p-p Scattering at Very High Energy and Massive Electrodynamics*

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We apply a formula derived recently in massive electrodynamics to p-p scattering. The $C=+1$ amplitude due to ω exchange gives rather interesting predictions at very high energy. The total cross section is found to be 31–32 mb; $\sigma_{el}\cong 4.5$ mb.

A CONSIDERABLE amount of work has been done in the past year in the area of massive quantum electrodynamics.[1–5] It is hoped that by summing certain sets of diagrams we may unveil some general features of strong interactions at high energy. The results have been interesting. The scattering amplitudes all have the general characteristics of (i) form factors or impact factors and (ii) multiple-scattering or quasiparticle (eikonal) exchange. The former aspect bears close resemblance to the work of Chou and Yang.[6,7] It is also observed that only transverse degrees of freedom appear in the exchange, which give rise to impact representation.

There are infinite sets of diagrams to be summed and none of these calculations are easy. It is natural for one then to seek some hint from the available data. In other words, we would like at this point to confront some of the theoretical results with experiments to see how we fare and then look for direction to proceed. This is why the present work has been undertaken.

A word of philosophy is perhaps proper. At some stage in such a model calculation, one must admit certain assumptions on the basis of phenomenological deduction. Theoretical justification may hopefully be put forward at a later time.

It is found that if vacuum polarization effects are neglected and if the soft-photon approximation is made to the t-channel exchanges[5] (or, if Z graphs are dropped[3,4]), then the invariant scattering amplitude for $p(p_1)+p(p_2)\to p(p_3)+p(p_4)$ can be written as

$$T=i(2\pi)^4\delta(p_1+p_2-p_3-p_4)\times[\delta_{\lambda_1\lambda_3}F_1(k^2)-i\chi_{\lambda_3}\sigma_2 k\chi_{\lambda_1}F_2(k^2)]\times[\delta_{\lambda_2\lambda_4}F_1(k^2)-i\chi_{\lambda_4}\sigma_2 k\chi_{\lambda_2}F_2(k^2)]\times(p\cdot p'/m^2)E(k^2),\quad(1)$$

where

$$E(k^2)=\int d^2x_\perp e^{-ik\cdot x_\perp}\times\left\{\exp\left[-ig^2\int\frac{d^2q_\perp}{(2\pi)^2}e^{iq_\perp\cdot x_\perp}\frac{1}{q^2+\mu^2}\right]-1\right\}.\quad(2)$$

$k=p_3-p_1$ is the momentum transfer and is assumed to be along the x direction; the notations are otherwise self-apparent. It is noticed that this amplitude contains both the $C=+1$ and $C=-1$ parts, and that they both give constant contribution at high energy to the differential cross section. It is deduced phenomenologically, however, that at high energy, exchanges with the quantum number of the vacuum should dominate. Thereupon, we here make a "to be proven" assumption that in a more realistic calculation where some other suitable sets of graphs are also summed, the $C=-1$ amplitudes will be suppressed in some Regge-like manner.

The vector particle we have in mind as the interaction carrier is the ω meson. The φ meson is neglected since it is known that its coupling with $N\bar{N}$ is small. Also, the Pauli term of $\omega N\bar{N}$ will be dropped, as it is also known to be small. The scattering amplitude then will be written as

$$T_\omega=i(2\pi)^4\delta(p_1+p_2-p_3-p_4)\delta_{\lambda_1\lambda_3}\delta_{\lambda_2\lambda_4}[F_1(k^2)]^2\times(p\cdot p'/m^2)\frac{2\pi}{\mu_\omega^2}\int_0^\infty xdx\,J_0\left(\frac{k}{\mu_\omega}x\right)\times\left\{\cos\left[\frac{g_\omega^2}{2\pi}K_0(x)\right]\cosh\left[\frac{g_\omega^2}{2\pi}\frac{\gamma_\omega}{2\mu_\omega}xK_1(x)\right]-1+i\sin\left[\frac{g_\omega^2}{2\pi}K_0(x)\right]\sinh\left[\frac{g_\omega^2}{2\pi}\frac{\gamma_\omega}{2\mu_\omega}xK_1(\mu)\right]\right\},\quad(3)$$

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[1] H. Cheng and T. T. Wu, Phys. Rev. Letters **22**, 666 (1969); **23**, 670 (1969); Phys. Rev. **182**, 1852 (1969); **182**, 1868 (1969); **182**, 1873 (1969); **182**, 1899 (1969); **184**, 1868 (1969); **186**, 1611 (1969); Phys. Rev. D **1**, 456 (1970); **1**, 459 (1970); **1**, 467 (1970); (to be published).

[2] S. J. Chang and S. K. Ma, Phys. Rev. Letters **22**, 1334 (1969); Phys. Rev. **188**, 2385 (1969); B. W. Lee, Phys. Rev. D **1**, 2361 (1970).

[3] H. Cheng and T. T. Wu, Phys. Rev. **184**, 1868 (1969); Phys. Rev. D **1**, 1069 (1970); **1**, 1083 (1970).

[4] S. J. Chang, Phys. Rev. D **1**, 2977 (1970).

[5] Y. P. Yao, Phys. Rev. D **1**, 1380 (1970); **1**, 2971 (1970).

[6] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965); see also, H. D. I. Arbarbanel, S. D. Drell, and F. Gilman, Phys. Rev. Letters **20**, 280 (1968).

[7] T. T. Chou and C. N. Yang, in *Proceedings of the Second International Conference on High-Energy Physics and Nuclear Structure, Rehovoth, Israel, 1967*, edited by G. Alexander (North-Holland, Amsterdam, 1967), pp. 348–359; Phys. Rev. Letters **20**, 1213 (1968); Phys. Rev. **170**, 1591 (1968); **175**, 1832 (1968).

in which we have given a width to the ω meson, i.e.,

$$\frac{1}{q^2+\mu^2}\rightarrow\frac{1}{q^2+(\mu-\frac{1}{2}i\gamma)^2}\cong\frac{1}{q^2+\mu^2}+\frac{i\mu\gamma}{(q^2+\mu^2)^2}. \quad (4)$$

J_0, K_0, and K_1 are the standard (modified) Bessel functions. We use the values[8]

$$g_\omega^2/4\pi=7,\quad \mu_\omega=780\text{ MeV},\quad\text{and}\quad \gamma_\omega=13\text{ MeV}. \quad (5)$$

The dotted curves in Figs. 1 and 2 are the results. The form factor used here is the one parametrized by Chou and Yang.[7] The curves are laid on Krisch's plots[9] in which the abscissa is $\beta^2p_\perp^2\rightarrow -t$ as $s\rightarrow\infty$. We choose

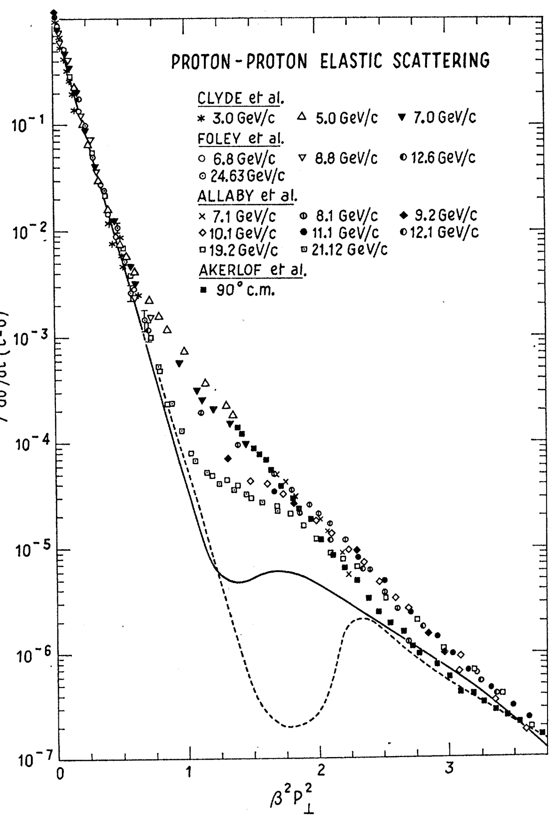

FIG. 1. Proton-proton elastic scattering cross sections. The dotted curve includes ω exchange alone. The solid curve also includes effects of $\rho^0\pi^0$ (1650) background. The abscissa is in units of $(\text{GeV}/c)^2$ (see Ref. 9).

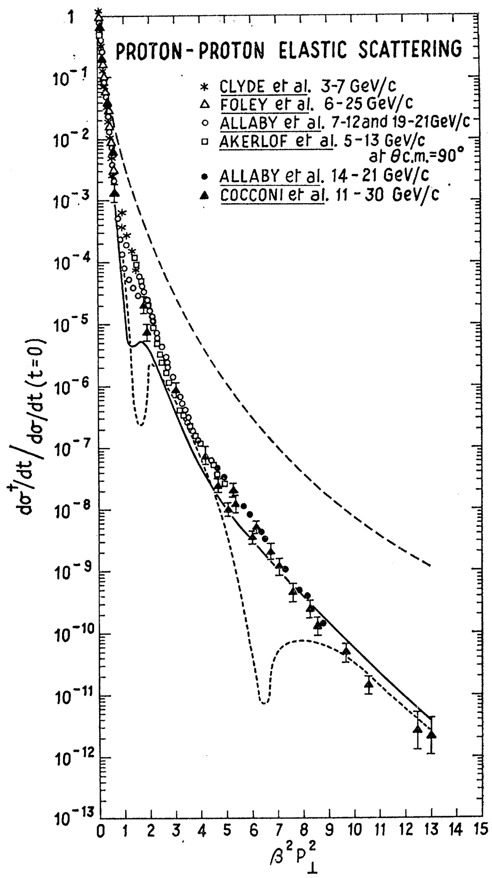

FIG. 2. Proton-proton elastic scattering cross sections. The dotted curve includes ω exchange alone, the solid curve also includes effects of $\rho^0\pi^0$ background. Also plotted is $[F_1(t)]^4$ (dashed curve). The abscissa is in units of $(\text{GeV}/c)^2$ (see Ref. 9).

these plots because it is observed that the data points at different energies have less scattering.

The dip at $t\cong -1.5$ has the general tendency of the experimental points; the dip at $t\cong -6$ reminds us of those in the Chou-Yang model.[7,10] In the other regions, the curve falls correctly.

One reason we cannot fit the data at present energies by using ω alone is its narrow width. Consequently, we have not generated enough real part to the amplitude. (The ratio of the real to the imaginary part is less than 2%.) There are many ways to introduce more real part. One way, due to Durand and Lipes,[10] is by putting in spin-orbit coupling. We shall indicate here another way. It is worth emphasizing that the real part must be somewhat energy dependent so that in the high-energy limit, the amplitude is purely imaginary.

We make use of a reported $\rho^0\pi^0$ enhancement around 1650 Mev which has the same internal quantum number as ω. We shall fix the center of the exchanged background at this value. Essentially then, there are two

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[8] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969). The value of $g_{\omega NN}^2/4\pi$ as determined from low-energy nucleon-nucleon scattering data ranges from 5 to 8. G. Köpp and P. Söding, Phys. Letters **23**, 494 (1966); T. Ueda and A. E. S. Green, Phys. Rev. **174**, 1304 (1968); Nucl. Phys. **B10**, 289 (1969). The high value is also given by vector-dominance models. I thank Dr. I. Kimel for a discussion on this point.

[9] A. D. Krisch, Phys. Rev. Letters **19**, 1149 (1967); unpublished. The latter plots are similar to the one in Phys. Rev. Letters except that (a) experimental total cross sections have been used in the form $d\sigma^+/dt(t=0)=\sigma_{\text{tot}}/4\pi$, and (b) $d\sigma^+/dt=d\sigma/dt$ is assumed for $-t<2.75$ $(\text{GeV}/c)^2$ (private communication with Professor A. D. Krisch and Dr. M. Marshak).

[10] L. Durand III and R. Lipes, Phys. Rev. Letters **20**, 637 (1968).

parameters (width and coupling) which will be varied in the subsequent calculation. The over-all effects[11] of including this "particle" are to make the following changes on the invariant amplitude in Eq. (3):

$$T_\omega \to T_{\omega+\rho^0\pi^0},$$

$$\frac{g_\omega^2}{2\pi}K_0(x) \to \frac{g_\omega^2}{2\pi}K_0(x)+\frac{g_{\rho\pi}^2}{2\pi}K_0\left(\frac{\mu_{\rho\pi}}{\mu_\omega}x\right),$$

$$\frac{g_\omega^2}{2\pi}\frac{\gamma_\omega}{2\mu_\omega}xK_1(x) \to \frac{g_\omega^2}{2\pi}\frac{\gamma_\omega}{2\mu_\omega}xK_1(x) + \frac{g_{\rho\pi}^2}{2\pi}\frac{\gamma_{\rho\pi}}{2\mu_{\rho\pi}}\left(\frac{\mu_{\rho\pi}}{\mu_\omega}x\right)K_1\left(\frac{\mu_{\rho\pi}}{\mu_\omega}x\right). \tag{6}$$

We use

$$g_{\rho\pi}^2/4\pi=16,\quad \mu_{\rho\pi}=1.65\text{ GeV},\quad \text{and}\quad \gamma_{\rho\pi}=\mu_{\rho\pi}. \tag{7}$$

Then the solid curves in Figs. 1 and 2 result. We have not made a χ^2 fit to the data. For small $-t$, we see that the dip structure is reproduced at $-t\cong 1.2$. The recent CERN data[12] may pull the points near this region towards the theoretical curve. For large $-t$, the agreement is good, taking into consideration that the p-p data are still quite poor. Also plotted on Fig. 2 is $(F_1)^4$.

We now discuss the results.

(a) We stress that the existence of the $\rho^0\pi^0$ resonance is not important in the above calculation. We prefer to regard it as a way of parametrizing the background. Upon including it, the ratio of the real to the imaginary part of the amplitude in the forward direction is about 15%. The sign contradicts the experimental value at the present available energy. This feature is shared by other multiple-scattering models.[13] A way to overcome this difficulty is by arguing that Regge terms still contribute near the forward direction at an energy about 20 GeV.

(b) As mentioned before, one should make the real part slightly energy dependent so that in the true asymptotic energy region the amplitude becomes purely imaginary.[14] In other words, it is the author's opinion that the curve due to ω exchange alone will eventually better approximate p-p scattering as the energy goes up.

(c) The total cross section comes out to be 31–32 mb in the two calculations, compared with the experimental value of $\cong$38 mb. $\sigma_{el}=4.5$ mb with ω exchange only, which is a bit lower than the experimental value of $\sigma_{el}/\sigma_{tot}\cong 0.25$ at 20 GeV.[15] Increasing $\omega N\bar{N}$ coupling will result in an increase of σ_{tot} and σ_{el}. For example, $\sigma_{tot}=36.2$ mb and $\sigma_{el}=5.5$ mb when $g_{\omega N\bar{N}}^2/4\pi=9$. The background will not change these values much. Since the form factor F_1 that we used throughout the calculation is the electric form factor, which is not precisely the strong form factor, we consider such agreement to be satisfactory.

(d) Our model shares some common features with that of Chou and Yang.[7] On the other hand, there are essential differences in detail. Loosely speaking, we have the following picture. The two protons interact in the region where they overlap each other spatially. There, they undergo multiple scattering. This picture is geometrical, in contradistinction to theirs, which is optical.

(e) This model differs from the optical models.[16] We consider only $C=+1$ exchanges. The damping here is mainly due to the form factor $F_1(k^2)$, although the eikonal function also provides some. The model is also different from the infrared model[17] in that soft-photon approximation is made on the t-channel exchanged photons only. The radiative photons are treated exactly. No question of renormalizability arises.

(f) We now turn to the theoretical side. If one agrees that the fit bears some physical truth, it is clear what should be done. We have to show that indeed the $C=-1$ amplitude is suppressed. Effects of hard-photon exchanges must also be investigated.

(g) If the trend of the Serpukhov data continues,[18] we have to see how this model can be modified to accommodate the new situation.

I would like to thank Professor A. D. Krisch for generously making available some of his unpublished plots. Encouragement from Professor Y. Tomozawa is appreciated. A discussion with Professor M. Ross has been most helpful.

[11] N. Armenise *et al.*, Phys. Letters **26B**, 336 (1968); Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969); A. M. Cnops *et al.*, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968). The spin-parity assignment of this resonance is not known.

[12] G. Cocconi (unpublished).

[13] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 857 (1967). For a review of multiple-scattering models see C. B. Chiu, Rev. Mod. Phys. **41**, 640 (1969).

[14] L. Van Hove, Phys. Letters **5**, 252 (1963); **7**, 76 (1963); Rev. Mod. Phys. **36**, 655 (1964).

[15] K. J. Foley *et al.*, Phys. Rev. Letters **11**, 423 (1963); **11**, 503 (1963).

[16] The models that use optical vector potentials are R. Serber, Phys. Rev. Letters **10**, 357 (1963); Rev. Mod. Phys. **36**, 649 (1964); M. M. Islam and J. Rosen, Phys. Rev. Letters **19**, 178 (1967); **22**, 502 (1969); Phys. Rev. **178**, 2135 (1969).

[17] E.g., H. M. Fried and T. K. Gaisser, Phys. Rev. **179**, 1491 (1969).

[18] G. G. Beznogikh *et al.*, Phys. Letters **30B**, 274 (1969).