Discrepancy between Soft-Pion Predictions and Experiment in $NN \to NN\pi$*

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Sources of a discrepancy between experiment and model-independent soft-pion predictions for the production of low-energy pions in the reaction $pp \to np\pi^+$ at 740 MeV are examined and discussed. These predictions deal only with the leading behavior of the production amplitude. However, higher-order terms, present because $\mu \neq 0$, seem to dominate because of resonance effects. An extended version of the Mandelstam resonance model appears sufficient to explain the experimental data, even for the lowest-energy pions. Ambiguities in pion production at threshold are also discussed.

I. INTRODUCTION

A RECENT experiment[1] has compared the production of low-energy pions in the reaction $pp \to np\pi^+$ at $T_{p,\mathrm{lab}} \approx 740$ MeV with soft-pion-emission calculations.[2] The preliminary cross sections are almost an order of magnitude larger than the presumably model-independent predictions.[3]

In view of this large discrepancy, the validity of the low-energy theorem used by GSS in their calculations might be questioned. The principal ingredient used in proving the theorem is the hypothesis of a partially conserved axial-vector current (PCAC). Because of the many successes of the PCAC hypothesis over the last decade,[4] this is a serious matter. One is impelled to see if an explanation of the discrepancy can be found elsewhere.

We have reexamined the soft-pion calculations and suggest that the discrepancy is probably due to the resonance effects of the $\Delta(1236)$ isobar. The outline of this paper is as follows: In Sec. II we briefly review the basis of the low-energy theorem and how it was applied in the calculations of GSS; the nature of the discrepancy with experiment is also presented. In Sec. III, what ambiguities there are in the calculation are examined in some detail and results for the charge ratio $pp \to pp\pi^0$ to $pp \to np\pi^+$ are also discussed. In Sec. IV, we investigate whether isobar effects[5,6] could unexpectedly be masking the soft-pion emission. It appears that they do, and, at least in our crude calculations, can account for all of the observed discrepancy. Finally, in Sec. V, we consider where, if anywhere, the soft-pion-emission theorem might be useful for this reaction in view of the strong resonance effects.

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[1] D. R. F. Cochran, P. N. Dean, P. A. M. Gram, E. A. Knapp, E. R. Martin, D. E. Nagle, R. B. Perkins, W. J. Schlaer, E. D. Theriot, and H. A. Thiessen (private communication).

[2] C. T. Grant, M. E. Schillaci, and R. R. Silbar, Phys. Rev. **184**, 1737 (1969), hereafter referred to as GSS.

[3] H. A. Thiessen, Bull. Am. Phys. Soc. **14**, 1198 (1969).

[4] See, e. g., S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[5] S. J. Lindenbaum and R. H. Sternheimer, Phys. Rev. **105**, 1874 (1957).

[6] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).

II. SOFT-PION-EMISSION THEOREM AND CALCULATION

The amplitude for the pion production reaction $pp \to np\pi^+$ (with four-momenta labeled as in Fig. 1) may be expanded in a Taylor's series about the soft-pion point $q_\alpha = 0$. Schematically,

$$T_{np\pi^+} = A + Bq + O(q^2). \tag{1}$$

The soft-pion-emission theorem,[7] in fairly close analogy[8] to the low-energy theorem for nucleon-nucleon bremsstrahlung,[9,10] provides a prescription for evaluating the leading term A in terms of on-shell nucleon-nucleon scattering quantities. Essentially, only those graphs in which the pion is emitted from an external line [Figs. 2(a) and 2(b)] contribute to A. The πNN coupling is axial vector, $\gamma \cdot q\gamma_5$, and this, together with the virtual-nucleon propagator, gives the leading behavior. For example, for Fig. 2(a), the πNN coupling and nucleon propagator contribute a factor

$$\gamma \cdot q\gamma_5 / [(p_1 - q)^2 - m^2] = -\gamma \cdot q\gamma_5 / (2p_1 \cdot q - q^2). \tag{2}$$

This is mathematically ambiguous as $q_\alpha \to 0$. Physically, a definite way of taking this limit is indicated: First let $\mathbf{q} \to 0$ (take the pion to rest) and then let $q_0 = \mu \to 0$. This gives a well-defined procedure for the evaluation of the A term from the external emission

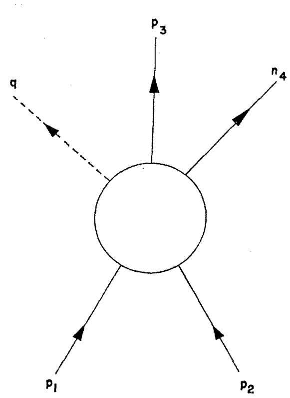

FIG. 1. Labeling of the four-momenta for the pion production reaction $pp \to np\pi^+$.

[7] S. L. Adler, Phys. Rev. **139**, B1638 (1965).

[8] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).

[9] F. E. Low, Phys. Rev. **110**, 974 (1958).

[10] E. Nyman, Phys. Rev. **170**, 1628 (1968).

graphs. Non-Born terms, such as those of Figs. 2(c) and 2(d), can contribute only to the higher-order terms in Eq. (1).

This theorem is most easily proved using the PCAC hypothesis directly.[7] That the pion-nucleon coupling to the external nucleon lines is axial-vector rather than pseudoscalar is, in this derivation, a result of the use of PCAC. Indeed, one can see how in a theory with pseudoscalar πNN coupling consistency with PCAC forces the leading behavior of internal- and external-emission contributions coalesce to give effective external emission with axial-vector coupling.[8] Alternatively, exact chirality conservation can be used to derive the theorem.[11]

In accord with this theorem, GSS calculated the leading behavior of the production amplitude by evaluating the graphs shown in Figs. 2(a) and 2(b). The identity of the incident protons requires appropriate antisymmetrization. The axial-vector $pn\pi^+$ coupling constant is $g\sqrt{2}/2m$, where $g^2/4\pi=14.8$. The $NN\rightarrow NN$ scattering vertices in the diagrams are specified by the usual five invariant functions,[12] which were computed[10] in terms of the NN scattering phase shifts.[13,14] GSS took

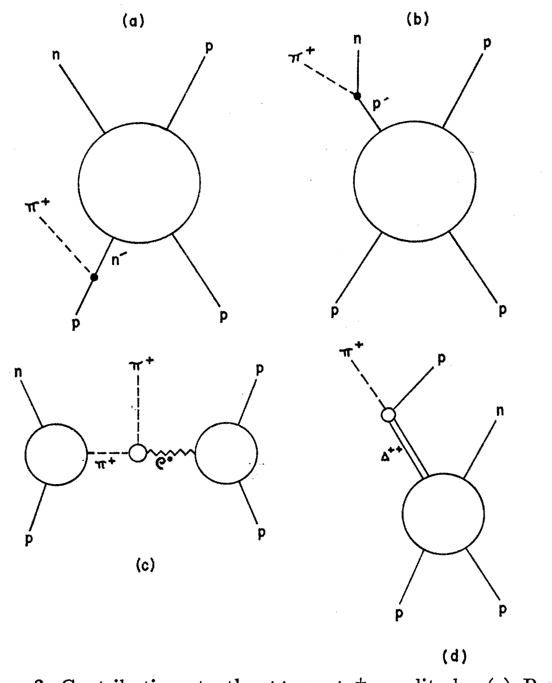

FIG. 2. Contributions to the $pp\rightarrow np\pi^+$ amplitude. (a) Pre-emission from an incoming, external nucleon line. (b) Post-emission from an outgoing nucleon line. (c) An internal-emission contribution. (d) Isobar resonance contribution.

[11] See Adler and Dashen, Ref. 4, p. 112, or the earlier original treatments of Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962); Y. Nambu and E. Shrauner, *ibid.* **128**, 862 (1962). The theorem has also been derived using a gauge condition for zero four-momentum pions by R. Baier and H. Kühnelt, Nuovo Cimento **63**, 135 (1969).

[12] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960).

[13] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. **169**, 1149 (1968); **173**, 1272 (1968).

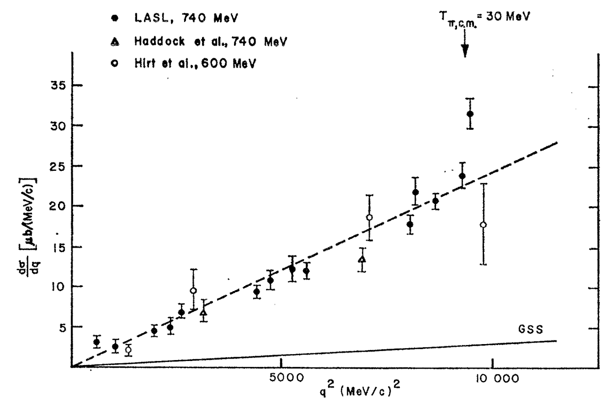

FIG. 3. Low end of the pion spectrum, $d\sigma/dq$, with q, the pion three-momentum in the over-all c.m. frame, plotted versus q^2. Experimental data are from counter experiments of Los Alamos (Ref. 1) and UCLA-Berkeley (Ref. 15) at $T_{p,\mathrm{lab}}\approx 740$ MeV and of CERN (Ref. 16) at 600 MeV. (The Los Alamos data are preliminary.) Soft-pion predictions of GSS (Ref. 2) are shown as a solid line; dashed line through the data has a slope about eight times that of the predictions.

the pion to be *physical and at rest* in the over-all c.m. frame $q_\alpha=(\mu,\mathbf{0})$. This forces the virtual nucleon to be off its mass shell. As in the case of bremsstrahlung $NN\rightarrow NN\gamma$,[10] an on-shell approximation is made; in GSS the NN invariant functions were evaluated at an energy which was the average of the two-nucleon energies before and after the off-shell scattering. By squaring this approximate production amplitude, summing over spins, and carrying out the phase-space integrations, GSS then obtained the cross section $d\sigma/dE_{\pi,\mathrm{c.m.}}$ near $E_{\pi,\mathrm{c.m.}}=\mu$ *with no free parameters.*

As stated in the Introduction, the results of the Los Alamos pion-production experiment[1] are in strong disagreement with these predictions. For the data on pions produced with kinetic energies less than 30 MeV in the c.m. system, the comparison between experiment and theory[3] is shown in Fig. 3. The reason for plotting the low end of the pion spectrum in this way—$d\sigma/dq$ versus q^2, with q the magnitude of the pion three-momentum in the over-all c.m. frame—is that if the production amplitude is independent of q, then phase-space factors predict a straight-line dependence. The slope of this line, which passes through the origin, is fixed by the squared production amplitude. The experimental data do lie approximately on a straight line, but with a slope about eight times that predicted by GSS.

Figure 3 also shows some unpublished data from other counter experiments at $T_{p,\mathrm{lab}}=740$ [15] and 600 MeV.[16] The experiment at a different energy involves

[14] Z. Janout, Yu. M. Kazarinov, F. Lehar, and A. M. Rozanova, Nucl. Phys. **A127**, 449 (1969).

[15] R. P. Haddock, M. Zeller, and K. M. Crowe, UCLA Report No. MPG-64-1P (unpublished).

[16] E. Heer, W. Hirt, M. Martin, E. G. Michaelis, C. Serre, P. Skarek, and B. T. Wright, in *Proceedings of the Conference on Intermediate Energy Physics, 1966* (College of William and Mary, Williamsburg, Va., 1967), p. 277; W. Hirt, thesis, Eidgenosse Technische Hochschule, Zurich, 1968 (unpublished).

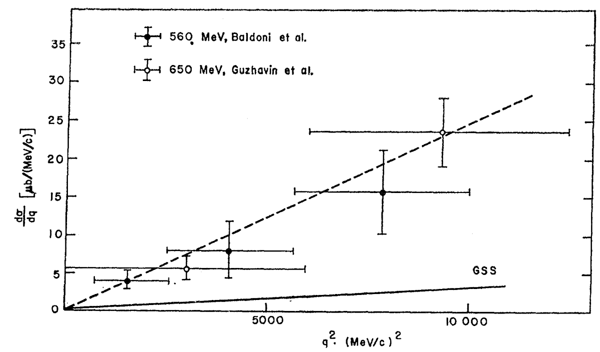

FIG. 4. Same as Fig. 3, but with experimental data from bubble-chamber experiments at 560 MeV (Ref. 17) and 650 MeV (Ref. 18). There is a total of about 30 events that constitute these data points.

no problems in this comparison, since the GSS predictions are quite insensitive to $T_{p,\mathrm{lab}}$ from 500 to 800 MeV. There also exist a few published bubble-chamber data for production of low-energy pions at $T_{p,\mathrm{lab}}=560$ [17] and 650 MeV.[18] These are shown in Fig. 4, again corroborating the Los Alamos experiment (but with very low statistics). The conclusion that experiment disagrees with the published soft-pion predictions is unavoidable.

III. AMBIGUITIES IN SOFT-PION CALCULATION

Let us consider now how sensitive the GSS soft-pion predictions are to the details of the calculation.

First, we note a number of small points regarding the calculation: (a) The GSS formulas, evaluating the helicity amplitudes for the reaction, can be applied to any incident energy. We have compared the GSS predictions for threshold, $T_{p,\mathrm{lab}}\approx 300$ MeV, with our earlier threshold calculation[19] that uses trace techniques for the spin-averaged squared amplitude. They agree with each other. (b) An important ingredient in the GSS calculation is the NN phase-shift information, which is not to be particularly trusted in this energy range well above the inelastic threshold. We have found that the predictions change by $\approx 15\%$ when the Dubna phase shifts[14] are used in place of the Livermore phase shifts.[13] (c) Other reasonable choices of the on-shell point for the evaluation of the NN invariant functions give, for these energies, typical variations in $d\sigma/dE_{\pi,\mathrm{c.m.}}$ of $\approx 15\%$. (Near threshold,[19,20] $T_{p,\mathrm{lab}}\approx 300$ MeV, the choice of on-shell point is rather more serious.) It appears from this that the discrepancy is not due to something so simple.

A more serious ambiguity of the GSS calculation is that it is frame dependent, the pion being taken at rest in the c.m. frame. The soft-pion-emission theorem, on the other hand, is frame independent, since a zero four-momentum pion remains such under all Lorentz transformations. It is more in the spirit of the theorem to evaluate the A term of the amplitude for a truly soft pion, $q_\alpha=(0,\mathbf{0})$. This can be easily done (in the sense of $\mathbf{q}\rightarrow 0$ first, then $q_0=\mu\rightarrow 0$) by setting $\mu=0$ in the formulas given by GSS. Figure 5 shows the comparison between the $\mu=0$ and 140 MeV cases of the squared, spin-averaged amplitude as a function of incident proton energy.

A first conclusion to be seen from Fig. 5 is that the difference between the two cases becomes small as the incident energy grows. At $T_{p,\mathrm{lab}}\approx 740$ MeV, this difference is $\approx 10\%$. Indeed, Fig. 5 would seem to show that, for now, the best place to check the soft-pion-emission theorem is at an energy like 740 MeV. Here the extrapolation from $\mu=0$ to 140 MeV is a small effect. At the same time, the energy is low enough that we still have input information from the NN phase-shift analyses[13,14] (which presently cut off at 750 MeV).

Incidentally, Fig. 5 also shows that, near threshold, the physical ($\mu=140$ MeV) squared amplitude is some 30 times larger than the $\mu=0$ squared amplitude.[21] This casts some doubt on the apparently successful threshold calculations of $NN\rightarrow NN\pi$.[19,20] On the other hand, a

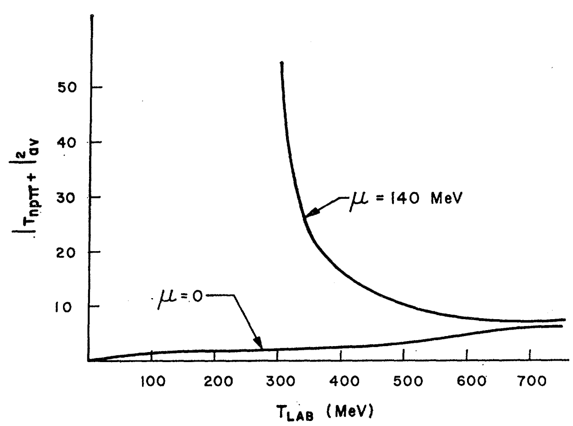

FIG. 5. Comparison of the squared, spin-averaged production amplitude, $(|T_{np\pi^+}|^2)_{\mathrm{av}}$, for zero pion mass, $\mu=0$, with that for physical pion mass, $\mu=140$ MeV, as a function of incident proton laboratory energy.

[17] B. Baldoni, S. Focardi, H. Hromadnik, L. Monari, F. Saporetti, S. Feminò, F. Mezzaranes, E. Bertolini, and G. Gialanella, Nuovo Cimento **26**, 1376 (1962).

[18] V. M. Guzhavin, G. K. Kliger, V. Z. Kolganov, A. V. Lebedev, K. S. Marish, Yu. D. Prokoshkin, V. T. Smolyankin, A. P. Sokolov, L. M. Soroko, and T. Wa-Ch'uang, Zh. Eksperim. i Teor. Fiz. **46**, 1245 (1964) [Soviet Physics. JETP **19**, 847 (1964)].

[19] M. E. Schillaci, R. R. Silbar, and J. E. Young, Phys. Rev. Letters **21**, 711 (1968); **21**, 1030(E) (1968); Phys. Rev. **179**, 1539 (1969).

[20] D. S. Beder, Nuovo Cimento **56A**, 625 (1968); **58A**, 908(E) (1968).

[21] The large difference between the curves in Fig. 5 near threshold can be simply understood. By Ref. 19, the squared amplitude here is proportional to $p_i|\mathfrak{F}|^2$, where p_i is the initial-nucleon c.m. three-momentum and $|\mathfrak{F}|^2$ is a certain quadratic combination of the NN invariant amplitudes. Because of the rapid falloff of the NN amplitudes near the elastic threshold, $|\mathfrak{F}|^2_{\mu=0}$ is less than $|\mathfrak{F}|^2_{\mu=140}$, simply because they are evaluated at different energies. At the same time, the factor p_i (which arises from the pion vertex) is finite at threshold when $\mu\neq 0$, yet forces $|T_{np\pi^+}|^2$ to zero as $T_{p,\mathrm{lab}}\rightarrow 0$ when $\mu=0$.

calculation by Young[22] indicates that for the Ss amplitude at least, the extrapolation from $\mu=0$ to 140 MeV might not be so drastic. At this point, we can only say that the threshold soft-pion calculations[19,20] must be viewed with caution.

The soft-pion theorem can also be applied to the reaction $pp \rightarrow pp\pi^0$. In particular, the predicted ratio $pp \rightarrow pp\pi^0/pp \rightarrow np\pi^+$ should be even less sensitive to the small ambiguities in these calculations discussed above. We have computed the low-energy π^0 production, following closely the procedure described in GSS. Experimental rates for such π^0 production are only very poorly known; cross sections (based on very few events) have been measured in two bubble-chamber experiments.[17,18] We find that, for $T_{\pi,\text{c.m.}} \approx 14$ MeV at $T_{p,\text{lab}}=560$ MeV, the predicted soft-π^0 production is smaller than experiment[17] by about a factor of 40. A similar statement holds at 650 MeV.[18] Moreover, the predicted π^0/π^+ charge ratio differs from the experimental ratio (poorly known at these low-pion energies) by a factor of about 5.

IV. RESONANCE EFFECTS

Perhaps the discrepancy between theory and experiment can be attributed to the higher-order terms in Eq. (1), which vanish in the soft-pion limit $q_\alpha \rightarrow 0$. Because the physical pion cannot have all its four-momentum components zero, the term Bq, for example, is always present. If the coefficient B were for some reason large, this term might even dominate the production amplitude. If this were true, then the soft-pion techniques, which only predict the A term, could well give a disagreement with experiment. In such a case, the low-energy theorem simply is not useful for this reaction, and therefore, nothing can be concluded regarding the assumptions made in proving it.

How, then, might the higher-order terms in Eq. (1) be large? Such a situation must arise from non-Born contributions to the amplitude. An obvious candidate for this is the resonance graph shown in Fig. 2(d). The threshold for $NN \rightarrow N\Delta(1236)$ is at $T_{p,\text{lab}}=650$ MeV. Near this energy, in fact, such a resonance, or isobar, model[5,6] provides a fairly good description of the production of the higher-energy pions.

In their soft-pion calculation, GSS ignored these resonance effects. This seemed reasonable on two counts. First, if $\mathbf{q}=0$, then the pion-nucleon subenergies are

$$W_{\pi p}=W_{\pi n}=(m^2+\mu^2+2\mu E_f)^{1/2} \approx m+\mu+\mu T_f/(m+\mu)\,, \qquad (3)$$

where $E_f=m+T_f$ is the c.m. energy of either of the final nucleons. For $T_{p,\text{lab}}=740$ MeV, the pion-nucleon subenergies are only ≈ 13 MeV above the πN threshold at $m+\mu$ and $\approx 2\frac{1}{2}$ half-widths below the resonance peak at 1236 MeV. In addition to being this far off resonance,

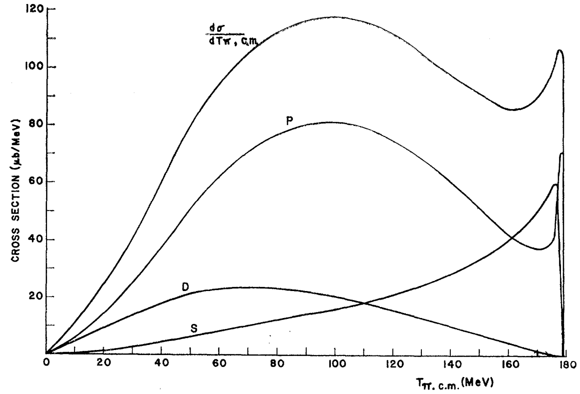

FIG. 6. Pion energy spectrum predicted by the extended Mandelstam model. Separate contributions of the s-, p-, and d-wave contributions are also shown.

the p-wave $\Delta \rightarrow N\pi$ vertex has a factor of k, the pion momentum in the πN c.m. frame, which will further tend to reduce the isobar contribution so close to the πN threshold. Secondly, the soft-pion theorem ignores such contributions because the denominator in the Δ propagator does not vanish in the soft-pion limit [compare with Eq. (2)].

However, if we do not take the $\mu \rightarrow 0$ limit, it can be easily seen that the isobar graphs can contribute in the same order as the nucleon pole graph. Referring to Fig. 2(d), we compare the following expression with Eq. (2):

$$\frac{\gamma\cdot q\gamma_5}{(p_3+q)^2-m^{*2}}=\frac{\gamma\cdot q\gamma_5}{m^2-m^{*2}+2p_3\cdot q+q^2} \approx \frac{\mu\gamma_0\gamma_5}{m^2-m^{*2}+2m\mu+\mu^2}. \qquad (4)$$

Here we have set $\mathbf{q}=0$ and we are ignoring spin complications of the Δ propagator and vertex. Since $m^2-m^{*2}\approx -4\mu m$, we see that the denominator is of the same order as in Eq. (2). We conclude from this that the resonance contribution to this process may be important.[22a]

A simple test of how important these resonance contributions are is provided by the π^0/π^+ charge ratio. The isobar model of Lindenbaum and Sternheimer,[5] for example, makes a definite prediction for this ratio, $d\sigma_{pp\pi^0}/dE_{\pi,\text{c.m.}}$ versus $d\sigma_{np\pi^+}/dE_{\pi,\text{c.m.}}$, independent of the pion energy. This prediction of 1:5 is in rough agreement with the available bubble-chamber data,[17,18] even at the lowest pion energies, in contrast with the soft-pion prediction discussed in Sec. III.

A rough calculation in the spirit of Ref. 5 is disappointing, however. We can obtain an upper limit to

[22] J. E. Young, Ann. Phys. (N. Y.) 56, 391 (1970).

[22a] *Note added in proof.* Indeed, if $m^*=m$ in a world where $\mu=0$, then applications of the soft-pion-emission theorem ought to include Δ poles right from the beginning. One must then worry about double counting, as in Fig. 8.

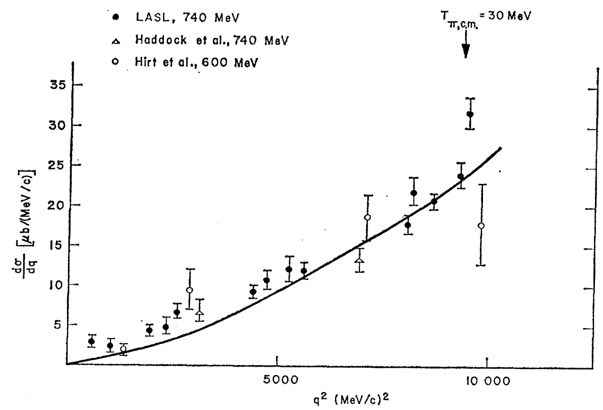

FIG. 7. Low-energy-pion spectrum predicted by the extended Mandelstam model compared with the experimental data (Refs. 1, 15, and 16).

the isobar contribution by ignoring all other contributions, including the nucleon pole graphs responsible for the soft-pion results of GSS. The NN $\rightarrow N\Delta$ amplitude is assumed to be a constant, fixed by the total inelastic cross section. The low end of the pion spectrum was found, in this model, to be comparable in magnitude with the GSS predictions. For example, for $T_{\pi,\text{c.m.}} \approx 12$ MeV, we obtained $d\sigma/dE_{\pi,\text{c.m.}} \approx 2$ μb/MeV, as compared with the soft-pion value of ≈ 3 μb/MeV and the experimental value of ≈ 24 μb/MeV.

Such a simple model, however, cannot be expected to provide an adequate description of the situation for low pion energies. For, in the limit $\mathbf{q} \rightarrow 0$, the final nucleons have their maximum momentum, which, at $T_{p,\text{lab}} = 740$ MeV, is about 450 MeV/c in the c.m. frame. Surely, at such a large relative momentum (between the produced Δ and the other nucleon), many partial waves must be present in the $NN \rightarrow N\Delta$ amplitude.[23] This implies a complicated momentum and angle dependence in this amplitude, which in the above calculation has just been taken as a constant. This is an important difference, moreover, since the various partial-wave amplitudes will contribute incoherently to the (angle-integrated) pion spectrum, roughly like $\sum |a_l|^2$. Thus, the predicted energy spectrum at low pion energies in this model ought to be rather larger than that given by the s-wave contribution alone (as in the calculation described above).[24]

This effect can already be seen in the somewhat more ambitious calculation by Mandelstam,[6] in which p-wave transitions are also included. In this model there are essentially two free parameters, one each for the s- and p-wave transitions, and they are fixed by the total inelastic cross section and the shape of the spectrum. The inclusion of the p-wave increases the cross section for the low-energy pions of interest here by about a factor of 3 over the s-wave calculation mentioned above. This is not yet enough by itself to account for the observed cross sections, but, as we have seen, it is reasonable (and perhaps even necessary) to include even higher partial waves in such calculations. We have extended Mandelstam's model by including d-wave effects in a rough way. In the notation of Ref. 6, neglecting spin complications, we simply added to the cross section a term

$$\sigma_l = C_l^2 p^{2l} |f(q)|^2 S(Q) \tag{5}$$

for $l=2$. Keeping the parameters for the s and p waves in the same ratio as given in Ref. 6, and arbitrarily assuming the d wave contributes about 15% to the total inelastic cross section (hence fixing C_2),[25] we get the predicted spectrum shown in Fig. 6.

A comparison with the experimental data[1,15,16] for low pion energies is shown in Fig. 7. We should emphasize that the curve here contains only isobar contributions; again the nucleon pole contributions have not been included. Contrary to our prior expectations, it appears that the low-energy-pion data can be quite easily explained in terms of this resonance mechanism. This statement is only made, however, in the context of a model which involves arbitrary (though reasonable) parameters.

One naturally wonders how close to the production threshold one must be before these resonance effects become negligible. As a check of this, we calculated the total production cross section at $T_{p,\text{lab}} = 320$ MeV using our extended Mandelstam model, keeping the parameters used at 740 MeV.[25] We find a value of 8 μb, as compared with the soft-pion prediction[19] of 20 μb. This is another difficulty in these threshold calculations,[26] in addition to the ambiguity discussed in connection with Fig. 5.

We conclude this section with some interpretive comments. Among the approximations made in GSS

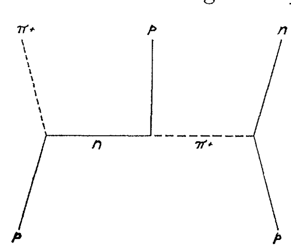

FIG. 8. Pre-emission graph with the NN scattering effected by one-pion exchange. This crossed-channel graph contributes to the isobar graph shown in Fig. 2(d).

[23] In the NN system at such c.m. momenta, the phase-shift analyses (Refs. 13 and 14) indicate that even g waves are important.

[24] At higher pion energies, the $N\Delta$ relative momentum will be rather smaller, and the higher partial waves will be less important.

[25] The $s{:}p{:}d$ parameter ratios for this case are 1:4.45:0.51, and the total inelastic cross section was taken to be 15 mb. The rough form given in Eq. (5) was checked, for $l=0$ and 1, against the various spin-dependent s- and p-wave contributions given by Mandelstam (Ref. 6). Except for the effect of the final-state NN interaction, which modifies the spectrum only at the highest $E_{\pi,\text{c.m.}}$, we found good agreement. We thus feel that Eq. (5) for $l=2$ gives the d-wave effect sufficiently well for our purposes here.

[26] Beder (Ref. 20) earlier recognized the possible trouble from these graphs but did not pursue the problem. One can check to see that Δ poles will contribute an amplitude whose propagator is $\approx\frac{1}{3}$ as large as that from N poles when $m^{*2} \approx 4\,\mu m$ is taken into account.

and in Refs. 19 and 20 was to set $\mathbf{q}=0$. As a result, terms proportional to $\boldsymbol{\sigma}\cdot\mathbf{v}_\pi$ were dropped while terms proportional to $\boldsymbol{\sigma}\cdot\mathbf{v}_{p_i}$ were retained. This is probably unreasonable because the small pion mass easily allows $|\mathbf{v}_\pi|$ to be comparable with $|\mathbf{v}_{p_i}|$. In fact, such a kinematic situation already becomes possible at $T_{p,\mathrm{lab}}=316$ MeV. In addition, such $\boldsymbol{\sigma}\cdot\mathbf{v}_\pi$ terms in the nucleon pole graphs correspond to p-wave pions and, indeed, contribute to the resonance effects described above. For example, if one assumes one-pion exchange for the off-shell NN scattering, as depicted in Fig. 8, it is well known that this crossed-channel graph contributes strongly to the isobar graph shown in Fig. 2(d).

V. CONCLUSIONS

We have seen that the production of low-energy pions in NN collisions at moderate energies is very probably dominated by the strong resonance effects of the $\Delta(1236)$. Thus the low-energy theorem obtained from PCAC is not applicable to this reaction in this energy region. Furthermore, such effects probably continue to be surprisingly important, even down to threshold energies.

An obvious question then is where, if anywhere, this soft-pion-emission theorem might be useful for this process. Can one devise a kinematical situation in which the resonance effects are minimal? For example, by an appropriate coincidence experiment on $pp\to np\pi^+$, perhaps the subenergy $W_{\pi p}$ can be set equal to $m+\mu$ while at the same time $W_{\pi n}$ is far above any πN resonance energy, say $W_{\pi n}\approx 5$ GeV. This does happen if $\mathbf{v}_\pi=\mathbf{v}_{p_f}$ and the incident lab momentum is ≈ 70 GeV/c. Needless to say, to do such an experiment with reasonable statistics would be difficult at this time. Theoretically, such a soft-pion calculation would also be difficult. For one thing, there is at present no NN scattering information that can be used as input. More seriously, the question of frame dependence, brought up in Sec. III, makes the calculation ambiguous, since to use the soft-pion techniques one would like to be in an appropriate frame, such as the pion rest frame.

This question was discussed to some extent by Shrauner[27] for the reaction $\pi N\to\pi N\pi_{\mathrm{soft}}$ in the GeV-energy region. He presented results for the soft pion at rest with respect to both the initial and final nucleon, but because of paucity of data, comparison was made in the latter case only. Incidentally, it is interesting to note that Shrauner's soft-pion predictions are also too low by about a factor of 7. Since this same ratio persists for both π^0 and π^+ production (from a π^-p initial state), Shrauner did not consider p-wave isobar effects (which would give a different charge ratio) to be responsible for this discrepancy. However, in this calculation the kinetic energy of the soft pion and final nucleon is about 10 MeV in their c.m. frame, and, as we have shown above, isobar effects are still significant in $NN\to NN\pi$ even this close to the threshold at $m+\mu$.

The importance of resonance effects in the reaction $NN\to NN\pi$ thus leads one to consider their role in $\pi N\to\pi\pi N$. Indeed, one can describe this process in terms of the reaction sequence $\pi N\to\pi\Delta\to\pi\pi N$.[28] With some fitted parameters, this provides a fairly good description of the experimental data near threshold. In a separate calculation, Chang[29] has obtained the pion-production cross section near threshold in a parameter-free way using soft-pion and current-algebra techniques. This calculation, which also agrees with the available data rather nicely, includes some effects of the Δ resonance, although their relative contributions are not given explicitly. However, it appears that certain tree graphs which include the Δ resonance and which might well be important in the $\pi N\to\pi\Delta\to\pi\pi N$ chain have been omitted. Further, the set of graphs which Chang did consider seems to involve some double counting. The relative importance for this reaction of nucleon pole graphs, resonance contributions, and other possible internal diagrams is now being investigated.

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