is rather mild, as remarked also by Lee,[15] it is quite possible that abnormal poles would not show up in tests of forward dispersion relations, unless the precision of the data were increased by an order of magnitude. The most fruitful approach might be, therefore, to search for abnormal poles in the partial-wave analysis of pion-nucleon scattering data. At present one can probably rule out, with some confidence, abnormal poles with $M\lesssim 1.5$ GeV, but there is no evidence concerning mass values greater than 2 GeV. In the intermediate region there are considerable data, all of which are consistent with the absence of abnormal resonances. These data are not conclusive, however, because in all the analyses, the explicit assumption has been made that abnormal resonances were absent and, moreover, the derived parameters of normal resonances differ greatly from group to group, depending on the assumptions that have been made about the background amplitude.[16] It may be noted that the region where data are at present being accumulated is especially significant from the point of view of the structure of neutron stars. A careful study of baryon resonances in the mass region $M\lesssim 2.5$ GeV (and also mesonic resonances with $M\lesssim 1.5$ GeV) is urged. Against the improbability of finding abnormal resonances through a more incisive investigation, the importance of knowing more accurately the properties (and number) of the normal resonances may be balanced; what is needed, besides more data, is the development and use of more refined methods of analysis which emphasize the analyticity properties of the amplitudes.

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[16] R. J. Plano, rapporteur's talk, in *Proceedings of the Lund International Conference on Elementary Particles* (Berlinska Boktryckeriet, Lund, Sweden, 1969), p. 321.

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Atomic Processes Involved in Matter-Antimatter Annihilation*

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Atomic processes are important in determining the particle-antiparticle annihilation rates in the matter-antimatter interaction. Consideration of these processes is given for a hydrogen-antihydrogen mixture. The atomic effects considered are the effect of the Coulomb field on the direct annihilations, radiative capture, and rearrangement collisions. Radiative capture and rearrangement reactions lead to particle-antiparticle bound states from which annihilation proceeds. The rearrangement cross sections are considerably greater than the direct-annihilation cross sections and therefore lead to a large increase in the total annihilation rates over a wide range of kinetic energies. The lifetimes of two types of hydrogen-antihydrogen mixtures are calculated and are found to differ significantly from the results obtained by others.

I. INTRODUCTION

THE basic symmetries between matter and antimatter have led to a consideration of the possibility that large amounts of antimatter may exist within the universe.[1–3] The existence of antimatter would have important and possibly detectable consequences if the antimatter should come in contact with matter and undergo annihilation.[4–6]

The matter-antimatter interaction occurs on three scales: the large-scale interaction involving the presence of magnetic, electric, and gravitational fields, the atomic-scale interaction involving Coulomb forces and the electromagnetic field, and the nuclear-scale interaction involving nuclear and electromagnetic forces wherein the particle-antiparticle annihilations occur. The most significant and best known of the processes that affect the state of a region of matter-antimatter contact are the particle-antiparticle annihilations. The annihilations cause a depletion in the amount of matter and antimatter present and through their release of energy may produce an increase in the temperature in the region of contact and in surrounding regions.

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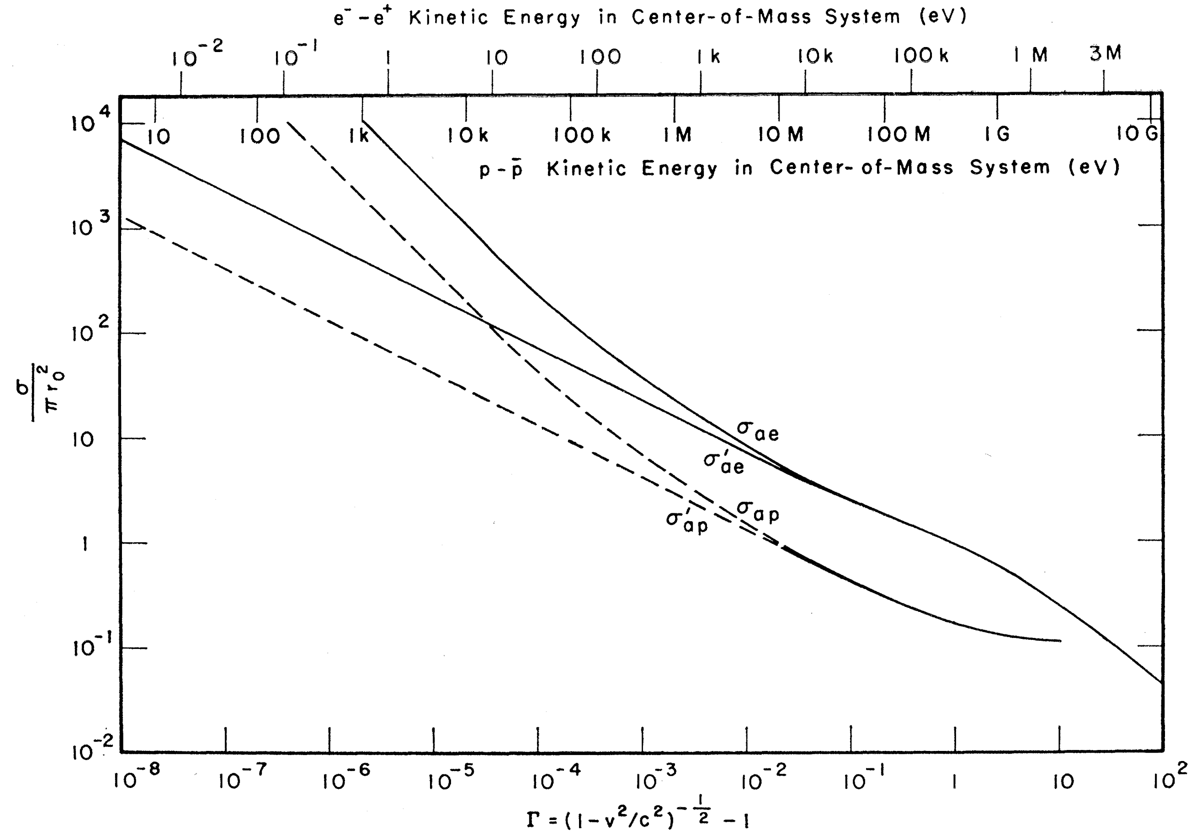

FIG. 1. e^--e^+ and p-$\bar{p}$ direct-annihilation cross sections, corrected for Coulomb attraction, σ_{ae} and σ_{ap}, and in the plane-wave approximation, σ'_{ae} and σ'_{ap}.

The effects of large-scale processes involving magnetic, electric, and gravitational fields on a matter-antimatter mixture have been treated by Alfvén and Klein.[1,2] They present a hypothesis concerning the origin of the universe in which they give consideration to the state of an ambiplasma, a mixture of ionized matter and antimatter at extremely low density. Most importantly, they find that the matter and antimatter will separate into different regions with the result that the annihilation rate is greatly reduced. The regions of matter and antimatter will be separated by "leidenfrost" layers of higher temperature and lower density in which the matter and antimatter mix and annihilate. In addition, Alfvén and Klein have calculated the lifetime as a function of density of an atomic-scale mixture of matter and antimatter. Teller has also considered large-scale processes occurring in a region of matter-antimatter contact in a hypothesis concerning the nature of quasars.[3]

Atomic-scale electromagnetic processes which under certain conditions have the important effect of governing the rate of particle-antiparticle annihilation in a matter-antimatter mixture have been largely neglected. Limited consideration to such processes has been given by Nauenberg and Ruderman[4] in their treatment of the processes that would occur if an antimeteor were to enter the earth's atmosphere, and by the present authors[7] in their treatment of atom-antiatom collisions. The purpose of this paper will be to give more detailed consideration to the atomic-scale interactions that are important in determining the state of a matter-antimatter mixture.

The principal feature of these interactions is that the atomic-scale components of matter will combine with the atomic-scale components of antimatter to form bound states of oppositely charged particles and antiparticles. Annihilation then occurs from the bound state. Since the cross sections for such reactions are under certain conditions considerably greater than the cross sections for direct particle-antiparticle annihilation, the atomic processes may govern and greatly enhance the annihilation rates.

The atomic processes considered are the effect of the Coulomb field on direct-annihilation cross sections

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(Sec. II), radiative capture (Sec. III), and rearrangement collisions (Sec. IV). These processes are considered for a mixture of molecular, atomic, and/or ionized hydrogen and antihydrogen for a wide range of kinetic energies between the matter and antimatter constituents. In Sec. V, the annihilation rates as well as the lifetime of an atomic-scale mixture of hydrogen and antihydrogen are determined.

Since the existence of antimatter is speculative, the physical circumstances for which the matter-antimatter interaction should be considered are uncertain. Sections II, III, and IV give the cross sections for the processes that lead to particle-antiparticle annihilation for kinetic energies between the matter and antimatter constituents from about 10^{-3} eV to roughly 1 GeV (about 10°K to roughly 10^{13} °K). In cases where these processes lead to the formation of particle-antiparticle bound states, the cross sections for the resulting annihilations are equal to the bound-state formation cross sections for number densities from zero to at least 10^{11} cm^{-3}. At densities above 10^{11} cm^{-3}, breakup of the particle-antiparticle bound states prior to annihilation may reduce the annihilation cross sections.

The problems of negative particles in matter and of antimuonium in matter are closely related to the problems of the matter-antimatter interaction (Sec. VI).

Except where otherwise indicated, colliding particles are considered in the center-of-mass (c.m.) system.

II. DIRECT-ANNIHILATION CROSS SECTIONS

In an ionized mixture of hydrogen and antihydrogen, electrons (e^-) may directly annihilate with positrons (e^+), and protons (p) with antiprotons ($\bar{p}$).

The principal e^--e^+ interaction that leads to direct annihilation is the electromagnetic interaction in which two photons are produced. The cross section for this reaction in the plane-wave approximation is[8,9]

$$\sigma'_{ae}=\frac{\pi r_0^2}{(\gamma+1)}\left(\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln[\gamma+(\gamma^2-1)^{1/2}]-\frac{\gamma+3}{(\gamma^2-1)^{1/2}}\right), \quad (1)$$

$$r_0=e^2/m_ec^2, \quad \gamma=(1-v^2/c^2)^{-1/2},$$

where r_0 is the classical electron radius, and v is the electron-positron relative velocity. A plot of σ'_{ae} is given in Fig. 1. In the nonrelativistic limit ($v\to 0$), Eq. (1) becomes

$$\sigma'_{ae}=(c/v)\pi r_0^2. \quad (2)$$

The p-$\bar{p}$ annihilation reaction is dominated by the strong interaction and produces principally positive, negative, and neutral pions. The cross section σ'_{ap} for this reaction has been determined experimentally for incident kinetic energies of $\bar{p}$ in the laboratory from 20 MeV to 6 GeV,[10–12] for which the plane-wave approximation applies. The portion of the curve of σ'_{ap} for the kinetic energy range in the c.m. system of 10 MeV to 2 GeV in Fig. 1 represents a smooth curve drawn through the experimental points. No reliable information that would allow extrapolation of σ'_{ap} to lower energies is available. However, an estimate of σ'_{ap} for lower energies may be obtained in the following way: In the energy range from 10 MeV to 1 GeV, the σ'_{ap} curve is approximately fitted by

$$\sigma'_{ap}(v)=0.19\sigma'_{ae}(v). \quad (3)$$

With the use of Eq. (2), Eq. (3) gives, in the nonrelativistic limit,

$$\sigma'_{ap}=0.19(c/v)\pi r_0^2. \quad (4)$$

Equation (4) is used to extrapolate σ'_{ap} to energies below 10 MeV. This extrapolation is shown by the

TABLE I. e^--e^+ and p-$\bar{p}$ radiative capture cross sections. (Kinetic energy of the particles is in the c.m. system.)

e^--e^+ Kinetic energy (eV)	$\sigma_{re}/\pi r_0^2$	p-$\bar{p}$ Kinetic energy (eV)	$\sigma_{rp}/\pi r_0^2$
6.80×10^{-9}	3.57×10^{13}	1.25×10^{-5}	1.05×10^{7}
6.80×10^{-8}	3.18×10^{12}	1.25×10^{-4}	9.42×10^{5}
6.80×10^{-7}	2.78×10^{11}	1.25×10^{-3}	8.27×10^{4}
6.80×10^{-6}	2.40×10^{10}	1.25×10^{-2}	7.17×10^{3}
6.80×10^{-5}	2.01×10^{9}	1.25×10^{-1}	5.97×10^{2}
6.80×10^{-4}	1.63×10^{8}	1.25	4.83
6.80×10^{-3}	1.24×10^{7}	12.5	3.69
1.70×10^{-2}	4.36×10^{6}	31.2	1.29
3.47×10^{-2}	1.91×10^{6}	63.7	0.565
6.80×10^{-2}	8.61×10^{5}	1.25×10^{2}	0.255
1.39×10^{-1}	3.69×10^{5}	2.55×10^{2}	0.109
4.25×10^{-1}	9.48×10^{4}	7.81×10^{2}	2.82×10^{-2}
1.70	1.59×10^{4}	3.12×10^{3}	4.73×10^{-3}
6.80	2.29×10^{3}	1.25×10^{4}	6.79×10^{-4}
27.2	2.11×10^{2}	5.00×10^{4}	6.25×10^{-5}
68.0	34.6	1.25×10^{5}	1.03×10^{-5}
6.8×10^{2}	0.206	1.25×10^{6}	6.11×10^{-8}

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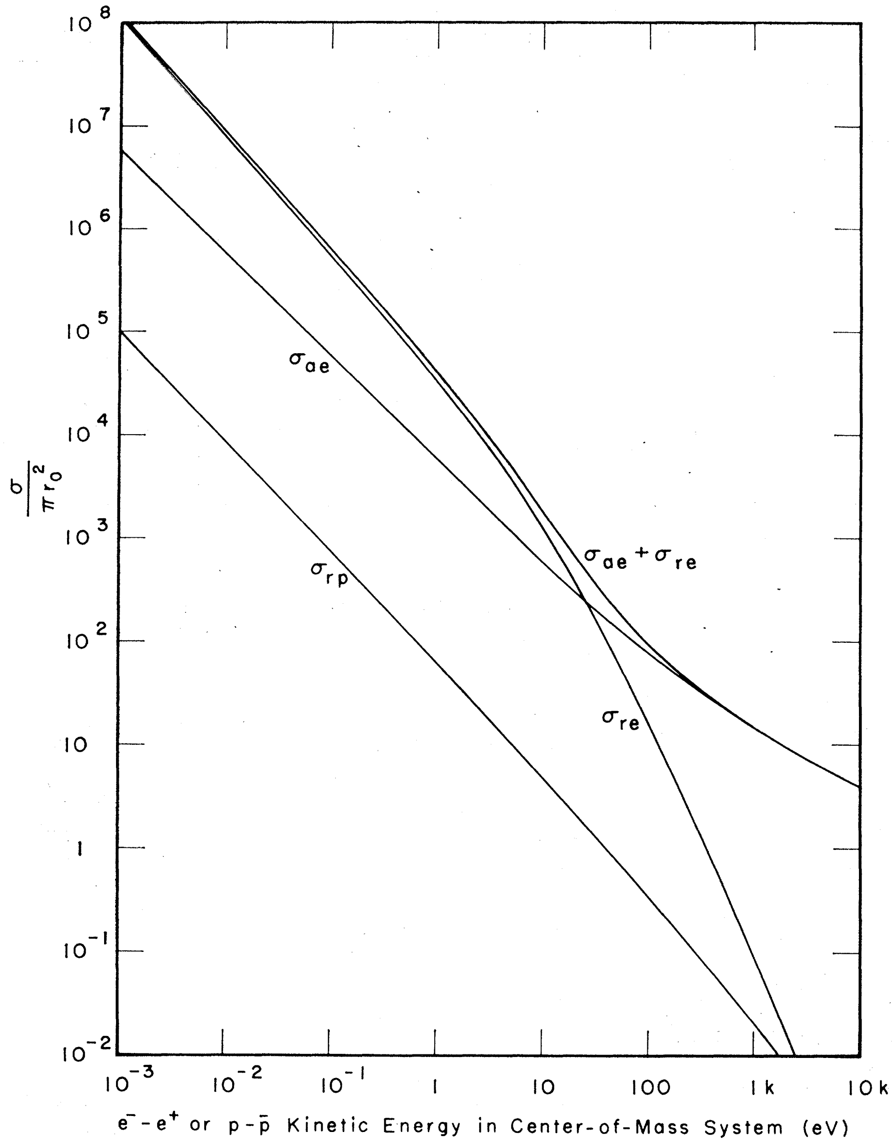

FIG. 2. e^--e^+ and p-$\bar{p}$ radiative capture cross sections σ_{re} and σ_{rp}, with the e^--e^+ direct-annihilation cross section σ_{ae} for comparison. The total e^--e^+ annihilation cross section in the absence of atoms and molecules, $\sigma_{ae}+\sigma_{re}$.

dashed portion of the σ'_{ap} curve in Fig. 1. In the limit $v\rightarrow 0$, Eq. (4) gives the proper v^{-1} form[13] for σ'_{ap}, although the numerical constant in Eq. (4) is only an estimate.

For an e^--e^+ kinetic energy of less than about 10 keV and for a p-$\bar{p}$ kinetic energy of less than about 10 MeV, the plane-wave approximation is invalid, and the effect of Coulomb attraction must be considered. The effect of the Coulomb field on the direct-annihilation cross sections σ_{ae} and σ_{ap} may be taken into account[9,13] by multiplying σ'_{ae} and σ'_{ap}, respectively, by the ratio of the square of the amplitude of the Coulomb wave function for zero positron-electron and proton-antiproton separation to the corresponding quantity in the plane-wave approximation. This gives

$$\sigma_{ae}=2\pi(\alpha c/v)(1-e^{-2\pi\alpha c/v})^{-1}\sigma'_{ae}, \tag{5}$$

and

$$\sigma_{ap}=2\pi(\alpha c/v)(1-e^{-2\pi\alpha c/v})^{-1}\sigma'_{ap}, \tag{6}$$

where α is the fine-structure constant. The direct-annihilation cross sections σ_{ae} and σ_{ap} are shown in Fig. 1.

III. RADIATIVE CAPTURE

In an ionized mixture of hydrogen and antihydrogen, an e^- may undergo radiative capture with an e^+ to form a bound state of positronium (Ps) and a p may undergo radiative capture with a $\bar{p}$ to form a bound

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state of protonium (Pn):

$$e^-+e^+ \rightarrow \mathrm{Ps}+h\nu, \tag{7}$$

$$p+\bar{p} \rightarrow \mathrm{Pn}+h\nu. \tag{8}$$

If the Ps and Pn so formed remain intact, each will annihilate. Estimates of the Ps [14] and Pn breakup cross sections yield the result that the breakup rates are negligible for particle number densities in the hydrogen-antihydrogen mixture that are less than 10^{11} cm^{-3}. Hence, the radiative capture reactions (7) and (8) provide additional channels for e^--e^+ and p-$\bar{p}$ annihilation for such densities. Further, the e^--e^+ and p-$\bar{p}$ annihilation cross sections resulting from radiative capture are therefore equal to the radiative capture cross sections for these densities.

In determining the radiative capture cross section σ_r for a nonrelativistic initial relative velocity v of the two particles involved, high accuracy may be obtained by considering only dipole transitions and first-order effects of the perturbing electromagnetic field. In this approximation[15] the total radiative capture cross section to all bound states of two particles with charges $+e$ and $-e$ may be shown to be given by the equation

$$\sigma_r(v)=(\mu_e^2/\mu^2)\sigma_r(v)_{e^-p}, \tag{9}$$

where μ is the reduced mass of the two particles, μ_e is the reduced e^--p mass, and $\sigma_r(v)_{e^-p}$ is the electron-proton radiative capture cross section. Using Eq. (9) and the formulas and values for $\sigma_r(v)_{e^-p}$ determined by Menzel and Pekeris[16] and Bates *et al.*,[17] we have calculated σ_{re} for e^--e^+ radiative capture and σ_{rp} for p-$\bar{p}$ radiative capture to an accuracy of $\pm 1\%$. The results of this calculation are given in Table I and are shown in Fig. 2. In the calculation, use was made of a formula for σ_r which we have determined:

$$\sigma_r=\frac{2^4}{3\sqrt{3}}\frac{e^2h}{\mu^2c^3}\lambda^2(\ln\lambda+0.20+0.25\lambda^{-2/3}), \tag{10}$$

where $\lambda=e^2/hv$. Equation (10) gives σ_r with an accuracy of $\pm 1\%$ for $\lambda \gtrsim 7$ and is exact in the limit $\lambda\rightarrow\infty$ $(v\rightarrow 0)$.

It is found that for all energies of interest σ_{rp} is considerably less than σ_{ap}, and hence radiative capture is insignificant for p-$\bar{p}$ annihilation. From Fig. 2, where σ_{ae} is also given, it is seen that the direct-annihilation and radiative capture cross sections have comparable values for e^--e^+. In addition, it is seen that σ_{re} exceeds σ_{ae} for kinetic energies below about 50 eV.

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IV. REARRANGEMENT COLLISIONS

When the hydrogen-antihydrogen mixture contains even a small fraction of the hydrogen and antihydrogen in the form of atoms or molecules, various rearrangement collisions occur that greatly influence the e^--e^+ and p-$\bar{p}$ annihilation rates. These rearrangement collisions include

$$\mathrm{H}+\bar{\mathrm{H}} \rightarrow \mathrm{Pn}+(\mathrm{Ps}\text{ or }e^-+e^+), \tag{11}$$

$$\mathrm{H}_2+\bar{\mathrm{H}} \rightarrow \mathrm{Pn}+(\mathrm{Ps}\text{ or }e^-+e^+)+(\mathrm{H}\text{ or }p+e^-), \tag{12}$$

$$\begin{aligned}\mathrm{H}_2+\bar{\mathrm{H}}_2 \rightarrow\ & 2\,\mathrm{Pn}+(2\,\mathrm{Ps}\text{ or }\mathrm{Ps}+e^-+e^+\text{ or }2e^-\\ &+2e^+),\text{ or}\rightarrow \mathrm{Pn}+(\mathrm{Ps}\text{ or }e^-+e^+)\\ &+(\mathrm{H}+\bar{\mathrm{H}},\ \mathrm{H}+\bar{p}+e^+,\ p+e^-+\bar{\mathrm{H}},\\ &\text{or }p+e^-+\bar{p}+e^+),\end{aligned} \tag{13}$$

$$p+\bar{\mathrm{H}} \rightarrow \mathrm{Pn}+e^+, \tag{14}$$

$$p+\bar{\mathrm{H}}_2 \rightarrow \mathrm{Pn}+e^++(\bar{\mathrm{H}}\text{ or }\bar{p}+e^+), \tag{15}$$

$$e^-+\bar{\mathrm{H}} \rightarrow \mathrm{Ps}+\bar{p}, \tag{16}$$

where H denotes a hydrogen atom and $\bar{\mathrm{H}}$ denotes an antihydrogen atom. If any one of the reactions (11)–(16) is not its own charge-conjugate analog, the reaction is understood to designate also the charge-conjugate analog which behaves in an exactly analogous manner. Thus, reaction (16) also denotes $e^++\mathrm{H}\rightarrow \mathrm{Ps}+p$.

By virtue of the fact that each of the reactions (11)–(16) involves the production of bound states of Ps and/or Pn, which will annihilate, these reactions provide additional means for e^--e^+ and p-$\bar{p}$ annihilation. Further, when Ps and Pn breakup may be neglected (Sec. III), the cross sections for these reactions are the cross sections for the annihilations to which these reactions lead.

In order to simplify the calculation of the cross sections for reactions (11)–(16), it will be assumed, as will usually be true in most cases of interest, that each of the atoms or molecules present initially is in the ground state.

A. H+$\bar{\mathrm{H}}$ Rearrangement Collision

The H+$\bar{\mathrm{H}}$ rearrangement reaction (11) is an inelastic scattering reaction which occurs for initial kinetic energies down to zero. There is no threshold energy for this reaction because the Pn formed is free to enter into a state with an adequate binding energy. On the other hand, for initial kinetic energies in excess of the difference between the initial and final binding energies of the e^- and e^+, reaction (11) occurs with a very low probability. This is because the transfer of a significant amount of kinetic energy to the light particles is improbable, and hence, for energies in excess of the initial minus the final binding energy of the e^- and e^+ (20.8 or 27.2 eV) it is highly improbable that the p and $\bar{p}$ enter into a bound state of Pn.

TABLE II. Form of interatomic potential energy $V(R)$ for H-$\bar{\text{H}}$.

Range of R	$V(R)$
$R \lesssim 0.5a_0$	Equation (20)
$0.5a_0 \lesssim R \lesssim 3a_0$	V uncertain
$3a_0 \lesssim R \lesssim 9a_0$	Equation (20)
$9a_0 \lesssim R \lesssim 11a_0$	Suitably weighted average of Eqs. (18) and (20)
$11a_0 \lesssim R$	Equation (18)

The cross section for reaction (11) for energies between about 10^{-3} and 1 eV may be determined by a semiclassical method that is similar to the semiclassical treatment of the H-H spin-exchange interaction by Purcell and Field[18] and has some features in common with Wightman's treatment of π^--H scattering.[19] To employ this method, it is necessary to know the fixed nuclei interatomic potential energy V for H-$\bar{\text{H}}$.

For values of the H-$\bar{\text{H}}$ internuclear separation R much less than 1 Bohr radius a_0, V is approximately the attractive p-$\bar{p}$ Coulomb potential,

$$V=-e^2/R. \tag{17}$$

For values of R greater than about $10a_0$, V is equal to the long-range van der Waals dispersion energy of H-H,[20]

$$V=-6.50(a_0/R)^6(e^2/a_0). \tag{18}$$

This is due to the facts that the dispersion energy is dependent upon the square of the Coulomb interaction energy $\mathbf{v}$ between the particles of one atom and the particles of the other atom or antiatom, and that $\mathbf{v}$ for an atom-antiatom pair is the negative of $\mathbf{v}$ for the corresponding atom-atom pair.

For values of $R \lesssim 10\ a_0$, a perturbation-theory determination of V may be made in which $\mathbf{v}$ is the perturbation potential energy. The Lennard-Jones–Brillouin–Wigner perturbation expansion[21] is used, principally because of its rapid convergence. When carried to the second power of $\mathbf{v}$, this expansion gives

$$V=\int \psi_0^*\mathbf{v}\psi_0 d\tau+\sum_{i\neq 0}\frac{A_{0i}A_{i0}}{V+E_0-E_i}, \tag{19}$$

where $A_{0i}=\int\psi_0^*\mathbf{v}\psi_i d\tau$, E_0 and ψ_0 are the unperturbed ground-state energy and wave function of H-$\bar{\text{H}}$, and E_i and ψ_i are the unperturbed excited-state energies and eigenfunctions of H-$\bar{\text{H}}$. If E_i is neglected in the last term of Eq. (19), closure may be applied to that term and the resulting equation may be solved for V, giving

$$V=\tfrac{1}{2}(A_{00}-E_0)-\frac{1}{2}\left[(E_0+A_{00})^2+4\left(\int\psi_0^*\mathbf{v}^2\psi_0 d\tau-A_{00}{}^2\right)\right]^{1/2}, \tag{20}$$

where $A_{00}=\int\psi_0^*\mathbf{v}\psi_0 d\tau$. For large R, Eq. (20) gives the long-range van der Waals dispersion energy of H-$\bar{\text{H}}$ to within 10%. It is estimated that the error in V as given by Eq. (20) is within about 15% for $R \gtrsim 3a_0$. In the limit $R\to 0$, V as given by Eq. (20) approaches V as given by Eq. (17), and it is estimated that the error in V as given by Eq. (20) is $\lesssim 25\%$ for $R \lesssim 0.5a_0$. The forms of V employed in the calculation for different ranges of R are given in Table II.

The H-$\bar{\text{H}}$ interatomic potential energy determined above may be used to compare the energy of the initial system of reaction (11), H-$\bar{\text{H}}$ (with fixed R), to the energy of the final system composed of stationary p and $\bar{p}$ separated by the distance R plus either free Ps with zero kinetic energy or free e^- and e^+ with zero kinetic energy. The values of R for which the energy of H-$\bar{\text{H}}$ is equal to the energies of the other systems are called critical radii. For the case involving Ps, the critical radius is $R_P=(0.5\pm0.2)a_0$, and for the case involving free e^- and e^+, the critical radius is $R_e=(0.2\pm0.1)a_0$. For R greater than the critical radius, the H-$\bar{\text{H}}$ energy is less than that of the final system, and for R less than the critical radius the H-$\bar{\text{H}}$ energy is greater than that of the final system.

The remainder of the semiclassical determination of the cross section for reaction (11) consists of a determination of the inner turning point R_0 of classical

TABLE III. Inner turning point R_0 versus impact parameter R_1 for H-$\bar{\text{H}}$.

R_1	R_0
$R_1>R_1'$	$R_0 \gg R_P,\ R_e$
$R_1<R_1'$	$R_0\simeq 0.1a_0\simeq 0.2R_P\simeq 0.5R_e$

TABLE IV. H-$\bar{\text{H}}$ rearrangement cross section $\sigma_{\text{H}+\bar{\text{H}}}$. (Kinetic energy of the two atoms is in the c.m. system.)

Kinetic energy (eV)	$\sigma_{\text{H}+\bar{\text{H}}}/10^{10}\pi r_0{}^2$
8.57×10^{-1}	0.483
5.44×10^{-1}	0.592
3.40×10^{-1}	0.718
2.18×10^{-1}	0.853
1.36×10^{-1}	1.01
8.57×10^{-2}	1.17
5.44×10^{-2}	1.35
3.40×10^{-2}	1.55
2.18×10^{-2}	1.77
1.36×10^{-2}	2.02
8.57×10^{-3}	2.24
5.44×10^{-3}	2.59
3.40×10^{-3}	2.94
2.18×10^{-3}	3.33
1.36×10^{-3}	3.79
8.57×10^{-4}	4.32
5.44×10^{-4}	4.94
3.40×10^{-4}	5.71
2.18×10^{-4}	6.56
1.36×10^{-4}	7.62

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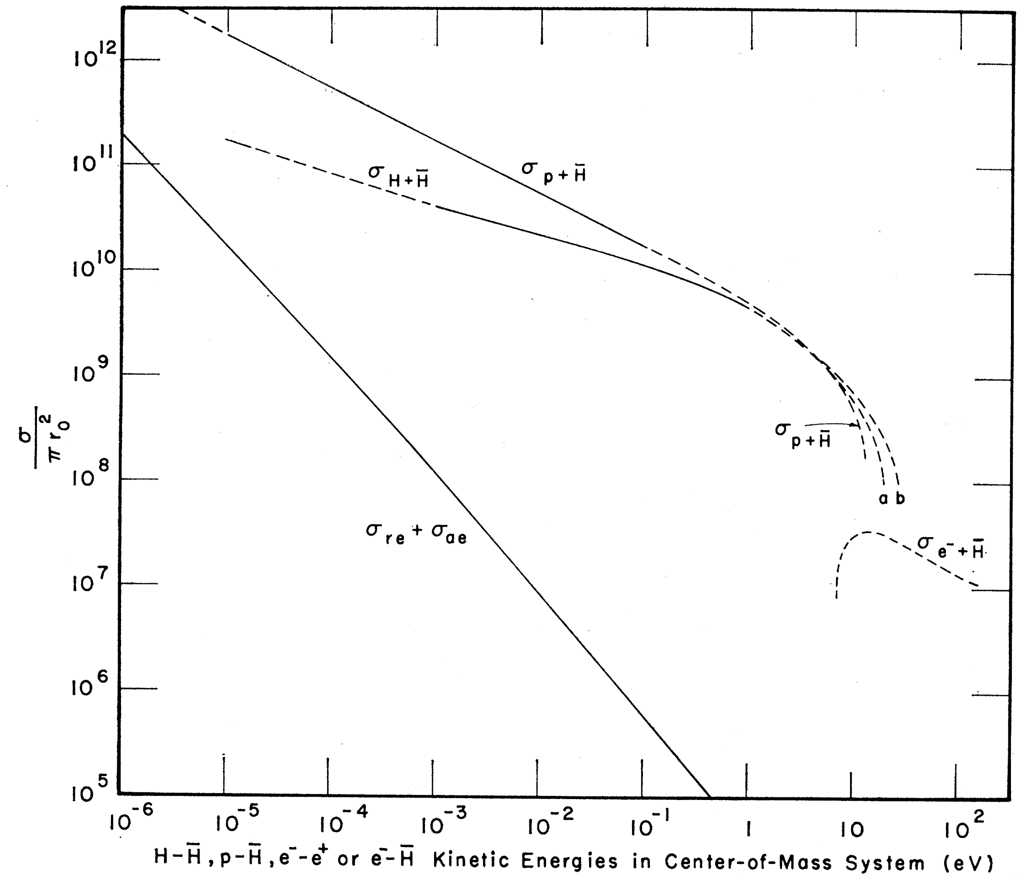

FIG. 3. Cross sections $\sigma_{H+\bar{H}}$, $\sigma_{p+\bar{H}}$, and $\sigma_{e^-+\bar{H}}$ for the H-$\bar{H}$, p-$\bar{H}$, and e^--$\bar{H}$ rearrangement collisions which lead to annihilation, with $\sigma_{re}+\sigma_{ae}$ for comparison.

orbits of H and $\bar{H}$ around their c.m. as a function of impact parameter R_1 and a comparison between R_0 and R_P or R_e. Classical orbits may be considered because the rationalized wavelength $\hbar/\mu v$ of the H-$\bar{H}$ relative motion is small compared to the relevant interaction distance for a wide range of kinetic energies. By the use of the determined form of V, it is found that as a function of R_1, R_0 is discontinuous at a value of R_1 equal to R_1'. The qualitative behavior of R_0 as a function of R_1 is given in Table III.

When R_1 is greater than R_1', R is always several times greater than R_P or R_e and no rearrangement takes place. When R_1 is less than R_1' and, as a consequence, R becomes less than R_P or R_e, free Ps or free e^--e^+ are rapidly produced. The Ps or e^--e^+ c.m. wave function then spreads out. This spreading occurs rapidly, and only a very small fraction of the wave function remains close enough to the p and $\bar{p}$ for the system to revert to this original state if R were to again become greater than R_P or R_e. Thus, for $R_1<R_1'$ the rearrangement occurs with near unity probability, and the cross section for reaction (11) is therefore equal to $\pi R_1'^2$. R_1' is a function of the initial kinetic energy and may be numerically determined from V. A plot of $\sigma_{H+\bar{H}}$ is given in Fig. 3 and a tabulation is given in Table IV. A fit to $\sigma_{H+\bar{H}}$ which is accurate to within about 20% for relative energies between 10^{-3} and 1 eV is given by

$$\sigma_{H+\bar{H}}=1.06\times10^7(c/v)^{0.64}\pi r_0^2.$$

The foregoing semiclassical treatment of reaction (11) is a poor approximation for incident energies less than about 10^{-3} eV, where the use of classical orbits for the H-$\bar{H}$ relative motion is invalid. For incident energies above about 1 eV, the values of R_1' are inaccurate because the range of R for which V is uncertain, $0.5a_0\lesssim R\lesssim 3a_0$, is important in determining R_0 as a function of R_1. Within the incident energy range 10^{-3} eV $\lesssim E\lesssim 1$ eV the semiclassical treatment gives $\sigma_{H\text{-}\bar{H}}$ with an accuracy of a few percent.

For energies above 1 eV, two estimates were made for $\sigma_{H+\bar{H}}$ and are shown by the two dashed portions labeled a and b of the $\sigma_{H+\bar{H}}$ curve in Fig. 3. The estimates were made under the assumptions that reaction (11) produces either Ps but no e^-+e^+ (curve a), or produces e^-+e^+ but no Ps (curve b). The estimates were based on the semiclassical approximation and an

TABLE V. Relation of interatomic potential energies and rearrangement cross sections of H-$\overline{\rm H}_2$ and $\rm H_2$-$\overline{\rm H}_2$ to those of H-$\overline{\rm H}$.

	$V/V_{\rm H\text{-}\overline{H}}$	$\sigma/\sigma_{\rm H+\overline{H}}$
H-$\overline{\rm H}_2$	1.29 ($R\gtrsim 5a_0$)	1.06
$\rm H_2$-$\overline{\rm H}_2$	1.47 to 1.62 ($10a_0\gtrsim R\gtrsim 5a_0$)	1.13

approximation for V provided by Eq. (20). The curves are also based on the fact that reaction (11) will occur with only a very low probability for initial kinetic energies in excess of the difference between the initial and final binding energies of the e^- and e^+.

It is difficult to determine the relative probabilities for Ps and free e^-+e^+ production in reaction (11). The relative probability for Ps production is made greater by the fact that $R_P>R_e$, but is made less by the greater restrictions imposed on the e^- and e^+ motion in Ps formation than in free e^-+e^+ formation.

Since the Pn and Ps formed by reaction (11) will not undergo breakup under most circumstances (as discussed in Sec. III), their subsequent annihilations provide additional means that lead to p-$\bar{p}$ and e^--e^+ annihilation when the hydrogen-antihydrogen mixture contains H and $\overline{\rm H}$. The resulting cross section for p-$\bar{p}$ annihilation is $\sigma_{\rm H+\overline{H}}$, and the resulting cross section for e^--e^+ annihilation is probably a significant fraction of $\sigma_{\rm H+\overline{H}}$. By comparing $\sigma_{\rm H+\overline{H}}$ with estimated values of σ_{ap} given by Eq. (4), it is seen that $\sigma_{\rm H+\overline{H}}$ is considerably greater than σ_{ap} for energies from about 10^{-4} to 30 eV. Depending on the amount of H and $\overline{\rm H}$ present, reaction (11) can therefore give a considerable increase in the total p-$\bar{p}$ annihilation rate over that due to direct annihilation alone. By comparing $\sigma_{\rm H+\overline{H}}$ with $\sigma_{ae}+\sigma_{re}$ in Fig. 3, it is seen that reaction (11) can similarly increase the total e^--e^+ annihilation rate over that due to direct annihilation and radiative capture alone. The direct e^--e^+ and p-$\bar{p}$ annihilations that occur in H+$\overline{\rm H}$ scattering are considerably less than σ_{ae} and σ_{ap}, due to the fact that the other particles present screen the e^--e^+ and p-$\bar{p}$ Coulomb potentials.

Reaction (11), which is a form of inelastic scattering, thoroughly dominates H+$\overline{\rm H}$ scattering for relative energies $\lesssim 1$ eV. Hence, H+$\overline{\rm H}$ scattering is predominately blackbody scattering for these energies, and the H+$\overline{\rm H}$ elastic scattering cross section is that which arises from reaction (11). Since several angular momentum waves are involved in reaction (11) for relative energies $\gtrsim 10^{-3}$ eV, the H+$\overline{\rm H}$ elastic scattering cross section is therefore about equal to $\sigma_{\rm H+\overline{H}}$ for relative energies between about 10^{-3} and 1 eV.

A more detailed discussion of the semiclassical method employed here has been given for the system $\overline{\rm M}$-H, where $\overline{\rm M}$ is an antimuonium atom.[22] In most respects $\overline{\rm M}$-H is equivalent to H-$\overline{\rm H}$.

[22] D. L. Morgan, Jr., dissertation, Yale University, 1966 (unpublished).

B. $\rm H_2+\overline{H}$ and $\rm H_2+\overline{H}_2$ Rearrangement Collisions

Arguments similar to those used at the beginning of Sec. IV A may be applied to reactions (12) and (13), which also occur for energies down to zero. In these reactions a significant amount of kinetic energy may be carried away due to the fact that particles or combinations of particles with a total mass equal to or greater than the proton mass may be produced. Thus, reactions (12) and (13) are likely to possess significant cross sections for initial kinetic energies in excess of the difference between the total initial and final e^- and e^+ binding energies.

The evaluation of V for $\rm H_2$-$\overline{\rm H}$ or $\rm H_2$-$\overline{\rm H}_2$ by the perturbation method of Sec. IV A involves integrals that cannot be readily evaluated. We have, therefore, made estimates of V for $\rm H_2$-$\overline{\rm H}$ and $\rm H_2$-$\overline{\rm H}_2$ which are based on the known long-range interatomic potential energies of $\rm H_2$-H and $\rm H_2$-$\rm H_2$.[23] Based on the ratios of V for $\rm H_2$-$\overline{\rm H}$ and $\rm H_2$-$\overline{\rm H}_2$ to V for H-$\overline{\rm H}$, corresponding ratios between the rearrangement cross sections can be approximately determined by the use of the semiclassical method. The ratios between the V's and the cross sections are given in Table V.

The form of reaction (12) in which H is produced is much more probable than the form in which free p and e^- are produced because of the fact that when the reaction occurs, the $\overline{\rm H}$ will approach to within a distance of about $0.1a_0$ from one H while remaining at a distance of about $1.4a_0$ (the H-H separation in $\rm H_2$) from the other H. It is, therefore, very unlikely that the other H will be sufficiently disturbed for its e^- to become unbound. Reaction (13) will occur with 2 Pn being produced with a much higher probability than Pn being produced, because of the fact that when one H approaches close to one $\overline{\rm H}$, it is very likely that the other H and $\overline{\rm H}$ will do the same.

Considerations regarding the relative probabilities of Ps and free e^-+e^+ production in reactions (12) and (13) are similar to the considerations regarding the relative probability of Ps and free e^-+e^+ production in reaction (11) given in Sec. IV A.

C. $p+\overline{\rm H}$ and $p+\overline{\rm H}_2$ Rearrangement Collisions

By arguments similar to those used for reaction (11) in Sec. IV A, reaction (14) may be shown to occur for energies down to zero and up to an energy equal to the 13.6-eV binding energy of $\overline{\rm H}$. Above 13.6 eV it will occur with a very low probability.

The semiclassical method of Sec. IV A may be used to calculate the cross section $\sigma_{p+\overline{\rm H}}$ for reaction (14). As an approximation to V for p-$\overline{\rm H}$, we take the monopole-induced dipole potential energy

$$V=-e^2\alpha_{\rm H}/2R^4 \qquad (21)$$

[23] H. Margenau and N. R. Kestner, *Theory of Intermolecular Forces* (Pergamon, New York, 1969), pp. 235, 244.

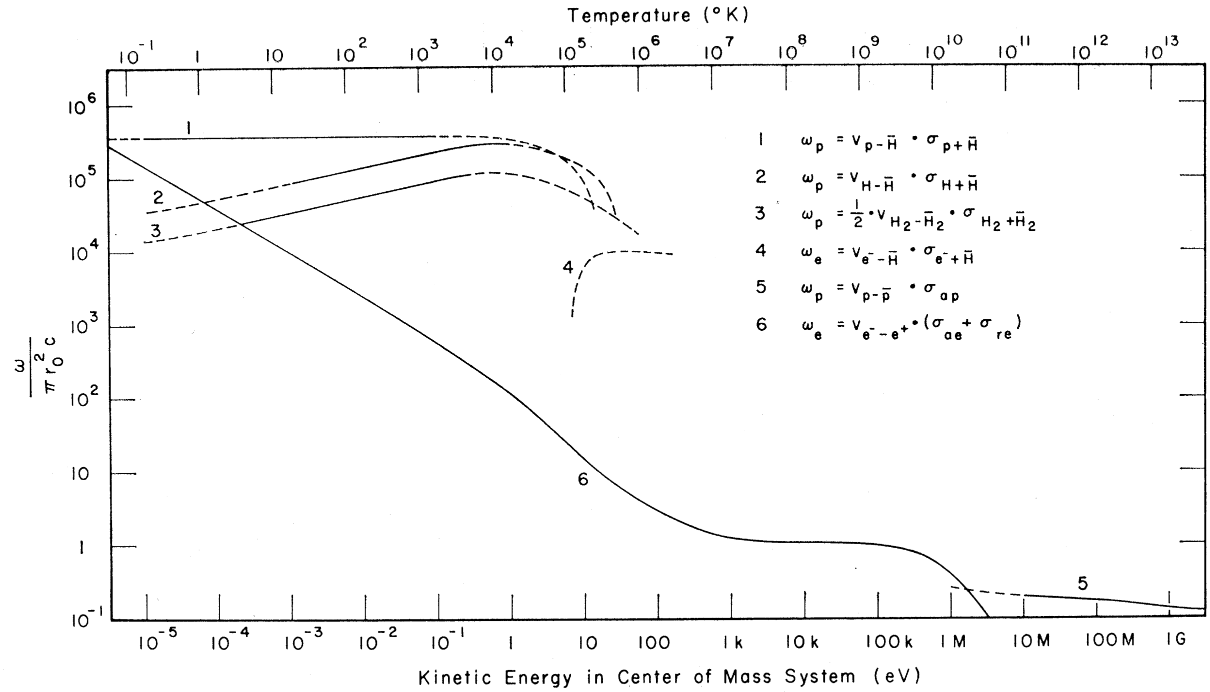

FIG. 4. Per particle per unit antiparticle number-density annihilation rates for various processes producing annihilation. The approximate temperature is shown.

for $R \gtrsim 5a_0$, where $\alpha_H = 4.5a_0^3$ is the dipole polarizability of H,[24] and we take

$$V = -e^2/R \tag{22}$$

for $R \ll a_0$. The critical radius for p-$\bar{H}$ is the same as that for π^--H, which is about $0.6a_0$.[19] The semiclassical method then gives results for reaction (14) which are qualitatively similar to those obtained for reaction (11) in Sec. IV A. Although the simple form used for V at long range, (21), restricts the higher-energy limit of validity for $\sigma_{p+\bar{H}}$, it permits an analytic solution for $\sigma_{p+\bar{H}}$ to be obtained:

$$\sigma_{p+\bar{H}} = 3.60 \times 10^5 (c/v) \pi r_0^2, \tag{23}$$

where v is the p-$\bar{H}$ relative velocity. Equation (23) is valid to within several percent for p-$\bar{H}$ relative energies between 10^{-5} and 0.1 eV. A plot of $\sigma_{p+\bar{H}}$ is given in Fig. 3. The dashed portions of the $\sigma_{p+\bar{H}}$ curve are estimates of $\sigma_{p+\bar{H}}$ determined in the same manner as the dashed portions of the $\sigma_{H+\bar{H}}$ curve.

By use of the semiclassical method, it is found that the cross section for reaction (15) is equal to 1.14 times the cross section for reaction (14) for energies from about 3×10^{-5} to 0.3 eV. Since a $\bar{p}$ is available to carry off energy, reaction (15) occurs with significant probability for initial kinetic energies greater than the difference between the total initial and final e^- and e^+ binding energies.

[24] L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill, New York, 1935), p. 185.

D. $e+\bar{H}$ Rearrangement Collision

Reaction (16) is an inelastic reaction with a threshold energy of 6.8 eV (binding energy of H minus the binding energy of Ps). Theoretical calculations have been made for reaction (16) in the form $e^+ + H \rightarrow Ps + p$, but the results are in considerable disagreement.[14,25] A rough order-of-magnitude estimate for the cross section $\sigma_{e^-+\bar{H}}$ of reaction (16) is shown in Fig. 3. This estimate indicates that $\sigma_{e^-+\bar{H}}$ is considerably greater than $\sigma_{ae}+\sigma_{re}$ and roughly comparable to the cross sections of reactions (11)–(13) in the energy range of 10 to at least 100 eV.

V. ANNIHILATION IN HYDROGEN-ANTIHYDROGEN MIXTURE

The results of the preceding sections make it possible to calculate the e^--e^+ and p-$\bar{p}$ annihilation rates in an atomic-scale mixture of hydrogen and antihydrogen. The single-particle annihilation rates per unit antiparticle number density, which are designated ω_e for e^--e^+ annihilation and ω_p for p-$\bar{p}$ annihilation and which apply to each of the processes leading to annihilation, discussed in the preceding sections, are

$$\omega_e = v\sigma_e, \tag{24}$$

$$\omega_p = v\sigma_p, \tag{25}$$

[25] B. H. Bransden and Z. Jundi, Proc. Phys. Soc. (London) **92**, 880 (1967); M. Fels and M. H. Mittleman, Phys. Rev. **163**, 129 (1967); I. M. Cheshire, Proc. Phys. Soc. (London) **83**, 227 (1964); R. J. Drachman, Phys. Rev. **171**, 110 (1968).

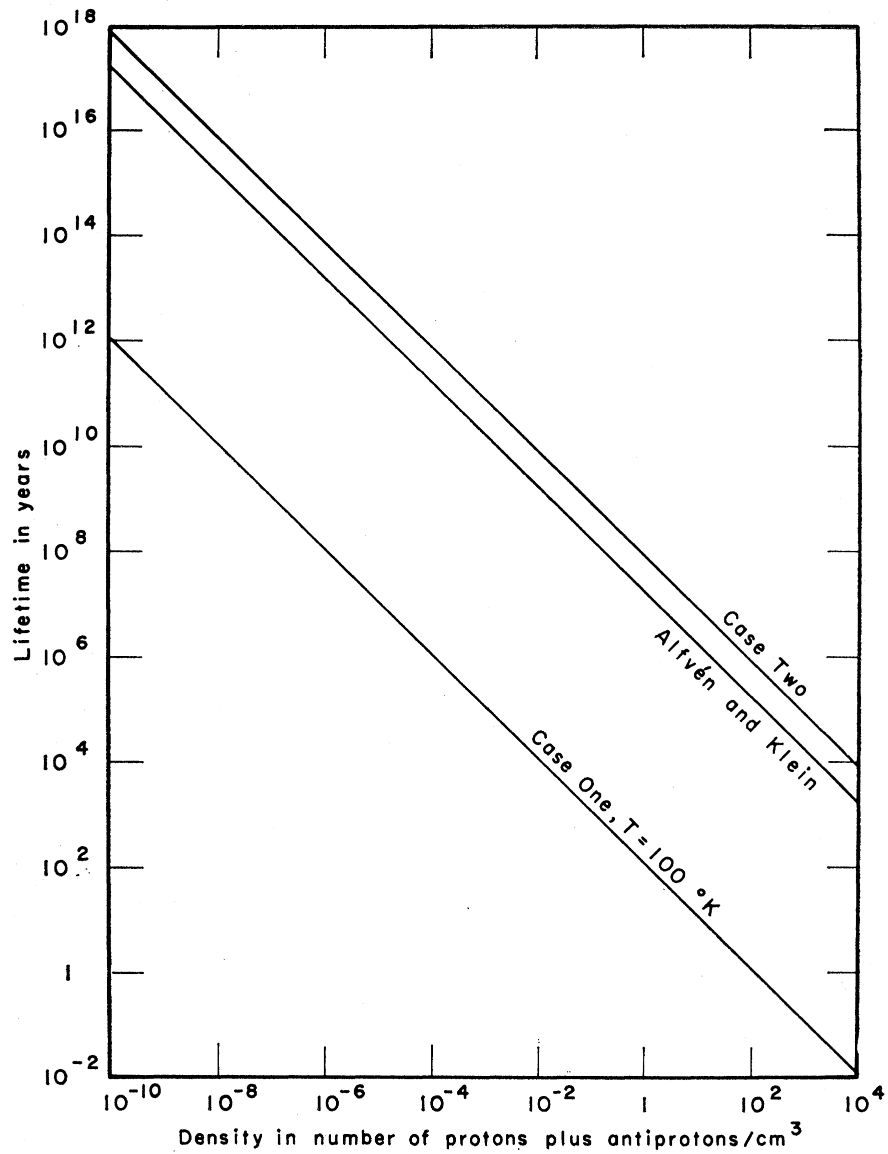

FIG. 5. Lifetimes of the p-$\bar{p}$ component of hydrogen-antihydrogen mixtures.

where σ_e is the e^--e^+ annihilation cross section and σ_p is the p-$\bar{p}$ annihilation cross section. The v's in Eqs. (24) and (25) are the relative velocities of interacting species that are involved. The choices to be made for σ_e and σ_p out of the annihilation cross sections given in the preceding sections depend upon the fractions of ionization of the hydrogen and antihydrogen and upon the relative velocities of the hydrogen and antihydrogen components. If more than one annihilation process is important, an appropriate linear combination of the ω must be taken. If distributions of relative velocities are involved, ω must be integrated over these distributions. In Fig. 4 we give ω_e for $\sigma_e=\sigma_{ae}+\sigma_{re}$, $\sigma_{e^-+\bar{\mathrm{H}}}$ and ω_p for $\sigma_p=\sigma_{ap}$, $\sigma_{p+\bar{\mathrm{H}}}$, $\sigma_{\mathrm{H}+\bar{\mathrm{H}}}$, $\frac{1}{2}\sigma_{\mathrm{H}_2+\bar{\mathrm{H}}_2}$. We have assumed that only the form of reaction (13) in which 2 Pn is produced occurs.

In choosing σ_e and σ_p, consideration must be given to the fact that the mixture may not be in thermal equilibrium. Therefore, molecules and atoms or free particles may be present for unusually high or low energies between the interacting species. In the case where atoms or molecules are involved and the energies are sufficiently high so that the wavelength of the relative motion is considerably less than 1 Bohr radius, σ_e will be $\sigma_{ae}+\sigma_{re}$ and σ_p will be σ_{ap}.

Alfvén and Klein[1,2] have chosen σ_e and σ_p to be equal to the nonrelativistic limit of σ'_{ae} which is given by Eq. (2). Their choices are correct within a factor of about 10 for energies between the interacting species greater than roughly 0.1–1 keV, depending upon the physical circumstances. An energy of 0.1 keV corresponds to a temperature of about 10^6 °K.

By making use of the results for σ_e and σ_p in Secs. II–IV, the lifetime of atomic-scale mixtures of hydrogen and antihydrogen may be determined. We have determined the lifetime of the p-$\bar{p}$ component of an equal-part hydrogen-antihydrogen mixture for two specific cases. Case 1 is an un-ionized H, $\bar{\mathrm{H}}$ mixture in thermal equilibrium at a temperature between $T=10°$K and $T=10\,000°$K. In addition, the mixture has a sufficiently small product of density times linear size, so that the annihilation products escape without loss of energy; hence T and ω_p remain constant. Case 2 is an ionized p, $\bar{p}$, e^-, e^+ mixture in which the p, $\bar{p}$ portion is in thermal equilibrium at an initial temperature between 10^{10} and $10^{13.5}$ °K. Within this temperature range, ${}_p\omega$ is approximately constant. No restriction is placed on the density and size of the mixture in this case, because at $10^{13.5}$ °K the total mass energy of the mixture is about equal to the kinetic energy, and hence no sigficant change in ω_p can occur.

The equation governing the decrease due to annihilations of the number densities n_p, $n_{\bar{p}}$, n_{e^-}, and n_{e^+} of p, $\bar{p}$, e^-, and e^+ under the assumption of an equal amount of hydrogen and antihydrogen present is

$$dn/dt=-\tfrac{1}{2}n^2\omega\,, \tag{26}$$

where t is the time, $\omega=\omega_e$ or ω_p, $n=n_{e^-}+n_{e^+}$ or $n_p+n_{\bar{p}}$, and, initially, $n_p=n_{e^-}=n_{\bar{p}}=n_{e^+}$. Equation (26) differs from the decay equation of an unstable particle or nucleus in that n^2 rather than n appears on the right-hand side. The solution of Eq. (26) for ω independent of time (which is true for cases 1 and 2) is

$$n/n_0=(1+\tfrac{1}{2}n_0\omega t)^{-1}\,, \tag{27}$$

where n_0 is the initial value of n. Because of the t^{-1} dependence of n in the limit $t\rightarrow\infty$, a mean lifetime cannot be defined in the ordinary sense. However, we may take a reasonable analog to the mean lifetime applicable to Eq. (27) to be $\tau_{1/3}$, the time for n/n_0 to decrease to $\frac{1}{3}$. Equation (27) then gives

$$\tau_{1/3}=4/n_0\omega\,. \tag{28}$$

For case 1, by using Eq. (28) and $\omega=\omega_p=v\sigma_{\mathrm{H}+\bar{\mathrm{H}}}$, we find the lifetime of the p-$\bar{p}$ component of the mixture to be

$$\tau_{1/3}=6.4\times10^{-5}(T/°\mathrm{K})^{-0.18}(\pi r_0^2cn_0)^{-1}\,, \tag{29}$$

within an accuracy of about $\pm25\%$. For case 2, the corresponding lifetime is given within a factor of 2 (owing to the only approximate constancy of ω_p) by

$$\tau_{1/3}=20(\pi r_0^2cn_0)^{-1}\,. \tag{30}$$

In Fig. 5, $\tau_{1/3}$, as given by Eqs. (29) and (30), and the mean lifetime, as given by Alfvén,[2] are shown. The mean lifetime given by Alfvén has been multiplied by a factor of 4 to conform with our definition of lifetime given by Eq. (28). The lifetimes for case 1 are several orders of magnitude smaller than the lifetimes found by Alfvén, owing to the neglect of atomic effects by Alfvén. The lifetimes for case 2 are greater than those of Alfvén because his choice of σ_p is too great by about a factor of 5.

VI. CONSEQUENCES FOR OTHER PROBLEMS

The results of Sec. IV are relevant to the problems of negative particles and antimuonium (bound μ^-,e^+) in matter.

The cross sections for nuclear capture in atomic and molecular hydrogen of thermal energy, negative particles with masses much greater than the electron mass, are obtained from the cross sections for the capture reactions (14) and (15). The cross section for the capture of such a negative particle is equal to that for the capture of a $\bar{p}$ for equal values of kinetic enrgye (in the c.m. system).

The rearrangement reaction in which antimuonium and a hydrogen atom react to produce the μ^-, p bound system plus Ps or e^-+e^+ is similar to reaction (11) and has the same cross section for equal kinetic energies in the thermal range. This reaction and the resulting values of the cross section are important to the possible muonium to antimuonium conversion.[22,26]

ACKNOWLEDGMENTS

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[26] V. W. Hughes, Bull. Am. Phys. Soc. 6, 301 (1961); J. J. Amato, P. Crane, V. W. Hughes, D. L. Morgan, Jr., J. E. Rothberg, and P. A. Thompson, Phys. Rev. Letters **21**, 1709; **21**, 1786E (1968).