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Regge Model for $Y_1^*(1385)$-Production Reactions*

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A model is constructed for the analysis of the reactions $K^-p \to \pi^- Y_1^{*+}(1385)$ and $\pi^+p \to K^+Y_1^{*+}(1385)$ in terms of the exchanges of two Regge trajectories [the $K^*(892)$ and $K^{**}(1420)$]. The present model is a natural extension of the one which has been successful in the description of the $\Delta(1238)$-production reactions.

REACTIONS in which the isobar $Y_1^*(1385)$ is produced provide an important source of information on the Regge trajectories corresponding to the $K^*(892)$ ($J^P=1^-$) and $K^{**}(1420)$ ($J^P=2^+$) mesons. Data on these reactions in π^+p and K^-p collisions at high energies (that is, at laboratory momenta of the order of 10 GeV/c or higher) have only recently become available. As the $Y_1^*(1385)$ and $\Delta(1238)$ belong to the same $SU(3)$ representation (the decuplet with $J^P=\frac{3}{2}^+$), it is reasonable to expect that the mechanisms by which they are produced are similar. It is thus very desirable to have a common framework for the analysis of the Y_1^*- and the corresponding Δ-production reactions. Such a combined analysis will have the advantage of testing $SU(3)$ symmetry in Regge-pole models and will provide, in addition, a basis for comparing the trajectory functions of various mesons belonging to the same nonet (e.g., ρ with K^*, and A_2 with K^{**}).

In the following, the available data on the reactions

$$\pi^+p \to K^+Y_1^{*+}(1385) \quad (1)$$

and

$$K^-p \to \pi^-Y_1^{*+}(1385) \quad (2)$$

are analyzed in terms of a conventional Regge-pole model. The structure of the Regge residues will be chosen to reflect the dominance of the magnetic dipole ($M1$) transition between the nucleon and the isobar, as formulated in an earlier work.[1] The hypothesis of $M1$ dominance was strongly indicated by the features of the experimental data on the Δ-production reactions

$$\pi^+p \to \pi^0\Delta^{++} \quad (3)$$

$$\to \eta\Delta^{++}, \quad (4)$$

$$K^+p \to K^0\Delta^{++}. \quad (5)$$

These features are (i) a minimum in the differential cross section of reaction (3) near $t=-0.6$ (GeV/c)^2; (ii) the consistency of the cross sections of all the reactions with a turnover for $-t$ near its minimum value, $(-t)_{\min}$; and (iii) the equality of the decay density-matrix elements ρ_{mn} of the $\frac{3}{2}^+$ isobar in all the reactions and their approximate equality with the Sakurai-Stodolsky values[2] (in the Gottfried-Jackson frame[3]) of

$$\rho_{33}=\tfrac{3}{8}, \quad \text{Re}\rho_{31}=0, \quad \text{Re}\rho_{3-1}=\tfrac{1}{8}\sqrt{3}, \quad (6)$$

independently of whether it is the ρ or the A_2 which is involved.

Differential cross-section data on reaction (1) are now available at laboratory momenta of 14.0,[4] 10.0,[4] 8.0,[5] 6.0,[4] 5.05,[6] and 4.0 [6] GeV/c. There are no data on the decay density-matrix elements for this reaction. As far as reaction (2) is concerned, differential cross-section data are now available for laboratory momenta of 16.0,[7] 10.1,[8] 8.0,[7] 6.0,[9] and 5.5 [10] GeV/c. There are data on the decay density-matrix elements of the Y_1^* for $p_{\text{lab}}=6.0,$[9] 5.5,[10] and 4.1 [10] GeV/c:

$$\rho_{33}=0.28\pm0.08,\ 0.22\pm0.06,\ 0.28\pm0.09,$$
$$\text{Re}\rho_{31}=-0.07\pm0.07,\ 0.06\pm0.05,\ 0.19\pm0.10,$$
$$\text{Re}\rho_{3-1}=0.13\pm0.08,\ 0.14\pm0.06,\ 0.15\pm0.09,$$

respectively. These values represent averages in the range $(-t)_{\min}\leq -t \leq 0.5$ (GeV/c)^2.

The 8.0- and 10.1-GeV/c differential cross-section data on reaction (2) clearly indicate a turnover near the forward direction. The data on reaction (1) do not extend to angles small enough to detect a turnover. The data on the decay density-matrix elements are con-

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sistent with those of the $\Delta(1238)$,[1] especially when consideration is taken of the facts that the Δ-decay elements rise gradually from small values in the near forward direction to the Sakurai-Stodolsky values and that the Y_1^*-decay elements quoted above are averaged over a large interval in t.[11]

Generally speaking, then, the production mechanisms for the Y_1^* and the Δ are similar, and we are justified in using the model of $M1$ dominance as a unified description of reactions (1)–(5). The cross section corresponding to the pure $M1$ transition is given by[12]

$$d\sigma/dt=\{32\pi[s-(m_1+\mu_1)^2][s-(m_1-\mu_1)^2]\}^{-1} \times[(16/3)\phi(M^2-t)|B|^2],$$

where $B=B_V+B_T$. In the particular case of reactions (1) and (2),

$$B_V=B_q(i+\tan\tfrac{1}{2}\pi\alpha_q)\alpha_q(\alpha_q+1)[(s-u)/2s_0]^{\alpha_q-1}$$
$$B_T=B_Q(i-\cot\tfrac{1}{2}\pi\alpha_Q)\alpha_Q(\alpha_Q+1)[(s-u)/2s_0]^{\alpha_Q-1}.$$

Here $\alpha_{q(Q)}$ is the trajectory on which lies the K^* (K^{**}). To relate these amplitudes to those entering reactions (3)–(5) requires an assumption concerning the application of $SU(3)$ symmetry, which is expected to be broken here. The functions B_q and B_Q will be assumed to be related to the corresponding functions for the ρ and A_2 by products of $SU(3)$ Clebsch-Gordan coefficients. All masses will be given their experimental values and the trajectories α_q and α_Q will be determined from the data on reactions (1) and (2). Thus

$$B_q=C^V B_{0V}, \quad B_Q=C^T B_{0T},$$

where B_{0V} and B_{0T} were determined to be[1]

$$B_{0V}=34.4e^{0.32t}, \quad B_{0T}=27.7e^{0.15t}.$$

The scale factor s_0 is chosen to be 1 GeV^2. The symmetry coefficients for reaction (1) are

$$C^V=-\sqrt{\tfrac{1}{6}}, \quad C^T=\sqrt{\tfrac{1}{2}}.$$

For reaction (2), C^V has a positive sign. The trajectory functions are assumed to be linear, and, therefore, the parameters which are free to vary are their intercepts and slopes:

$$\alpha_q=\alpha_{0q}+\alpha_q{}'t, \quad \alpha_Q=\alpha_{0Q}+\alpha_Q{}'t.$$

No assumptions of exchange degeneracy will be made.

In attempting to fit all the existent data, it has become apparent that, relative to this kind of model, the various sets of data are somewhat inconsistent with one another. Some of this inconsistency is surely due to the fact that the errors quoted for the experimental points do not always reflect sources of uncertainty other than statistical ones. Furthermore, the sequence of nucleon isobars may include resonances with masses as large as 3.23 GeV ($p_{\rm lab}=5.08$ GeV/c).[13] Such direct-channel resonances might make the application of the Regge model somewhat suspect in the lower-energy region. Thus, data on both reactions with laboratory momenta less than 8.0 GeV/c will not be included in the actual fitting. Of the higher-energy data, the 8.0-GeV/c data[5] on reaction (1) are based on a very small number of events. Moreover, the 10.1-GeV/c data[8] on reaction (2) will not greatly affect the fitting because of the relatively large errors quoted. For these reasons, only the 10.0- and 14.0-GeV/c data[4] on reaction (1) and the 8.0- and 16.0-GeV/c data[7] on reaction (2) will be retained for fitting.

The values of the trajectory parameters are

$$\alpha_q=(0.39\pm0.02)+(0.98\pm0.07)t,$$
$$\alpha_Q=(0.29\pm0.01)+(1.06\pm0.03)t,$$

corresponding to a maximum χ^2 of 168 for the 36 data points. The 10.0-GeV/c data on reaction (1) and the 8.0-GeV/c data on reaction (2) are fitted very well. The large value of the minimum in χ^2 has its origin in the relatively flat experimental t distributions at the higher energies, especially at 16.0 GeV/c. The model predicts that, when $\alpha_q(t)=0$ [$t\simeq-0.4$ (GeV/c)^2 using the parameters above], the differential cross sections for reactions (1) and (2), evaluated at the same energy, are equal.[14] In general, then, if one reaction has a flat t distribution, the other has a steep one. The data at 6.0 GeV/c are in disagreement with this, while the data at 8.0 and approximately 10.0 GeV/c are not, except at the highest values of t in the case of the 10.0- and 10.1-GeV/c data. The 14.0- and 16.0-GeV/c data are not directly comparable, but if the factor $(p_{\rm lab\,1}/p_{\rm lab\,2})^{2[1-\alpha(0)]}\approx0.8$ is applied (this is the approximate decrease in magnitude of the differential cross section as the momentum is increased from $p_{\rm lab\,1}$ to $p_{\rm lab\,2}$), it is seen that the data are inconsistent with this prediction. These experimentally flat t distributions greatly constrain the parameters, especially the slopes of the trajectories. Thus, while the fit is poor, the errors calculated for the parameters are small.

Because the errors quoted for the experimental points chosen for fitting here are purely statistical in origin, one might expect that inclusion of uncertainties arising from such sources as the subtraction of back-

[11] In fitting the data of Ref. 10 with the exchange of only the elementary $K^*(892)$, J. Mott [Nucl. Phys. **B13**, 565 (1969)] found it necessary to invoke absorption-model corrections in order to explain the observed values of the density-matrix elements.

[12] This corresponds to model 1 of Ref. 1.

[13] There is a possible recurrence of the $\Delta(1238)$ with this mass [Particle Data Group, Rev. Mod. Phys. **42**, 87 (1970)].

[14] The same feature is not expected to persist when meson-baryon hypercharge exchange reactions are considered, however, because the contribution arising from the exchange of the vector-meson nonet cannot be well represented by one amplitude, vanishing at $\alpha(t)=0$ (see Ref. 18). Thus, the corresponding data on such reactions [e.g., $\pi p\to K(\Sigma,\Lambda)$ and $\bar{K}p\to\pi(\Sigma,\Lambda)$] are not expected to exhibit a crossover. In this regard, see the data cited in the last two works of Ref. 20.

ground events[15] might lead to rather different values of the parameters, in addition to lowering the minimum value of χ^2. It will be assumed here that the statistical errors and the errors from other sources simply add. Thus, if e_s is the statistical error and e_r is the total error from other sources, the total error e is assumed to be

$$e=e_s+e_r.$$

This prescription is tantamount to saying that, given the measured value x, another experimental value x' could be expected to lie in the interval

$$x-e_r\leq x'\leq x+e_r$$

and that the error in x' is equal to e_s. For the purpose of examining the effects of such uncertainties, we will assume that e_r is 10% of the measured values for reaction (1) and 20% for reaction (2).[15] The following trajectories result from the increase in the errors:

$$\alpha_q=(0.37\pm0.05)+(0.94\pm0.11)t,$$
$$\alpha_Q=(0.29\pm0.02)+(1.03\pm0.05)t,$$

giving a χ^2 of 55 for the 36 data points. The trajectory parameters are very stable against such modifications

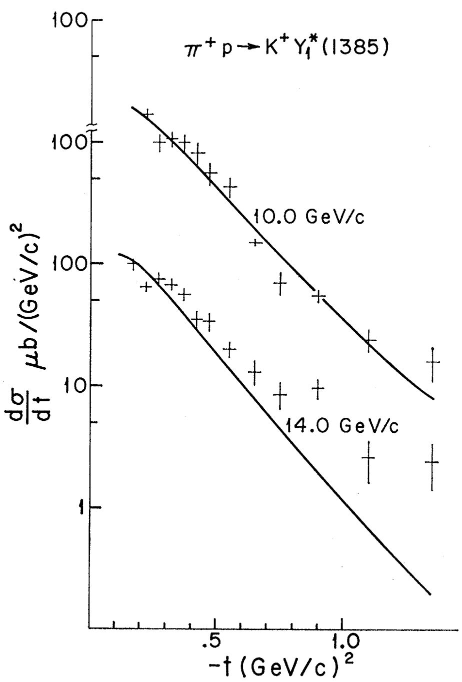

FIG. 1. Theoretical curves at 10.0 and 14.0 GeV/c for reaction (1), and corresponding data of Ref. 4.

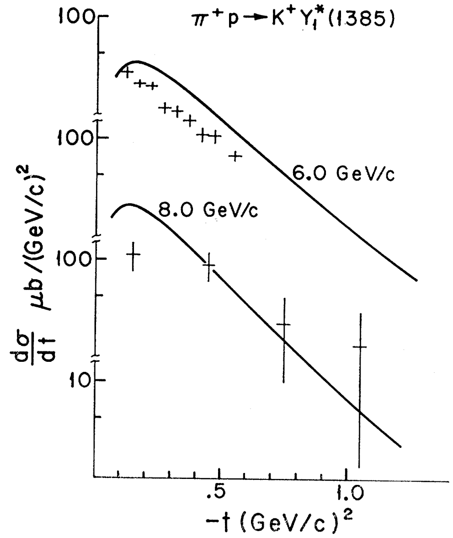

FIG. 2. Theoretical curves at 6.0 and 8.0 GeV/c for reaction (1). The data points shown (Refs. 6 and 5, respectively) were not used in obtaining the fit.

of the errors.[16] The effects of the experimentally flat t distributions at the higher energies have been reduced somewhat and the errors determined for the parameters have thereby been increased to values which probably more accurately reflect the quality of the fit.

The theoretical curves calculated from the second set of trajectory parameters are shown in Figs. 1–4, together with the corresponding experimental data. The errors shown in these figures are the original, unmodified ones. Integrated cross sections for reactions (1) and (2) can be calculated. These are plotted against laboratory momentum in Fig. 5. These cross sections are not strictly proportional to inverse powers of $p_{\rm lab}$, but can be approximated by the form

$$\sigma=a(p_{\rm lab})^{-n}.$$

For reaction (1), the parameters are $a_1=420\,\mu$b and $n_1=1.79$, while for reaction (2) they are $a_2=690\,\mu$b and $n_2=1.85$. If the identification

$$n=2(1-\langle\alpha\rangle)$$

is made ($\langle\alpha\rangle$ is some average of the effective trajectory for the reaction), then

$$\langle\alpha_1\rangle=0.11,\quad\langle\alpha_2\rangle=0.08.$$

Here α_1 and α_2 are the effective trajectories of reactions (1) and (2).[17]

[15] A. B. Clegg [Nucl. Phys. **B13**, 222 (1969)] has discussed the problem of estimating background contributions to the total mass spectrum. In considering reactions related by isospin invariance, he found that large errors may be made in the partial differential cross sections if standard procedures of separating the resonance production from the background are used. See Ref. 7 for the authors' estimates of their uncertainties in removing the background from their data.

[16] If the lower-energy data alone are fitted with the same model, the intercept of the Q trajectory is approximately half the value found here, the q intercept remaining essentially unchanged. Such a fit underestimates the higher-energy experimental cross sections by a large factor.

[17] The effective trajectory is defined through the equation $d\sigma/dt=As^{\alpha(t)}$. In the present case, $\alpha(t)$ will not equal either the q or Q trajectory.

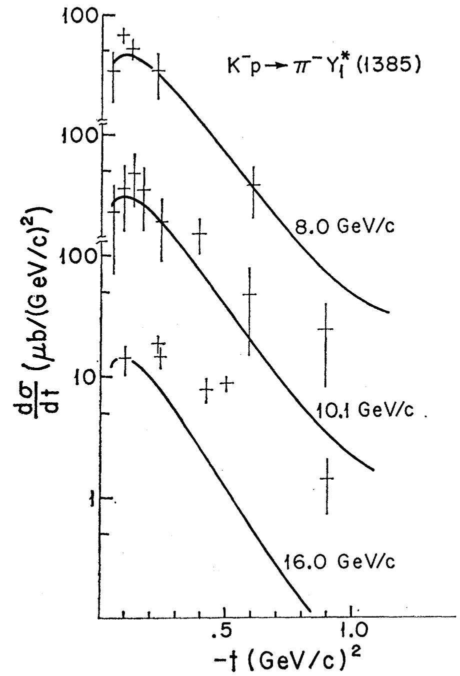

FIG. 3. Theoretical curves at 8.0, 10.1 and 16.0 GeV/c for reaction (2). The 8.0- and 16.0-GeV/c data of Ref. 7 were used in obtaining the fit; the 10.1-GeV/c data of Ref. 8 were not.

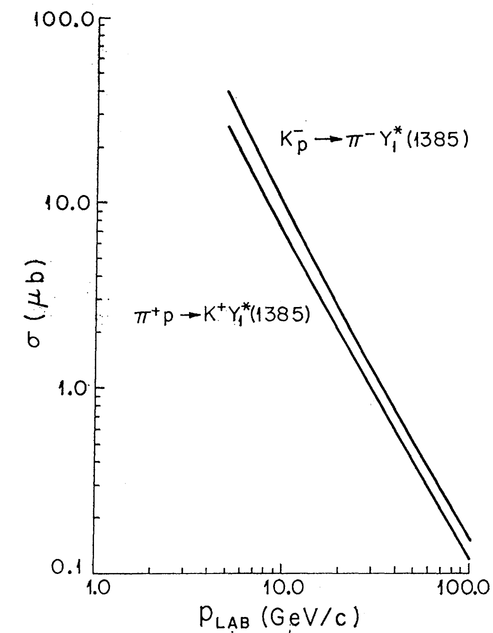

FIG. 5. The integrated theoretical cross sections of reactions (1) and (2). The dependence on p_{lab} is approximately $p_{\text{lab}}^{-1.85}$ for the upper curve and $p_{\text{lab}}^{-1.79}$ for the lower one.

DISCUSSION

There is no reaction in which only the q trajectory or only the Q trajectory can be regarded *a priori* as the dominant exchange, and the information on these hypercharge-exchange trajectories has consequently to be obtained indirectly, or under special hypothesis, such as that of exchange degeneracy. The determination of these trajectories therefore depends to some extent on the details assumed for the structure of the residue functions. In particular, the slopes of the Regge trajectories are sensitive to the t dependence assumed for the residues. Such questions can only be resolved when the variation of the logarithm of the energy within a set of data to be fit is large compared to the energy-independent parameters, typically of the order of several inverse $(\text{GeV}/c)^2$, which enter the residue functions.

The trajectory intercepts are, on the other hand, ascertainable usually with greater reliability than the slope parameters. The values of the intercepts obtained in the present analysis are consistent with those obtained from other sources. The analysis of hypercharge

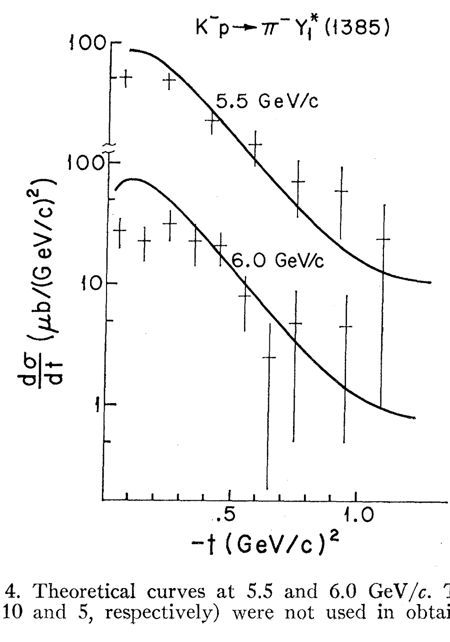

FIG. 4. Theoretical curves at 5.5 and 6.0 GeV/c. The data (Refs. 10 and 5, respectively) were not used in obtaining the fit.

TABLE I. Summary of various determinations of the intercepts of the q and Q trajectories.

Intercept	Present analysis Y^*	Hyperon prod. Ref. 18	Total cross sections Ref. 19
α_{0q}	0.37 ± 0.05	0.35[a]	0.24 ± 0.15
α_{0Q}	0.29 ± 0.02	0.24[a]	0.24[a,b]

[a] No errors are available for these numbers.
[b] $\alpha_{0Q}=0.77\pm0.04$ if no assumption of $SU(3)$ symmetry breaking is made.

exchange reactions[18] ($\pi^+ p \to K^+\Sigma^+$, $\pi^- p \to K^0\Lambda^0$, etc.) gives the intercepts $\alpha_{0q}=0.35$ and $\alpha_{0Q}=0.24$ (with uncertain errors). The intercepts resulting from an analysis of total cross-section data are also consistent with the values of the present analysis provided we postulate[19] that the Pomeranchuk trajectory has a small $I=0$ octet component in addition to the usual $SU(3)$ singlet component. Table I summarizes the situation on the intercepts of the q and Q trajectories.

In conclusion, the following comments may be made: Although the quality of the fits in the present case is not comparable with those which can be made with the Δ-production data, it nevertheless demonstrates that $SU(3)$ symmetry for Regge vertices and Regge behavior are consistent with the data. Further, the same mechanism seems to be operative in the production of these members of the $\frac{3}{2}^+$ decuplet. The q and Q trajectories do not seem to be degenerate,[20] and the values determined from the analysis of the Y_1*(1385)-production reactions are consistent with earlier determinations from other reactions.

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[20] K. W. Lai and J. Louie [Nucl. Phys. **B19**, 205 (1970)] have examined reactions (1) and (2) with a view to testing the exchange degeneracy of the K^* and K^{**} exchanges. They find that exchange degeneracy is not indicated in these reactions. D. J. Crennell *et al.* [Phys. Rev. Letters **23**, 1347 (1969)] and P. R. Auvil *et al.* [Phys. Letters **31B**, 303 (1970)] have found that the data on meson-baryon hypercharge exchange reactions similarly do not indicate exchange degeneracy for these exchanges.

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Weak Interactions with Lepton-Hadron Symmetry*

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We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Milis theory is discussed.

INTRODUCTION

Weak-interaction phenomena are well described by a simple phenomenological model involving a single charged vector boson coupled to an appropriate current. Serious difficulties occur only when this model is considered as a quantum field theory, and is examined in other than lowest-order perturbation theory.[1] These troubles are of two kinds. First, the theory is too singular to be conventionally renormalized. Although our attention is not directed at this problem, the model of weak interactions we propose may readily be extended to a massive Yang-Mills model, which may be amenable to renormalization with modern techniques. The second problem concerns the selection rules and the relationships among coupling constants which are carefully and deliberately incorporated into the original phenomenological Lagrangian. Our principal concern is the fact that these properties are not necessarily maintained by higher-order weak interactions.

Weak-interaction processes, and their higher-order weak corrections, may be classified[2] according to their dependence upon a suitably introduced cutoff momentum Λ. Contributions to the S matrix of the form

$$\sum_{n=1}^{\infty} A_n(G\Lambda^2)^n$$

(where G is the usual Fermi coupling constant and A_n are dimensionless parameters) are called zeroth-order

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