Pseudoscalar-Meson–Nucleon Scattering and Regge Cuts*

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A simultaneous fit to the data on the four charge-exchange reactions $\pi^- p \to \pi^0 n$, $K^- p \to \bar{K}^0 n$, $K^+ n \to K^0 p$, and $\pi^- p \to \eta n$ is made, using a Regge-pole model modified by the inclusion of the lowest-order Regge cuts produced, by means of the impact-parameter representation.

I. INTRODUCTION

RECENTLY, many attempts have been made to modify the usual Regge-pole model, which has been so useful in studying many of the features of experimental differential cross sections. These modifications consist of the inclusion of Regge cuts. Although it had been suspected for a long time that Regge cuts were import ant in explaining the finer features of the experimental data, such as final-particle polarization effects,[1] a prescription for actually introducing these singularities was lacking. Several authors,[2–7] however, have now introduced various ways of doing this. These methods are, in fact, very similar, although the points of view from which they are arrived at differ somewhat. All of the prescriptions have as their basis the idea of accounting for certain effects of unitarity in the Regge amplitudes, without actually making the amplitudes unitary. These effects of unitarity are made to enter through the partial-wave amplitudes or their equivalent. In one group of models,[2–4] use is made of the Glauber formalism[8] to convert the partial-wave sum to the impact-parameter representation.[9] There, the eikonal is assumed to be a matrix in spin and internal symmetry [$SU(2)$ or $SU(3)$] space. Together with the assumption that the usual Regge amplitude determines the eikonal, this prescription gives the basis for the introduction of Regge cuts. A second group of models[5,6] also uses unitarity in a multichannel way through the Sopkovitch formula.[10] To second order, at least, these two attempts give the same results. A third attempt[7] has used unitarity in a more direct way.

In the present paper the formalism developed for the first group will be followed in order to obtain a model with which to simultaneously study the reactions of Table I. Excepting Reaction 3, traditional Regge models have been fitted to these reactions.[11] These models have failed in several important and by now well-known ways to explain the data. In a reaction in which only one Regge trajectory is exchanged, the various helicity amplitudes all have the same phase. All polarization effects are therefore calculated to be zero. This difficulty was circumvented by introducing interference with direct-channel resonances, which cannot explain polarization effects at very high energies,[12] by adding a second trajectory with the same quantum numbers,[13] or by modifying the usual Regge amplitude.[14] Further, various factors which vanish at certain points had to be inserted in order to explain the shapes of some differential cross sections. These are the nonsense wrong-signature and crossover zeros.[11] Because of various objections to these resolutions of the difficulties, the Regge-cut models have been used to explain the experimental features.[2,4–6,15–17]

Section II reviews the formalism starting from Glauber's work and lists the further assumptions made

TABLE I. Reactions and associated $SU(3)$ Clebsch-Gordan coefficients.

Reaction	Reaction number	η_ρ	η_A
$\pi^- p \to \pi^0 n$	1	1	0
$K^- p \to \bar{K}^0 n$	2	$-1/\sqrt{2}$	1
$K^+ n \to K^0 p$	3	$1/\sqrt{2}$	1
$\pi^- p \to \eta n$	4	0	$\frac{1}{3}\sqrt{2}$
$\pi^- p \to \pi^- p$	...	$-1/\sqrt{2}$	0
$\pi^+ p \to \pi^+ p$	...	$1/\sqrt{2}$	0

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[1] V. M. de Lany, D. J. Gross, I. J. Muzinich, and V. L. Teplitz, Phys. Rev. Letters **18**, 149 (1967); C. B. Chiu and J. Finkelstein, Nuovo Cimento **48A**, 820 (1967).

[2] R. C. Arnold, Phys. Rev. **153**, 1523 (1967).

[3] C. B. Chiu and J. Finkelstein, Nuovo Cimento **57**, 649 (1968).

[4] S. Frautschi and B. Margolis, Nuovo Cimento **56**, 1155 (1968).

[5] F. Henyey, G. L. Kane, J. Pumplin, and M. Ross, Phys. Rev. Letters **21**, 946 (1968).

[6] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Nuovo Cimento **48A**, 1075 (1967); J. M. Drouffee and H. Navelet, in Proceedings of the 1969 Regge-Cut Conference, University of Wisconsin, Madison, Wisc., edited by P. M. Fishbane and L. M. Simmons, Jr. (unpublished).

[7] R. Henzi, Nuovo Cimento **46A**, 370 (1966).

[8] R. J. Glauber, *Lectures in Theoretical Physics* (Interscience, New York, 1959), Vol. 1, p. 315.

[9] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).

[10] N. J. Sopkovitch, Nuovo Cimento **26**, 186 (1962).

[11] F. Arbab and C. Chiu, Phys. Rev. **147**, 1045 (1966); R. J. N. Phillips and W. Rarita, *ibid.* **139**, B1336 (1965); D. D. Reeder and K. V. L. Sarma, *ibid.* **172**, 1566 (1968).

[12] R. J. N. Phillips, Nuovo Cimento **45A**, 245 (1966); R. K. Logan and L. Sertorio, Phys. Rev. Letters **17**, 834 (1966); B. R. Desai, D. T. Gregorich, and R. Ramachandran, *ibid.* **18**, 565 (1967); G. Altarelli *et al.*, Nuovo Cimento **48A**, 245 (1967).

[13] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1966); A. Ahmadzadeh and W. B. Kaufman, Phys. Rev. **188**, 2438 (1969).

[14] R. Kreps and J. W. Moffat, Phys. Rev. **175**, 1942 (1968).

[15] M. L. Blackmon and G. Goldstein, Phys. Rev. **179**, 1480 (1969).

[16] M. L. Blackmon, Phys. Rev. **178**, 2385 (1969).

[17] R. C. Arnold and M. L. Blackmon, Phys. Rev. **176**, 2082 (1968).

in order to keep the model manageable. Section III contains the results of the simultaneous fit to the differential cross sections and polarization data of the reactions of Table I and comparisons with the elastic data. Section IV contains a summary of what the model does and does not explain.

II. FORMALISM

As in Ref. 2, Regge cuts can be introduced through a modification of the Glauber formalism. In that formalism, the phase shifts in the various partial waves are replaced by a continuous function of the impact parameter b,

$$2\delta_l(k)\rightarrow \chi(kb)\,,$$

where k is the momentum in the c.m. system, χ is the eikonal, and the impact parameter is defined by $l=kb$. For particles with spin, the eikonals can be easily introduced through the helicity amplitudes of Jacob and Wick.[18] In the present case of pseudoscalar-meson–nucleon scattering, the s-channel helicity amplitudes have the partial-wave expansions[19]

$$T_{\lambda_c0;\lambda_a0}=\sum_J (J+\tfrac{1}{2})T_{\lambda_c0;\lambda_a0}{}^J d_{\lambda_a\lambda_c}{}^J(\theta)\,, \tag{1a}$$

$$T_{+0;+0}{}^J\equiv T_{++}{}^J=[-4\pi(\sqrt{s})/2imk]\{\exp[2i\delta_{(J+1/2)-}] +\exp[2i\delta_{(J-1/2)+}]-2\}\,, \tag{1b}$$

$$T_{+-}{}^J=[-4\pi(\sqrt{s})/2imk]\{\exp[2i\delta_{(J-1/2)+}] -\exp[2i\delta_{(J+1/2)-}]\}\,. \tag{1c}$$

Here, m is the mass of the nucleon and the $\pm$ subscript on the phase shifts indicates what total angular momentum J the state has $(J=l\pm\frac{1}{2})$. As in the traditional approach to the impact-parameter representation, it is assumed that the energy in the c.m. system is large enough so that the sum over l can be converted into an integral over the impact parameter. Further, the well-known approximate relation for the Legendre functions of large order and small argument is used[20]:

$$d_{1/2,1/2}{}^J\simeq\cos\tfrac{1}{2}\theta\; J_0(b\sqrt{-t})\simeq J_0(b\sqrt{-t})\,,$$
$$d_{-1/2,1/2}{}^J\simeq J_1(b\sqrt{-t})\,.$$

Two eikonals can now be introduced:

$$\delta_{l+}+\delta_{(l+1)-}\rightarrow\chi_0\,,\quad \delta_{l+}-\delta_{(l+1)-}\rightarrow\chi_f\,.$$

The helicity-nonflip and helicity-flip amplitudes then become

$$T_{++}=4\pi i(\sqrt{s})(k/m)\int_0^\infty bdb\; J_0(b\sqrt{-t}) \times\sum_I\,[\exp(i\chi_0{}^I)\cos(\chi_f{}^I)-1]P_I\,, \tag{2a}$$

$$T_{+-}=-4\pi(\sqrt{s})(k/m)\int_0^\infty bdb\; J_1(b\sqrt{-t}) \times\sum_I \exp(i\chi_0{}^I)\sin(\chi_f{}^I)P_I\,. \tag{2b}$$

Isospin has been introduced in these expressions, and the operator P_I is the projection operator of the respective total isospin I in the s channel. The function $\chi_{0,f}{}^I$ is the corresponding isospin eikonal.

The modification to the original formalism can now be stated.[2] The amplitudes expanded to first order in the eikonals are identified with the amplitudes possessing the usual Regge energy dependence. By performing an inverse Fourier-Bessel transformation, the eikonals may be explicitly calculated in terms of these "primitive" Regge amplitudes. The complete amplitude can then be obtained by using the resulting eikonals in (2a) and (2b). If the complete amplitudes are expressed as series in powers of the various χ, the terms of order N may be thought of as the simultaneous contribution of N Regge poles or, in other words, as a cut arising from the mutual action of N Regge trajectories.[3,4] Because we will treat the Regge poles with quantum numbers different from those of the vacuum to first-order only (thereby preserving the linearity leading to factorizability) and treat the vacuum trajectory, the Pomeranchuk trajectory, not as a pole, but as a way of parametrizing the main features of elastic scattering, we will carry out such an expansion and retain terms only through second order.

To carry out this program, it is necessary to give the dependence of the "primitive" Regge amplitudes on the momentum-transfer variable t, and it is here that further assumptions must enter. In order to perform the integrations analytically and, at the same time, keep the results in a relatively simple form, the signature factors will be written in the rotating phase form

$$(1-e^{-i\pi\alpha})/\sin\pi\alpha=a_-(t)ie^{-i\pi\alpha/2}$$

for odd-signature trajectories, and

$$\alpha(1+e^{-i\pi\alpha})/\sin\pi\alpha=a_+(t)e^{-i\pi\alpha/2}$$

for the even ones. To this same end, the trajectories will all be assumed to be linear functions of t:

$$\alpha=\alpha_0+\alpha' t\,.$$

The functions $a_\pm(t)$ will be absorbed into the unknown residue functions, about which further assumptions must be made.

At this point, the question of nonsense wrong-signature (nws) zeros must be reviewed. It had been found by Gribov and Pomeranchuk[21] that there are essential singularities at the nws points, arising from the third double-spectral function. Mandelstam,[22] how-

[18] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[19] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).
[20] L. Durand III and Y. Chiu, Phys. Rev. **139**, B646 (1965).
[21] V. N. Gribov and I. Ya. Pomeranchuk, Phys. Letters **2**, 239 (1962).
[22] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).

ever, pointed out the existence of a Regge cut which removes the essential singularity from the physical sheet, leaving only a simple pole. This pole then cancels the zero associated with the nws point, leaving the amplitude nonzero and finite.[23–26] The dip observed in Reaction 1 had been achieved by the use of a nws zero. According to the above, however, such a zero was not legitimate, and it thus appeared that Regge theory was failing to explain an important experimental feature which had been counted one of its major triumphs. It was suggested, by way of resolving this apparent contradiction, that this could indicate the smallness of the third double-spectral function in this reaction, implying the practical absence of the Gribov-Pomeranchuk essential singularities and the cut discussed by Mandelstam, and thereby reinstate the zero.[23] Another possibility discussed was that the nws pole was "additive," with an amplitude having one part vanishing and another remaining finite at the nws point, which would still permit a dip in the differential cross section. Theoretical arguments have been made,[26] however, which indicate that the presence of "additive" and the absence of "multiplicative" poles is an unlikely situation.

If the dip in the differential cross section could arise from the interference of the first- and higher-order terms, the experimental data could be explained without the necessity of forcing an extra nws zero into the model. No such zero, therefore, will be included in the residues here. Furthermore, the unknown residue functions (which include part of the signature factor) will be assumed to have the t dependence e^{Ct}. Because of the reactions we are considering, the trajectories involved are those of the ρ and A_2 mesons and of the Pomeranchukon. Since the Pomeranchukon will be treated as the main part of elastic scattering, its trajectory will be assumed to be flat with an intercept at $t=0$ of 1.0. The somewhat unorthodox parameter ξ appearing in the "primitive" Pomeranchuk amplitude allows for the fact that the ratio of the real to imaginary parts of the elastic amplitudes in the forward direction is nonzero even at high energies.[27] We will take this parameter to be constant in the energy range considered here, although, in more realistic models of the elastic amplitude, the ratio of the real to imaginary parts may be expected to decrease, for example, as $(\ln s)^{-1}$ in a Regge-cut model of the vacuum amplitude, or as $s^{\alpha_{P'}(0)-\alpha_P(0)}$ in a model containing only the P and P' trajectories.[28] With these assumptions, the "primitive" amplitudes for these exchanged states are

$$^1T_{++}{}^P=-i\beta_{++}{}^P s\exp(C_P t)\exp(i\xi), \quad (3a)$$

$$^1T_{++}{}^\rho=i\beta_{++}{}^\rho\exp[(\ln s-\tfrac{1}{2}i\pi)\alpha_\rho]\exp(C_\rho{}^0t), \quad (3b)$$

$$^1T_{+-}{}^\rho=i[(\sqrt{-t})/2m]\beta_{+-}{}^\rho\exp[(\ln s-\tfrac{1}{2}i\pi)\alpha_\rho]\times\exp(C_\rho{}^ft), \quad (3c)$$

$$^1T_{++}{}^A=\beta_{++}{}^A\exp[(\ln s-\tfrac{1}{2}i\pi)\alpha_A]\exp(C_A{}^0t), \quad (3d)$$

$$^1T_{+-}{}^A=[(\sqrt{-t})/2m]\beta_{+-}{}^A\exp[(\ln s-\tfrac{1}{2}i\pi)\alpha_A]\times\exp(C_A{}^ft). \quad (3e)$$

It will be observed that the further assumption of a zero helicity-flip amplitude for the Pomeranchuk trajectory has been made. With each trajectory, except for the Pomeranchuk, there are associated the parameters α_0 and α', and C and β for the flip and the nonflip amplitudes. The vacuum amplitude contains the parameters C, $\beta_{++}{}^P$, and ξ. From these primitive amplitudes, the helicity amplitudes through second order can be generated. The helicity-nonflip amplitude is

$$T_{++}=T_{++}{}^\rho+T_{++}{}^A. \quad (4)$$

Similarly, the helicity-flip amplitude is

$$T_{+-}=T_{+-}{}^\rho+T_{+-}{}^A. \quad (5)$$

Here

$$T_{++}{}^e=\epsilon_e\zeta_{++}{}^e\left\{\exp(\mu\alpha_{0e}+D_e{}^0t)+\frac{i}{2\sqrt{6}}\frac{\nu_P}{(D_e{}^0+C_P)}\times\exp[\mu(\alpha_{0e}+1)+D_{e+P}{}^0t]\right\} \quad (6a)$$

and

$$T_{+-}{}^e=\epsilon_e\zeta_{+-}{}^e\left\{\exp(\alpha_{0e}\mu+D_e{}^ft)+\frac{i}{2\sqrt{6}}\frac{C_P\nu_P}{(D_e{}^f+C_P)^2}\times\exp[\mu(\alpha_{0e}+1)+D_{e+P}{}^ft]\right\}, \quad (6b)$$

and e stands for either ρ or A (for the A_2 meson). In the above expressions, the following abbreviations have been used:

$$\epsilon_\rho=i,\quad \epsilon_A=1,$$
$$\mu=\ln s-\tfrac{1}{2}i\pi,$$
$$\zeta_{++}{}^e=(1/\sqrt{2})\eta_e\beta_{++}{}^e,$$
$$\zeta_{+-}{}^e=(1/\sqrt{2})[(\sqrt{-t})/(2m)]\eta_e\beta_{+-}{}^e,$$
$$D_e{}^{0,f}=\alpha_e{}'\mu+C_e{}^{0,f},$$
$$D_{e+P}{}^{0,f}=D_e{}^{0,f}C_P/(D_e{}^{0,f}+C_P),$$
$$\nu_P=(m/4\pi k\sqrt{s})e^{i\xi}\beta_{++}{}^P,$$

and $\eta_{\rho,A}$ is an appropriate ratio of $SU(3)$ Clebsch-Gordan coefficients (see Table I). The first-order terms are just the primitive amplitudes, while the second-order terms are the Regge-cut amplitudes involving

[23] S. Mandelstam and L. L. Wang, Phys. Rev. **160**, 1490 (1967).
[24] C. E. Jones and V. L. Teplitz, Phys. Rev. **159**, 1271 (1967).
[25] A. H. Mueller and T. L. Trueman, Phys. Rev. **160**, 1296 (1967).
[26] R. Roskies, Phys. Rev. **175**, 1933 (1968).
[27] K. J. Foley *et al.*, Phys. Rev. Letters **14**, 862 (1965); **19**, 193 (1967); **19**, 857 (1967).
[28] See Ref. 4 for the Regge-cut model mentioned, and the work of R. J. N. Phillips and W. Rarita [Phys. Rev. **139**, B1336 (1965)] for the model containing the P and P' trajectories.

both the vacuum and either the ρ or A_2. The unpolarized differential cross section, the total cross section, and the polarization of the final nucleon are

$$\frac{d\sigma}{dt}=\frac{m^2}{16\pi^2 sk^2}(|T_{++}|^2+|T_{+-}|^2)\,, \tag{7a}$$

$$\sigma_{\text{tot}}=(-m/k\sqrt{s})\,\text{Im}T_{++}{}^{\text{el}}(\theta=0)\,,$$

$$P=-2n\frac{\text{Im}(T_{++}T_{+-}{}^*)}{(|T_{++}|^2+|T_{+-}|^2)}\,, \tag{7b}$$

where n is a unit vector in the direction of $q_i \times q_f$ and q is the c.m. momentum of the meson.

It is now possible to review some of the important features of the model. If two states A and B, characterized by a t dependence $e^{D_{A},{}_{B}t}$, respectively, are exchanged, the cut arising in second order will have the t dependence $e^{D_{A+B}t}$, where $(D_{A+B})^{-1}=(D_A)^{-1}+(D_B)^{-1}$. Thus, if the D's are positive, the cut has a less marked t dependence than either first-order amplitude. Although the states A and B dominate at low values of t, as $-t$ increases, the cut becomes dominant. In the particular case being examined here, the Pomeranchuk residue is such that the ρ amplitude and its cut are almost completely out of phase. Thus, in the region in which the magnitudes of these terms become nearly equal, there should be a destructive interference apparent in the resulting differential cross section. Such a cancellation could produce the dip structure of Reaction 1. As $-t$ is increased through this region of cancellation, the cut dominates over the pole. Since these two amplitudes are approximately 180° out of phase, a large phase change occurs in the neighborhood of this momentum transfer. This will give rise to structure in the calculated polarization.

III. DISCUSSION AND RESULTS

Because a simultaneous fit to the data on the elastic reactions corresponding to those of Table I has not been undertaken, there is some indeterminacy in the signs of the residues. There are, however, well-known constraints arising from $SU(3)$ symmetry which may, when coupled with experimental information, allow some of the residues to be fixed in sign. For example,

$$A(\pi^-p\to\pi^-p)=-A_\rho+A_P\,,$$
$$A(\pi^+p\to\pi^+p)=A_\rho+A_P\,,$$

or

$$A_\rho=\tfrac{1}{2}[A(\pi^+p\to\pi^+p)-A(\pi^-p\to\pi^-p)].$$

Here, of course, A_ρ is the ρ amplitude and A_P is the vacuum-exchange amplitude. By the optical theorem [see Eq. (7b)],

$$\sigma_{\text{tot}}(\pi^+p)-\sigma_{\text{tot}}(\pi^-p)=-(m/k\sqrt{s})\beta_{++}{}^{\rho}e^{\alpha_{0\rho}\ln s}\,\text{Im}(ie^{-i\pi\alpha_{0\rho}/2})\,.$$

Only the first-order term is used, an approximation based on the dominance of the "primitive" amplitudes near $t=0$. If $-1<\alpha_{0\rho}<1$, the right-hand side of this expression has its sign opposite that of $\beta_{++}{}^{\rho}$. Experimentally, the left-hand side is negative. Thus, $\beta_{++}{}^{\rho}$ is positive. In a similar way, the nonflip A_2 residue can be related to the K-meson–nucleon reactions:

$$\sigma_{\text{tot}}(K^-p)+\sigma_{\text{tot}}(K^+p)-\sigma_{\text{tot}}(K^+n)-\sigma_{\text{tot}}(K^-n)=-(\sqrt{2}m)/(k\sqrt{s})\beta_{++}{}^{A}e^{\alpha_{0A}\ln s}\,\text{Im}(e^{-i\pi\alpha_{0A}/2})\,.$$

For $0<\alpha_{0A}<1$, the right-hand side has the same sign as $\beta_{++}{}^{A}$. The left-hand side is known experimentally to be positive, and thus $\beta_{++}{}^{A}$ is also positive.

The sign of the flip amplitude of the ρ meson is then known from the sign of the experimental polarization of Reaction 1 and the fact that total cross sections are positive (thereby giving the sign of $\beta_{++}{}^{P}$ through the optical theorem). No such conclusive polarization data exist from which the sign of the flip amplitude of the A_2 meson could be determined. However, fitting the data on Reaction 3 turns out to be possible for only one sign of that residue.

The best fits result in the values of the parameters given in Table II. The obvious comments about these values are that the linear trajectories of the ρ and the A_2 mesons are similar to what has been found in the simple Regge models.[11] The ρ trajectory does not pass through the proper mass, while the A_2 trajectory passes quite near the mass of the higher A_2 meson.[29] The value of ξ gives agreement with the observation that the real part of the forward elastic amplitude is approximately -0.2 times the imaginary part.[27]

The individual reactions will now be discussed.

$\pi^-p\to\pi^0n$. The ρ trajectory and the ρ-vacuum cut control this reaction. Here, as in all the reactions (see Table II), the helicity-flip amplitude is dominant. This is necessary in order that the turnover at low t be explained (see Fig. 1).[30] Only for the 13.3-GeV/c data are the low-t points missed.[31] The dip near $t=-0.6$ (GeV/c)^2 is caused by the cut-pole interference. The model is not completely successful in explaining the apparent s dependence of the dip structure. The data suggest that the dip becomes deeper and moves to higher values of $-t$ as the incident energy increases. The model, how-

TABLE II. Values of the parameters giving the best fit.

Exchange	β_{++}	β_{+-}	α_0	α'	C^0	C^f	ξ
ρ	28.3	160.1	0.50	1.10	0.36	1.80	···
A_2	23.3	−145.0	0.38	0.94	−0.20	0.42	···
P	−115.4	···	···	···	3.60	···	0.21

[29] Particle Data Group, Rev. Mod. Phys. **41**, 109 (1969).
[30] A. V. Sterling *et al.*, Phys. Rev. Letters **14**, 763 (1965); P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966); M. A. Wahlig and I. Mannelli, Phys. Rev. **168**, 1515 (1968).
[31] R. J. N. Phillips and W. Rarita (see Ref. 11) note that these data are apparently in conflict with a smooth energy dependence of the cross sections.

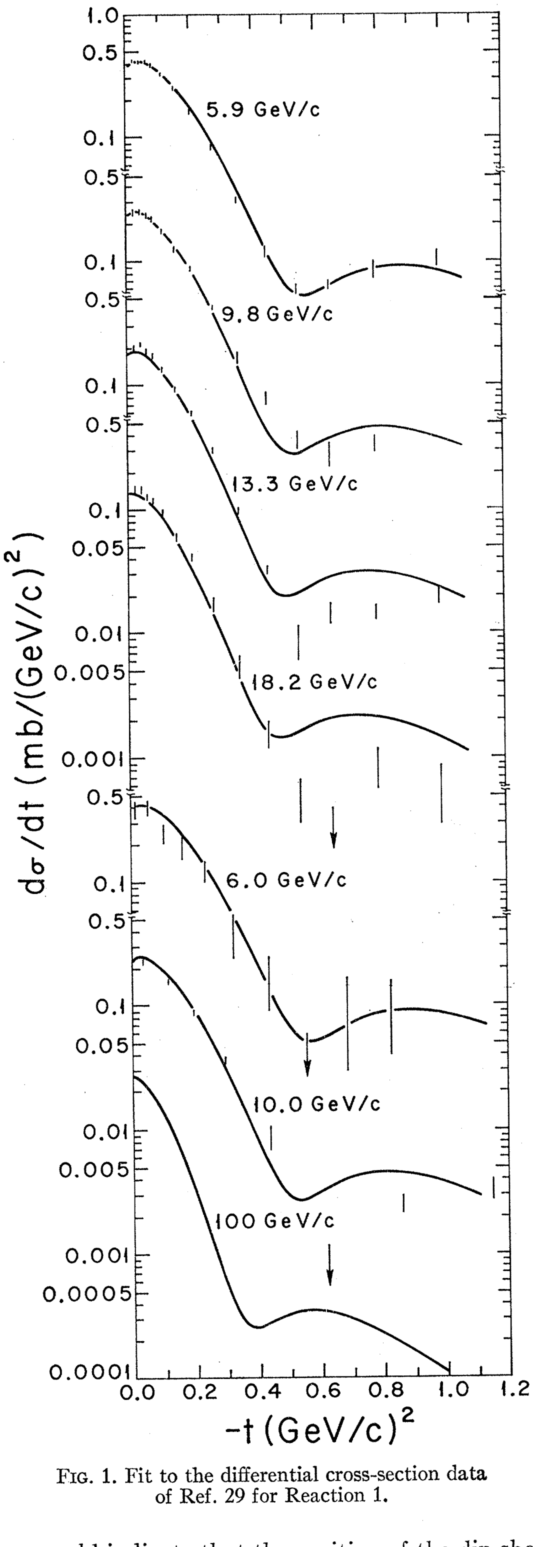

FIG. 1. Fit to the differential cross-section data of Ref. 29 for Reaction 1.

ever, would indicate that the position of the dip should move toward lower values of $-t$, as the energy increases, because of the shrinkage of both the pole and the cut. The data at the higher energies are, however, not sufficiently accurate in the dip region for any conclusive judgement to be made about the shape of the dip. The nonflip amplitude has an interference minimum in the region of $t=-0.35$ (GeV/c)^2. This is not apparent in the differential cross section because of the overwhelming size of the flip amplitude. This interference minimum does, however, appear indirectly in the elastic reactions, giving rise to the crossover effect.[11] As in the case of the dip near $t=-0.6$ (GeV/c)^2, this minimum had been arranged for in the simple Regge models by the insertion of a zero in the t-channel nonflip amplitude of the ρ meson.

The fit to the polarization data[32] is shown in Fig. 2. The qualitative features of the polarization are quite well described by the model. The fit is not quantitatively a good one, but the data are of such a quality that this is not unsatisfactory. At the lowest energies, moreover, direct-channel resonances, although no longer affecting the differential cross sections, may be causing as much as 15% of the polarization.[33] As mentioned at the end of Sec. II, the model predicts a structure in the polarization in the region of the cancellations. This is the source of the large negative polarization in the region of $t=-0.5$ (GeV/c)^2. The two regions of cancellation, the one in the flip amplitude near $t=-0.6$ and the one in the nonflip amplitude near $t=-0.35$ (GeV/c)^2, cause a large rotation of the phase of each of these amplitudes. Because of the dominance of the flip amplitude, however, this large polarization occurs just before

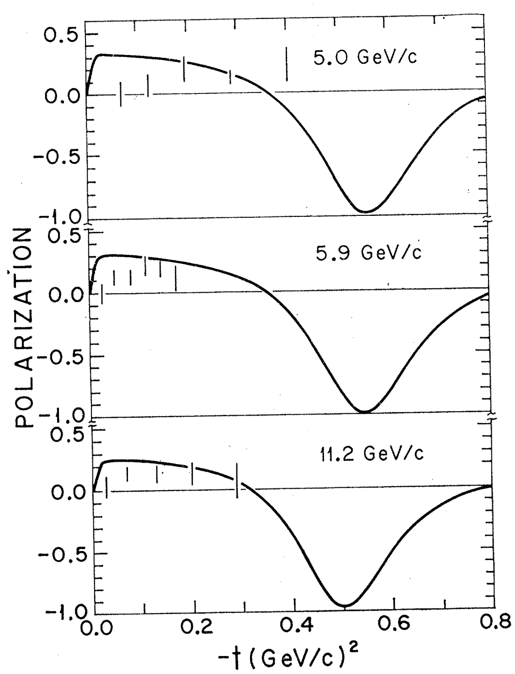

FIG. 2. Fit to the polarization data of Ref. 31 for Reaction 1.

[32] P. Bonamy *et al.*, Phys. Letters **23**, 501 (1966); D. D. Drobnis *et al.*, Phys. Rev. Letters **20**, 274 (1968).
[33] This is discussed in the first two works of Ref. 12.

the dip in the differential cross section so that the measurable quantity, $P(d\sigma/dt)$, has no such spectacular structure. This large polarization is, therefore, not now an experimentally testable feature of the model.

$K^-p \rightarrow \bar{K}^0 n$. Here, in addition to the ρ trajectory and the ρ-vacuum cut, the A_2 trajectory and the A_2-vacuum cut must be considered. The fit to the data[34] is presented in Fig. 3. Both the helicity-flip amplitudes of the ρ and A_2 exchanges have minima associated with the cancellation of the respective poles and attendent cuts. Because the residue of the A_2 pole has a flatter t dependence, however, the destructive interference occurs at a larger value of $-t$. Thus, the cancellation in the ρ amplitude is filled in by the amplitude of the A_2. Furthermore, the cancellation in the A_2 is not as complete as it is for the ρ exchange, because the pole and cut are not so nearly out of phase. Thus, rather than the dip-bump structure of Reaction 1, the differential cross section here simply levels off.

Predictions for the neutron polarization are given in Fig. 4. The presence of the A_2 trajectory, whose amplitude is approximately 90° out of phase with the ρ amplitude, makes the polarization very large near the forward direction. The polarization then decreases and goes through zero near $t=-0.4$ $(\text{GeV}/c)^2$ because of the cut-pole cancellations in the nonflip amplitude.

$K^+n \rightarrow K^0 p$. This reaction is essentially the line-reversed partner of Reaction 2. The effect of line reversal is that the amplitudes for the ρ exchange become negative relative to those of Reaction 2. If we were to assume that the ρ and A_2 mesons are strongly exchange degenerate (their residue functions and trajectories are equal), the amplitudes would be 90° out of phase, and the differential cross sections for Reactions 2 and 3 would be identical. If, on the other hand, exchange degeneracy is assumed for the trajectories, but not for the residue functions, the primitive amplitudes would still be 90° out of phase, but the cut terms would have a relative phase dependent on t. There is some evidence that the differential cross sections are not the same and we have therefore not made any assumptions of exchange degeneracy. Although some of the data on this reaction[35] are at a rather low mo-

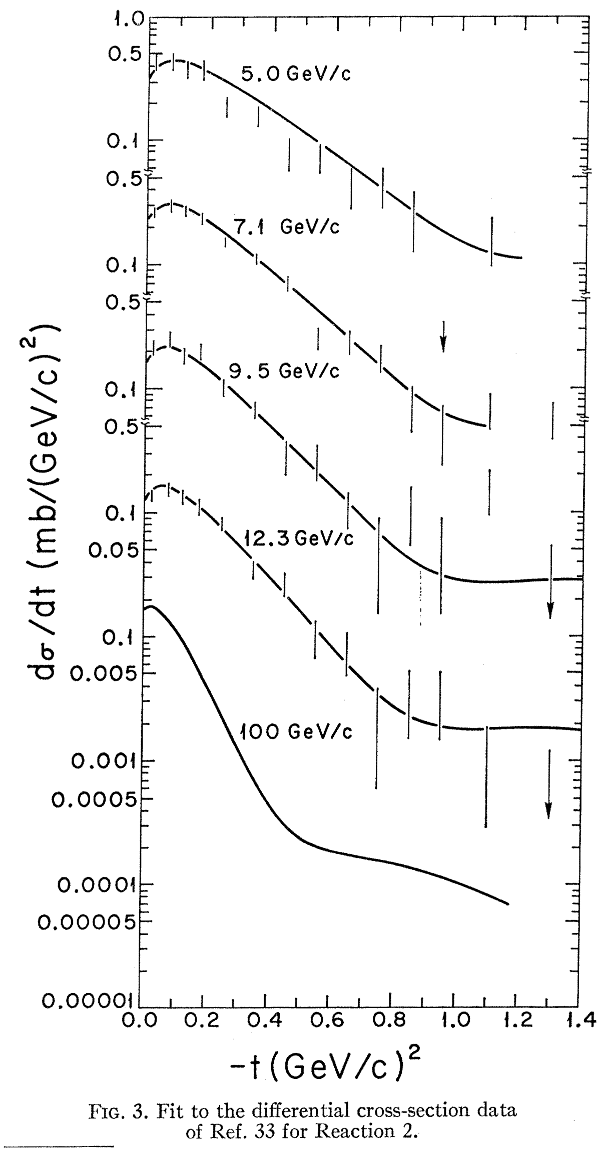

FIG. 3. Fit to the differential cross-section data of Ref. 33 for Reaction 2.

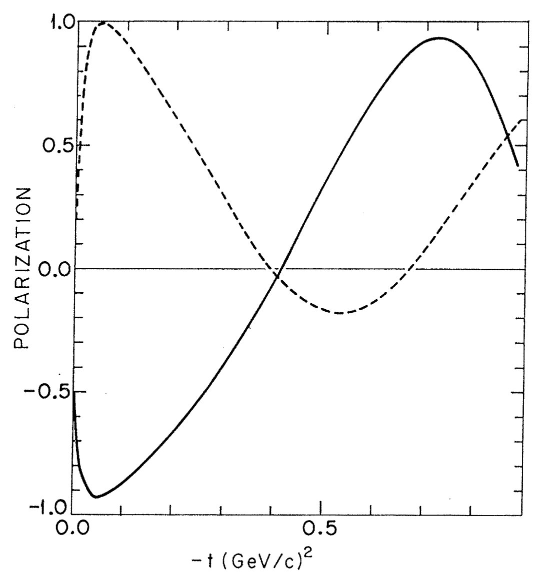

FIG. 4. Predictions of the final nucleon polarization for Reaction 2 (solid curve) and Reaction 3 (broken curve) at 12.3 GeV/c.

[34] P. Astbury *et al.*, Phys. Letters **23**, 396 (1966).

[35] $P_{\text{lab}}=2.27$ GeV/c: I. Butterworth *et al.*, Phys. Rev. Letters **15**, 734 (1965). We use the data only above $t=-0.15$ $(\text{GeV}/c)^2$ in order to avoid the effects due to the deuteron. $P_{\text{lab}}=3.0$ GeV/c: Y. Goldschmidt-Clermont *et al.*, Phys. Letters **27B**, 602 (1968). $P_{\text{lab}}=5.5$ GeV/c: D. Cline, J. Matos, and D. D. Reeder, Phys. Rev. Letters **23**, 1318 (1969). Only the 2.27-GeV/c data were used in the fit.

mentum, namely, 2.27 GeV/c, there appear to be no direct-channel resonances for such processes, and a Regge analysis should be applicable. If a Regge fit to Reaction 2 is extrapolated to this momentum, the data on Reaction 3 are higher by a factor of about 2. Barring gross errors in the data on Reaction 3, the conclusion is that the two differential cross sections are not identical. The fit gives a manifestly non-exchange-degenerate result for both the trajectories and residues. As mentioned above, a fit (see Fig. 5) can be obtained only for one sign of the A_2 flip amplitude. This sign gives a qualitatively acceptable fit to Reaction 3, for all three values of the lab momentum. The opposite

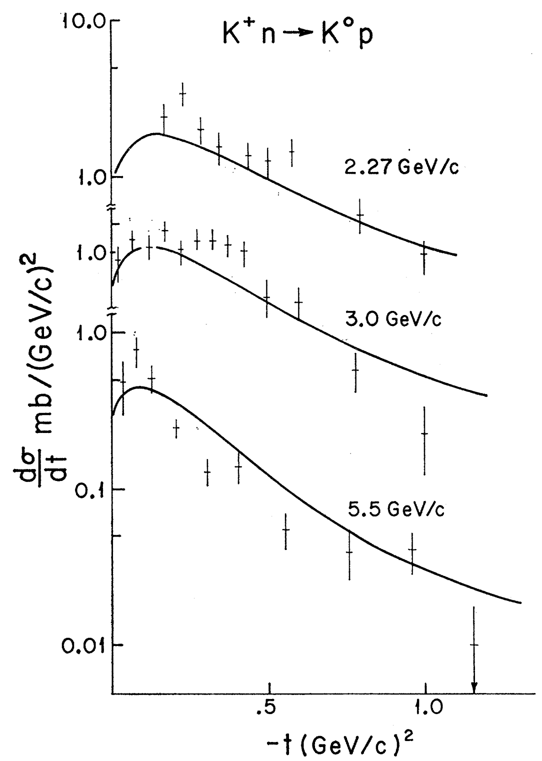

FIG. 5. Fit to the differential cross-section data of Ref. 34 for Reaction 3.

sign gives much too steep a t dependence. The behavior of the polarization is similar to that in Reaction 2 except for the change in the sign of the ρ amplitudes. One may compare this with the results of Hartley, Moore, and Moriarty,[36] who used an absorptive model with $U(6)\times O(3)\times U(6)$ symmetric Regge residues.

$\pi^- p \rightarrow \eta n$. Only the A_2 pole and the cut contribute to this reaction. The fit to the data[37] is presented in Fig. 6. The model gives a shrinkage which is slightly more rapid than the data indicate, although the fit is in general satisfactory. As in Reactions 2 and 3, there is a leveling of the differential cross section. The polarization is similar to that in the ρ-dominated Reaction 1. Its average value from $t=0.0$ to $t=-0.4$ (GeV/c)^2 is about -10.0%, which is consistent with the small amount of data available.[38]

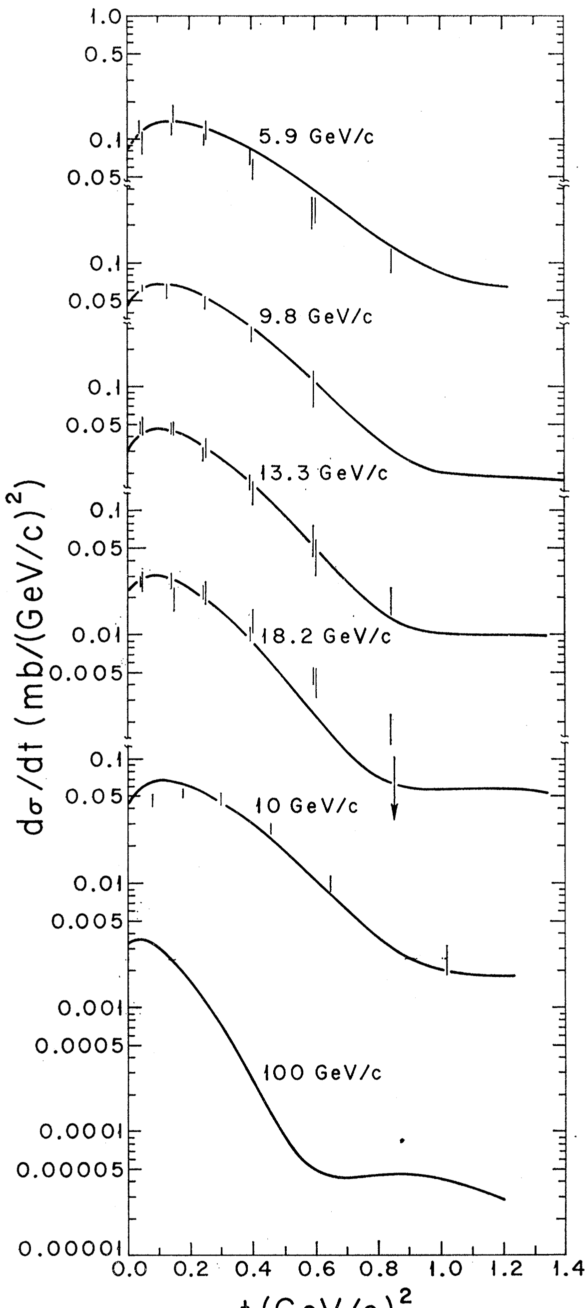

FIG. 6. Fit to the differential cross-section data of Ref. 35 for Reaction 4.

[36] B. J. Hartley, R. W. Moore, and K. J. M. Moriarty, Phys. Rev. D 1, 954 (1970); Phys. Rev. 187, 1921 (1969).

[37] O. Guisan *et al.*, Phys. Letters 18, 200 (1965); see also the last work of Ref. 30.

[38] P. Bonamy *et al.*, in *Proceedings of the International Conference on Elementary Particles, Heidleburg, 1967*, edited by H. Filthuth (Wiley, New York, 1968); see also the last work of Ref. 30.

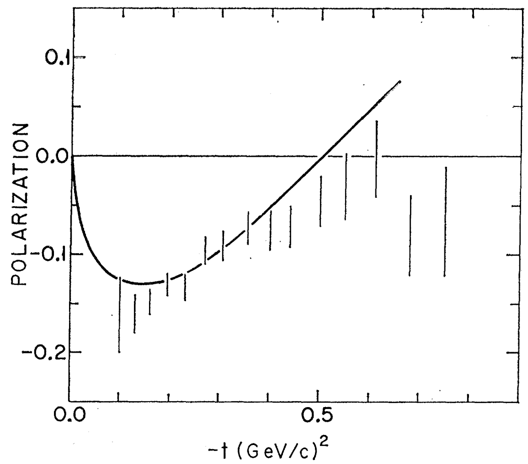

FIG. 7. Calculation of final proton polarization in $\pi^- p$ elastic scattering and data of Ref. 37 at 10 GeV/c.

Elastic reactions. In order to treat simultaneously the elastic reactions, the primitive Pomeranchuk amplitude would have to be included, at the very least. Since we do not propose actually to fit the elastic reactions, we do not worry about the inclusion of other trajectories. By including the primitive Pomeranchuk amplitude and using the parameters found in fitting the charge-exchange reactions, we will simply calculate differential cross sections and polarizations for some of the elastic reactions, in order to examine how widely applicable the model is. When this is done, it is found that the calculated cross sections are too high by a factor of approximately two.[39] In Ref. 5 it was necessary to include a parameter multiplying the Pomeranchuk residue (which was there taken from the size of the elastic cross section). This extra parameter was attributed there to the effect of inelastic intermediate states which can be reached by the exchange of the Pomeranchuk trajectory. The value which the parameter had to have in order to give a fit was approximately 1.5. This is directly related to the calculations above, being a factor of two too high. From our point of view this indicates a serious incompleteness in the model. If we make use of such a factor, the calculated curves for the differential cross sections and the polarizations are in good qualitative agreement with the data. An example of this agreement is given in Fig. 7 for $\pi^- p$ elastic polarization data. The zero in the polarization at $t = -0.6$ (GeV/c)^2 illustrates the effect of the cut-pole interference in the helicity-flip ρ amplitude.

[39] M. Borghini *et al.*, Phys. Letters 21, 114 (1966).

IV. CONCLUSION

A satisfactory simultaneous fit of a Regge-cut model to the available differential cross section and polarization data on the four charge-exchange reactions of Table I has been obtained. This was accomplished without the introduction of a nws zero into the helicity-flip amplitude of the ρ meson, without the introduction of a crossover zero in the nonflip amplitude, and without any assumption of exchange degeneracy. All the residues are determined in both sign and magnitude by appealing to experimental information and fitting the line-reversed Reaction 3.

The distinguishing features of this model are the following.

1. All of the characteristics of the experimental data are described.
2. A fit including Reaction 3 is possible.

The model is apparently unsatisfactory in the following ways.

1. An extra parameter is needed in the Pomeranchuk residue if that term is to represent elastic scattering.
2. It predicts that the dip in Reaction 1 moves to lower values of $-t$ as the energy increases.

The model suggests the following.

1. Beyond the point where the polarization goes through zero, there should be a large polarization effect in all of the reactions.
2. In Reactions 2–4, beyond $t = -0.5$ (GeV/c)^2, the differential cross sections should level off before again decreasing.

These structures in the polarization and differential cross section, along with the crossover effect, are all related to cut-pole interferences and therefore occur at corresponding values of the momentum transfer. Both of these features are at present experimentally untestable. Since this behavior persists at high energies, however, it might be possible to test them in the future.

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