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Eikonal Model for High-Energy Elastic p-p Scattering at Large Angles

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A simplified form of a relativistic generalization of the eikonal approximation for wide-angle potential scattering provides a semiquantitative description of p-p scattering at large energy and momentum transfer over a variation of more than five orders of magnitude in $d\sigma/dt$. The model contains two adjustable parameters.

I. INTRODUCTION

RELATIVISTIC generalizations of the eikonal approximation have been discussed largely in the context of quantum electrodynamics[1,2] and of field theories of scalar particles.[1,3] It is desirable to attempt to apply the method to some hadronic processes to see whether it can be relevant to strong interactions. The purpose of this paper is (a) to point out that the neutral-soft-vector-meson (NSVM) exchange model previously applied to elastic nucleon form factors and high-energy p-p scattering[4] and to pion-nucleon scattering[5] is a relativistic generalization of the eikonal expansion appropriate for wide-angle scattering, and (b) to show that correct application of a simplified form of the model leads to an improved fit to wide-angle elastic p-p scattering at high energies.[6]

One derives a relativistic version of the eikonal approximation by neglecting the recoil of a scattered particle after emission of a soft meson. This involves neglecting terms in the propagator of the scattered particle that are quadratic in the momenta of the exchanged mesons. It then becomes possible, either explicitly[7] or by using functional techniques,[1] to sum all generalized ladder diagrams in which soft mesons are exchanged in all possible ways between the scattered particles. The expression so obtained is not necessarily appropriate without correction for wide-angle scattering.

Schiff[8] has shown that in potential scattering at high energies the dominant contribution to the nth-order term in the amplitude for wide-angle scattering comes from the integration region in which there is a single scattering through virtually the whole angle and $n-1$ near forward scatterings. A generalization of this picture of wide-angle scattering has been used by Cardy[9] to obtain fixed-angle high-energy behavior in scalar perturbation theory and in quantum electrodynamics.

The NSVM exchange model of Ref. 4 is a generalization of the relativistic eikonal model ideally suited to describing wide-angle scattering. It gives a form for the proton-proton amplitude that is the product of two terms in configuration space:

$$M(x_1,x_2;\,y_1,y_2)=M_H(x_1,x_2;\,y_1,y_2)\times\exp[\Phi(x_1,x_2;\,y_1,y_2)]. \quad (1)$$

The exponential factor is the result of summing over exchanges of neutral soft-vector mesons between pairs of proton legs in all possible ways in all three channels. M_H is the hard part of the amplitude, i.e., what is left over after summing the soft exchanges.[10] It is M_H that gives rise to the large-angle sattering. By analogy with Schiff's analysis of wide-angle potential scattering, we choose a relativistic Born approximation for M_H.

We first compare Eq. (1) with the nonrelativistic approximation of Schiff[8] and with other relativistic eikonal models. We then use a simplified version of the amplitude to fit the high-energy, wide-angle p-p elastic scattering data with various simple choices for the Born factor M_H. We find significantly improved agreement with the data in this way as compared to the results of Ref. 4 in which M_H was taken to be independent of t and was implicitly assumed to give the correct forward cross section. Finally we discuss possible implications of the present model as well as a feature that distinguishes it from other eikonal approaches; we also point out that the application of the model to the elastic nucleon form factors[4] gives results that depend only on the assumption of vector dominance of the current but not on any simplification of the soft-exchange corrections.

II. EIKONAL NATURE OF MODEL

The form obtained for Φ in Eq. (1) by summing all NSVM exchanges is

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[1] Henry D. I. Abarbanel and Claude Itzykson, Phys. Rev. Letters **23**, 53 (1969).
[2] S. J. Chang and S. Ma, Phys. Rev. Letters **22**, 1334 (1969).
[3] Maurice Lévy and Joseph Sucher, Phys. Rev. **186**, 1656 (1969).
[4] H. M. Fried and T. K. Gaisser, Phys. Rev. **179**, 1491 (1969).
[5] H. M. Fried, Phys. Rev. D **1**, 596 (1970).
[6] Some other applications of eikonal-type models to hadronic interactions are listed in footnote 2 of Ref. 15.
[7] Maurice Lévy, Phys. Rev. **130**, 791 (1963); Steven Weinberg, *ibid.* **140**, B516 (1965); and Lévy and Sucher, Ref. 3. References to early work on soft photons are cited by these authors.
[8] L. I. Schiff, Phys. Rev. **103**, 443 (1956).
[9] J. L. Cardy, Nucl. Phys. **B17**, 493 (1970).
[10] See Ref. 4 for details. Equation (1) is exact until some specific simple form is chosen for M_H.

$$\Phi(x_1,x_2;y_1,y_2)$$
$$=ig^2\int_0^\infty d\xi\int_0^\infty d\eta\{v_1\cdot v_2\Delta_C(x_1-x_2-\xi v_1+\eta v_2)$$
$$+v_1'\cdot v_2'\Delta_C(y_1-y_2+\xi v_1'-\eta v_2')$$
$$+v_1\cdot v_1'\Delta_C(x_1-y_1-\xi v_1-\eta v_1')$$
$$+v_2\cdot v_2'\Delta_C(x_2-y_2-\xi v_2-\eta v_2')$$
$$+v_1\cdot v_2'\Delta_C(x_1-y_2-\xi v_1-\eta v_2')$$
$$+v_2\cdot v_1'\Delta_C(y_1-x_2+\xi v_1'+\eta v_2)\}\,, \quad (2)$$

where $v_{\mu i}\equiv p_{\mu i}/m$ and we consider the process $p_1+p_2\to p_1'+p_2'$. The first pair of terms comes from s-channel exchanges, the second from t-channel exchanges, and the last from u-channel exchanges. The factors $v_1\cdot v_2$, $v_1'\cdot v_1$, and $v_1'\cdot v_2$ are essentially s, t, and u, respectively. The appearance of these factors is a characteristic feature of vector exchange that removes the inherent $1/s$ behavior of the scalar propagator so that each power in an expansion in powers of g^2, the nucleon-NSVM coupling constant, has the same s dependence. Soft scalar exchanges, on the other hand, do not modify the hard amplitude significantly.

The momentum-space amplitude is

$$(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')M(s,t)$$
$$=\int d^4x_1\,e^{ip_1\cdot x_1}\int d^4x_2\,e^{ip_2\cdot x_2}$$
$$\times\int d^4y_1\,e^{-ip_1'\cdot y_1}\int d^4y_2\,e^{-ip_2'\cdot y_2}M(x_1,x_2;y_1,y_2)\,. \quad (3)$$

Therefore the choice of a neutral vector-meson exchange Born term for M_H leads to

$$(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')M(s,t)$$
$$=g^2\int d^4x_1\,e^{i(p_1-p_1')\cdot x_1}\int d^4x_2\,e^{i(p_2-p_2')\cdot x_2}$$
$$\times\exp[\Phi(x_1,x_2;x_1,x_2)]\bar{u}(p_1')\gamma_\mu u(p_1)\delta_{\mu\nu}$$
$$\times\bar{u}(p_2')\gamma_\nu u(p_2)\Delta_C(x_1-x_2)-(p_1'\leftrightarrow p_2')\,. \quad (4)$$

The t-channel contributions to $\Phi(x_1,x_2;x_1,x_2)$ do not depend on x_1 or x_2 and so factor outside the x integration completely. The remaining propagators depend only on x_1-x_2. Thus

$$M(s,t)=g^2\int d^4x\,e^{iq\cdot x}\Delta_C(x)\bar{u}(p_1')\gamma_\mu u(p_1)$$
$$\times\bar{u}(p_2')\gamma_\mu u(p_2)e^{\Phi(x)}-(p_1'\leftrightarrow p_2')\,, \quad (5)$$

where $q=p_1-p_1'-p_2'=p_2$, and

$$\Phi(x)=ig^2\int_0^\infty d\xi\int_0^\infty d\eta[v_1\cdot v_2\Delta_C(x-\xi v_1+\eta v_2)$$
$$+v_1'\cdot v_2'\Delta_C(x+\xi v_1'-\eta v_2')$$
$$+v_1'\cdot v_1\Delta_C(-\xi v_1-\eta v_1')$$
$$+v_2'\cdot v_2\Delta_C(-\xi v_2-\eta v_2')$$
$$+v_1\cdot v_2'\Delta_C(x-\xi v_1-\eta v_2')$$
$$+v_2\cdot v_1'\Delta_C(x+\xi v_1'+\eta v_2)]\,. \quad (6)$$

The function Φ is our relativistic generalization of the eikonal.

To retrieve the nonrelativistic eikonal approximation from Eq. (5), it is necessary to neglect the t-channel exchanges, which represent an intrinsically relativistic effect: emission and reabsorption of quanta by the scattered particle. But there is no reason to exclude such terms from a relativistic eikonal model, and they prove to be crucial in fitting the wide-angle p-p data. The expression $\Phi(x)$ *without* the t-channel terms is the relativistic eikonal χ [Eq. (3.23)] of Ref. 3, here written for vector exchange.

Following Lévy and Sucher, we can take the static limit of Eq. (5) by assuming nucleon 2 to be infinitely heavy so that $v_2=(1,\mathbf{0})$, and by neglecting the exchange term. The use of the identity

$$g^2\int_{-\infty}^{\infty}dx_0\,\Delta_C(x_0,\mathbf{x})=-\frac{g^2}{4\pi}\frac{e^{-\mu|\mathbf{x}|}}{|\mathbf{x}|}\equiv V(\mathbf{x}) \quad (7)$$

in Eq. (5) then leads to the result

$$f(\mathbf{p},\mathbf{p}')=-\frac{m}{2\pi}\int d^3r\,e^{i\mathbf{q}\cdot\mathbf{r}}V(\mathbf{r})$$
$$\times\exp\left\{i\frac{m}{|\mathbf{p}|}\int_0^\infty d\xi[V(\mathbf{x}-\xi\hat{p})+V(\mathbf{x}+\xi\hat{p}')]\right\}\,, \quad (8)$$

provided the projectile nucleon is also nonrelativistic. The scattering amplitude is related to the invariant amplitude by

$$f=-\frac{m^2}{\sqrt{s}}\frac{1}{2\pi}M\,,$$

and spin factors have been suppressed in writing Eq. (8). Equation (8) is identical to Eq. (13) of Schiff.[8] The expression (5) is thus a relativistic generalization of the eikonal model appropriate for wide-angle scattering.

It is instructive to compare the small-angle form of Eq. (5) with other relativistic eikonal models. In this limit we have $v_1\sim v_1'$ and $v_2\sim v_2'$ so that, when spin factors are neglected,

$$M(s,t)\sim g^2\int d^4x\, e^{iq\cdot x}\Delta_C(x)\frac{p_1\cdot p_2}{m^2}\times\exp\left\{ig^2\frac{p_1\cdot p_2}{m^2}\int_{-\infty}^{\infty}d\xi\int_{-\infty}^{\infty}d\eta\,\Delta_C(x-\xi v_1-\eta v_2)\right\},\quad(9)$$

the exchange term contributing little for small angles. (Again, for purposes of comparison with other models, the t-channel contributions have been neglected.) Since[11] $q\cdot(p_1+p_1')=-q\cdot(p_2+p_2')=0$, we have, in the near forward direction, $q\cdot v_1\sim q\cdot v_2\sim 0$. Thus the change of variable $x=b-\sigma v_1-\sigma' v_2$, where $b\cdot v_1=b\cdot v_2=0$, enables two integrations to be carried out leaving an integral over impact parameters. Then, since

$$\bar{u}(p_1')\gamma_\mu u(p_1)\delta_{\mu\nu}u(p_2')\gamma_\nu u(p_2)\sim(p_1\cdot p_2/m^2)\delta_{s_1s_1'}\delta_{s_2s_2'}$$

near $t=0$,

$$M(s,t)=\delta_{s_1s_1'}\delta_{s_2s_2'}\frac{s}{2m^2}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{q}}F(b)e^{F(b)},\quad(10)$$

where

$$F(b)\equiv -ig^2\int\frac{d^2p}{(2\pi)^2}\frac{e^{i\mathbf{p}\cdot\mathbf{b}}}{\mathbf{p}^2+\mu^2}.$$

Abarbanel and Itzykson,[1] on the other hand, find

$$M(s,t)=\delta_{s_1s_1'}\delta_{s_2s_2'}\frac{s}{2m^2}\int d^2b\, e^{i\mathbf{b}\cdot\mathbf{q}}[e^{F(b)}-1],\quad(11)$$

as do Chang and Ma.[2]

The difference here is clearly one of counting and is discussed by Lévy and Sucher.[3] In deriving Eq. (5) the hard exchange was counted separately from the soft exchanges, whereas near the forward direction all exchanges are soft and correct counting would lead to Eq. (11). Lévy and Sucher choose the correct counting for near-forward scattering but as a consequence get a result that differs from Schiff's for wide-rangle potential scattering. On the other hand, the present counting is correct for wide-angle scattering if the wide-angle scattering is dominated by a single hard exchange.

III. COMPARISON WITH EXPERIMENT

A simplified form of Eq. (5) emerges if hard and soft exchanges are decoupled by a dipolelike approximation as discussed in Ref. 4. This approximation amounts to neglecting the x dependence of $\Phi(x)$. It should be emphasized that this is a drastic approximation: In the nonrelativistic case [Eq. (8)] the result of this separation is that the phase vanishes entirely and nothing is left but the Born term. In the relativistic case, however, significant s and t dependence is retained in the exponential that modifies the Born term, so that it is worthwhile to compare the simplified form with experiment.

[11] This argument follows that of Ref. 1.

As shown in Ref. 4, after appropriate renormalization,

$$\Phi(x=0)\to\Phi(s,t,u)=2\gamma[F(t)+F(u)-F(s)],$$

where

$$F(t)=1-\frac{2x+1}{[x(x+1)]^{1/2}}\ln[\sqrt{x}+\sqrt{(x+1)}],$$

$$x\equiv -t/4m^2\quad\text{for } t<0$$

and

$$\mathrm{Re}F(s)=F(4m^2-s),\quad s>4m^2.$$

Thus in the no-correlation limit,

$$\frac{d\sigma}{dt}=\left(\frac{d\sigma}{dt}\right)_{\mathrm{Born}}e^{4\gamma[F(t)+F(u)-F(4m^2-s)]}.\quad(12)$$

The parameter γ is a combination of nucleon-NSVM coupling constant and integration cutoff separating soft from hard exchanges. It is discussed in Ref. 4 and is treated as a free parameter in fitting the data. In applying Eq. (12) to the experiments there is no reason to assume that the only "potential" responsible for the wide-angle scattering is vector exchange. Accordingly, we have tried single ρ_0 exchange with $g_\rho A_{\rho\mu}\bar{\psi}\gamma_\mu\psi$ coupling and single π^0 exchange with γ_5 coupling. Other choices of coupling and exchanged object are possible but are not expected to alter the theoretical curves significantly. For single ρ^0 exchange,

$$\left(\frac{d\sigma}{dt}\right)_{\mathrm{Born}}=\frac{2\pi}{s(s-4m^2)}\left(\frac{g_\rho^2}{4\pi}\right)^2[\ ],\quad(13)$$

where

$$[\ ]=\left[\frac{(s-2m^2)^2+(u-2m^2)^2+4m^2t}{(t-m_\rho^2)^2}+\frac{(s-2m^2)^2+(t-2m^2)^2+4m^2u}{(u-m_\rho^2)^2}+2\frac{(s-2m^2)^2-4m^2s+8m^4}{(t-m_\rho^2)(u-m_\rho^2)}\right];$$

whereas for single π^0 exchange,

$$\left(\frac{d\sigma}{dt}\right)_{\mathrm{Born}}=\frac{3\pi}{s(s-4m^2)}\left(\frac{g_\pi^2}{4\pi}\right)^2.\quad(14)$$

In writing Eq. (14) we have neglected m_π^2 since the comparison is to be made only away from $t=0$. Similarly, for large $|t|$ the value used in Eq. (13) for m_ρ^2 is unimportant.

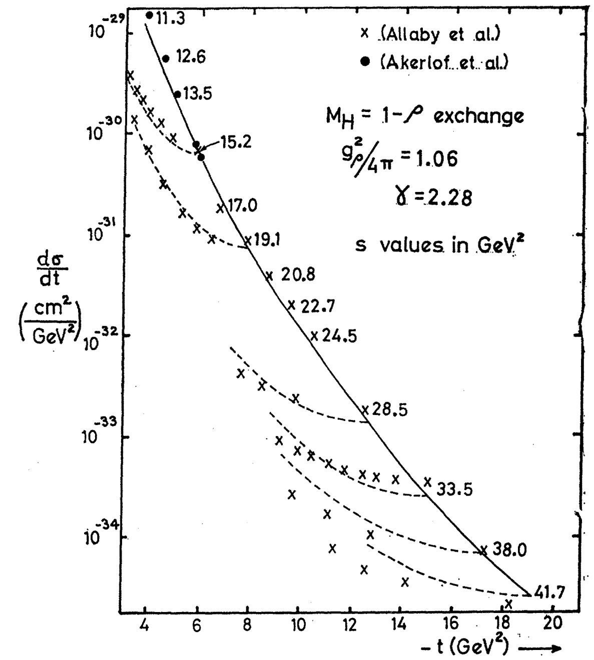

FIG. 1. Comparison of theory with wide-angle p-p scattering for single ρ exchange in M_H. Solid line is the theoretical 90° curve and dashed lines are predictions at constant s in GeV².

Figure 1 shows a comparison of single ρ^0 exchange theory with wide-angle data[12] for $g_\rho^2/4\pi=1.06$ [13] and $\gamma=2.28$. The corresponding comparison for single π^0 exchange is shown in Fig. 2 with the same value of γ and with $g_\pi^2/4\pi=3.75$. The single π exchange gives somewhat flatter t dependence and hence better agreement with the experimental t dependence at higher s values. On the other hand, single ρ exchange fits the angular dependence somewhat better at lower energies. Combining ρ and π exchanges does not alter the fit in any significant way. Since

$$\left(\frac{d\sigma}{dt}(\theta=90°)\right)_{\text{Born}}=\frac{\text{constant}}{s(s-4m^2)},$$

either exchange gives the same results for 90°.

Because ρ exchange gives a constant total cross section, it is worth comparing the hard-ρ exchange model with experiment down to $t=0$, though it is not expected to be valid there. Figure 3 shows the results of such a comparison. For

$$``m_\rho^2"=\tfrac{2}{3}(0.765\text{ GeV})^2,$$

$(d\sigma/dt)_0$ corresponds to $\sigma_{\text{tot}}=22$ mb provided the amplitude is purely imaginary at $t=0$.

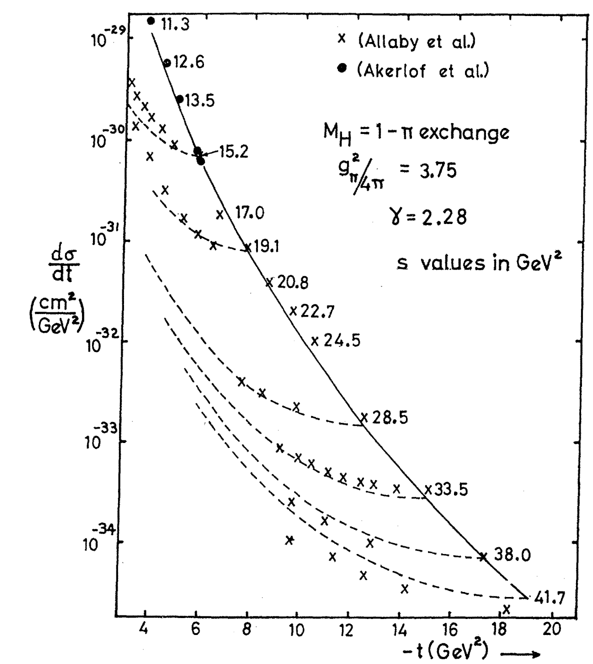

FIG. 2. Comparison of theory with wide-angle p-p scattering for single π exchange in M_H. Solid line is the theoretical 90° curve and dashed lines are predictions at constant s in GeV².

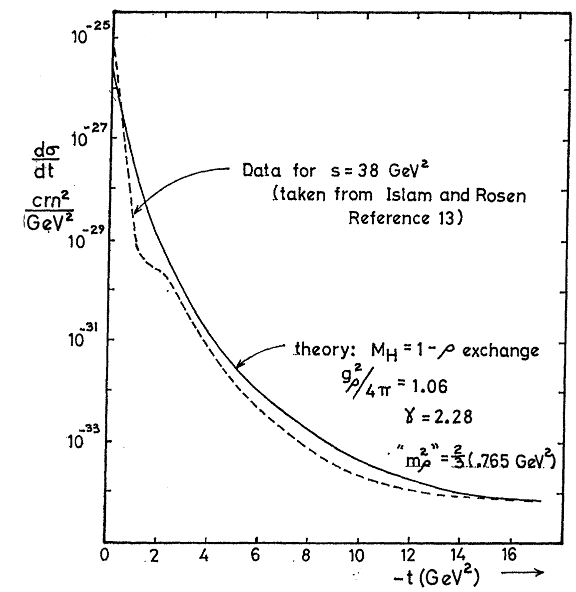

FIG. 3. Comparison of theory (solid line) with experiment (dashed line) for p-p scattering at all angles at $s=38$ GeV². Here M_H corresponds to single hard-ρ exchange. Model is not expected to be valid for small $|t|$, say for $-t<6$ GeV² at $s=38$ GeV².

[12] Wide-angle data are taken from J. V. Allaby, A. N. Diddens, A. Klovning, E. Lillethun, E. J. Sacharidis, K. Schlüpmann, and A. M. Wetherall, Phys. Letters **27B**, 49 (1968); C. W. Akerlof, R. H. Hieber, A. D. Krisch, K. W. Edwards, L. G. Ratner, and K. Ruddick, Phys. Rev. **159**, 1138 (1967). The data for all angles at $s=38$ GeV² are from the compilation of M. M. Islam and Joe Rosen, *ibid.* **185**, 1917 (1969).

[13] The value given by R. D. Peccei, Phys Rev. **176**, 1812 (1968), in a current-algebraic context is $g_\rho^2/4\pi\sim0.83$.

IV. CONCLUSION

The preceding analysis indicates the importance of including t-channel exchanges in a relativistic eikonal model. These have so far been neglected in discussions of such models.[1–3,9] We recall that Krisch[14] finds that $d\sigma/dt$ is a function of the combination of variables ut/s. Since

$$F(t)\sim 1-\ln(-t/m^2)$$

for large negative t, the combination

$$\exp\{4\gamma[F(t)+F(u)-F(4m^2-s)]\}\to (ut/s)^{-4\gamma},$$

when all variables become large. The t-channel exchanges are crucial in obtaining this result.

The agreement of the model with experiment, while not perfect, is encouraging enough to suggest that wide-angle proton-proton scattering at high energies may indeed take place by a single hard exchange modified by all possible soft exchanges. In a potential-theory eikonal approximation for scattering from a composite target, single hard exchange begins to dominate only as the number of consitituents becomes large.[15] Parton models of deep-inelastic electron-proton scattering[16,17] also indicate that the proton is a bound state of an infinite number of parts.

The most severe limitation of the present calculation as a test of the relevance of relativistic eikonal model to hadronic interactions is the neglect of hard-soft correlations. This shortcoming has to a large extent been removed by Fried[5] in his analysis of π-N scattering. He is able to get dip-bump structure and polarization effects from the model. A calculation of NSVM corrections to the Compton amplitude that includes coupling between hard and soft effects is in progress.[18]

[14] A. D. Krisch, Phys. Rev. Letters **19**, 1149 (1967).
[15] R. L. Sugar and R. Blankenbecler, Phys. Rev. **183**, 1387 (1969).
[16] J. D. Bjorken and E. A. Paschos, Phys. Rev. **185**, 1975 (1969).
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[18] H. M. Fried and T. K. Gaisser, Phys. Rev. (to be published); H. M. Fried and Hector Moreno, Phys. Rev. Letters **25**, 625 (1970).

Finally we note that the application of the present model to the elastic nucleon form factors[4] does not suffer from the neglect of hard-soft correlations provided we assume simple vector dominance of the current. For the photon-nucleon-nucleon vertex function with all possible NSVM exchanges between nucleon legs, the relativistic eikonal of Eq. (2) is

$$\Phi=ig^2\int_0^\infty d\xi\int_0^\infty d\eta(v\cdot v')\Delta_C(u-w-\xi v-\eta v'),$$

so that when the Fourier transform is taken in Eq. (10) of Ref. 4 [the analog of Eq. (3) above], Φ becomes independent of the remaining integration variables[19] provided the "hard" part of the coordinate-space vertex function has the structure

$$\Gamma_\mu{}^H(w,u;z)=g_{\rho\gamma}\bar{u}(p')G_\mu u(p)\int d^4x \times\Delta_C(z-x;m_V{}^2)\delta^4(w-x)\delta^4(u-x),$$

where G_μ is a collection of constants, isotopic-spin matrices and, Dirac matrices appropriate for the particular vertex function (e.g., isovector, proton magnetic, etc.) under consideration, and m_V is a vector-meson mass. The good fit obtained in this way in Ref. 4 for the elastic nucleon form factors is thought to reflect the fact that the soft exchanges have been handled exactly.

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[19] See the remark after Eq. (4) above.