64A**, 189 (1969).

It is interesting to note that even when we increase our kernel strength by a factor of 5 to get the output Pomeranchuk pole at 1.0, the average multiplicity is only 1.04 ln$s$ [see (4.3e)]. Our model seems to indicate that the average multiplicity for N-N scattering is closer to 1 ln$s$ than to 2 ln$s$ (see Ref. 15).

Note added in proof. The Michigan experiment of Ref. 15 has now been completed. They obtained an average multiplicity per inelastic collision to be 1.14 ln$s$. We wish to thank Dr. Donald E. Lyon for informing us of this result before publication.

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Multichannel Model of the $K^*(890)$

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The inelastic channels $K^*\pi$ and K are included in a three-channel model of the $I=\frac{1}{2}$ p-wave $K\pi$ amplitude. Feynman diagrams for vector-meson exchange are used as input to multichannel N/D equations, which are solved to obtain the scattering amplitude. Coupling constants which cannot be measured experimentally or calculated using $SU(3)$ are obtained by employing $SU(6)_W$ in the rest system of one of the particles at a given vertex. The $K^*\pi$ channel, neglected in previous calculations, is observed to influence the amplitude strongly, but the width of the $K^*(890)$ is calculated to be 210 MeV or about four times the experimental value.

I. INTRODUCTION

COMPARATIVELY little work has been done on the effect of inelastic channels on the parameters of resonances in the K_π system. Those channels which lie lowest and should therefore be considered first are $K^*\pi$ and $K\eta$ with threshold energies of 1030 and 1042 MeV, respectively. The $K\eta$ channel has been included in a two-channel model of the p-wave $K\pi$ system in an article by Fulco, Shaw, and Wong.[1] Aside from this latter and an article by Gupta, Saxena, and Mathur[2] which presents the results of a single-channel calculation of s-, p-, and d-wave $K\pi$ phase shifts, the literature on the $K\pi$ interaction seems relatively sparse.

By contrast, multichannel models of the $\pi\pi$ interaction have been studied in great detail. Two-channel models of the ρ meson which have been studied include a treatment of the $\pi\pi$-$\pi\omega$ system by Zemach and Zachariasen,[3] and of the $\pi\pi$-$K\bar{K}$ system by Balázs.[4] A complete comparison of the several possibilities $\pi\pi$-$\pi\omega$, $\pi\pi$-$K\bar{K}$, and $\pi\pi$-$\pi\omega$-$K\bar{K}$ was also performed by Fulco, Shaw, and Wong.

There are, however, difficulties in a calculation of the p-wave $K\pi$ interaction using $K^*\pi$ as an inelastic channel which are not encountered in an analysis in which $K\eta$ is the only inelastic channel or in any of the multichannel $\pi\pi$ calculations mentioned. In the latter cases, all coupling constants may be determined either directly from experimental measurements of decay rates or indirectly by $SU(3)$. In a model of the $K\pi$ interaction which includes $K^*\pi$ as an inelastic channel, one encounters coupling constants which must be evaluated either by appeal to higher symmetries or by making assumptions about ϕ-ω mixing in the vector-meson octet. We have chosen the former approach and evaluate such coupling constants by using $SU(6)_W$ in the rest system of one of the particles at a given vertex. The model discussed in this paper is a three-channel one with $K^*\pi$ and $K\eta$ taken as inelastic channels.

The single-particle exchange contributions are calculated as Feynman diagrams and are used as input into a multichannel N/D equation. The diagrams used and their partial-wave analysis are discussed in Sec. II. In Sec. III we describe in greater detail the N/D formalism used and the exact method of solution employed. The results of the calculation are summarized in Sec. IV.

II. INPUT DIAGRAMS

We use as input terms the set of diagrams shown in Fig. 1. These consist of all permissible t- or u-channel exchanges of pseudoscalar or vector mesons. Since we

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[1] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[2] K. C. Gupta, R. P. Saxena, and V. S. Mathur, Phys. Rev. **141**, 1479 (1966).

[3] C. Zemach and F. Zachariasen, Phys. Rev. **128**, 849 (1962).

[4] L. A. P. Balázs, Phys. Rev. **137**, B168 (1965).