does not work simply because of the interference terms.[10] As an example, in $pp \to pp\pi^0$ at threshold, where the π^0 can be preemitted from either incoming proton, the combination of pp invariant functions which appears is[7]

$$3|F_S+F_V|^2-2\,\mathrm{Re}(F_S+F_V)(F_T^*+F_A^*)+11|F_T+F_A|^2, \quad (6)$$

quite different from the elastic combination given in Eq. (5). A similar statement holds for a process such as $p\alpha \to p\alpha\pi^0$ at higher energies, where the π^0 can be emitted by the proton either before or after the $p\alpha$ scattering vertex. Again, the interference term between the pre- and post-emission graphs gives a different combination of the invariant functions than is measured in an unpolarized $p\alpha$ elastic-scattering experiment.

[10] That the interference terms do not cause any trouble in the photon case, even in $O(k^0)$, can be seen quite explicitly in Eq. (14) of Ref. 4.

To summarize, we have seen that in contrast to the case of soft-photon emission the measurement of an unpolarized pion-radiative cross section can, as seen in the comparison of Eq. (6) with Eq. (5), give information on the elastic-scattering amplitudes that is unavailable from unpolarized elastic-scattering experiments.

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Regge Poles and the Pauli Principle*

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In a Regge-pole model of large-angle pp scattering, Huang and Pinsky take the Pauli principle into account by adding t- and u-channel Regge poles. We point out that this procedure is consistent with the finite-energy sum rules. In a recent note, Pinsky points out some regularities in the experimental data and contends that they favor a different model in dealing with the Pauli principle, namely, some kind of "duality" between the t and u channels. We show that the regularities he noticed are also reproduced by the original "additive" model.

In their model of large-angle pp scattering, Huang and Pinsky[1] take the Pauli principle into account by adding the contributions of a t- and u-channel Regge pole, in analogy with Feynman graphs. It is known, however, that a simple addition of Regge-pole contributions from two different channels may be counting the same contribution twice, as is the case in the so-called interference model for πp scattering, in which the scattering amplitude is taken to be the sum of t- and s-channel Regge-pole contributions. That double counting is committed and revealed by examining the finite-energy sum rules (FESR).[2] In that case, there is a duality between t- and s-channel Regge poles, in the sense that a t-channel Regge pole already includes some contributions of the s-channel Regge poles and vice versa. It was argued in Ref. 1 that the criticism of the interference model may not apply to the t-u additivity, because the third channel (s channel) has no resonances. We wish to supplement that argument here and to comment on a paper by Pinsky,[3] which contains a criticism of the model of Ref. 1.

The criticism of the interference model based on FESR is not relevant to the model of Ref. 1, because the latter is in fact consistent with FESR. To demonstrate this in a simplified form we write a Veneziano representation[4] for a scalar amplitude $A(t,u)$, which is antisymmetric in t and u, and which has no pole in the variable $s=4m^2-t-u$:

$$A(t,u)=\beta(\alpha_t-\alpha_u)\Gamma(1-\alpha_t)\Gamma(1-\alpha_u)/\Gamma(2-\alpha_t-\alpha_u)$$
$$=\beta\left[\frac{\Gamma(2-\alpha_t)\Gamma(1-\alpha_u)}{\Gamma(2-\alpha_t-\alpha_u)}-\frac{\Gamma(2-\alpha_u)\Gamma(1-\alpha_t)}{\Gamma(2-\alpha_t-\alpha_u)}\right], \quad (1)$$

where β is a constant. The fact that this is consistent with FESR can be shown in a manner similar to that employed in Ref. 4. Note that the first term in (1) alone contains the leading t-channel Regge pole α_t, and the second term alone contains the leading u-channel Regge pole α_u, but the daughter Regge poles α_t-k α_u-k ,($k=1, 2, 3, \ldots$) enter into both terms. In

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the model of Ref. 1, all secondary trajectories are neglected by neglect of terms of order z_t^{-1} and z_u^{-1}, where z_t and z_u are, respectively, the cosines of the scattering angles in the t channel and the u channel. Thus the additivity assumed in Ref. 1 is supported by the Veneziano model.

In a recent letter Pinsky[3] noted a regularity of the pp differential cross section; namely, in plots of $\ln(sd\sigma/d\Omega)$ versus $\ln(s-u)$ at fixed t, the data points seem to fall on approximately straight lines, for a range of large t values. He takes this to mean that a good fit is

$$sd\sigma/d\Omega=f(t)(s-u)^{\alpha(t)}, \tag{2}$$

where the functions $f(t)$ and $\alpha(t)$ are determined by the

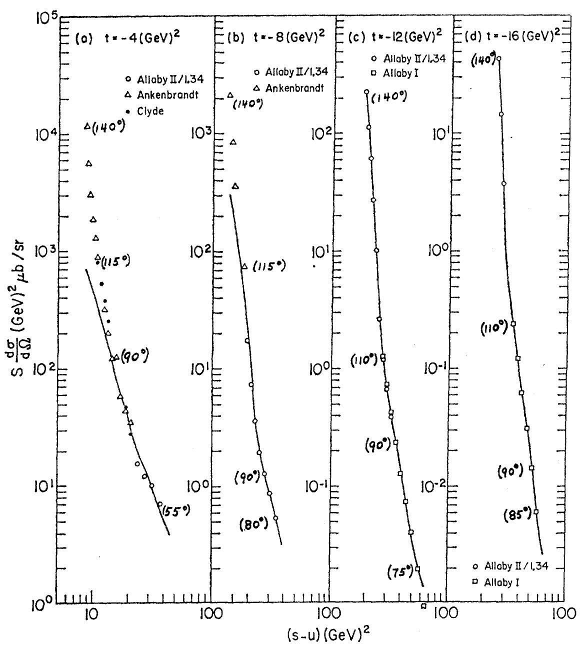

FIG. 1. Elastic pp scattering. The experimental points are interpolations of data taken from Refs. 6–9. Those from Ref. 9 are uniformly divided by 1.34, as explained in the text. Statistical errors are a few percent. Numbers in parentheses give c.m. scattering angles. The solid curves are theoretical curves calculated from the model of Ref. 1.

data. Since (2) looks like the contribution from a t-channel Regge pole alone, it is taken to be a proof of t-u duality, and a disproof of the model of Ref. 1. It is clear, however, that since the latter model is successfully fitted to experiments, it cannot be disproved by plotting the data in a special way. An explicit calculation shows that the model of Ref. 1 also produces approximately straight lines on this special plot and gives a good fit to the data for large $-t$ and large $s-u$, as is shown in Fig. 1. The theoretical curves (solid lines) are calculated using Eqs. (49), (75), and (79)[5] of Ref. 1. The experimental points are obtained by four-point Lagrange interpolations in both θ and t of the logarithm of the cross section, the data being taken from Refs. 6–9. The data in Ref. 9 were not used in Ref. 1 to determine model parameters (of which there are eight), because the cross sections of Ref. 9 lie systematically higher than those of Ref. 8 by an average factor of 1.34. After uniform division by this factor, however, they are consistent with the theoretical model.

It should be emphasized that the main point illustrated in Fig. 1 is that a theoretical model with additive Regge poles can produce a cross section that looks like (2). The comparison with data cannot be taken too seriously, and is subject to change, because sets of existing data are inconsistent among themselves. By changing the absolute normalization of the data from Ref. 9, we do not imply that its normalization is in error. This is done merely for expediency, for the theoretical model was fitted to data with a different absolute normalization.

In conclusion, we think that the approximate straight lines in Fig. 1 have no fundamental significance, and that the uniqueness of (2) is illusory.

[5] See Erratum, Ref. 1, for important corrections.

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