Next we integrate [skipping steps similar to those in (C10)]

$$J_2=3\int_0^{2\pi}\int_{-1}^{1}\int_0^{2\pi}\int_{-1}^{1} d\cos\theta_A\, d\phi_A d\cos\theta_B\, d\theta_B\, f(\theta_{AB})(\hat{q}_A{}^B\cdot\hat{z})(\hat{q}_B{}^A\cdot\hat{z})$$

$$=12\pi\int_{-1}^{1} d\cos\theta_{AB}\frac{f(\theta_{AB})}{q_A{}^Bq_B{}^A}\left\{\int_{-1}^{1} d\cos\theta_A\int_{\cos(\theta_{AB}+\theta_A)}^{\cos(\theta_{AB}-\theta_A)} d\cos\theta_B\, J(\phi_{AB}|\cos\theta_{AB})\right.$$

$$\times[q_Aq_B(1+\mathbf{X}_A{}^B\mathbf{X}_B{}^A)\cos\theta_A\cos\theta_B+\mathbf{X}_B{}^Aq_A{}^2\cos^2\theta_A]$$

$$\left.+\int_{-1}^{1} d\cos\theta_B\int_{\cos(\theta_{AB}+\theta_A)}^{\cos(\theta_{AB}-\theta_A)} d\cos\theta_A\, J(\phi_{AB}|\cos\theta_{AB})\mathbf{X}_A{}^Bq_B{}^2\cos^2\theta_B\right\}$$

$$=8\pi^2\int_{-1}^{1} d\cos\theta_{AB}\frac{f(\theta_{AB})}{q_A{}^Bq_B{}^A}[q_Aq_B(1+\mathbf{X}_A{}^B\mathbf{X}_B{}^A)\cos\theta_{AB}+\mathbf{X}_B{}^Aq_A{}^2+\mathbf{X}_A{}^Bq_B{}^2]$$

$$=8\pi^2\int_{-1}^{1} d\cos\theta_{AB}\, f(\theta_{AB})\hat{q}_A{}^B\cdot\hat{q}_B{}^A=J_1. \qquad \text{(C11)}$$

Similarly, one obtains $J_1=J_2$ when $\hat{t}_A$ or $\hat{t}_B$ is $\hat{q}_A$ or $\hat{q}_B$ instead of $\hat{q}_A{}^B$ or $q_B{}^A$. The integral in (C1) is twice the integral J_1, and only the integral over $\cos\theta_{AB}$ remains to be performed numerically.

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Some Comments on Baryonic States

R. P. Feynman
California Institute of Technology, Pasadena, California 91109

AND

S. Pakvasa* and S. F. Tuan*
University of Hawaii, Honolulu, Hawaii 96822
(Received 15 June 1970)

We discuss some regularities in the baryon mass spectrum which have been suggested by one of us and possible experimental verification of them.

In this paper we should like to call attention to certain approximate regularities among the square masses of the baryons with the hope that future research can establish whether they are real or are the result of numerical accidents in the limited data available.

Our classification of states will be guided by the three-quark model of "baryons" and the principle of Regge recurrence. The states[1] of three quarks each of spin $\frac{1}{2}$ depend on the symmetry character of the state. If it is symmetric, it is a **56** (consisting of a spin-quartet unitary-spin decimet, $^4 10$, and a spin-doublet unitary-spin octet, $^2 8$). If it is antisymmetric it is a **20** (spin-doublet unitary-spin octet, $^2 8$, and a spin-quartet singlet, $^4 1$). For the intermediate symmetry, we have the double representation of a $\mathbf{70}={}^2 1, {}^2 8, {}^4 8, {}^2 10$.

We next suppose that the over-all state is entirely symmetric. If we add internal degrees of freedom, we suppose that the lowest states are the s states, themselves symmetric and of zero angular momentum. Thus our lowest states are

$$(\mathbf{56},0^+)={}^2 8_{1/2}{}^+,\ {}^4 10_{3/2}{}^+,$$

where the ${}^a b_j{}^p$ give spin multiplicity a, unitary spin multiplicity b, parity p, and angular momentum j of the states. These, of course, are taken to be the fundamental octet and the lowest decimet (with $\Delta=1236$).

We may expect this to recur on a Regge trajectory[2] by adding 2, 4, ... units of angular momentum (which

* Work supported in part by the U. S. Atomic Energy Commission.

[1] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[2] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **7**, 394 (1961); **8**, 41 (1962).

TABLE I. Particles of unknown spin are predicted by guesswork (Gell-Mann–Okubo plus Regge recurrence) to complete a multiplet and carry an asterisk. The established states are taken where possible from the Review of Particle Properties, A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970). After each particle is the square of the mass, in GeV^2.

$J^P=(1/2)^-$	$(1/2)^+$	$(3/2)^-$	$(3/2)^+$	$(5/2)^-$	$(5/2)^+$	$(7/2)^-$	$(7/2)^+$
8	**8**	**1**	**10**	**8**	**8**	**1**	**10**
N(1535) 2.36 Λ(1670) 2.79	N(938) 0.88 Λ(1115) 1.24 Σ(1190) 1.42 Ξ(1314) 1.74	Λ(1520) 2.30	Δ(1236) 1.53 Σ(1385) 1.92 Ξ(1530) 2.34 Ω^-(1672)* 2.80	N(1670) 2.79 Σ(1765) 3.12 Λ(1830) 3.37 Ξ(1930)* 3.72	N(1688) 2.85 Λ(1815) 3.30 Σ(1915) 3.65	Λ(2100) 4.41	Δ(1950) 3.80 Σ(2030) 4.12
1 Λ(1405) 1.97		**8** N(1520) 2.31 Λ(1690) 2.86 Σ(1670) 2.79			**10** Δ(1890) 3.57	**8** N(2190) 4.80	**8** [N(1990) 3.96]
10 Δ(1650) 2.72 Σ(1750) 3.06	**8** N(1470) 2.16	Ξ(1820)* 3.31	**8** [N(1860) 3.46]				
8 N(1700) 2.89	**8** N(1780) 3.17	**10** Δ(1670) 2.79					
	10 Δ(1910) 3.65	**8** [N(2040) 4.16]					

States of unknown spin and parity: N(2650) 7.02, N(3030) 9.18
Δ(2420) 5.86 (1/2+); Δ(2850) 8.12; Δ(3230) 10.4
Λ(2350) 5.86
Σ(2250) 5.06; Σ(2455) 6.03; Σ(2595) 6.73
Ξ(2030) 4.12; Ξ(2750) 5.06; Ξ(2500) 6.25

we shall call orbital angular momentum). Depending on how this adds to the spin, we get various total angular momenta and expect

$$(\mathbf{56},2^+)={}^2\mathbf{8}_{5/2}{}^+,\ {}^2\mathbf{8}_{3/2}{}^+;\ {}^4\mathbf{10}_{7/2}{}^+,\ {}^4\mathbf{10}_{5/2}{}^+,\ {}^4\mathbf{10}_{3/2}{}^+,\ {}^4\mathbf{10}_{1/2}{}^+.$$

As we shall see (cf. Tables I and II), we have candidates for most of these states, and they seem to lie (in mass squared) about 2.1 GeV^2 above the corresponding $(\mathbf{56},0^+)$ states.

The odd-parity states are not understood by simply adding an "external" odd L to the $(\mathbf{56},0^+)$, but rather by supposing that this L is internal and combines with the spin and unitary symmetry to make a totally symmetric state. For example, the first $L=1^-$ state is undoubtedly of intermediate symmetry and must therefore combine with the **70** to form a totally symmetric state, so the lowest odd-parity states are

$$(\mathbf{70},1^-)={}^2\mathbf{1}_{3/2}{}^-,\ {}^2\mathbf{1}_{1/2}{}^-;\ {}^2\mathbf{8}_{3/2}{}^-,\ {}^2\mathbf{8}_{1/2}{}^-;\ {}^4\mathbf{8}_{5/2}{}^-,\ {}^4\mathbf{8}_{3/2}{}^-,\ {}^4\mathbf{8}_{1/2}{}^-;\ {}^2\mathbf{10}_{3/2}{}^-,\ {}^2\mathbf{10}_{1/2}{}^-.$$

These will be compared with the data in Table II.

TABLE II. The position in the vertical boxes is guided by mass and parity [and $SU(6)$ representations with appropriate orbital L]. The numbers given for the **8** octets are the masses of the nucleon member squared (because that is most often available experimentally); for **10** the masses are the $(M_\Delta)^2$; for **1**, of course, the mass is $(M_\Lambda)^2$.

$(SU(6), L^P)\diagdown J$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
(56, 0^+)	8 0.88	10 1.53			
(70, 1^-)	1 1.97	1 2.30			
	8 2.36	8 2.31			
	8 2.89	8	8 2.79		
	10 2.72	10 2.79			
(56, 2^+)		8	8 2.85		
	10 3.65	10	10 3.57	10 3.80	
(70, 3^-)			1	1 4.41	
			8	8	
		8	8	8 4.80	8
			10	10	

They also should have Regge recurrences by combining **70** with 3^- or 5^-, etc.

If orbital angular momentum is supposedly distributed among the three quarks so that the total state is symmetrical, there are additional states with double excitation of orbital momentum [for example, the first to arise along with the above-mentioned recurrence $(\mathbf{56},2^+)$ is another $(\mathbf{56},0^+)$, $(\mathbf{70},2^+)$, $(\mathbf{70},0^+)$, and $(\mathbf{70},1^-)$, but we have not identified these as necessary from the data, as yet]. (Except see comments on the Roper resonance below.)

We have looked for empirical relations among the squares of the masses so that we could better see to what extent linear Regge trajectories[2] might fit the data.

This has the disadvantage that the nearly equal spacing of the decimet masses will have to be taken as an accident, replaced by an empirical formula, say, that the square masses are equally spaced, which is a less satisfactory fit. The first approximation of the quark model with an extra mass associated with the strange quark is that the mass of the Σ and Λ are equal. We shall suppose that the Σ, Λ splitting is mysterious and use the empirical relation

$$\tfrac{1}{2}(\Sigma^2+\Lambda^2)=\tfrac{1}{2}(N^2+\Xi^2) \qquad (1)$$

for the masses of the octet. This is numerically nearly the same as the Gell-Mann–Okubo formula for the mass, $\frac{1}{4}(\Sigma+3\Lambda)=\frac{1}{2}(N+\Xi)$ (because of numerical coincidences).

The squares of the masses of the particles and resonances for which the spin and parity are known or can be surmised are indicated in Table I. In Table II we have associated these particles with the states that we have expected. There are some nucleon excited states of high energy for J^P, which have not yet been established as well as the others. They do not fit at all well into our scheme, and we have omitted them from Table II. In Table I they are in brackets [ ].

There is no way experimentally to tell if a Λ is a singlet **1** or an octet **8** (by just looking at the mass); it is often easy using the Gell-Mann–Okubo mass formula for octets, $\frac{1}{2}(M_N+N_\Xi)=\frac{3}{4}M_\Lambda+\frac{1}{4}M_\Sigma$. Likewise, Σ and Ξ appear in both **8** and **10**, but it is usually easy to guess to which a particular one belongs.

In Table I, particles of unknown spin parity located by guesswork[3–5] (Gell-Mann–Okubo plus Regge recurrence) carry an asterisk.

Let us define[4] the spread of multiplet per unit strangeness $=\Delta m^2/\Delta S=c$; thus, for octet and decuplet,

$$c_8=\tfrac{1}{2}(\Xi^2-N^2)\cong\tfrac{1}{2}(\Sigma^2+\Lambda^2)-N^2, \qquad (2)$$

$$c_{10}=\tfrac{1}{3}(\Omega^2-\Delta^2)=\tfrac{1}{2}(\Xi^2-\Delta^2)=(\Sigma^2-\Delta^2). \qquad (3)$$

This last relation is not perfect for the 1236 decimet, giving $c_{10}=0.42$, 0.40, and 0.39, respectively. It has been observed that c may be a universal constant,[4,6] at least for a given kind of multiplet; for instance,

$$c_8\approx\tfrac{1}{2}(\Sigma^2+\Lambda^2)-N^2$$
$$=0.45\ (\tfrac{1}{2}^+),\ 0.51\ (\tfrac{3}{2}^-),\ 0.45\ (\tfrac{5}{2}^-),\ 0.61\ (\tfrac{5}{2}^+); \qquad (4)$$

$$c_{10}\approx\Sigma^2-\Delta^2=0.39\ (\tfrac{3}{2}^+),\ 0.32\ (\tfrac{7}{2}^+),\ 0.34\ (\tfrac{1}{2}^-). \qquad (5)$$

REMARK

The universality-of-c_8 ansatz is more consistent with a solution for the $J^P=\frac{5}{2}^+$ octet [$N(1688)$, $\Lambda(1815)$, $\Xi(1960)$, $\Sigma(1844)$?] than the currently fashionable solution [$N(1688)$, $\Lambda(1815)$, $\Sigma(1915)$, $\Xi(1992)$?]. The experimental possibility of two Ξ resonances in the 1930–1960-MeV region with $J^P=\frac{5}{2}^+$ and $\frac{5}{2}^-$ is certainly not ruled out.[7] It seems reasonable at least to discuss the possible detachment of $\Sigma(1915)$ away from the $J^P=\frac{5}{2}^+$ octet and reassign it to another $SU(3)$ multiplet. Unfortunately, assignment of $\Sigma(1915)$ to the $J^P=\frac{5}{2}^+$ decimet does not seem natural either, since taken together with $\Delta(1880)$ (cf. Table I), $c_{10}\approx0.10$. *We urge therefore that an accurate spin-parity measurement be made for the $\Sigma(1915)$ state.*[8]

$SU(6)$-PLUS-ORBITAL-ANGULAR-MOMENTUM CLASSIFICATION

In Table II we present a chart of baryon multiplets in terms of spin J versus [$SU(6)$,L^P] classification. The entries are taken from *established* N, Δ, and Λ baryonic states of Table I, representing, respectively, the **8**, **10**, and **1** $SU(3)$ representations of given spin parity. The omissions from Table II of states regarded as "established" in Table I are the $J^P=\frac{1}{2}^+$ states $N(1460)$, $N(1780)$. These are *not* expected on the basis of the simplest hypothesis given by the chart of Table II. Dalitz[3] proposed that the P_{11} Roper resonance $N(1460)$ and an experimentally hinted P_{33} $\Delta(1688)$ state be regarded as the radial excitation $(\mathbf{56},L=0^+)_{N=2}$ of the familiar ground state $(\mathbf{56},L=0^+)$ with $N=0$ typified by $N(939)$ and $\Delta(1236)$. Likewise, the $N(1785)$ state together with a hinted[9] P_{33} $\Delta(2160)$ can be part of a further radial excitation $(\mathbf{56},L=0^+)_{N=4}$. Here $N=0$, 2, 4, ... denotes the number of quanta of excitation in the harmonic-oscillator picture. There now exists evidence for *possible* partners to the Roper $N(1460)$ state, for instance, the $\Sigma(1620)$.[10] However, one might hope that these secondary classifications involving other internally excited states for the **56**, **70**, and **20** will exhibit sufficiently different characteristics in terms of production, elasticity, and even mass relations[11] that they can be sorted out of the superimposed primary classifications given by Table II.

Among the "primary" states of the $SU(6)$ quark model with L excitation (but without "radial" excitations) given by Table II, the following rules give a good accounting of things.

(a) Regge corresponding multiplets which are equal in parity, but differ by the addition of two units of J, add close to 2.10 GeV^2 in M^2, i.e., the slope of the Regge trajectories is close to $1/1.05=0.95$. This is exactly the same as the slope of the ρ-meson trajectory. For instance, the Regge recurrence of the $J^P=\frac{3}{2}^-$ $N_\gamma(1520)$ is *not* the $N(2190)$ with $J^P=\frac{7}{2}^-$ according to this rule. Rather, a new nucleon state with $J^P=\frac{7}{2}^-$ is expected at mass 2100 MeV ($M^2\cong4.4$) to be the Regge recurrence of $N_\gamma(1520)$; the known $N(2190)$ is then the Regge recurrence of a nucleon D_{13} state at about 1670 to 1700 MeV, for which there already exists some experimental evidence.[3,12]

(b) Possible multiplets of the same type and same parity (i.e., the same horizontal line in Table II) are *degenerate*, with one notable exception: $\mathbf{1}(J^P=\frac{1}{2}^-)\neq\mathbf{1}(J^P=\frac{3}{2}^-)$. This observation, stressed particularly by one of us,[4] leads to a rather concrete physical picture in the quark model[3,13] with very specific experimental predictions. We expect the following.

[3] R. H. Dalitz, in *Proceedings of the International Conference on Symmetries and Quark Models* (Gordon and Breach, New York, 1970).
[4] R. P. Feyman, Caltech Lecture Notes, 1969 (unpublished).
[5] S. Pakvasa and S. F. Tuan, Nucl. Phys. **B8**, 95 (1968).
[6] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **3**, 125 (1966) [Soviet Phys. JETP Letters **3**, 79 (1966)]; S. Matsuda and S. Oneda, Phys. Rev. D **1**, 944 (1970).
[7] See, e.g., S. Apsell *et al.*, Phys. Rev. Letters **24**, 777 (1970).
[8] The Σ state here may well be related to a D_{13} $\Sigma(1940)$ claimed recently [see A. Barbaro-Galtieri, in *Proceedings of Hyperon Resonances—70* (Moore, Durham, N. C., 1970), p. 173. See also V. E. Barnes *et al.*, Phys. Rev. Letters **22**, 479 (1969).
[9] A. Barbaro-Galtieri *et al.*, Rev. Mod. Phys. **42**, 87 (1970).
[10] D. J. Crennell *et al.*, Phys. Rev. Letters **21**, 648 (1968); A. C. Ammann *et al.*, *ibid.* **24**, 327 (1970).
[11] Indeed, the experimental evidence for a $\Xi(1630)$ with $T=\frac{1}{2}$ [another partner for the Roper $N(1460)$?] does suggest an altered mass relation as emphasized recently by S. Meshkov [in *Proceedings of Hyperon Resonances—70* (Moore, Durham, N. C., 1970), p. 471].
[12] A. Brody *et al.*, Phys. Rev. Letters **22**, 1401 (1969); C. Lovelace and F. Wagner (unpublished).
[13] R. H. Dalitz, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics* (Hawaii U. P., Honolulu, Hawaii, 1968), p. 325.

(i) An octet $J^P=\frac{3}{2}^-$ nucleon resonance N, nearly mass degenerate with the $J^P=\frac{1}{2}^-$ $N(1700)$ and the $J^P=\frac{5}{2}^-$ $N(1670)$ octet counterparts. As mentioned earlier, there does exist preliminary evidence for a D_{13} nucleon resonance at about 1670–1700 MeV.[12,3] Clearly, establishment of this state at this particular energy is of *substantial interest.*

(ii) An octet nucleon state with $J^P=\frac{3}{2}^+$, nearly mass degenerate with the $J^P=\frac{5}{2}^+$ $N(1690)$, is suggested. The present experimental data[9] claim a P_{13} state, but its mass is far away at about 1860 MeV.

(iii) A Δ state with $J^P=\frac{3}{2}^+$, nearly mass degenerate with $J^P=\frac{1}{2}^+$ $\Delta(1910)$, $J^P=\frac{5}{2}^+$ $\Delta(1890)$, and $J^P=\frac{7}{2}^+$ $\Delta(1950)$. The search for such a state in the 1900-MeV region is of prime importance.

(iv) A series of nucleon states N, with $J^P=\frac{3}{2}^-$, $J^P=\frac{5}{2}^-$, and $J^P=\frac{9}{2}^-$ at about 2200 MeV to complement the $J^P=\frac{7}{2}^-$ $N(2190)$.

We might add here also that in the absence of strong mixing in particle-mass spectra, the same type of near mass degeneracies is expected for the hyperon partners of the nucleon states. For instance, a recently found S_{01} Λ state[14] at 1850–1860 MeV can be associated with the $J^P=\frac{5}{2}^-$ Λ state at 1830 MeV [the Λ partner of $N(1670)$ in (i)]; the search for the D_{03} Λ member in the 1800–1860-MeV mass region is evidently of interest. Likewise, a search for $J^P=\frac{1}{2}^-$ and $J^P=\frac{3}{2}^-$ states in the Σ system to go with $J^P=\frac{5}{2}^-$ $\Sigma(1765)$ [the Σ partner of $N(1675)$ in (i)] is evidently desirable in the 1765–1800-MeV region. Some of these states are likely to be highly inelastic with small coupling to the initial $\bar{K}N$ channel; hence an *extensive* search from partial-wave analysis is very likely needed.

The physical content of statements (i)–(iv) becomes much more apparent when one recognizes that they correspond, respectively, to the **8** ^4P of **70**, $L=1^-$; **8** ^2D of **56**, $L=2^+$; **10** ^4D of **56**, $L=2^+$; and **8** ^4F of **70**, $L=3^-$, in the language of the $SU(6)$ quark model.[13] The near degeneracy of members belonging to ^4P, ^2D, ^4D, and ^4F is then *the statement that the spin-orbit contribution to the mass splitting in the quark model is small.*[15]

The exceptional case of $\mathbf{1}(J^P=\frac{1}{2}^-)\neq\mathbf{1}(J^P=\frac{3}{2}^-)$ [and hence perhaps also of the Regge recurrent states $\mathbf{1}(J^P=\frac{5}{2}^-)$ and the $\mathbf{1}(J^P=\frac{7}{2}^-)$, the latter possibly the $\Lambda(2100)$] is somewhat of a puzzle. Here the $\Lambda(1519)$ and $\Lambda(1405)$ spin-orbit splitting is 114 MeV, rather more substantial than what would be expected from our conjecture. Perhaps it is related to the substantial lack of purity of these states as $SU(3)$ singlets, as has been suggested from decay considerations.[16] The problem is evidently one deserving further study.[17]

We must emphasize again that the nucleon excited states $P_{13}(1860)$, $F_{17}(1990)$, and $D_{13}(2040)$, although fairly well established, do not fit at all well into our scheme.

We have no rule for the Σ, Λ mass difference, except possibly that the sign of $\Sigma^2-\Lambda^2$ is that of the parity of the state.

The states in this system can be represented approximately by a nonrelativistic harmonic-oscillator model. Walker[18] has emphasized that the photoelectric matrix elements calculated from the wave functions of this model (with two arbitrary constants other than the masses) agree qualitatively to a surprising degree. One of us (R. P. F.) has made a relativistic modification which leaves no arbitrary constants other than the masses of the resonances, and finds the agreement is, if anything, slightly enhanced.

CONCLUSION

We emphasize again that the assumption of no strong mixing in baryonic mass spectra remains consistent with empirical data. In particular, the regularities emphasized here offer specific experimental tests of mass degeneracies in the baryonic states due to the absence of large spin-orbit effects in the quark model picture. It appears possible that all hyperon Regge trajectories have the same slope, 0.95 GeV^{-2}.

ACKNOWLEDGMENT

One of us (R. P. F.) would like to thank Professor George Zweig for discussions concerning regularities in baryon mass spectra.

[14] D. Plane, in *Proceedings of Hyperon Resonances—70* (Moore, Durham, N. C., 1970), pp. 148–149. We are indebted to Professor R. H. Dalitz for this observation.

[15] As emphasized in Ref. 13, there is not yet an understanding of these spin-orbit contributions (at least of the simple $\mathbf{L}\cdot\mathbf{S}$ type) in detail. The current data are, however, in reasonable accord with the *assumption* of *small* spin-orbit contributions to mass splitting.

[16] G. Yodh, Phys. Rev. Letters **18**, 810 (1967); see also R. D. Tripp, in *Proceedings of the Third Hawaii Topical Conference in Particle Physics* (Western Periodicals, Los Angeles, 1970).

[17] In terms of the realistic heavy-quark model (sometimes called the naive quark model), a question has been raised whether the $\Lambda(1405)$ should or should not be classified under the quark model. See the review by R. H. Dalitz, in paper presented at Duke Hyperon Resonance Conference, 1970 (unpublished).

[18] R. L. Walker, in Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969, p. 23 (unpublished).