Inelastic e-p Scattering Data and a New Parton Model*

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We elaborate on the application of the parton concepts to inelastic lepton-nucleon scattering. We propose a series of sum rules connecting the structure function $\nu W_2(\nu,Q^2)$ and the properties of the partons. In particular, a sum rule is derived relating the structure function of the neutron and the proton which depends only on the charge distribution of the partons. We then attempt to generalize the quark-parton model of Bjorken and Paschos (BP) so as to fit the data on inelastic e-p scattering which are presently available. We find that we are unable to fit the data with this model. A new model is therefore developed in which the partons are not identified with quarks, and the minimum number of partons is four (whereas in the BP model it is three). It is found that this model gives an excellent fit to the data in the approximation that the scale invariance is true. It is pointed out that the minimum number of partons plays an important role in parton models. Our fits also show that the mean square charge per parton is very small, which rules out any reasonable possibility of associating individual quarks with partons.

THIS paper will examine the details of the e-p scattering data[1] from the parton-model[2,3] concepts. The data available at present seem to be good enough to test models for inelastic electron-nucleon scattering. With this object in mind, we first generalize the quark-parton model of Bjorken and Paschos[2] (BP). Then we propose a slightly different model for the partons and show that the parton concepts explain the data very well.

The basic idea of the parton model is that at large c.m. energies of the e-p system, the proton may be thought of made up of fundamental pointlike constituents called partons from which the electrons scatter instantaneously and *incoherently*. One further assumes that to a good approximation the momentum of a parton is given by a fraction of the proton momentum. It can then be derived that the structure function[4] W_2 is given by

$$\nu W_2(\nu,Q^2)=\sum_N P(N)\langle\sum_1^N Q_i^2\rangle_N x f_N(x)=F(x)\,, \quad (1)$$

where $x=Q^2/2M\nu$. In this equation, ν is the invariant energy transfer and $-Q^2$ is the squared four-momentum transfer to the electron. The other expressions are defined by the following: $P(N)$ is the probability of finding N partons in the proton; $\langle\sum_1^N Q_i^2\rangle_N$ is the average value of the sum of the squared charges of the partons in a configuration of N partons; and $f_N(x)$ is the probability of finding a parton with longitudinal fraction x of the proton's four-momentum. A more detailed explanation of these expressions[5] can be found in Ref. 2. Equation (1) shows how the parton model embodies the scale-invariance[6] property of the inelastic lepton-nucleon scattering from the very beginning. Now one must satisfy

$$\sum_{N_0}^{\infty} P(N)=1\,, \quad (2)$$

where N_0 is the minimum number of partons in the proton; and

$$\int_0^1 f_N(x)dx=1\,. \quad (3)$$

A series of sum rules can be derived from Eq. (1) by multiplying it by a function $\phi(x)$ and integrating over x:

$$\int_0^1 F(x)\phi(x)dx =\sum_N P(N)\langle\sum_1^N Q_i^2\rangle_N\int_0^1 x\phi(x)f_N(x)dx\,. \quad (4)$$

The left-hand side can be measured experimentally for a given $\phi(x)$ while the right-hand side will be the predictions of a parton model. If we put $\phi(x)=1$, and assume a symmetric distribution of momenta among the partons, we obtain the BP result for the mean square charge per parton:

$$\sum_N P(N)\langle\sum_1^N Q_i^2\rangle_N N^{-1}=\int_0^1 F(x)dx\,. \quad (5)$$

From the e-p inelastic scattering data, the right-hand side of Eq. (5) is found to be approximately 0.18. We shall parametrize the charge-square distribution of the

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[1] M. Breidenbach *et al.*, Phys. Rev. Letters **23**, 935 (1969); E. D. Bloom *et al.*, *ibid.* **23**, 930 (1969).
[2] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).
[3] R. P. Feynman, in *Proceedings of the Third International Conference on High Energy Collisions at Stony Brook, 1969* (Gordon and Breach, New York, 1969).
[4] The structure functions W_1 and W_2 are related to the cross section for electron-proton inelastic scattering in the laboratory frame by

$$\frac{d^2\sigma}{d\Omega dE'}=\frac{\alpha^2}{4E^2\sin^4(\frac{1}{2}\theta)}[W_2\cos^2(\tfrac{1}{2}\theta)+2W_1\sin^2(\tfrac{1}{2}\theta)],$$

where E and E' are, respectively, the incident and scattered electron energies and θ is the scattering angle.
[5] Unless otherwise stated, out notation is that of Ref. 2.
[6] J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

partons in the following way:

$$\langle \sum_1^N Q_i^2 \rangle_N = \delta(N+\delta'), \tag{6}$$

where δ and δ' are arbitrary parameters. We note that setting $\phi(x)=1/x$, and assuming the same δ for neutron and proton, we find

$$\int_0^1 [F^p(x)-F^n(x)]\frac{dx}{x} = \delta(\delta'^p-\delta'^n). \tag{7}$$

Here p and n refer to proton and neutron, respectively. This sum rule is true for any $P(N)$ and $f_N(x)$ and depends only on the charge distribution of the partons.

We now investigate the quark-parton model of BP, in which the partons are identified with quarks. In this model the first three partons are three quarks (to make up the correct baryon number), and the remaining partons are quarks and antiquarks in *equal numbers*. Thus the number of partons takes on only the values 3, 5, 7, ... By assuming the usual quark-model structure of the nucleons for the first three quarks, it can be derived that $\delta\delta'=1-3\delta$ for the proton, and $3\delta\delta'=2-9\delta$ for the neutron. Here δ is the mean square charge per parton of the cloud of quarks and antiquarks, which in this model must lie between $\frac{1}{9}$ and 4/9.

To study the model more closely we shall keep the above properties and, following BP, shall assume for simplicity that

$$f_N(x)=(N-1)(1-x)^{N-2}, \tag{8}$$

which can be derived by assuming a constant joint momentum probability distribution for the N partons. If we now impose, following experimental indications, that $\lim_{x\to 0}F(x)\neq 0$, then it is necessary that the maximum number of partons be infinite, and a finite nonzero value of $F(0)$ may be obtained by requiring $P(N)\sim(\text{const})/N^2$ for large N. A simple form for P_N which guarantees this is

$$P(N)=C/(N+\alpha)(N+\beta), \tag{9}$$

where α and β are arbitrary parameters. Note that $\alpha=0$, $\beta=-1$, and $\delta=2/9$ give the original BP model, which does not agree with the data. With our generalization of the BP model, in which we have three free parameters, one might expect that we could fit the data very well. However, our attempts to do so did not have much success. We obtain qualitative fits, but in all of these δ is about 0.01, which is an order of magnitude lower than this model's predicted minimum δ of $\frac{1}{9}$. This means, in effect, that at least 90% of the partons are not behaving as quarks, but rather as neutral particles. This may be an effect of $q\bar{q}$ binding, or perhaps reflects the possibility that partons and quarks are different things. Moreover, the fits tend to give too high a value of $F(x)$ between $x=1$ and $x=0.2$. We have tried to remedy this by summing from $N_0=5$ to ∞, i.e., assuming that there are always at least 5 partons, but this on the other hand underestimates $F(x)$ between $x=1$ and $x=0.2$. Thus we conclude that the modifications we have made to the BP model are not sufficient to give a realistic model.

We now propose a slightly different parton model. In this model the partons are not identified with quarks, but will be assumed to have charges 1, -1, or zero. There is a minimum number of partons N_0, and *the number of partons can be any integer from N_0 onward.* In this model δ is the mean square charge of the $N-N_0$ partons, and for integrally charged partons is the ratio of the number of charged particles to the total number of particles in the $(N-N_0)$ partons.[7] δ' is related to the structure of the first N_0 partons. We shall take $P(N)$, $f_N(x)$, and $\langle\sum Q_i^2\rangle_N$ to be given as before by Eqs. (8), (9), and (6), respectively. Equations (2), (5), and (1) then give

$$C=(\beta-\alpha)/[\psi(N_0+\beta)-\psi(N_0+\alpha)], \tag{10}$$

$$0.18\simeq\delta-\frac{C\delta\delta'^p}{\alpha\beta}\times\left[\psi(N_0)+\frac{\beta}{\alpha-\beta}\psi(N_0+\alpha)-\frac{\alpha}{\alpha-\beta}\psi(N_0+\beta)\right], \tag{11}$$

and

$$F(x)=C\delta(1-x)^{N_0-2}\times\left[1+x\frac{(1+\beta)(\delta'-\beta)}{\beta-\alpha}J_{N_0+\beta}(1-x)+\alpha\leftrightarrow\beta\right], \tag{12}$$

where[8] $\psi(x)=d\ln\Gamma(x)/dx$ and

$$J_n(y)=\sum_{m=0}^{\infty}\frac{y^m}{n+m}. \tag{13}$$

The behavior of $F(x)$ near $x=1$ is proportional to $(1-x)^{N_0-2}$, and a glance at the data seems to indicate that $F(x)$ behaves like $(1-x)^2$ near $x=1$. We have therefore fixed N_0 to be 4. (We have also looked at $N_0=3$, 5 and found that these do not work). Thus in our model the summation over N goes through 4, 5, 6, ..., ∞. Next we set $(\delta\delta')^p=1-4\delta$. In the integrally charged parton picture this corresponds[9] to only one charged parton in the $N=4$ configuration of the proton.

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[7] The assumption of integrally charged partons is not absolutely necessary: Any model which allows δ to be sufficiently small, and gives the formulas for δ' in terms of δ, will obviously give the same result.

[8] For an exposition on the $\psi(x)$ functions, see *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (U. S. Department of Commerce, National Bureau of Standards, Washington, D. C., 1964) Appl. Math. Ser. 55.

[9] The other possibility for the proton, namely $\delta\delta'=3-4\delta$, corresponds to too many charged particles in the $N=4$ configuration.

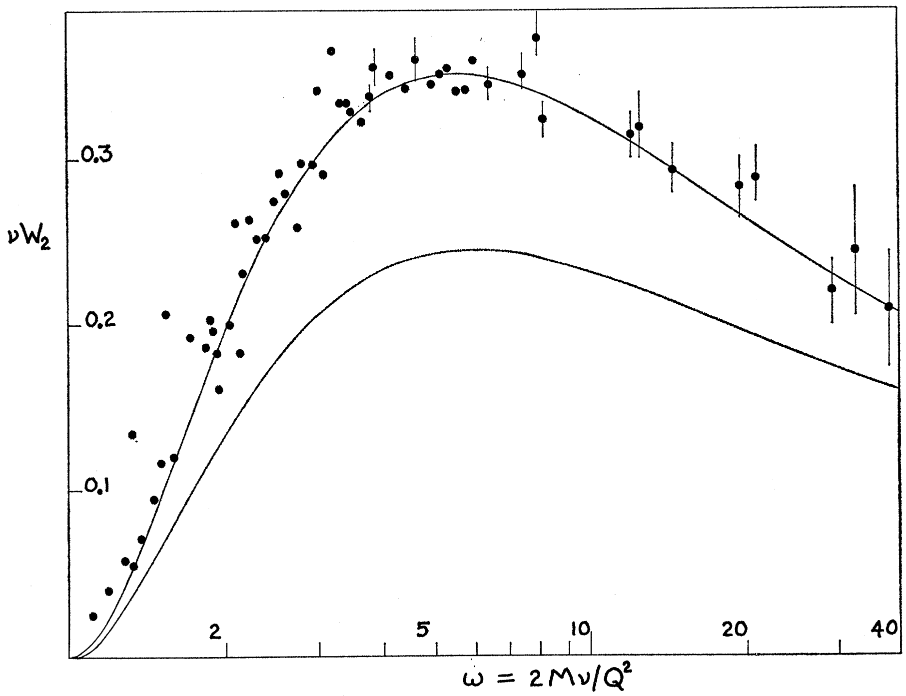

FIG. 1. Data and predictions for νW_2. The points are a representative selection of the e-p inelastic scattering data from Ref. 1, using $\theta=6°$, $R=0$, and incident electron energy 10.0, 13.5, and 16.0 GeV. Where error bars are not given, the error is of the same order as the size of the circle. The upper curve is our fit to the data using Eq. (14) with $\delta=0.042$. The lower curve is our prediction for νW_2 for inelastic e-n scattering.

Attempts to fit the e-p data[10] indicate that this model is very flexible. It is not really necessary to have two free parameters in $P(N)$; one can easily fix either of α and β and still obtain a very good fit by varying the other and δ. (This is quite the opposite of the situation for our modified BP model, where we could not fit the data even by varying all three parameters.) Moreover, in this model it is found that for all the fits the value of δ is much the same, and is approximately 0.04. For simplicity we have chosen the fit corresponding to $\alpha=-1$, $\beta=-2$, and $\delta=0.042$ to plot in Fig. 1. In this case $F(x)$ can be expressed in terms of elementary functions as

$$F(x)=2\delta[(1-x)^2-x(\delta'+2)(\ln x+1-x)]. \quad (14)$$

The fit for the proton is seen to be excellent. The sum rule of Eq. (7) may be helpful in distinguishing different models. The right-hand side of the sum rule is $\frac{1}{3}$ for the BP model. With the same reasoning which led us to the proton $N=4$ configuration, the neutron can have two competing configurations, namely one with all four neutral partons and one with two charged and two neutral partons. Presumably the real neutron is some mixture of these. To obtain the trend of the neutron curve we have set $(\delta\delta')^n=\frac{2}{3}-4\delta$ which corresponds[11] to a relative weight of the neutral configuration twice that of the charged one. The plot for the neutron is shown in Fig. 1.

Finally, we make the following remarks:

(i) The parton concept gives a useful description of the inelastic electron-proton scattering data.

(ii) δ is very small, implying that most partons are neutral particles. This rules out any reasonable possibility of associating individual quarks with partons.

(iii) The minimum number of partons plays an important role in a parton model. We may trace the success of our model to the fact that we have $N_0=4$, and the lack of success of our generalization of the quark-parton model of BP to the fact that the minimum number of partons in this model cannot be four.

(iv) An approximate estimate of $F(x)$ at very high energies $(x\to 0)$ is of the order of 0.08 for both the proton and the neutron.

(v) A measurement of the neutron data will be invaluable in constructing a more realistic model of the partons.

We would like to thank Professor K. C. Wali for helpful discussions.

[10] We have chosen the $\theta=6°$ data for our fits, because the value of νW_2 is then less sensitive to the precise value of R. In particular we consider only the case for $R=0$, since experimentally R seems to be small. See E. Bloom *et al.*, in Proceedings of the Third Conference on Particle Physics at Honolulu, Hawaii, 1969 (unpublished).

[11] Note that if one considers the $N=4$ configuration for the nucleon to be constituted of three quarks and one neutral particle, then $(\delta\delta')^p=1-4\delta$ and $(\delta\delta')^n=\frac{2}{3}-4\delta$. Bearing in mind Ref. 7, we see that it is impossible to determine the precise nature of the first N_0 partons from inelastic e-p scattering alone.