so that

$$f(x,\Delta)\approx\int dp\, e^{ip(\Delta-\xi S)}e^{-\xi p^2 T}=\left(\frac{\pi}{\xi T}\right)^{1/2}e^{-(\Delta-\Delta_0)^2/4\xi T}, \quad (29)$$

where we have defined

$$\Delta_0=\xi S. \quad (30)$$

Thus, without doing any explicit calculations, we see that for large ξ we expect the distribution of energies of particles emerging from a slab of thickness x to follow a normal curve centered around Δ_0.[7] In Fig. 3 we show the results of explicit calculations of Eq. (27) for $f(x,\Delta)$, and we see that for the two cases considered ($\xi=6.75\times10^5$ eV, corresponding to a 10-cm track in hydrogen at STP), this expectation is indeed realized.

We also note that the original problem which we posed, the question of whether straggling might lead a pair to produce a faint cloud-chamber track, can now be answered in the negative, since the probability of finding $\Delta<\frac{1}{2}\Delta_0$ is very small indeed, and, since high-energy pairs are not produced often, we can rule out this possibility.

[7] It is amusing to note that if we were calculating the straggling for a single particle, where $\omega(\epsilon)=A/\epsilon^2$, we would find $\Delta_0=\xi/\epsilon_{\min}$ and $T=\epsilon_{\max}$. For the case where $\epsilon_{\max}$ is restricted only by the classical upper bound discussed in Sec. II, the spread of energies will be maximized, while for the case of the restricted energy-loss problem considered in this paper, $\epsilon_{\max}$ will be less than its highest allowable value. This means that for the restricted problem, we would expect a narrower spread of energies, a circumstance remarked upon in Ref. 3.

V. CONCLUSION

The net result of the foregoing study of the ionization loss of charged pairs in matter is that in future quark searches, the production of faintly ionizing tracks by such an effect can be safely ruled out, and that, in the absence of other explanations of such tracks, they will have to be interpreted as evidence for the existence of fractionally charged particles.

PHYSICAL REVIEW D VOLUME 2, NUMBER 1 1 JULY 1970

Two-Pion Decay Mode of the ω and ρ-ω Mixing*

ROBERT G. SACHS AND JORGE F. WILLEMSEN†
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois, 60637
(Received 22 December 1969; revised manuscript received 6 April 1970)

Recent experiments reporting the 2π decay mode of the ω meson and its interference with that mode of the ρ meson are analyzed in terms of the propagator method for mixing of particle states. Both vector mixing and mass mixing are included, and the energy dependence of the ρ width is explicitly taken into account. It is shown that the effect as observed in the reaction $e^+e^-\to\pi^+\pi^-$ can be accounted for in terms of available parameters. The consequent determination of the parameters makes possible an analysis of strong-interaction phenomena of the same type, giving information on the relative phases of ω and ρ production amplitudes. This is applied to the ρ-ω interference observed in $\pi^+p\to\pi^+\pi^-\Delta^{++}$. Other experiments are suggested.

I. INTRODUCTION

DIRECT evidence for the 2π decay mode of the ω meson produced in the reaction

$$e^+e^-\to\pi^+\pi^- \quad (1)$$

has recently been reported.[1] There are also reports of evidence for this decay mode in other processes, in particular, in the reaction[2]

$$\pi^+p\to\pi^+\pi^-\Delta^{++}. \quad (2)$$

In both cases, it is the interference between the 2π mode of the ρ meson and the 2π mode of the ω that is observed. Therefore, the process is sensitive to the phase of the mixing of the "bare" ρ and ω states of definite G parity.

Reaction (1) is particularly suitable for determining the parameters of this mixing since, in addition to these parameters, only known quantities are needed to describe the interference. This is a direct consequence of the absence of any strong-interaction effect in reaction (1) other than the coupling of ρ and ω to the 2π state. It lends a special significance to the reaction because, once the mixing parameters are determined in this way, they can be used to analyze other processes, such as reaction (2), to obtain valuable information concerning the phases of the amplitudes for production of the vector mesons in strong interactions.

* Supported in part by the U. S. Atomic Energy Commission.
† NDEA Predoctoral Fellow.
[1] J. E. Augustin, D. Benaksas, J. Buon, F. Fulda, V. Gracco, J. Haissinski, D. Lalanne, F. Laplanche, J. Lefrancois, P. Lehmann, P. C. Marin, J. Perez-y-Jorba, F. Rumpf, and E. Silva, Nuovo Cimento Letters 2, 214 (1969).
[2] G. Goldhaber, W. R. Butler, D. G. Coyne, B. H. Hall, J. N. MacNaughton, and G. H. Trilling, Phys. Rev. Letters 23, 1351 (1969).

Gourdin, Stodolsky, and Renard[3] have analyzed reaction (1) on the basis of the phenomenological theory of mixing[4] and have noted that, to some extent, the required phases are known *ab initio* just from a knowledge of the ratio of the ρ width to the $\omega\rho$ mass difference. They conclude that the phase of the mixing predicted by the phenomenological theory is in strong disagreement with the experimental phase reported in Ref. 1.

The experiments that have been reported probably do not have the precision that would justify any such definitive statement about agreement or disagreement with theory. They do provide a strong indication of the qualitative effects to be observed and they set the stage for much more precise work in the near future. Because of the potential value of these phenomena as a means for measuring phases of vector-meson amplitudes, it is informative to make use of the available data, crude as they are, to demonstrate the relationship with general theoretical considerations and to show how the different phenomena may be interrelated.

In this connection, we wish to show that one cannot establish the disagreement between theory and experiment claimed in Ref. 3 even if one takes the experimental results to have a precision comparable to the stated errors. Our purpose in showing this in not only to do away with the specter of the failure of the phenomenological theory but also to introduce some improvements in the method of Ref. 4 which will provide a better method for analysis of the more precise experiments anticipated for the future.

Our disagreement with the conclusions of Ref. 3 arises from several causes.

(a) There is a discrepancy[5] between the phase conventions of Ref. 1 and Ref. 3. The experimental result in the more conventional phase would be $\alpha=164°\pm28°$ (rather than $-164°$) which is to be compared to the two alternatives of Ref. 3, $\alpha=112°$ or $\alpha=-89°$. Because of the change of sign (from $\alpha=-164°$ to $\alpha=+164°$) it is the first rather than the second that is to be compared and the discrepancy is reduced from a difference of 75° to a difference of 52°, which is hardly significant.

(b) The relatively large width of the ρ resonance implies that a more careful treatment of the energy dependence of the width is required.

(c) Since the ρ^0 and ω^0 are vector mesons, there are in principle two complex mixing parameters at our disposal rather than just the one assumed in the method of Ref. 4.

The changes wrought in the analysis by points (b) and (c), while small, manage to accumulate in such a way as to allow for a possible shift in the theoretical value of $\alpha=112°$ to a maximum of $\alpha=132°$, which serves to reduce even further any discrepancy between theory and experiment. The limitations on the small corrections are based on estimates of the contributions to the mixing parameters. A much more precise experimental value of α may make it possible to turn the argument around and obtain the values of the real and imaginary parts of the mixing parameters.

Specific information concerning the mixing parameters is needed to make a precise analysis of strong-interaction phenomena such as described by Eq. (2). The required relationships and the present limitations on them are set forth for reactions of this kind in Sec. V.

[3] M. Gourdin, L. Stodolsky, and F. M. Renard, Phys. Letters **30B**, 347 (1969).

[4] J. Harte and R. G. Sachs, Phys. Rev. **135**, B459 (1964).

[5] We are indebted to Dr. Perez-y-Jorba for confirmation of the fact that the phase convention in the analysis of Ref. 1 had the opposite sign from the usual convention.

II. VECTOR-MESON PROPAGATOR METHOD

The processes in which we are interested may be described in terms of the diagrams shown in Fig. 1. The double line is symbolic for the $\omega\rho$ propagator matrix[4] $[\Delta_F'(k)]_{\mu\nu}$, where μ and ν are the space-time labels for the 4-vector fields describing the ρ and ω. It is well known[6] that a spectral representation of the propagator of a vector meson leads to

$$[\Delta_F'(k)]_{\mu\nu}=\lim_{z\to k^2+i\epsilon}\{G(z)\delta_{\mu\nu}+z^{-1}[G(z)-G(0)]k_\mu k_\nu\}\,,$$

where $G(z)$ has the form

$$G(z)=(zK^2-M_0^2)^{-1}. \qquad (3)$$

The quantities K^2, M_0^2, and G are symmetric[4] 2×2 matrices with matrix elements given in terms of the "unmixed" states, i.e., states of definite G parity, $|\rho^0\rangle$ and $|\omega^0\rangle$. (It is assumed that $\omega\phi$ mixing has been taken into account in $|\omega^0\rangle$.) The fact that $G(z)$ is determined by two matrices, K^2 and M_0^2, means that two kinds of mixing must be considered[7]: "vector mixing" (or "current mixing"),[8] due to off-diagonal elements in K^2, and "mass mixing", due to off-diagonal elements in M_0^2. In effect, only mass mixing was considered in Refs. 3 and 4.

The form of $G(z)$, Eq. (3), also serves to include the energy dependence of the widths of the states $|\rho^0\rangle$ and $|\omega^0\rangle$. Since these are vector-meson states, the partial widths contain the kinematic factors

$$(k^2-k_T^2)^{3/2}(k^2)^{-1/2}=k^2(1-k_T^2/k^2)^{3/2}\,,$$

where k_t is the threshold for the decay mode contributing to the width. For values of k^2 near the resonance value m_ρ^2, the variation in the second factor is small and the widths may be taken to be proportional to k^2.

Since $z=k^2$ in Eq. (3), we see that the energy dependence of the widths, which normally would appear as imaginary terms in M_0^2, can be included by shifting the imaginary contributions to K^2.

[6] K. Johnson, Nucl. Phys. **25**, 435 (1961).

[7] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

[8] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

Assuming now that there is no other energy dependence in K^2 and $M_0{}^2$ in the neighborhood of the resonances, we take them to have the following form:

$$K^2=\begin{pmatrix}1+i\Gamma_\rho/m_\rho & 2\lambda\\ 2\lambda & 1\end{pmatrix}, \tag{4}$$

$$M_0{}^2=\begin{pmatrix}m_\rho{}^2 & \eta m_\rho{}^2\\ \eta m_\rho{}^2 & m_\omega{}^2\end{pmatrix}, \tag{5}$$

where Γ_ρ is the total width of the ρ at resonance and the width of the ω has been neglected. The small parameters λ and η are of the order of the G-violating (electromagnetic) interactions and they determine the mixing. Since all absorptive terms are included in K^2, η is real but λ will have imaginary contributions.

The solution of the mixing problem may now be carried out in terms of these parameters by means of methods analogous to those developed in Ref. 4. Equation (3) is rewritten in the form

$$G(z)=K^{-1}(z-W)^{-1}K^{-1}, \tag{6}$$

with

$$W=K^{-1}M_0{}^2K^{-1}. \tag{7}$$

By treating the factor $(z-W)^{-1}$ by the method of Ref. 4, we are led to the result

$$G(z)=\sum_\alpha \frac{|\alpha\rangle\langle\tilde{\alpha}|}{z-z_\alpha}, \tag{8}$$

where the two mixed states $|\rho\rangle$ and $|\omega\rangle$ are denoted by $|\alpha\rangle$ and are given by

$$|\alpha\rangle=K^{-1}|\alpha'\rangle, \tag{9}$$

with $|\alpha'\rangle$ the solution of

$$W|\alpha'\rangle=z_{\alpha'}|\alpha'\rangle, \tag{10}$$

$z_{\alpha'}$ being the roots of

$$\det(W-z_{\alpha'})=0. \tag{11}$$

The states $\langle\tilde{\alpha}|$ are simply the transpose of $|\alpha\rangle$:

$$\langle\tilde{\alpha}|j\rangle=\langle j|\alpha\rangle, \tag{12}$$

and the normalization of the states is

$$\langle\tilde{\alpha}|K^2|\beta\rangle=\delta_{\alpha\beta}. \tag{13}$$

For all practical purposes, the roots of Eq. (11) may be taken to be

$$z_\rho=m_\rho{}^2-im_\rho\Gamma_\rho,$$
$$z_\omega=m_\omega{}^2-im_\omega\Gamma_\omega, \tag{14}$$

where the distinction from the masses and widths of the unmixed states $|\rho^0\rangle$ and $|\omega^0\rangle$ is negligible.

We are now in a position to express the matrix element for a physical process, such as one of those described by Fig. 1, in terms of the mixed states. If the vector fields are always coupled to conserved currents,

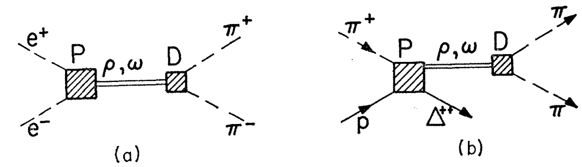

FIG. 1. Schematic diagrams of the processes leading to Eq. (15). The double line is symbolic for the propagator matrix.

insertion of the expression for the propagator leads to the matrix element

$$T_{FI}(k^2)=\sum_{ij}\sum_\mu D_{Fi}{}^\mu\langle i|G(k^2)|j\rangle P_{jI}{}^\mu, \tag{15}$$

where $|i\rangle$, $|j\rangle=|\rho^0\rangle$ or $|\omega^0\rangle$. $P_{jI}{}^\mu$ is the production amplitude for the vector-meson state $|j\rangle$ of polarization μ and $D_{Fi}{}^\mu$ is the corresponding decay amplitude into the final mode F, which in our case will be the 2π mode. The meaning of $G(k^2)$ is

$$G(k^2)=\lim_{z\to k^2+i\epsilon} G(z),$$

and k^2 is the square of the effective mass of the final state F.

By inserting Eq. (8) for $G(z)$ into Eq. (15), we find

$$T_{FI}(k^2)=\sum_\alpha\left(\sum_\mu f_{F\alpha}{}^\mu g_{\alpha I}{}^\mu\right)(k^2-z_\alpha)^{-1}, \tag{16}$$

where the effective coupling for production of the mixed state $|\alpha\rangle$ is given by virtue of Eq. (12) as

$$g_{\alpha I}{}^\mu=\sum_j\langle j|\alpha\rangle P_{jI}{}^\mu, \tag{17}$$

and the effective coupling for decay is

$$f_{F\alpha}{}^\mu=\sum_i D_{Fi}{}^\mu\langle i|\alpha\rangle. \tag{18}$$

III. EXPLICIT SOLUTION

In order to determine the coefficients $\langle i|\alpha\rangle$, we consider only first-order terms in λ and η and make use of the fact that Γ_ρ/m_ρ is small, as in Eq. (14). Then W, given by Eq. (7) and determined from Eqs. (4) and (5), becomes

$$W=m_\rho{}^2\begin{pmatrix}(1+i\Gamma_\rho/m_\rho)^{-1} & q\\ q & m_\omega{}^2/m_\rho{}^2\end{pmatrix}, \tag{19}$$

with

$$q\approx(\eta-2\lambda e^{-i\theta})e^{-i\theta} \tag{20}$$

serving as the effective mixing parameter. We have introduced

$$\theta\equiv\Gamma_\rho/2m_\rho, \tag{21}$$

and will use $1+i\theta=e^{i\theta}$ whenever the second- and higher-order terms make no significant difference, i.e., where it appears as a factor of the small mixing parameter λ or η.

The solutions of Eqs. (9) and (10) then lead to the following values of the coefficients $\langle j|\alpha\rangle$ [9]:

$$\langle\rho^0|\rho\rangle=(1+i\Gamma_\rho/m_\rho)^{-1/2},\quad \langle\omega^0|\rho\rangle=s-\lambda e^{-i\theta},\qquad \langle\rho^0|\omega\rangle=-(s+\lambda)e^{-i\theta},\quad \langle\omega^0|\omega\rangle=1,\tag{22}$$

where

$$s\equiv -qm_\rho^2e^{2i\theta}/(m_\omega^2-m_\rho^2+im_\rho\Gamma_\rho),\tag{23}$$

and we have set $m_\omega=m_\rho$ in factors of Γ_ρ.

We are now in a position to write down the two-pion amplitude in resonance form. Since $\omega\to 2\pi$ is forbidden by the G-parity selection rule, $D_{2\pi,\omega}$ must arise from electromagnetic or other isospin-violating effects. Hence it is expected to be small, of the same order as η or λ. For convenience, we define a small parameter λ', of this order of magnitude, by[10]

$$D_{2\pi,\omega}{}^\mu=-\lambda' e^{-i\theta}D_{2\pi,\rho}{}^\mu.\tag{24}$$

Then from Eqs. (18) and (22) we have, to first order in mixing,[11]

$$f_{2\pi,\rho}{}^\mu=(1+i\Gamma_\rho/m_\rho)^{-1/2}D_{2\pi,\rho}{}^\mu,\qquad f_{2\pi,\omega}{}^\mu=-(s+\lambda+\lambda')e^{-i\theta}D_{2\pi,\rho}{}^\mu.\tag{25}$$

Similarly, from Eqs. (17) and (22),

$$g_{\rho I}{}^\mu=(1+i\Gamma_\rho/m_\rho)^{-1/2}P_{\rho I}{}^\mu+(s-\lambda e^{-i\theta})P_{\omega I}{}^\mu,\qquad g_{\omega I}{}^\mu=P_{\omega I}{}^\mu-(s+\lambda)e^{-i\theta}P_{\rho I}{}^\mu.\tag{26}$$

Use of Eq. (14) now leads to the following expression for the amplitude, Eq. (16):

$$T_{2\pi,I}(k^2)=T_0\left[\frac{1}{k^2-m_\rho^2+ik^2\Gamma_\rho/m_\rho}+\frac{a_\omega(I)}{k^2-m_\omega^2+im_\omega\Gamma_\omega}\right],\tag{27}$$

where T_0 is a constant,

$$a_\omega(I)=-b(I)(s+\lambda+\lambda')e^{-i\theta},\tag{28}$$

with

$$b(I)\equiv\gamma(I)-(s+\lambda)e^{-i\theta}-\gamma^2(I)(se^{-i\theta}-\lambda)\tag{29}$$

and

$$\gamma(I)\equiv\sum_\mu D_{2\pi,\rho}{}^\mu P_{\omega,I}{}^\mu\Big/\sum_\mu D_{2\pi,\rho}{}^\mu P_{\rho,I}{}^\mu.\tag{30}$$

At this point, we compare these results to those obtained by the method of Ref. 4. In the first term of Eq. (27), the energy dependence of the width appears explicitly while the corresponding term would have a constant width in the approximation of Ref. 4. (However, it has been customary to insert the energy dependence at this point, without explicit justification.) This change arises from the difference between $\langle\rho^0|\rho\rangle$ given in Eq. (22) and the value $\langle\rho^0|\rho\rangle=1$ associated with the other treatment.

The other changes are buried in $a_\omega(I)$. The addition of $\lambda\neq 0$ due to vector mixing affects Eqs. (28) and (29) explicitly as does the appearance of the phase θ, which is another consequence of the energy dependence of the width. These changes also affect quite directly the value of s through Eqs. (20) and (23). We note, however, that in the absence of vector mixing ($\lambda=0$), an imaginary contribution to η in Eq. (20) would be required to take account of the absorptive mixing effects that we have been including in an energy-dependent form by inserting them in λ.

Clearly, these many changes could have a substantial effect on predictions concerning the phase of $a_\omega(I)$, and we shall find that they are numerically significant but not large.

IV. ANALYSIS OF $e^++e^-\to\pi^++\pi^-$

The determination of $b(I)$ depends on the initiating mechanism and it will vary from one process to another both in magnitude and phase. For this reason, the relative amplitude $a_\omega(I)$ will vary markedly with the reaction considered, as already noted in Ref. 4. We consider first the case of electron-positron annihilation in order to make an analysis of the results reported in Ref. 1. The production of the vector mesons is assumed to take place through an intermediary photon and, in the spirit of vector dominance, the photon–vector-meson couplings $f_{\gamma\rho}$ and $f_{\gamma\omega}$ are taken to be real. Then $\gamma(e^+e^-)=f_{\gamma\omega}/f_{\gamma\rho}$ and this ratio may be taken from experiment[12] to be

$$\gamma(e^+e^-)\approx\tfrac{1}{3}.\tag{31}$$

Since $|s|$ will turn out to be small and $|\lambda|$ even smaller, Eq. (29) may be put in the form

$$b(e^+e^-)\approx\tfrac{1}{3}-se^{-i\theta},\tag{32}$$

whence (28) becomes

$$a_\omega(e^+e^-)\approx-(\tfrac{1}{3}-se^{-i\theta})(s+\lambda+\lambda')e^{-i\theta}.\tag{33}$$

This may now be compared with the amplitude extracted from Ref. 1:

$$a_\omega(e^+e^-)\approx 0.02e^{i\alpha},\tag{34}$$

with[13]

$$\alpha=164°\pm 28°.\tag{35}$$

[9] The normalization condition, Eq. (13), has been ignored here since it leads to a common factor which will not be of interest.

[10] Since only one 4-vector is available in the 2π state to give the dependence on polarization, λ' is independent of μ.

[11] In Ref. 3 it is noted that the unitarity relation $2\,\mathrm{Im}\lambda m_\rho^2=\sum_F D_{\rho F}{}^*D_{\omega F}$ leads to a relationship between Imλ and λ' which happens to lead to a cancellation of λ' in $s+\lambda'$ by virtue of the connection between s and λ given by Eqs. (20) and (23). Thus, in their case $s+\lambda'$ can be replaced by a "mutilated" version of s, but this procedure offers no advantage here.

[12] J. E. Augustin, D. Benaksas, J. C. Bizot, J. Buon, B. Delcourt, V. Gracco, J. Haissinski, J. Jeanjean, D. Lalanne, F. Laplanche, J. Lefrancois, P. Lehmann, P. Marin, H. Nguyen Ngoc, J. Perez-y-Jorba, F. Richard, F. Rumpf, E. Silva, S. Tavernier, and D. Treille, Phys. Letters **28B**, 503 (1969).

[13] Note that the sign convention for the phase in Ref. 1 is opposite to ours. See Ref. 5.

Comparison of Eq. (33) with Eq. (34) yields

$$|s|\approx 0.06. \quad (36)$$

The parameters describing the resonances may be taken directly from experiment and, for purposes of direct comparison, we shall use the values given in Ref. 1:

$$m_\rho=773\text{ MeV},\quad \Gamma_\rho=111\text{ MeV},\qquad m_\omega=783\text{ MeV},\quad \Gamma_\omega=12\text{ MeV}. \quad (37)$$

Then we find

$$s=(|q|/0.15)e^{-i(\phi_0-\phi-\theta)}, \quad (38)$$

with

$$\theta=4° \quad (39)$$

and

$$\phi_0=80°. \quad (40)$$

We have set

$$q=-|q|e^{i(\phi-\theta)} \quad (41)$$

in place of Eq. (20), and the phase ϕ is thereby defined by

$$\sin\phi=2(\text{Im}\lambda-\theta\,\text{Re}\lambda)/|q| \quad (42)$$

and

$$\cos\phi=(2\,\text{Re}\lambda-\eta)/|q|. \quad (43)$$

More information about the mechanism of mixing would be needed to make a theoretical determination of ϕ. However, we may use the result

$$|q|\approx 9\times 10^{-3}, \quad (44)$$

which follows from Eqs. (36) and (38), and the upper limit on $|\text{Im}\lambda|$ given by Eq. (7) of Ref. 3,

$$2|\text{Im}\lambda|\lesssim 2\times 10^{-3}, \quad (45)$$

to obtain conditions on ϕ. From Eq. (42) it follows that

$$-13°\leq\phi\leq 13° \quad (46a)$$

or

$$-167°\leq\phi\leq 193°. \quad (46b)$$

Since $|s|$ is small compared to $\gamma=\frac{1}{3}$, we find from Eq. (33)

$$\alpha\approx\pi-\phi_0+\phi+\phi'+3|s|\cos\phi, \quad (47)$$

where use has been made of the approximation

$$s+\lambda+\lambda'\approx se^{i\phi'}, \quad (48)$$

which follows from the small size of $|\lambda|/|s|$ implied by Eqs. (38) and (20). In fact, we have

$$2|\lambda|/|s|\leq 0.15, \quad (49)$$

and if the same condition applies to λ',

$$-9°\leq\phi'\leq 9°. \quad (50)$$

Clearly the choice allowed by Eq. (46b) for ϕ is excluded by the experimental result Eq. (35). Hence we use Eqs. (46a), (50), and the values of ϕ_0 and θ to place the following limits on α:

$$88°\leq\alpha\leq 132°. \quad (51)$$

This is clearly not excluded by the experimental value $\alpha=164°\pm 28°$ [Eq. (35)]. The comparison between the two does suggest that α lies in the upper half of the range given by Eq. (51), but a more precise experiment will be required to allow a more definite statement.

The difference between our result, Eq. (51), and the result of Gourdin, Stodolsky, and Renard[3] (who only wrote down their estimate of the upper limit on α) is due to an accumulation of small effects which may be understood explicitly from Eq. (47). In both treatments, the dominant phase is ϕ_0. Our term ϕ depends on both the vector-mixing and mass-mixing parameters λ and η as shown by Eqs. (42) and (43), but the basis of the estimate is the same although we obtain a somewhat larger range of ϕ. The meaning of ϕ' is shown by Eq. (48); the contribution of the λ' term is common to both treatments[11] but the additional term λ is new here and alters the result appreciably. It is due to the fact that vector mixing requires the manipulation indicated by Eq. (7). Finally, the addition of the $3|s|\cos\phi$ term in Eq. (47), which apparently corresponds to $\varphi_{\omega\gamma}$ in the other treatment, arises from the fact that the physical $|\omega\rangle$ state may be produced through the mechanism of $|\rho^0\rangle$ production, as indicated in Eq. (26).

It is worth noting that an accurate determination of α will make it possible to be more specific about ϕ and ϕ'. In general, one cannot hope to determine them separately unless it happens that the measured value of α lies at one end of the range (51). The ends of the range correspond, of course, to the extreme values of ϕ and ϕ' allowed by Eqs. (46a) and (50).

Since, in the amplitude for strong-interaction processes ϕ and ϕ' appear in combinations other than Eq. (47), as we shall see in Sec. V, it is necessary to have as much independent information about them as possible in order to give a precise interpretation to the strong-interaction data.

V. ANALYSIS OF $\pi^++p\rightarrow\pi^++\pi^-+\Delta^{++}$

The mixing parameters that have been determined from the e^+e^- experiment may now be used to analyze the reaction (2) using the data of Ref. 2. The important feature of the data is that destructive interference between the ω and ρ resonances appears to occur at $k^2=m_\omega^2$. For this value of the energy, Eq. (27) may be simplified by using Eqs. (28), (23), and (41), with the result

$$T_{2\pi,I}(m_\omega^2)=T_0\left[\frac{1+i(m_\rho/\Gamma_\omega)|q|b(I)e^{i(\phi+\phi')}}{m_\omega^2-m_\rho^2+im_\rho\Gamma_\rho}\right]. \quad (52)$$

The requirement for destructive interference is

$$\Phi+\phi+\phi'=\tfrac{1}{2}\pi, \quad (53)$$

where

$$\Phi\equiv\arg b(I). \quad (54)$$

Application of Eqs. (46a) and (50) then leads to the following range of values for Φ:

$$68°\leq\Phi\leq 112°, \quad (55)$$

where the lower limit corresponds to the upper limits on ϕ and ϕ' and vice versa.

Now $b(I)$ is given by Eq. (29), which may be adequately approximated by neglecting λ compared to s and dropping θ. Then

$$b(I)=\gamma(I)-s[1+\gamma^2(I)]. \quad (56)$$

The results presented in Ref. 2 indicate that

$$|b(I)|\approx 0.75 \quad (57)$$

for the data presented. Since $|s|$ is small compared to this, we may write

$$b(I)\approx|\gamma(I)|e^{i\Phi}, \quad (58)$$

with

$$\Phi\approx\beta-[|s|/|\gamma(I)|][\sin(\phi-\beta-76°)+|\gamma(I)|^2 \times\sin(\phi+\beta-76°)], \quad (59)$$

where $\beta=\arg\gamma(I)$. Equation (55) then leads to the estimate

$$65°\leq\beta\leq 115°. \quad (60)$$

As an illustration of the application of this result, we may consider the proposal made by Goldhaber, Fox, and Quigg,[14] which leads to the prediction $\beta=90°$.

The result, Eq. (60), is clearly in agreement with this prediction. A more severe test of the value $\beta=90°$ is provided by including the information gleaned from the e^+e^- experiment.[1] Under the assumption $\beta=90°$, we have

$$\Phi=90°+2°\cos(\phi-76°), \quad (61)$$

so from Eq. (53)

$$\phi+\phi'=-2°\cos(\phi-76°). \quad (62)$$

This expression may be inserted into Eq. (47) for the phase α, which is measured in the e^+e^- experiment. We have remarked that the comparison between Eq. (51) and the experimentally indicated Eq. (35) implies that ϕ and ϕ' should be chosen so as to maximize α. This condition and the restrictive Eq. (62) then lead to

$$\phi\approx 0,\quad \phi'\approx 0, \quad (63)$$

and

$$\alpha\approx 110°. \quad (64)$$

This value of α differs from the reported value, Eq. (35), by just twice the estimated error.

This serves to illustrate the nature of the analysis, and it is clear that when improved phase measurements are made, specific information concerning the interaction vertices may be extracted from them.

VI. CONCLUDING REMARKS

It is clear that a more accurate measurement of the $\omega\rho$ interference in the 2π mode produced in electron-positron annihilation would be very useful. It would provide a more severe test of the adequacy of the existing phenomenological theories of particle mixing. It would also yield a tool for the determination of relative phases of ω and ρ production vertices in many strong-interaction processes, such as reaction (2), which has been treated here.

From Eq. (30) it is clear that the interference phenomenon will be quite sensitive to differences between the ρ and ω in regard to the polarization dependence of the production amplitudes. This suggests that the measurement of the spin-density matrix elements of vector mesons produced in strong reactions as a function of $M_{\pi\pi}^2$ $(=k^2)$ through the ω mass region may provide a wealth of information concerning the production mechanism. One would expect in general to see a strong dependence of the spin-density matrix on $M_{\pi\pi}^2$ as the result of the interference between terms involving the ρ^0 production amplitude $P_{\rho I}^{\mu}$ and terms involving the ω^0 production amplitude $P_{\omega I}^{\mu}$. This interference phenomenon would be limited to a range in $M_{\pi\pi}^2$ of order $m_\omega\Gamma_\omega$, so high resolving power will be required to carry out such an experiment.

[14] A. S. Goldhaber, G. C. Fox, and C. Quigg, Phys. Letters **30B**, 249 (1969).