Polarization Theorems in Pion Photoproduction*

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(Received 22 May 1970)

Stichel's theorem relating the state of linearly polarized photons and spin-parity of t-channel exchanges in pion photoproduction is reexamined and generalized. For two specific initial and final nucleon polarization configurations, an exact theorem valid for all photon energies is proved.

INTRODUCTION

LINEARLY polarized photons of low energies (<1 BeV) have for a long time been used as an effective method to pin down the spin-parity of s-channel nucleon resonances in pion photoproduction.[1] At higher energies, the description of the process as proceeding through a series of direct-channel states becomes very complicated. But at sufficiently high photon energies we enter the region where we believe we can understand the pion production as being initiated by Regge exchanges in the t channel.

As first pointed out by Stichel,[2] there again exists a simple relation between the polarization of the incoming photon and the spin parity exchanged, now in the t channel. With the photon polarized normal to the production plane, the amplitude is, to the leading power in s, dominated by natural parity states $P=(-1)^J$; with polarization parallel to the plane, unnatural parity $P=(-1)^{J+1}$ dominate.

Today it is possible to obtain high-energy linearly polarized photons from electrons impinging on suitably oriented diamond crystals.[3] With this in mind, we have reexamined Stichel's theorem. Using the helicity formalism of Jacob and Wick,[4] we prove that the theorem is independent of the spin-parity of the final nucleon state to the leading power in s. Also we show that it is valid to *all* orders in s when both initial and final nucleons are polarized in the same direction normal to the production plane, or when they have opposite helicities. The only assumption made in this later case is that m_π^2 can be neglected with respect to m_N^2, which is very reasonable.

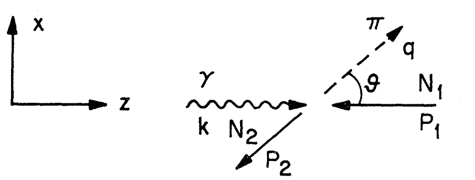

Fig. 1. Center-of-mass coordinates for photoproduction process.

In the last part of this paper we derive the same theorems now making use of the more transparent Feynman–van Hove model[5] for the process. Besides the simplicity of this approach, we also obtain a physical understanding of the approximations involved in proving the theorems, an understanding partially lost in the complicated crossing of helicity amplitudes between the s and t channels.

HELICITY FORMALISM

In the pion photoproduction process $\gamma N_1 \to \pi N_2$, where N_1 is a spin-$\frac{1}{2}$ nucleon and the final nucleon N_2 can have any spin-parity J^P, we let the incoming photon with helicity ± 1 and momentum k move along the z axis in the c.m. system. The produced π meson comes out at an angle ϑ and with momentum q as in Fig. 1.

The helicity states of the photon have polarization vectors

$$\boldsymbol{\varepsilon}_\pm = \mp(\sqrt{\tfrac{1}{2}})(\boldsymbol{\varepsilon}_x \pm i\boldsymbol{\varepsilon}_y). \tag{1}$$

Plane-polarized photons with the electric vector perpendicular or parallel to the production plane have polarization vectors $\boldsymbol{\varepsilon}_\perp$ and $\boldsymbol{\varepsilon}_\parallel$, where

$$\begin{aligned} \boldsymbol{\varepsilon}_\perp &= \boldsymbol{\varepsilon}_y = i(\sqrt{\tfrac{1}{2}})(\boldsymbol{\varepsilon}_+ + \boldsymbol{\varepsilon}_-), \\ \boldsymbol{\varepsilon}_\parallel &= \boldsymbol{\varepsilon}_x = -(\sqrt{\tfrac{1}{2}})(\boldsymbol{\varepsilon}_+ - \boldsymbol{\varepsilon}_-). \end{aligned} \tag{2}$$

In terms of the s-channel helicity amplitudes $f_{0\lambda_2,\lambda_k\lambda_1}{}^s(\vartheta)$, where $\lambda_k = \pm 1$ and $\lambda_1 = \pm\frac{1}{2}$, the corresponding cross sections for unpolarized initial and final nucleons can be written as

$$\begin{aligned} \sigma_\perp(\vartheta) &= \tfrac{1}{2} \sum_{\lambda_1,\lambda_2} |f_{0\,\lambda_2,1\,\lambda_1}{}^s + f_{0\,\lambda_2,-1\lambda_1}{}^s|^2, \\ \sigma_\parallel(\vartheta) &= \tfrac{1}{2} \sum_{\lambda_1,\lambda_2} |f_{0\,\lambda_2,1\,\lambda_1}{}^s - f_{0\,\lambda_2,-1\,\lambda_1}{}^s|^2. \end{aligned} \tag{3}$$

In order to Reggeize, we need the corresponding t-channel helicity amplitudes $f_{\lambda_2\lambda_1,0\lambda_k}{}^t(\theta)$, where θ is the scattering angle in this channel. Using the crossing matrices of Trueman and Wick,[6] which take a relatively simple form since the photon is massless and the pion

* Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

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[2] P. Stichel, Z. Physik **180**, 170 (1964).

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[5] L. van Hove, Phys. Letters **24B**, 183 (1967); R. P. Feynman, Caltech lectures, 1967 (unpublished).

[6] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) **26**, 322 (1964).

spinless, we get

$$f_{0\lambda_2,\lambda_k\lambda_1}{}^s(\vartheta)=d_{\lambda_1',\lambda_1}{}^{1/2}(\chi_1)d_{\lambda_2',\lambda_2}{}^J(\chi_2)f_{0-\lambda_k,\lambda_2'\lambda_1'}{}^t(\theta). \quad (4)$$

Inserting this into Eq. (3) and using the orthogonality of the d functions, the expressions for the two cross sections take the following form:

$$\sigma_\perp(\vartheta)=\tfrac{1}{2}\sum_{\lambda_1,\lambda_2}|f_{0\,1,\lambda_2\lambda_1}{}^t+f_{0\,-1,\lambda_2\lambda_1}{}^t|^2 =\sum_{\lambda_2}|f_{0\,1,\lambda_2\frac{1}{2}}{}^t+f_{0\,-1,\lambda_2\frac{1}{2}}{}^t|^2, \quad (5)$$

$$\sigma_{\|}(\vartheta)=\tfrac{1}{2}\sum_{\lambda_1,\lambda_2}|f_{0\,1,\lambda_2\lambda_1}{}^t-f_{0\,-1,\lambda_2\lambda_1}{}^t|^2 =\sum_{\lambda_2}|f_{0\,1,\lambda_2\frac{1}{2}}{}^t-f_{0\,-1,\lambda_2\frac{1}{2}}{}^t|^2. \quad (6)$$

In the last step of these two equations, we have made use of parity invariance.

Application of the parity operator to the pion-photon helicity states gives

$$P|JM;\lambda_k0\rangle=(-1)^{J+1}|JM;-\lambda_k0\rangle. \quad (7)$$

Using this relation, we can find states with natural parity $P=(-1)^J$,

$$|JM;10\rangle_+=|JM;10\rangle-|JM;-10\rangle, \quad (8)$$

and states with unnatural parity $P=(-1)^{J+1}$,

$$|JM;10\rangle_-=|JM;10\rangle+|JM;-10\rangle. \quad (9)$$

A Jacob-Wick[4] expansion in the t channel,

$$f_{0\,-1,\lambda_2\frac{1}{2}}{}^t(\theta)=\sum_J(2J+1)F_{0\,-1,\lambda_2\frac{1}{2}}{}^J d_{\lambda 1}{}^J(\cos\theta), \quad (10)$$

where $\lambda=\lambda_2-\frac{1}{2}$, gives partial waves F^J which now can be expressed in terms of amplitudes carrying natural and unnatural parity from Eqs. (8) and (10),

$$F_{0\,1,\lambda_2\frac{1}{2}}{}^J=\tfrac{1}{2}(F_{0\,1,\lambda_2\frac{1}{2}}{}^{J-}+F_{0\,1,\lambda_2\frac{1}{2}}{}^{J+}),$$
$$F_{0-1,\lambda_2\frac{1}{2}}{}^J=\tfrac{1}{2}(F_{0\,1,\lambda_2\frac{1}{2}}{}^{J-}-F_{0\,1,\lambda_2\frac{1}{2}}{}^{J+}). \quad (11)$$

The helicity amplitudes entering the cross sections can now be written as

$$f_{0\,1,\lambda_2\frac{1}{2}}{}^t+f_{0\,-1,\lambda_2\frac{1}{2}}{}^t =\tfrac{1}{2}\sum_J(2J+1)[F_{0\,1,\lambda_2\frac{1}{2}}{}^{J+}(d_{\lambda\,-1}{}^J-d_{\lambda\,1}{}^J) +F_{0\,1,\lambda_2\frac{1}{2}}{}^{J-}(d_{\lambda\,-1}{}^J+d_{\lambda\,1}{}^J)],$$
$$f_{0\,1,\lambda_2\frac{1}{2}}{}^t-f_{0\,-1,\lambda_2\frac{1}{2}}{}^t =\tfrac{1}{2}\sum_J(2J+1)[F_{0\,1,\lambda_2\frac{1}{2}}{}^{J-}(d_{\lambda\,-1}{}^J-d_{\lambda\,1}{}^J) +F_{0\,1,\lambda_2\frac{1}{2}}{}^{J+}(d_{\lambda\,-1}{}^J+d_{\lambda\,1}{}^J)]. \quad (12)$$

Now, using the recurrence relations for the d functions,[4]

$$d_{M\pm1}{}^J(\cos\theta)=[J(J+1)]^{-1/2}\times\left(-\frac{M}{\sin\theta}\mp\frac{\partial}{\partial\theta}\right)d_{M0}{}^J(\cos\theta), \quad (13)$$

and letting $\cos\theta$ be very large, we find

$$d_{M\,-1}{}^J-d_{M1}{}^J=-2[J(J+1)]^{-1/2}\times\sin\theta d_{M0}{}^{J\prime}\sim(\cos\theta)^J,$$
$$d_{M\,-1}{}^J+d_{M1}{}^J=+2[J(J+1)]^{-1/2}\times(M/\sin\theta)d_{M0}{}^J\sim(\cos\theta)^{J-1}. \quad (14)$$

Since $\cos\theta\sim s$, we see from Eq. (12) that *to the leading power in s the cross section for photon polarization normal to the production plane is dominated by natural-parity exchange, and for parallel polarization, unnatural parity dominates.* This is Stichel's theorem. *It also follows that the theorem holds for pion photoproduction with any nucleon isobar in the final state.*

It is obvious that the theorem would be exact to all orders in s if the nucleons had instead been scalar particles.

In case the final nucleon has spin-parity $J^P=\frac{1}{2}^+$ and the same mass as the target nucleon, we can prove a more restrictive theorem. Inserting the crossing relation Eq. (4) into the amplitudes entering $\sigma_\perp$ and $\sigma_{\|}$, and carrying out the summations, we get

$$f_{0\,\frac{1}{2},1\,\frac{1}{2}}{}^s\pm f_{0\,\frac{1}{2},-1\,\frac{1}{2}}{}^s =+\cos\tfrac{1}{2}(\chi_1\mp\chi_2)(f_{0\,-1,\frac{1}{2}\,\frac{1}{2}}{}^t\pm f_{01,\frac{1}{2}\,\frac{1}{2}}{}^t) +\sin\tfrac{1}{2}(\chi_1\mp\chi_2)(f_{0\,-1,\frac{1}{2}\,-\frac{1}{2}}{}^t\pm f_{01,\frac{1}{2}\,-\frac{1}{2}}{}^t), \quad (15)$$
$$f_{0\,\frac{1}{2},1\,-\frac{1}{2}}{}^s\pm f_{0\,\frac{1}{2},\,-1\,-\frac{1}{2}}{}^s =-\sin\tfrac{1}{2}(\chi_1\mp\chi_2)(f_{0\,-1,\frac{1}{2}\,\frac{1}{2}}{}^t\pm f_{01,\frac{1}{2}\,\frac{1}{2}}{}^t) +\cos\tfrac{1}{2}(\chi_1\mp\chi_2)(f_{0\,-1,\frac{1}{2}\,-\frac{1}{2}}{}^t\pm f_{01,\frac{1}{2}\,-\frac{1}{2}}{}^t).$$

The crossing angles χ_1 and χ_2 are given by Wang.[7] Making the approximation $m_\pi{}^2/m_N{}^2=0$, we find the following relation between them:

$$\chi_1+\chi_2=\pi. \quad (16)$$

Looking back at Eq. (15), we see that this implies that not all the previous t-channel helicity amplitudes will contribute. In particular, the s-channel combination corresponding to parallel photon polarization and nucleons with opposite helicities is given by a pure unnatural spin-parity combination in the t channel. This is our new theorem, valid to all orders in s. *With photon polarization parallel to the production plane and with initial and final nucleons of opposite helicities, only unnatural spin-parity exchanges contribute to the cross section.*

FEYNMAN–VAN HOVE DESCRIPTION

According to this model,[5] the amplitude for the relevant process (Fig. 2) is given as a sum over spin-J meson exchanges,

$$A(s,t)=\sum_J A^J(s,t), \quad (17)$$

where

$$A^J(s,t)=G_J{}^{\gamma\pi}G_J{}^{NN}V_{\mu_1\cdots\mu_J}{}^{\gamma\pi}\times\Pi_{\mu_1\cdots\mu_J,\nu_1\cdots\nu_J}{}^J V_{\nu_1\cdots\nu_J}{}^{NN}. \quad (18)$$

[7] Ling-Lie Wang, Phys. Rev. 142, 1187 (1966).

Here $\Pi_{(\mu)(\nu)}{}^J$ is the propagator for a spin-J meson, completely symmetric in its indices (μ) and (ν), contracting the vertex functions $V_{(\mu)}{}^{\gamma\pi}$ and $V_{(\nu)}{}^{NN}$, which are also completely symmetric in their indices. $G_J{}^{\gamma\pi}$ and $G_J{}^{NN}$ are coupling constants.

$V_{(\mu)}{}^{\gamma\pi}$ is constructed from the photon polarization ϵ_μ and the momenta k_α and q_β and takes the following forms dependent on the spin-parity of the exchanged meson:

$$P=(-1)^J: \quad V_{(\mu)}{}^{\gamma\pi}=\{\epsilon_{\mu_1\alpha\beta\delta}k_\alpha q_\beta \epsilon_\delta (k_{\mu_2}+q_{\mu_2})\cdots (k_{\mu_J}+q_{\mu_J})\}_{\text{symm}}, \quad (19)$$

$$P=(-1)^{J+1}: \quad V_{(\mu)}{}^{\gamma\pi}=\{[\epsilon_{\mu_1}(kq)-k_{\mu_1}(\epsilon q)] \times (k_{\mu_2}+q_{\mu_2})\cdots(k_{\mu_J}+q_{\mu_J})\}_{\text{symm}}. \quad (20)$$

Both are gauge invariant, as they should be. The other vertex function $V_{(\nu)}{}^{NN}$ is also easily constructed and, in the simple case of a final $\frac{1}{2}^+$ nucleon, takes the form

$$P=(-1)^J: \quad V_{(\nu)}{}^{NN}=\{\bar{u}_2[\alpha(p_{\nu_1}{}^1+p_{\nu_1}{}^2)\cdots(p_{\nu_J}{}^1+p_{\nu_J}{}^2) +\beta\gamma_{\nu_1}(p_{\nu_2}{}^1+p_{\nu_2}{}^2)\cdots(p_{\nu_J}{}^1+p_{\nu_J}{}^2)]u_1\}_{\text{symm}}. \quad (21)$$

In case of unnatural-parity exchange $P=(-1)^{J+1}$, it contains an additional γ_5. Here, α and β are form factors.

Since the following proofs hold for each J separately, we will only consider the case when $J=1$. We also choose to work in the $\gamma\pi$ Breit frame, defined, as the coordinate system where $\mathbf{q}=-\mathbf{k}$, and we let x be the direction of the incoming photon as in Fig. 3. Then $k_\alpha=(k; k,0,0)$ and $q_\beta=(\omega; -k, 0, 0)$.

In case of natural-parity exchange, the $\gamma\pi$ vertex function Eq. (19) then reduces to

$$V_\mu{}^{\gamma\pi}=(\omega\epsilon_{\mu x t\delta}-k\epsilon_{\mu t x\delta})k\epsilon_\delta. \quad (22)$$

This vector has to be contracted with a vector on the nucleon side. From Eq. (21) we have two possibilities for this vector:

$$a_\mu=\bar{u}_2 u_1(p_1+p_2)_\mu, \qquad b_\mu=\bar{u}_2\gamma_\mu u_1. \quad (23)$$

The vector a_μ contributes only by its y index which, in turn, implies that the photon polarization ϵ_δ will contribute only through its component in the z direction, normal to the production plane. However, the second vector b_μ has a nonzero z component allowing ϵ_δ in the production plane $\epsilon_{\parallel}=\epsilon_y$ to contribute. But the part of the amplitude coming from $\epsilon_{\parallel}$ is small as seen when writing out the spinor matrix elements,

$$|(\epsilon_{\parallel}b)/(\epsilon_\perp a)|\simeq s^{-1}\sqrt{(-t)}, \quad (24)$$

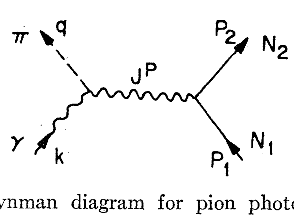

FIG. 2. Feynman diagram for pion photoproduction.

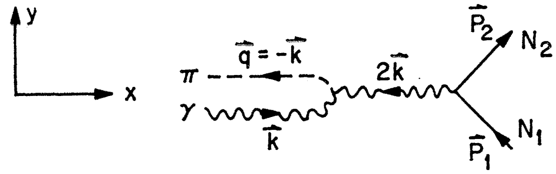

FIG. 3. Coordinates in $\gamma\pi$ Breit frame.

so that at high energies $\epsilon_\perp$ will dominate the amplitude for natural-parity exchange.

Similarly, we find that in the case of unnatural-parity exchange, $\epsilon_{\parallel}$ dominates the amplitude for large s.

For natural-parity exchange, the contribution from $\epsilon_{\parallel}$ came from b_z. In the approximation $m_\pi{}^2/m_N{}^2=0$, where $|\mathbf{p}_1|=|\mathbf{p}_2|=p$ and $E_1=E_2=E$, we have

$$b_z=\bar{u}_2\gamma_z u_1 =\chi_2{}^\dagger[(\boldsymbol{\sigma}\cdot\mathbf{p}_2)\sigma_z+\sigma_z(\boldsymbol{\sigma}\cdot\mathbf{p}_1)]\chi_1. \quad (25)$$

If now the nucleons have opposite helicities, e.g.,

$$(\sigma\cdot\hat{p}_1)\chi_1=-\chi_1, \quad (\boldsymbol{\sigma}\cdot\hat{p}_2)\chi_2=+\chi_2, \quad (26)$$

where $\hat{p}_{1(2)}=\mathbf{p}_{1(2)}/p$, then $b_z=0$. Also, if both nucleons are polarized normal to the production plane with spins in the same direction,

$$\sigma_z\chi_1=\chi_1, \quad \sigma_z\chi_2=\chi_2, \quad (27)$$

then again $b_z=0$ from Eq. (25) since $\mathbf{p}_1$ and $\mathbf{p}_2$ both are in the plane.

So when the nucleons have opposite helicities or are both polarized in the same direction normal to the production plane, $\epsilon_{\parallel}$ contributes only to unnatural-parity exchanges, independently of the photon energy.

Obviously this last theorem can also be proved in the helicity formalism but is not so easily come upon as in the Feynman-van Hove description.

CONCLUSION

We have reexamined Stichel's theorem connecting the photon polarization and the spin-parity of t-channel exchanges in pion photoproduction. In the particular case of photons polarized parallel to the production plane together with certain initial and final nucleon polarizations, we have proved that only states with unnatural spin parity will contribute to the cross section. In contrast to Stichel's theorem, this new observation is valid to all orders in s.

From an experimental point of view, this kind of photoproduction setup is difficult, but in principle there is nothing preventing it from being done with the present-day facilities. Hopefully, it would provide us with new reliable information about the strong and electromagnetic interactions.

ACKNOWLEDGMENT

We would like to thank Professor S. C. Frautschi for many helpful discussions.