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Branching Ratios for $K^+ \to 3\pi$ Decays*

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The branching ratios $\tau'^+/(K^+ \to \text{all})$ and τ'^+/τ^+ were obtained from a sample of 12 976 K^+ decays including 198 τ'^+ and 693 τ^+ decays. It was found that $\tau'^+/(K^+ \to \text{all}) = 0.0153 \pm 0.0011$, and that $\tau'^+/\tau^+ = 0.286 \pm 0.023$. The $\tau^+/(K^+ \to \text{all})$ branching ratio and the π^+ energy spectrum in τ'^+ decay were also obtained, and were found to be in good agreement with the more precise determinations of the other experiments. The branching-ratio data suggest a small $\Delta I = \frac{3}{2}$ admixture in the $K \to 3\pi$ decay amplitudes. Comparison with various theoretical models shows fair to good agreement between experiment and theory.

I. INTRODUCTION

THE branching ratios (partial rates) and secondary energy spectra of the three-pion decay modes of the K mesons have received a considerable part of the theoretical and experimental interest that has been shown in these decays in recent years. The increasing precision in their experimental determination, using larger numbers of events with improved statistics, provides tests of gradually greater strictness for a variety of theoretical predictions.

This paper describes an experiment to determine directly the branching ratios $(K^+ \to \pi^+\pi^0\pi^0)/(K^+ \to \text{all})$ and $(K^+ \to \pi^+\pi^0\pi^0)/(K^+ \to \pi^+\pi^+\pi^-)$. For this purpose, the τ'^+ $(K^+ \to \pi^+\pi^0\pi^0)$ and τ^+ $(K^+ \to \pi^+\pi^+\pi^-)$ decays were identified in a sample consisting of a total of 12 976 K^+ meson decays.[1,2] As checks on the data, the $\tau^+/(K^+ \to \text{all})$ branching ratio was also obtained and an analysis of the π^+ energy spectrum in τ'^+ decay was carried out.

The experimental procedures are discussed in Sec. II. Section III contains a description of the data. The details of the analysis and the results obtained are given in Sec. IV. In Sec. V the results are discussed and compared to predictions of certain theoretical models and to other experiments involving $K \to 3\pi$ decays. In particular, the $\Delta I = \frac{1}{2}$ rule is tested by comparing τ'^+ to τ^+ and K_L^0 rates. There have been indications of violation of this rule in the $K \to 3\pi$ rates[3] and significant violations have been reported to occur in the decay spectra.[4,5]

II. EXPOSURE AND SCANNING

A. Exposure

An 84-pellicle stack of 6-in.×8-in.×600-μm Ilford G-5 emulsion was exposed to a 400-MeV/c separated K^+ beam at the Bevatron of the Lawrence Radiation Laboratory of the University of California.[6] The beam kaons came to rest near the center of each pellicle, in an area $\sim 1.5 \times 4$ cm. It was desired that the density of stopped kaons be relatively high in order that the scanning time required to find a kaon decay event be fairly short. Therefore, the stack was inserted in the beam after only one stage of separation and a background of approximately 10 beam pions for each stopping K^+ was present. These pions were of minimum ionization and traversed the entire stack. The density of kaon endings in the stopping region of the exposed stack was $\sim 2 \times 10^4$ K^+/cm^3. The individual pellicles were aligned for scanning and track following by the method outlined in Ref. 7.

B. General Scanning Procedure

The middle region of the stack was systematically area scanned for K^+-meson endings. The scanning

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[1] Includes 4906 events from Ref. 2.

[2] S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).

[3] See, for example, J. W. Cronin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[4] T. S. Mast, L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, J. J. Murray, F. T. Solmitz, and R. D. Tripp, Phys. Rev. 183, 1200 (1969).

[5] J. Grauman, S. Taylor, E. L. Koller, D. Pandoulas, S. Hoffmaster, O. Raths, L. Romano, P. Stamer, A. Kanofsky, and V. Mainkar, Phys. Rev. Letters 23, 737 (1969).

[6] G. Goldhaber *et al.*, LRL Report No. BEV-483, 1960 (unpublished).

[7] S. Taylor. G. Harris, J. Orear, P. Baumel, and J. Lee, Rev. Sci. Instr. 30, 244 (1959).

efficiency for finding kaon or kaonlike endings was calculated on the basis of a careful rescan of randomly selected areas and was found to be $\sim 97\%$. Except for τ^+ decays, all K^+ secondaries were grain counted (or followed to their endings) for the purpose of detecting "heavy" secondaries, i.e., secondaries with grain counts 1.56 times minimum or greater, corresponding to 55 MeV for charged pions and 41.3 MeV for muons.[8] The maximum charged-pion kinetic energy expected from τ'^+ is 53.3 MeV.[9,10] Hence, the entire τ'^+ spectrum, together with a background from the low-energy part of the $K_{\mu 3}^+$ ($K^+ \rightarrow \mu^+ + \nu_\mu + \pi^0$) spectrum, should be included. Scanning efficiency for the detection of secondaries with grain counts 1.56 times minimum or greater is believed to be very high ($\geq 97\%$) for both steep and flat secondaries. The choice of the middle region of the stack for scanning maximized the number of secondaries coming to rest in the emulsion stack.

C. Identification of Primaries

The detailed scanning procedure was as follows: All heavily ionizing track endings were initially recorded. Endings that were within 25 μm of the pellicle surfaces and/or formed a dip or plane angle of 45° or greater with respect to the beam direction were excluded from the sample. The former criterion eliminated to a great extent the bias resulting from the difficulty in seeing K^+ secondaries near the pellicle surfaces, while the latter excluded from the sample a considerable number of tracks due to protons, pions, or muons that did not originate in the beam.

The existence of three heavy secondaries associated with the primary track ending served to identify τ^+ decays. Care was taken that such events appeared to be coplanar and momentum conserving. The rest of the primaries were followed a few hundred microns back from their endings and were inspected qualitatively. Pion and muon tracks were identified from their multiple scattering and rate of change of ionization with residual range characteristics, while other primaries were found to originate in interaction "stars" and were assumed to be proton tracks. Tracks that remained unidentified at this stage were assumed to be due to protons or K^+ mesons. A search was then made for secondaries associated with the endings of these tracks. The establishment of a secondary track served to identify the primary as a K^+ meson. If no secondary could be seen, the track was grain counted at 1 cm residual range, usually counting at least 500 grains. In order to separate K^+ mesons from protons, the count was compared to the grain count distribution obtained by grain counting previously identified K^+ meson and proton tracks at the same residual range. If the unknown primary gave a K grain count, a further search was made for the secondary and, if again no such track could be found, it was classified as a "K with no secondary" ($K_{\rm NS}$). It is assumed that in these cases a lightly ionizing secondary existed but was difficult to observe. A meaningful grain count at 1 cm residual range was sometimes unavailable because of "geometrical" difficulties (cracking or distortion of the emulsion, proximity of the primary to the pellicle surface where it ionizes less than normally, etc). To handle such cases, points at 0.8- and 0.6-cm residual range were "calibrated" against known K^+ meson and proton tracks and used instead of the 1-cm point whenever necessary.

D. Identification of Secondaries

τ^+ decays were identified visually as described above. All other secondaries that were found, except those whose ionization clearly exceeded 1.56 times minimum (automatically classified as heavy) were then grain counted. Depending on whether the average value of their grain count was less than, overlapped, or exceeded 1.56 times minimum within 2 standard deviations, they were classified as "light" ($K_{\rm L}$), "doubtful," or "heavy," respectively. The $K_{\rm NS}$'s mentioned above are assumed to belong to the $K_{\rm L}$ category. Doubtful secondaries were either followed or further grain counted, and heavy secondaries were followed. If the secondary stopped in the emulsion, it was identified by its decay product as a π^+ or as a μ^+ and its range was measured. Energies obtained from these ranges are accurate to $\sim 3\%$.

Heavy and doubtful secondaries that interacted strongly while in flight were identified as pions and their energies were determined from their grain counts relative to minimum in the vicinity of the strong interaction in conjunction with their observed ranges in the emulsion. The result was checked by grain counting near the K^+ ending. In those cases where the secondary track left the emulsion, grain counts relative to minimum at the point of leaving the stack and also at the K^+ decay point were carried out. This information, together with the measured range between these two points, served to identify it as either a π^+ or a μ^+ and to establish its energy. Depending on the amount of available data, energies obtained by grain-counting methods are accurate to $\sim$6–10%.

Finally, a significant deviation from the procedure for treating secondaries outlined above should be mentioned. At the beginning of the experiment, all scanners had been instructed to judge a secondary qualitatively as light, doubtful, or heavy before grain counting it. One of the scanners was proven to be consistently correct in his judgments by the subsequent grain count.

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[8] For grain count versus energy and range versus energy data, see W. Barkas and D. Young, LRL Report No. UCRL-2579 Rev. (unpublished). These data have been used throughout this experiment.

[9] All particle masses, Q values, maximum kinetic energies, etc. were taken from Ref. 10.

[10] A. Barbaro-Galtieri, S. E. Derenzo, L. R. Price, A. Rittenberg, A. H. Rosenfeld, N. Barash-Schmidt, C. Bricman, M. Roos, P. Söding, and C. G. Wohl, Rev. Mod. Phys. 42, 87 (1970).

Including rechecking at the end of the experiment, it was found that among 2095 K^+ mesons, 47 of which had single heavy secondaries, this scanner judged an actually heavy secondary as light only once. He was then permitted to grain count only those secondary tracks that he considered to be doubtful. About 4500 events were treated in this manner. No appreciable bias is expected to have resulted from this relaxation of the scanning procedure.

III. DATA

The sample of 8070 K^+ mesons obtained in this experiment consisted of 7425 K_L's, 448 τ^+ decays, 126 τ'^+ decays, and 71 $K_{\mu 3}^+$ decays with secondary energy less than 41.3 MeV (see Table I).

Primaries for which no secondaries could be found were identified by the grain-counting procedure described in Sec. II, yielding 608 K_{NS}'s (included in the number of K_L's above) and 417 protons. A certain number of these primaries, whose grain count was close to the K^+ meson-proton cutoff, may have been misidentified. It is expected, however, that the error in the number of K_{NS}'s due to such misidentification does not exceed 10%. Three primaries were classified solely on the basis of their qualitative appearance. Despite intensive searching, no secondaries were found on these tracks. Meaningful grain counts were unavailable, in one case, because of a strong interaction of the primary near its ending and, in the other two, because of cracks in the emulsion near the endings past which the tracks could not be followed. One of these tracks was classified as K_{NS} and is included in the K_L's above, while the other two were classified as protons. Clearly, no significant bias can result from these three cases. Of the primaries that were identified through grain counting, only two gave counts too low to be considered K-like, and were thus recognized as pion or muon tracks. Hence, the assumption that pion and muon primaries could be recognized from their qualitative features alone should not be a source of any significant bias in this experiment.

The τ'^+ category includes three events in which the secondaries interacted in flight. These secondaries were determined to be π^+'s from τ'^+ decays on the basis of the procedure outlined in Sec. II. Two other K^+ secondaries interacted in flight sufficiently near the K^+ decay point so that a conclusive identification of the tracks as light, doubtful, or heavy was prevented by the large statistical error in the available grain count. For one of these tracks, the grain count was between 1 and 2 standard deviations below 1.56 times minimum; for the other, the grain count was slightly less than 1 standard deviation below 1.56 times minimum. This evidence, together with the fact that the occurrence of a pion from $K_{\pi 2}^+$ ($K^+ \rightarrow \pi^+ + \pi^0$) decay is more probable by about two orders of magnitude than that of a maximum energy π^+ from τ'^+ decay, led to the classification of these two events as $K_{\pi 2}^+$ decays and to their inclusion in the K_L category. The number of $K_{\mu 3}^+$'s includes one event in which the secondary left the emulsion stack. The application of the grain-counting procedure previously described, together with the observed range of the track in the emulsion, led to results which favored the assumption that the secondary was a muon, although a pion whose energy was within the τ'^+ range could not be excluded.

TABLE I. Event types.

	K_L	K_{NS}	τ	τ'	$K_{\mu 3}$	Total K
Ref. 2	4553	217[a]	245	72	36	4906
This experiment	7425	608[a]	448	126	71	8070
Total	11 978	825[a]	693	198	107	12 976

[a] Included in K_L category.

Among the secondary tracks that were followed to their endings and identified by their decay products, one pion was observed to have a range of 5.0 cm, corresponding to a kinetic energy of 62 MeV. Since the $K_{\pi 2}^+$ has a π^+ secondary of 109 MeV and since this event had no obvious inelastic scatter, it was assumed to be a radiative $K_{\pi 2}^+$ decay ($K^+ \rightarrow \pi^+ + \pi^0 + \gamma$) and was classified as K_L. Finally, the sample of τ'^+ decays includes one event in which the range of the muon from the decay of the secondary pion is abnormally small (366 μm, corresponding to 3.1 MeV, instead of 660 μm, corresponding to 4.2 MeV). It is presumed that this is due to the occurrence of the radiative mode ($\pi^+ \rightarrow \mu^+ + \nu_\mu + \gamma$), since the $\pi^+ \rightarrow \mu^+$ decay configuration was very distinct in this case.

The data reported in Ref. 2 and further analyzed in Ref. 11 have been added to the sample in obtaining all results for this experiment (see Table I).

The determination of the energy spectrum of the π^+ from τ'^+ decay utilizes also the τ'^+'s from Ref. 12. This is permissible, since the probability of observing a τ'^+ decay with an associated Dalitz pair is independent of the energy of the charged pion secondary. For one of the 33 τ'^+'s reported in Ref. 12, the supposed Dalitz pair is now judged to be spurious. Therefore, this event was excluded from the sample. The four unidentified K^+ decays of Ref. 12 were all cases in which the heavy secondary left the stack and were treated by the methods previously described. The data favored the assumption of a pion from τ'^+ decay for one of these events, which was therefore added to the 32 events above. The assumption of a $K_{\mu 3}^+$ decay was favored in the other three cases. For one of these, the π^+ from τ'^+ energy range was excluded within 2 standard deviations of the grain-count data, while, for the other two, the evidence was less compelling. The total number of τ''s for the energy spectrum is then the 198 events of Table

[11] S. Bjorklund, E. L. Koller, and S. Taylor, Phys. Rev. Letters 4, 424 (1960); 4, 475(E) (1960).

[12] P. Stamer, S. Taylor, E. L. Koller, T. Huetter, J. Grauman, and D. Pandoulas, Phys. Rev. 151, 1108 (1966).

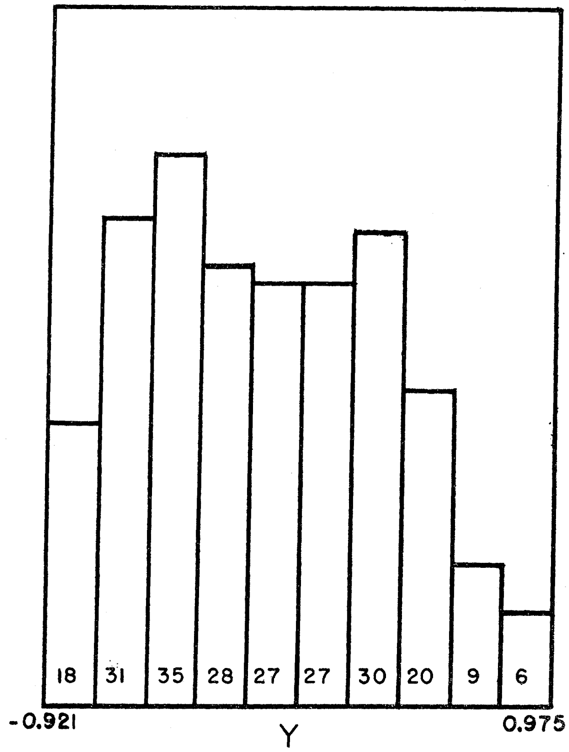

FIG. 1. Histogram of the π^+ energy distribution in 231 τ'^+ decays. Number of events in each division is indicated. Variable Y is linearly related to the π^+ energy T_3 through Eqs. (1) and (4) of the text.

I plus the 33 events discussed above. A histogram of these events as a function of T_3, the kinetic energy of the positive pion, is shown in Fig. 1.

IV. ANALYSIS OF DATA AND RESULTS

A. Branching Ratios

The branching ratios $R_{\tau'^+}$ and R_{τ^+} for τ'^+ and τ^+ decay to all K^+ meson decays, respectively, were determined directly by using the total number of τ'^+'s, $N_{\tau'^+}$, and τ^+'s, N_{τ^+}, as well as the total number of K^+ mesons, N_{K^+}, in the sample (see Table I). The values obtained were

$$R_{\tau'^+}=N_{\tau'^+}/N_{K^+}=0.0153\pm0.0011$$

and

$$R_{\tau^+}=N_{\tau^+}/N_{K^+}=0.0534\pm0.0021.$$

The errors associated with the values given above were obtained by combining the statistical error in each case with the error that would arise from misidentification of 10% of the K^+ meson primaries listed in the $K_{\rm NS}$ category (see Sec. III). The over-all error is dominated by the statistical error in both cases and is quite insensitive to the amount of the above-mentioned misidentification.

The branching ratio $R_{\tau'^+/\tau^+}$ for τ'^+ decay relative to τ^+ decay was also directly determined and found to be

$$R_{\tau'^+/\tau^+}=N_{\tau'^+}/N_{\tau^+}=0.286\pm0.023,$$

where the quoted error is the statistical error only.

B. π^+ Energy Spectrum in τ'^+ Decay

The final-state kinematics of $K\to 3\pi$ decays may be described in terms of two independent variables, such as the quantities X and Y.[13] These are related to the Lorentz-invariant Mandelstam variables for K decay at rest,

$$S_i=(M_K-m_i)^2-2M_KT_i \quad (i=1,2,3), \tag{1}$$

$$3S_0=\sum_{i=1}^{3} S_i=\sum_{i=1}^{3} (M_K-m_i)^2-2M_KQ, \tag{2}$$

by

$$X=-\sqrt{3}(S_1-S_2)/2M_KQ, \tag{3}$$

$$Y=-3(S_3-S_0)/2M_KQ, \tag{4}$$

where the index $i=3$ denotes the "odd" pion, M_K is the mass of the charged kaon, m_i is the mass of the ith pion, T_i is the kinetic energy of the ith pion, and Q is the Q value for the decay.

The physical range in Y was divided into ten equal bins (see Fig. 1). The "theoretical" number of events in the ith bin is given by

$$N_i^{\rm th}=N_0\int_{Y_{i\,\rm min}}^{Y_{i\,\rm max}} w(Y)dY \bigg/ \int_R w(Y)dY \quad (i=1,\ldots,10), \tag{5}$$

where N_0 is the total number of events, $Y_{i\,\rm min}$ and $Y_{i\,\rm max}$ are the lower and upper coordinates of the ith bin, respectively, R is the entire physical range of the Y variable, and $w(Y)$ is given by

$$w(Y)=|M(Y)|^2\left[\int_0^{X_{\rm max}(Y)} \phi(X,Y)dX\right].$$

Here $\phi(X,Y)$ is the invariant phase space and $|M(Y)|^2$ is the square of the matrix element averaged over X.

TABLE II. Fits to $|M(Y)|^2\propto 1+\alpha_{\tau'}(M_KQ/m_\pi^2)Y+\beta_{\tau'}(M_KQ/m_\pi^2)^2Y^2$.

	$\alpha_{\tau'}$	$\beta_{\tau'}$	Degrees of freedom	χ^2	Probability (%)
Linear fit	-0.344 ± 0.049	···	8	4.61	79.8
Quadratic fit	-0.351 ± 0.068	0.008 ± 0.055	7	4.59	70.9

[13] The variable X is identical to the Dalitz x; Y differs from the Dalitz y be a small term which would vanish if the charged and neutral pion masses were equal. See R. H. Dalitz, Phil. Mag. **44**, 1068 (1953).

Following Weinberg,[14] $|M(Y)|^2$ may be expanded in a power series in the variable Y defined above,

$$|M(Y)|^2 \propto 1+\alpha_{\tau'}\left(\frac{M_K Q}{m_\pi^2}\right)Y+\beta_{\tau'}\left(\frac{M_K Q}{m_\pi^2}\right)^2 Y^2+\cdots, \quad (6)$$

where m_π is the charged-pion mass.

The dependence of the observed spectrum on the variable Y is found by fitting the N_i^{th} to the observed frequencies N_i^{obs} (see Fig. 1) by means of a χ^2 test. This was done both by using the first two terms and first three terms of Eq. (6). The parameters $\alpha_{\tau'}$ and $\beta_{\tau'}$ determined by the fitting process are presented in Table II.

For the purpose of display, graphical approximations to $|M(Y)|^2$, both experimental (points with errors) and theoretical (solid curve), are presented in Fig. 2. These approximations were obtained by dividing the experimental and theoretical numbers of events in each bin by the appropriate integral over phase space and renormalizing to an average ordinate of unity. In calculating the theoretical numbers of events the linear approximation for $|M(Y)|^2$ was used with the fitted value of $\alpha_{\tau'}$.

V. DISCUSSION

Table III presents a compilation of τ'^+ and τ^+ branching-ratio data,[15–23] including the results of the present experiment.[24] The absolute τ^+ branching ratio R_{τ^+} was calculated for this experiment in order to provide a check on the data through comparison with the more precise published results.[20,21,23] It is seen that the values of $R_{\tau'^+}$, R_{τ^+}, and $R_{\tau'^+/\tau^+}$ obtained are consistent with the results of other experiments.

TABLE III. Compilation of τ'^+ and τ^+ branching-ratio data.

Authors	Ref.	R_{τ^+}(%)	$R_{\tau'^+}$ (%)	$R_{\tau'^+/\tau^+}$
Birge *et al.*	15	5.6±0.4	2.1±0.5	···
Alexander *et al.*	16	6.8±0.4	2.2±0.4	0.317
Giacomelli *et al.*	17	···	···	0.296±0.027
Roe *et al.*	18	5.7±0.3	1.7±0.2	···
Shaklee *et al.*	19	5.1±0.2	1.8±0.2	0.350±0.039
Callahan *et al.*	20	5.54±0.12	···	···
De Marco-Trabucco *et al.*	21	5.71±0.15	···	···
Young *et al.*	22	6.0±0.4	2.3±0.6	0.39±0.10
Ford *et al.*	23	5.552±0.045[a]	···	···
This experiment		5.34±0.21	1.53±0.11	0.286±0.023

[a] Assumes the K^+ lifetime to be $(12.35\pm0.06)\times10^{-9}$ sec.

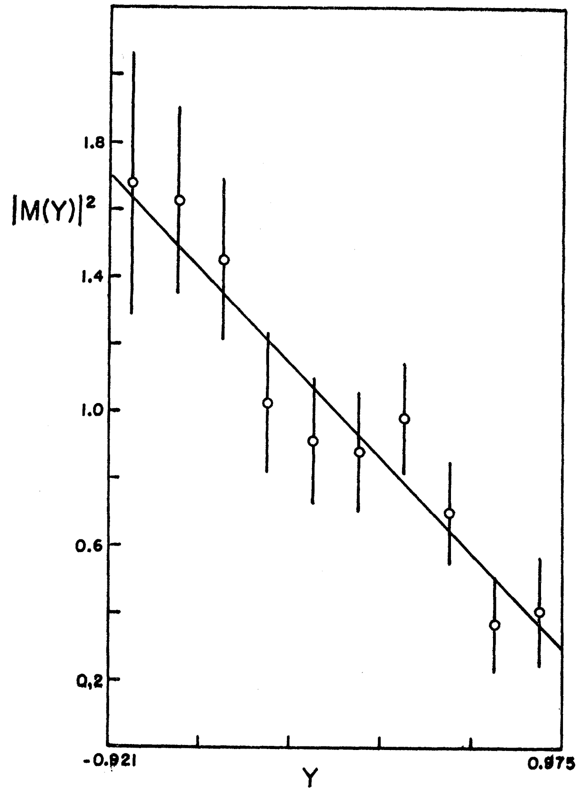

FIG. 2. Y dependence of $|M|^2$. Experimental (points with errors) and "theoretical" (solid curve) distributions were obtained by dividing the observed frequencies from Fig. 1 and the "theoretical" frequencies of Eq. (5), respectively, by the appropriate integrals over phase space. In calculating the theoretical frequencies, the linear approximation was assumed together with the best-fit value for the slope, $\alpha_{\tau'} = -0.344\pm0.049$.

The fit of the energy spectrum of π^+ from τ'^+ decay to linear and quadratic forms of the squared decay matrix element, carried out as a further check on the data,[25] is in good agreement with previous experi-

TABLE IV. Compilation of experimental τ'^+ slope parameters.

Authors	Ref.	Number of events	$\alpha_{\tau'}$
Kalmus *et al.*	26	1792	−0.320±0.027
Bisi *et al.*	27	1874	−0.400±0.067
Davison *et al.*	28	4048	−0.344±0.012
This experiment		231	−0.344±0.049

[14] S. Weinberg, Phys. Rev. Letters **4**, 87 (1960); **4**, 585(E) (1960).

[15] R. W. Birge, D. H. Perkins, J. E. Peterson, D. H. Stork, and M. N. Whitehead, Nuovo Cimento **4**, 834 (1956).

[16] G. Alexander, R. H. W. Johnson, and C. O. O'Cealleigh, Nuovo Cimento **6**, 478 (1957).

[17] G. Giacomelli, D. Monti, G. Quareni, A. Quareni-Vignudelli, W. Puschel, and J. Tietge, Phys. Letters **3**, 346 (1963).

[18] B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters **7**, 346 (1961).

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[21] A. De Marco-Trabucco, C. Grosso, and G. Rinaudo, Phys. Rev. **140**, B1430 (1965).

[22] P. S. Young, W. Z. Osborne, and W. H. Barkas, Phys. Rev. **156**, 1464 (1967).

[23] W. T. Ford, A. Lemonick, U. Nauenberg, and P. A. Piroué, Phys. Rev. Letters **18**, 1214 (1967).

[24] The branching-ratio results are consistent with those of Ref. 2 alone.

[25] The results of this analysis are consistent with those of a previous analysis (see Ref. 11) of the 72 events of Ref. 2 alone.

TABLE V. Experimental test of $\Delta I = \frac{1}{2}$ rule predictions.

Quantity tested	$\Delta I = \frac{1}{2}$ prediction	Experiment	Assumptions[a]
$\gamma^{+-0}/2\gamma^{+00}$	1.00	0.86 ± 0.07	$\Gamma^{+-0} = (2.345 \pm 0.099) \times 10^6$ sec^{-1} $\Gamma(K^+ \to \text{all}) = (0.810 \pm 0.003) \times 10^8$ sec^{-1}
$\gamma^{000}/(\gamma^{++-} - \gamma^{+00})$	1.00	0.80 ± 0.04	$\Gamma^{000} = (3.99 \pm 0.20) \times 10^6$ sec^{-1} $\Gamma^{++-} = (4.513 \pm 0.029) \times 10^6$ sec^{-1} $\Gamma(K^+ \to \text{all}) = (0.810 \pm 0.003) \times 10^8$ sec^{-1}
$\gamma^{++-}/4\gamma^{+00}$	1.00	1.01 ± 0.08	...
$\gamma^{000}/\frac{3}{2}\gamma^{+-0}$	1.00	0.99 ± 0.04	(Not tested in the experiment; value from Ref. 3)

[a] All values in this column are constrained-fit values from Ref. 10.

ments.[26–28] No evidence for a quadratic term was found (see Table II). A compilation of experimental τ'^+ slope parameter values is presented in Table IV, where the result of the present experiment is also listed.

For the partial rates in $K \to 3\pi$ decays, the $\Delta I = \frac{1}{2}$ rule predicts[29]

$$\gamma^{+-0}/2\gamma^{+00} = 1, \tag{7a}$$

$$\gamma^{000}/(\gamma^{++-} - \gamma^{+00}) = 1. \tag{7b}$$

With the additional assumption of the linear approximation,[14] the $\Delta I = \frac{1}{2}$ rule also predicts[29]

$$\gamma^{++-}/4\gamma^{+00} = 1, \tag{7c}$$

$$\gamma^{000}/\tfrac{3}{2}\gamma^{+-0} = 1. \tag{7d}$$

In (7a)–(7d) above, γ^{ijk} are the "idealized" partial rates for the various $K \to 3\pi$ decays, where i, j, and k are the pion charges. These are calculated under the assumptions (a) that there are no mass differences among members of the same isotopic spin multiplet, (b) that second- (and higher-) order terms in the power-series expansion of the squared decay matrix element can be neglected, and (c) that Coulomb interactions among the final-state pions are also neglected. Experimentally, assumption (b) above has been found to be valid within the limits of experimental error. However, mass differences within the same isospin multiplet and final-state Coulomb interactions are not negligible and affect appreciably the observed rates Γ^{ijk}. In comparing relations (7a)–(7d) above with experiment, these effects must be taken into account.[4,30] This may be done by taking

$$\gamma^{ijk} = \Gamma^{ijk}/\Phi^{ijk}, \tag{8}$$

where Φ^{ijk} are phase-space factors "corrected" to account for the above effects. Φ^{+00}, Φ^{+-0}, and Φ^{000}, normalized so that Φ^{++-} is unity, are 1.155, 1.268, and 1.451, respectively.[4]

Relations (7c) and (7d) would also result directly from assuming that the three pions are in an $I = 1$ final isospin state. Experimental agreement with (7c) and (7d) can then exclude $\Delta I \geq \frac{5}{2}$ contributions to the decay amplitudes, but it cannot exclude $\Delta I = \frac{3}{2}$ contributions. On the other hand, relations (7a) and (7b), which involve different kaons (K^+ and $K_L{}^0$), are sensitive to the presence of $\Delta I > \frac{1}{2}$ admixtures in the decay amplitudes. This point has been discussed in detail by Dalitz[31] and Barton *et al.*[32]

Table V presents a comparison between experiment and the predictions (7a)–(7d), using the results of this experiment for $R_{\tau'^+/\tau^+}$ and $R_{\tau'^+}$, together with the constrained fit values for $K_L{}^0 \to 3\pi$ and τ^+ partial rates and for the K^+ lifetime from Ref. 10. It is seen that the data are in fair agreement with predictions (7a) and (7c). The deviation in the case of prediction (7b) suggests the presence of a small $\Delta I = \frac{3}{2}$ amplitude in addition to the dominant $\Delta I = \frac{1}{2}$ amplitude. Evidence to this effect has been previously cited.[3]

Various authors[33–38] have proposed models for $K \to 3\pi$ decay that utilize the algebra of currents together with the assumption of partial conservation of axial-vector current (PCAC), utilize the $\Delta I = \frac{1}{2}$ rule and the linear approximation, and neglect final-state interactions. The predictions of these models regarding the rates of $K \to 3\pi$ decays are identical with (7a)–(7b).

Several authors have proposed models allowing for $\Delta I = \frac{1}{2}$ rule violations. Bouchiat and Meyer,[39] as well as Holstein,[40] have proposed models similar to the above, which allow both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ contributions to the decay amplitudes. In their predictions of partial rates,

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the two models are identical.[40] Matsuda and Oppo[41] consider $\Delta I=\frac{1}{2}$ rule violations through electromagnetic mechanisms. They suggest the possibility of interference occurring between the amplitude for the weak decay $K_L{}^0 \to 3\pi$, assumed to obey the $\Delta I=\frac{1}{2}$ rule, and the $\Delta I=\frac{1}{2}$-rule-violating amplitude for the weak-plus-electromagnetic process $K_L{}^0 \to \eta^0 \to 3\pi$. They also consider the case where $\eta^0 = X^0$ mixing is included, which introduces the additional weak-plus-electromagnetic amplitude for $K_L{}^0 \to X^0 \to 3\pi$. In connection with the $\Delta I=\frac{1}{2}$ rule, this could explain an apparent depression of the $K_L{}^0 \to 3\pi$ relative to the $K^+ \to 3\pi$ rates,[3] since no similar $\Delta I=\frac{1}{2}$-rule-violating processes occur for K^+ decays. A similar model, proposed by Greenberg,[34] suggests that violations of the $\Delta I=\frac{1}{2}$ rule might arise from the weak-plus-electromagnetic processes $K_L{}^0 \to \eta^0 \to 3\pi$ and $K_L{}^0 \to A_1{}^{(8)} \to 3\pi$. A depression of the neutral rates relative to the charged is also predicted by Neih,[42] using a phenomenological model with $\Delta I=\frac{3}{2}$ admixtures. Clavelli[43] applies current algebra with PCAC to a pole model. Retaining terms of the order of $m_\pi{}^2/M_K{}^2$ in the decay amplitudes, he arrives at deviations from the $\Delta I=\frac{1}{2}$ rule predictions. The $\Delta I=\frac{1}{2}$ results are then obtained if mass differences within isotopic spin multiplets are neglected. Using a pole model, but not making use of current algebra techniques, Graham and Yun[44] arrive at predictions for the rates, both under the assumption that $\Delta I=\frac{1}{2}$, and by allowing $\Delta I=\frac{3}{2}$ admixtures. A compilation of predictions on rates by models that allow for deviations from relations (7a)–(7d) is presented in Table VI. It is seen that the experimental results (Table V) are in good to fair agreement with these models, while they certainly cannot differentiate between them.

Predictions concerning the $K \to 3\pi$ rates are also made by models that are based on the strong interactions of the pions in the final state.[45–47] These models have at least one free parameter and any one of them should fit the experimental data reasonably well for some value or range of values of the parameters. Chiu *et al.*[48] extend the current-algebra-plus-PCAC approach to take into account final-state π-π interactions. They then show that introduction of even a large amount of interaction in the final state does not significantly change the description of $K \to 3\pi$ decays.

In conclusion, the branching-ratio data (a) suggest a small $\Delta I=\frac{3}{2}$ admixture in the $K \to 3\pi$ decay amplitudes, and (b) are in good to fair agreement with theoretical models that introduce $\Delta I=\frac{1}{2}$ rule violations.

Table VI. Predictions of $\Delta I=\frac{1}{2}$ rule violations by various models.

Author(s)	Ref.	$\gamma^{+-0}/2\gamma^{+00}$	$\gamma^{000}/(\gamma^{++-}-\gamma^{+00})$	$\gamma^{++-}/4\gamma^{+00}$	$\gamma^{000}/\frac{3}{2}\gamma^{+-0}$
Bouchiat and Meyer	39	0.815	0.815	1.000	1.000
Holstein	40	0.815	0.815	1.000	1.000
Matsuda and Oppo[a]	41	0.95	0.91	1.00	1.12
		0.82	0.79	1.00	0.97
Greenberg	34	0.80	0.84	1.00	1.05
Nieh	42	0.87	0.87	1.00	1.00
Clavelli	43	0.876	0.972	0.979	1.020
Graham and Yun	44	0.83	0.83	1.00	1.00

[a] Upper row and lower row are, respectively, without and with $\eta^0 = X^0$ mixing (see text).

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