Impulse Model for the Reactions $K^-+He^4 \rightarrow Y+\pi+H^3$ †

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An impulse model is presented for the reactions $K^-+He^4 \rightarrow Y+\pi+H^3$. The model incorporates the low-energy $\bar{K}$-nucleon parametrization of Kim. The calculation is done for capture at rest from both s and p atomic orbital states and also for kaon interactions in flight. The results are compared with experiment for the final states $\Lambda\pi^-He^3$ and $\Sigma^+\pi^-H^3$ from at-rest kaon interactions.

I. INTRODUCTION

IN a paper accompanying this one,[1] there are reported the results of an experimental investigation of the process

$$K^-+He^4 \rightarrow \Lambda+\pi^-+He^3. \quad (1)$$

The experimental results are compared with an impulse-model calculation of that process. The purpose of the present work is to record some details of the calculation and to discuss further the comparison with experiment. In addition, we present some results of calculations of the process[2]

$$K^-+He^4 \rightarrow \Sigma^{\pm}+\pi^{\mp}+H^3. \quad (2)$$

A slight modification of the calculation reported upon here has already been used[3] to discuss the process

$$K^-+d \rightarrow \Lambda+\pi^-+p. \quad (3)$$

There have already been several impulse-model calculations of the reactions (1)–(3) reported in the literature.[4–7] The only new feature of the present calculation is the inclusion of an s- and p-wave parametrization of the low-energy $\bar{K}$-nucleon scattering amplitude. This is in contrast with the results of previous work,[6,7] where the impulse amplitude is constructed from "direct" and "resonant" contributions that bear an unknown relationship to one another. Consequently, the present calculation, within the limitations of the impulse model itself, is completely free of arbitrary parameters. It follows then that the comparison between the predictions of the calculation and the experimental results will permit us to draw inferences concerning our understanding of tke atomic phenomena, the nuclear phenomena, and the "elementary particle" ($\bar{K}N$) phenomena that play a role in the reactions that we study.

It is certainly true that our calculated predictions would be much more convincing if they were accompanied by a rigorous justification of the use of the impulse approximation. Since we are unable to provide such justification, our confidence in the calculation must rely primarily upon experimental clues that it is giving reasonable results. However, in the case of capture from rest, which is our primary interest here, it is possible to argue that many of the predictions may be better than the model from which they are obtained. In order to see how this might come about, let us first note that the obvious correction to be made to the impulse approximation is to include the initial- and final-state scattering effects.

Now the important feature of capture from rest is that the capture takes place from a definite atomic state of the kaon $+\alpha$-particle system. The primary effect of initial-state scattering corrections will be to modify the small-distance part of the atomic wave function from the hydrogenic form that is used in our calculations. Such modification certainly cannot change the orbital state from which capture occurs and might be expected to affect only the nuclear overlap and, hence, the total capture rate. Since our main interest is in the shape of the Dalitz plot, such corrections are not of great concern.

Final-state scattering effects, on the other hand, are not a serious problem because we believe that they may be identified and corrected for, at least qualitatively. For example, it is to be expected that some of the events found in reaction (1) are best described as originating from "direct" $\Sigma^+\pi^-$ (or $\Sigma^0\pi^-$) production with the Σ converting to a Λ in the course of a "final-state scattering." Such events are, in fact, labeled by having a pion momentum characteristic of Σ production rather than

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[1] K. Bunnell, M. Derrick, T. Fields, L. G. Hyman, and G. Keyes, preceding paper, Phys. Rev. D **1**, 98 (1970).
[2] J. G. Fetkovich, in *Proceedings of the International Conference on Hypernuclear Physics*, edited A. Bodmer and L. G. Hyman (Argonne National Laboratory, Argonne, Ill., 1969), p. 451.
[3] K. Bunnell, D. Cline, R. Laumann, J. Mapp, and J. Uretsky, Nuovo Cimento Letters **3**, 224 (1970).
[4] J. Leitner and S. Lichtman, Nuovo Cimento **15**, 719 (1960).
[5] M. M. Block, Nuovo Cimento **20**, 716 (1961).
[6] J. Sawicki, Nuovo Cimento **33**, 361 (1964); Phys. Rev. **152**, 1246 (1966); Nucl. Phys. **B1**, 183 (1967).
[7] P. Said and J. Sawicki, Phys. Rev. **139**, B991 (1965).

Λ production and are readily disregarded in a comparison between experiment[1] and the simple-minded calculations done here. Other final-state scatterings involving the final nucleus should often lead to nuclear breakup; experiment suggests[2] that this does not often happen and such effects are unlikely to play a significant role. We do expect to see some distortion of the Dalitz plot due to strong "final-state forces" between pairs of particles. Generally speaking, we know where to look for these and take them qualitatively into account. It is to be hoped that future investigators will include such effects in more refined calculations. (See, however, Ref. 7.)

Having now disposed for the time being of the question of the validity of the impulse approximation, we proceed to discuss the calculation. The first step is to select an initial-state wave function. We look to experiment for guidance and conclude in Sec. II that in the case of stopping K mesons, the calculation should be done for capture from a $1s$ or $2p$ orbital. In Sec. III, we discuss briefly the choice of nuclear wave functions. The formulas used in the calculation are derived in Sec. IV, which should be skipped by those who do not wish to check our calculations. Section V consists of some brief remarks concerning the construction of a resonant P_{13} amplitude that is strongly coupled to the $\bar{K}N$ system. Section VI describes the results, and Sec. VII contains our conclusions.

II. INITIAL STATE FOR CAPTURE FROM REST

For at-rest kaons, it is of importance to know the orbital atomic state from which the capture originates. In hydrogen, the capture is generally thought to be from s states of high n (principal quantum number) in agreement with the atomic cascade time which has been experimentally determined.[8] In deuterium, the capture is also generally accepted to be from orbital s states.

In helium, the situation is at present very confused. An early theoretical estimate[9] assumed that the Stark effect, which played an important role in capture in hydrogen, also was important in helium so that, again, most capture would occur from high s states. This estimate gave a cascade time which was two orders of magnitude smaller than that inferred from the experiment.[10] Later theoretical estimates[11] of the cascade time in which the Stark effect was not dominant (so that capture would occur from low-lying states such as $1s$ or $2p$) gave a closer agreement with experimental measurement but was still a factor of 5 too small. It has been suggested[12,13] that the observed apparent cascade time results from a few kaons being "trapped" in circular orbits for $n=30$, while the remainder are captured rapidly, via the Stark mechanism in high s states. A recent calculation[14] has shown that the Stark effect should be small for the "trapped" kaons, and, hence, such a "trapping" mechanism is plausible.

The most direct way to determine the nature of the cascade process and, hence, the capture state is to look for the x rays emitted from the K^--helium system. Such an experiment has been done quite recently[15] and no x-ray lines were observed. An upper limit was obtained for the L_α intensity of 0.03 ± 0.07 x rays per stopping K^-. This result is in disagreement with an earlier experiment[16] in which an appreciable L_α intensity was reported. Using a model including Auger transitions, collisional mixing, and radiative transitions, the relative amounts of (mostly $n>3$) s-, p-, and d-state captures were estimated[15] such that the results were consistent with the later null measurement. The rates were 60–70% p state, 20–25% s state, and 10–15% d state, but the results were, of course, model-dependent. The model can reproduce the apparent long cascade time if approximately 1% of stopped kaons were "trapped" in the high-n orbits. As a result of the uncertainty concerning the detailed captured mechanism in helium, the s and p states are separately considered in this report.

III. NUCLEAR FORM FACTOR

As will be demonstrated, the impulse-approximation matrix elements for reactions (1)–(3) depend upon a form factor

$$F(q)=\int d^3r\,\tilde{F}(r)e^{i\mathbf{q}\cdot\mathbf{r}}. \tag{4}$$

Here $\mathbf{q}$ is the momentum transfer between the initial and final nuclear states and $\tilde{F}(r)$ represents the overlap of these states. In the case at hand then, $\tilde{F}(r)$ is to be interpreted as the probability amplitude (wave function) for He^4 to dissociate into a neutron and He^3 separated by a distance r. The same form factor enters into the impulse-model matrix elements for other reactions where a single nucleon is blasted out of the helium nucleus. Examples of such reactions may be of the form $He^4(X,X'p)H^3$, where X could be an electron, proton, or pion. It follows that a test of the impulse model, or rather its range of validity, consists of comparing the nuclear form factors determined from these different production experiments. We thereby concur with the suggestion of Jackson[17] that the careful correlation of a

[8] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959); Phys. Rev. **118**, 864 (1960).
[9] T. B. Day, Nuovo Cimento **18**, 381 (1960).
[10] M. M. Block, J. B. Kopelman, and C. R. Sun, Phys. Rev. **140**, B143 (1965); K. Bunnell, M. Derrick, T. Fields, L. G. Hyman, G. Leyes, J. G. Fetkovich, J. McKenzie, and I.-T. Wang (unpublished).
[11] J. E. Russell, Proc. Phys. Soc. (London) **85**, 245 (1965).
[12] A. S. Wightman, thesis, Princeton University, 1949 (unpublished).
[13] G. T. Condo, Phys. Letters **9**, 65 (1964).
[14] J. E. Russell, Phys. Rev. Letters **23**, 63 (1969).
[15] G. Burleson, in Ref. 2, p. 639.
[16] G. Burleson, D. Cohen, R. C. Lamb, D. N. Michael, and R. A. Schluter, Phys. Rev. Letters **15**, 70 (1965).
[17] D. F. Jackson, Advan. Phys. **17**, 481 (1968).

wide class of such experiments through the impulse approximation is a necessary first step in the study of short-range correlations in nuclei.

Ideally then, the proper procedure for us to follow is to do the calculation with a form factor taken from an appropriate inelastic scattering experiment. Unfortunately, it appears that the requisite data do not yet exist, so that the kaon absorption experiments may be the first to probe the single-particle structure of helium over the momentum range of interest to us (about 500 MeV/c). Therefore, in order to make a prediction of the experimental results, it is necessary for us to guess at a form factor.

The most obvious way to guess is to take a simple model of the helium wave function and incorporate experimental information that is already available.[18] We do this by trying an independent-particle Gaussian wave function (for both He^3 and He^4) that gives a charge form factor in agreement with that determined from *elastic* electron scatterings.[19] Such a wave function seems to give a reasonable fit[20] to experiments on π^- absorption by He^4. A Hulthén wave function for the two-body n-He^3 system has also been tried.[21] The point to be emphasized is that it is only through an act of great faith that one can believe that either choice corresponds to that of nature.

IV. DERIVATION OF FORMULAS

Although the impulse-approximation formulas have often been written down before,[22] we recount the derivation here for completeness and to permit verification of our normalization. All particles except the pion are treated nonrelativistically, and $\hbar=c=1$ throughout. For the sake of clarity, the following derivation is for reaction (1).

We define a matrix element

$$M=(2\pi)\delta(E_f-E_i)\langle S_\Lambda,S_{He^3};\mathbf{P}',\mathbf{Q}'|T|i\rangle \equiv(2\pi)\delta(E_f-E_i)T_{fi}, \tag{5}$$

with E_f (E_i) the final (initial) total energy, and S_Λ (S_{He^3}), the spin projection of the Λ-hyperon (He^3 nucleus) along a suitable axis. The relative momentum in the final state between the pion and Λ is $\mathbf{P}'$, and $\mathbf{Q}'$ is the momentum of the outgoing He^3 nucleus with respect to the Λ-π system. The initial state $|i\rangle$ may be either an atomic bound state or scattering state of the kaon-He^4 system. In the latter case, the relative momentum is (in an obvious notation)

$$\nu=(m_{He^4}\mathbf{P}_K-m_K\mathbf{P}_{He^4})(m_{He^4}+m_K)^{-1}. \tag{6}$$

The normalization of the states has been chosen so that the absorption rate from the state $|i\rangle$ into a state $\langle f|$ is just

$$\Gamma_{fi}=2(2\pi)^3\int(E_\Lambda E_{He^3}E_\pi)|T_{fi}|^2dt_{He^3}dt_\pi, \tag{7}$$

where t_i is the kinetic energy of the relevant final particle and the integral is over the observed portion of the Dalitz plot. Here E_i is the total (relativistic) energy of the particle in question. In the case of capture in flight, the cross section becomes

$$\frac{d^4\sigma}{dt_{He^3}dt_\pi d\Omega_\pi}=(2\pi)^6\beta_K^{-1}(E_\Lambda E_{He^3}E_\pi)|T_{fi}|^2, \tag{8}$$

with β_K^{-1} the velocity of the incoming kaon in the barycentric system.

We consider the cases where the initial state is either an atomic s or p state (of principal quantum number n) or a free-scattering state. Then a convenient way to proceed is to use a field-theoretic formalism and write (omitting spin indices for the moment)

$$\langle f|T|i\rangle=\langle\mathbf{P}_{He^3}\mathbf{P}_\Lambda|j_\pi|i\rangle =\int\langle\mathbf{P}_{He^3}\mathbf{P}_\Lambda|j_\pi|\mathbf{P}_{He^3}\mathbf{P}_n'\mathbf{P}_K'\rangle \times d^3P_{H^3}'d^3P_n'd^3P_K'\langle P_{He^3}'P_n'P_K'|i\rangle, \tag{9}$$

where we have expanded in a "complete set" of "in" states and used Lehmann-Symanzik-Zimmermann (LSZ) reduction[23] on the outgoing pion. Here j_π denotes the source "current" for the pion field evaluated at the (arbitrary) origin of space-time coordinates and the subscript n denotes the struck neutron. It is, of course, a consequence of the impulse approximation that the sum in Eq. (9) consists of just a single term. Actually, this is not quite correct; we must remember later to sum over both neutrons in the helium nucleus. The impulse approximation states that the matrix element of j_π in Eq. (9) must be proportional to $\delta^3(\mathbf{P}_{He^3}'-\mathbf{P}_{He^3})$, thereby giving

$$\langle f|T|i\rangle=\int\langle\mathbf{P}_\Lambda|j_\pi|\mathbf{P}_n'\mathbf{P}_K'\rangle d^3P_n'd^3P_K' \times\langle\mathbf{P}_{He^3}\mathbf{P}_n'\mathbf{P}_K'|i\rangle. \tag{10}$$

The advantage of the present formulation is that it now exhibits explicitly the amplitude for the "elementary" process $\bar{K}+n\to\Lambda+\pi$. In fact, in the barycentric

[18] We belabor this somewhat trivial point because it does not seem to be well understood. See, for example, the discussion following the talk by one of us (JLU) reported in Ref. 2, p. 493.

[19] This form factor used is of the form $F(q)=Ne^{-\beta q^2}$ with $\beta=0.59$ F. This corresponds to a He^4 and He^3 charge radius of 1.44 F. See R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956); also, R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearian, Phys. Rev. **160**, 874 (1967); and Ref. 5.

[20] S. G. Eckstein, Phys. Rev. **129**, 413 (1963).

[21] The results from the Hulthén form factor are given in the previous paper. The form factor is identical to that used in Ref. 7, $F(q)=N_H*[1/(q^2+m^2)-1/(q^2+n^2)]$ with $n=1.25$ F and $m=0.8$ F.

[22] A convenient reference is A. B. Clegg, *High Energy Nuclear Reactions* (Clarendon, Oxford, England, 1965).

[23] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento **1**, 425 (1955).

system, we have for this process

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{2}(2\pi)^7\left(\frac{P_\Lambda}{P_n}\right)\times(E_KE_\Lambda E_n)S_{\pi\Lambda}^{-1}|\langle\mathbf{P}_\Lambda|j_\pi|\mathbf{P}_n\mathbf{P}_K\rangle|^2, \quad (11)$$

where $S_{\pi\Lambda}$ is the square of the π-Λ barycentric energy. If we now write the differential cross section in the standard form[24]

$$\frac{d\sigma}{d\Omega}=|\langle f|F+i\boldsymbol{\sigma}\cdot\hat{P}_\Lambda\times\hat{P}_nG|i\rangle|^2 \quad (12)$$

(where $|f\rangle$ and $|i\rangle$ now stand for the Λ and neutron spin states), then the expansion of the matrix element of j_π into properly normalized partial-wave amplitudes is immediately accomplished. In fact, using the K-matrix parametrization of Ross and Shaw,[25] we insert in place of the matrix element $\langle\mathbf{P}_\Lambda|j_\pi|\mathbf{P}_n\mathbf{P}_K\rangle$, an expansion in terms of the partial-wave amplitudes

$$f_l\equiv(2\pi)^{-7/2}(2S_{\pi\Lambda}E_KM_\Lambda M_n)^{1/2}\times\{k_f{}^l[M-i(k_f'k_i')^{l+1/2}]^{-1}k_i{}^l\}_{\bar{K}n\to\pi\Lambda}, \quad (13)$$

where k_f (k_i) is the diagonal matrix of final (initial) barycentric channel momenta, and the subscript denotes the appropriate matrix element of the matrix expression in brackets.

There is still one difficulty left to be resolved, since the impulse model requires the use of Eq. (13) at energies below the physical $\bar{K}$-nucleon threshold. We resolve this by interpreting the k^l factors *outside* of the square brackets as angular momentum barrier factors which are given by the real initial and final collision momenta (which are defined by insisting upon momentum conservation at each "vertex"). On the other hand, it is to be expected that the entire quantity *inside* the square brackets should be an analytic function of the total energy $(S_{\pi\Lambda})^{1/2}$. Thus, the $k_{i,f}$ inside the brackets will have imaginary components and will be different, in some components, from the corresponding factors outside the brackets since they are to be expressed as functions of $S_{\pi\Lambda}$ and the physical particle masses. For this reason, we have used the prime ($'$) to denote the difference.

We discuss next the last factor in Eq. (10), which is the amplitude for the K^--He^4 atomic or scattering state to dissociate into free K^--He^3-n, K^- (virtual) particles. It is consistent with our approximation to write (in the over-all barycentric system)

$$\langle\mathbf{P}_{He^3}\mathbf{P}_n'\mathbf{P}_K'|i\rangle=\delta^3(\mathbf{P}_K'+\mathbf{P}_n'+\mathbf{P}_{He^3})F_n\times\left(\mathbf{P}_{He^3}-\frac{M_{He^3}}{M_{He^4}}\boldsymbol{\nu}\right)F_A(\boldsymbol{\nu}), \quad (14)$$

where F_A and F_n are, respectively, the atomic and nuclear form factors. Then the matrix element in Eq. (9) becomes [with $\boldsymbol{\nu}$ defined in Eq. (6)]

$$\langle f|T|i\rangle=\int d^3\nu\, F_n\left(\mathbf{P}_{He^3}-\frac{M_{He^3}}{M_{He^4}}\right)\times F_A(\boldsymbol{\nu})\langle P_\Lambda|j_\pi|\mathbf{P}_n'\mathbf{P}_K'\rangle,$$

with

$$\mathbf{P}_n'+\mathbf{P}_K'+\mathbf{P}_{He^3}=0. \quad (15)$$

Using the δ-function approximation for the s- and p-state atomic wave functions gives

$$\langle f|T|i\rangle=\pi(\beta/n)^{3/2}F_n(\mathbf{P}_{He^3})\langle P_\Lambda|j_\pi|\mathbf{P}_n'\mathbf{P}_K'=0\rangle, \quad (16a)$$

with $\mathbf{P}_n'+\mathbf{P}_{He^3}=0$ (s wave);

$$\langle f|T|i\rangle=i\pi(\beta/2n)^{3/2}(n^2-1)^{1/2}\boldsymbol{\varepsilon}^m\cdot\boldsymbol{\nabla}_\nu\times[F_n(\mathbf{P}_{He^3}-\tfrac{3}{4}\boldsymbol{\nu})\langle\mathbf{P}_\Lambda|j_\pi|\mathbf{P}_n'\mathbf{P}_K\rangle], \quad (16b)$$

with $\mathbf{P}_n'+\mathbf{P}_{He^3}+\mathbf{P}_K=0$ and $\mathbf{P}_K=\boldsymbol{\nu}=0$ (p wave);

$$\langle f|T|i\rangle=F_n(\mathbf{P}_{He^3}-\tfrac{3}{4}\mathbf{P}_K)\langle P_\Lambda|j_\pi|P_n'\mathbf{P}_K\rangle, \quad (16c)$$

with $\mathbf{P}_n'+\mathbf{P}_K+\mathbf{P}_{He^3}=0$ (plane wave). Here n is the principal quantum number, $\boldsymbol{\varepsilon}^m$ is the initial p-state polarization vector, and β^{-1} is $\frac{1}{2}$ the Bohr radius for the atomic system.

The nuclear form factor is defined to be

$$F_n(\mathbf{q})=(2\pi)^{-3/2}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}q_n(\mathbf{r}), \quad (17)$$

where $q_n(\mathbf{r})$ is either the He^3—He^4 overlap or the relative He^3-n wave function, as described in Sec. III.

The calculation outlined here is used for $Y\pi$ final states with energy above and also below the $\bar{K}N$ threshold. The computation of distributions and cross sections was done on a CDC 3600 computer. The program starts by defining a Dalitz-plot boundary and for a given T_π and T_{He^3} inside this boundary, the integration over $\cos\alpha_{K^-\pi}$ (in the in-flight case) is done numerically. This is done for all regions within the boundary and the values are summed across to get the pion and He^3 kinetic-energy distributions.

The program is set up in a manner such that any interaction of the type K^-+nucleus $\to Y+\pi+$(spectator) can be investigated using the impulse approximation. The relevant quantities that are calculated are: (1) a Dalitz plot of T_π versus $T_{(\text{spectator})}$, (2) the projections of the Dalitz plot, (3) the momentum spectra P_π and $P_{(\text{spectator})}$, (4) the invariant mass spectra of M (Y +spectator) and M ($Y\pi$), and (5) the decay angle of the $Y\pi$ system. The cross section or the absorption for the reaction is also calculated by summing over the whole Dalitz plot. In order to allow for spreading in the beam momentum, the program is arranged so that an experimental beam distribution can be inserted and the final distributions weighted accordingly.

[24] J. V. Lepore, Phys. Rev. **79**, 137 (1950). The usual reference given is G. F. Chew *et al.*, *ibid.* **106**, 1337 (1957).
[25] M. Ross and G. Shaw, Ann. Phys. (N. Y.) **13**, 147 (1961).

V. $SU(3)$ RESONANT AMPLITUDE FOR P_{13} WAVE

The calculation that we are reporting upon here was based upon the Kim[26] parametrization of the M matrix introduced in Eq. (13). This particular parametrization is characterized by the nearly complete decoupling of the $Y_1^*(1385)$ from the $\bar{K}$-nucleon channel. However, recent evidence from K^--d experiments suggested that the decoupling does not really occur in nature. It seemed resonable, as a result, to replace the Kim P_{13} amplitude by one that had the rather strong coupling of $Y_1^*(1385)$ to $\bar{K}N$ that is predicted by $SU(3)$. We then tested the modified amplitude against experiment with results that have been reported elsewhere.[3] In this section, we present the details of our construction of an $SU(3)$ P_{13} amplitude.

To define a T matrix for the P_{13} wave in the neighborhood of a resonance, the following method is employed.[27] Define

$$T=K(1-i\rho K)^{-1}$$

and

$$S=1+2i\rho^{1/2}T\rho^{1/2},$$

where ρ is a many-channel momentum matrix

$$\rho=\begin{pmatrix} k_{11}{}^3\theta(k_{11}) & & \\ & k_{22}{}^3\theta(k_{22}) & \\ & & \cdots \end{pmatrix},$$

where

$$\theta(k_{ii})=1 \text{ for } k_{ii} \text{ above threshold}$$
$$=0 \text{ for } k_{ii} \text{ below threshold}.$$

Now define ϕ near a resonance of energy w^* by

$$K^{-1}(w)\phi=0,$$

and if K^{-1} varies linearly with w,

$$K^{-1}(w)=A-Bw.$$

Now perturbing K^{-1} and retaining first-order terms,

$$K^{-1}(w^*+\Delta w)(\phi+\Delta\phi)=\Delta\lambda(\phi+\Delta\phi)$$

and

$$\Delta\lambda=-(\phi,B\phi)\Delta w,$$

then, since $K^{-1}(w)=A-Bw$,

$$-B=\delta K^{-1}(w)/\delta w$$

and

$$\Delta\lambda=(\phi,\delta K^{-1}(w)/\delta w,\phi)\Delta w.$$

To find the behavior of K around the resonance, we have $K^{-1}(w^*)\phi=(\Delta\lambda)\phi$, from which it is easily seen that

$$K(w\approx w^*)=\frac{\frac{1}{2}\gamma}{w^*-w}\phi\phi^\dagger,$$

where

$$\tfrac{1}{2}\gamma=-(\phi,\delta K^{-1}(w)/\delta w,\phi)^{-1},$$

[26] J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).
[27] R. L. Warnock (private communication); and F. J. Ernst, R. L. Warnock, and K. C. Wali, Phys. Rev. **141**, 1354 (1966).

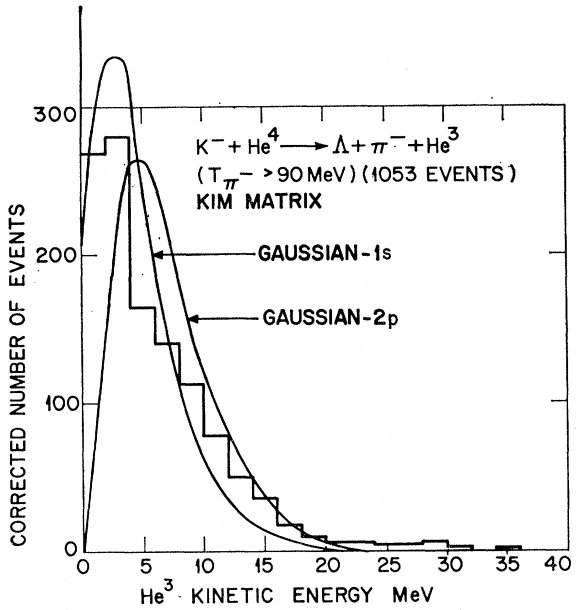

FIG. 1. He^3 kinetic-energy spectrum in the reaction $K^-He^4\rightarrow\Lambda\pi^-He^3$ for at-rest kaons from Refs. 1 and 2. The events have a pion energy greater than 90 MeV and hence have most of the Σ conversion events removed. Impulse model curves for the product Gaussian form factor using $1s$ and $2p$ capture are shown normalized to the total area.

and therefore

$$T=\tfrac{1}{2}\gamma\phi\phi^\dagger/[w^*-w-i(\phi,\rho\phi)^{1/2}].$$

To first order the ϕ's are now taken to be the vector of isoscalar factors for a given group.[28] The reduced width $\frac{1}{2}\gamma$ is found by evaluating the expression

$$\tfrac{1}{2}\Gamma=\tfrac{1}{2}\gamma(\phi,\rho\phi)$$

at the resonance where the width Γ is known. Hence, for the problem $\bar{K}N\rightarrow Y_1^*\rightarrow\Lambda\pi$, we use the isoscalar factors for $\mathbf{8}\times\mathbf{8}\rightarrow\mathbf{10}$, where **8** and **8** are the groups of pseudoscalar mesons and baryons and **10** is the decuplet of which the $Y_1^*(1385)$ is a member. The normalization to find $\frac{1}{2}\gamma$ is taken at the resonance $Y_1^*(1385)$ where the full width Γ is taken as 36 MeV. Note that this is in the same units as Kim's T matrix without the barrier factors k_i and k_σ.

VI. RESULTS

The results of the computation are presented and discussed in the preceding paper[1] and elsewhere.[2,29] There are only a few additional remarks that we add here.

Figures 1–3 show the energy distributions of the final state He^3 or H^3 in reactions (1) and (2) for at-rest kaons. Also shown are the fitted curves, as in the accompanying paper, for s- and p-state capture as calculated using

[28] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
[29] J. L. Uretsky, in Ref. 2, p. 493.

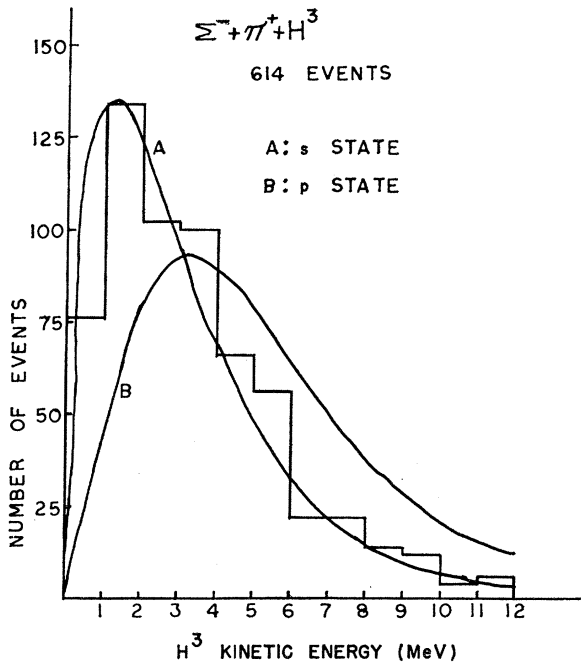

FIG. 2. H^3 kinetic-energy spectrum in the reaction $K^-He^4 \rightarrow \Sigma^-\pi^+H^3$ for at-rest kaons from Ref. 2. Impulse curves use the product Gaussian form factor with both $1s$ and $2p$ capture.

product Gaussian wave functions for the initial and final nuclei.[30] The important difference, in each case, between the s- and p-state cases is in the region of very low energy of the final nucleus. The depletion of events in this region in the case of p-state capture is a manifestation of the working of the angular momentum barrier. As such, this prediction is essentially independent of the nuclear model used. We therefore have a high degree of confidence in the conclusion that most of the captures come from atomic s states. Implications of this conclusion are discussed in Sec. VII.

The difference in the shapes of the impulse peaks in Figs. 2 and 3 is also of considerable interest. Detailed analysis shows that the difference arises from the interference between the resonating S_0 [$Y_0^*(1405)$] and "background" S_1 Σ-π production amplitudes. The interference turns out to be constructive in one case and destructive in the other. As a result, the position of the apparent resonance peaks are shifted from the actual resonance position, the direction depending upon the sign of the interference term. Since the resonance position is very close to the "impulse peak," the total effect is to modify slightly the shape of the peak.

VII. CONCLUSIONS

From the fact that the calculated cross sections are in reasonable agreement[1] with those measured in this experiment for the in-flight capture, we can conclude that the impulse model is not patently ridiculous. It is this fact, plus the related fact that the model gives reasonable absorption rates for s-wave and p-wave captures,[29] that encourages us to attempt to draw conclusions from fits to the experimental Dalitz plots.

To take things in sort of chronological order, let us consider first the capture process. We have argued in Sec. VI that most of the captures take place from atomic s states. Now the recent measurement[15] of x rays from K^- capture in He^4 suggests that the kaons make very few $K\alpha$ or $L\alpha$ transitions. From this, it is possible to infer that the captures are predominantly from states with principal quantum number $n \gtrsim 3$. We are immediately led into difficulty, since calculations[15] show that the resulting moderation time will be appreciably shorter than one infers from experiment. Our resolution of the disagreement is to conclude that the experimental determination of the moderation time is affected by trapping in circular orbits of large principal quantum number as proposed by Condo[12,13] and in more detail, by Russell.[14]

Let us turn next to the nuclear physics of the reaction. This bears chiefly upon the detailed shape of the He^3 and H^3 momentum spectra, appropriately corrected for final-state effects (especially $\Sigma-\Lambda$ conversion in the He^3 spectrum). Within the context of the impulse model, the shape of the spectrum shows the probability of a He^3 (H^3) and a neutron (proton) occurring within the nucleus as a function of their relative momentum. The fits to the data are notable in two respects. At low momenta, the dropoff is faster than is predicted by using Gaussian product wave functions (fitted to elastic electron scattering data). On the other hand, the high-momentum components are appreciably underestimated by the Gaussian form factor and, possibly, also by the

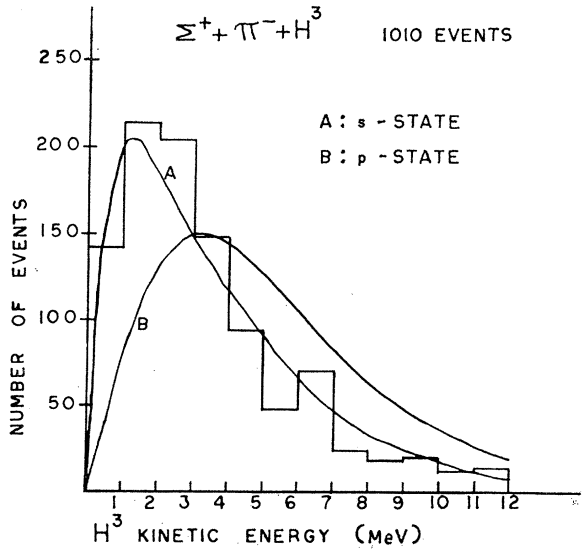

FIG. 3. H^3 kinetic-energy spectrum in the reaction $K^-He^4 \rightarrow \Sigma^+\pi^-H^3$ for at-rest kaons from Ref. 2. Impulse curves use the produce Gaussian form factor with both $1s$ and $2p$ capture.

[30] The data shown here are those presented in Ref. 2. While only the product Gaussian wave functions are shown here, the Hulthén form factor is shown for reaction (1) in Ref. 1.

two-body Hulthén fit. It would seem that product wave functions, not surprisingly, do not describe the helium nucleus very well. On the other hand, the three-body bound states seem to retain their identity inside the α particle surprisingly well, even for recoil momenta in excess of 400 MeV/c. We await with interest the explanation of these features by nuclear-structure experts.

Lastly, we consider some implications of our analysis concerning the "elementary-particle" features of the interaction, namely, the $\bar{K}+N \rightarrow Y+\pi$ reactions. It has been demonstrated elsewhere[3] that K^- absorption on nuclei seems to provide a meaningful way to study such reactions at energies below the physical threshold. An outcome of the earlier investigation was the discovery that the $\bar{K}NY_1^*(1385)$ coupling was much larger than had previously been suspected. A corollary conclusion that we offer here is that all the existing analyses of the low-energy $\bar{K}N$ scattering parameters are suspect and need to be checked in the "below-threshold" energy region. We have argued elsewhere[3] that this is probably done best with deuterium rather than more complex nuclei as the targets. As a consequence of the latter conclusion, we now have considerable doubts as to the validity of an earlier analysis[29] made by one of us (JLU) of the relative reaction rates and final-state mechanisms operative in K^- capture in helium.

In summary, then, we conclude the following.

(1) The impulse approximation gives a reasonable account of the capture process.

(2) The atomic capture is predominantly from s states with principal quantum number greater than or equal to two; therefore, the Condo mechanism is probably operative.

(3) An independent-particle model of the mass-three and mass-four nuclei, fitted to elastic electron-scattering data, is not suitable to describe the results of this experiment; in particular, there is an appreciable probability for $n-He^3$ relative momenta in excess of 400 MeV/c.

Having pursued our studies this far, it has become evident that there is a considerable amount of additional analysis that needs to be done. It is certainly not evident, for example, why the impulse model seems to work as well as it does. Some insight into the reason why may be attained by studying the effect of initial- and final-state distortions. These are, intuitively, the most important corrections to the impulse model.[17] Analysis of $\bar{K}$-deuterium reactions from the viewpoint of the three-body problem may also be a tractable way of feeding our intuition. At any rate, it is important to the experimentalist to see quantitatively the effect of final-state distortions. These have only been guessed at in the accompanying analysis.

A possibly important omission from the present calculation is the effect of Coulomb corrections throughout the interaction process. We do not expect these to be large, but the availability of high-statistics experiments is now making it important to calculate such "second-order" effects.

Finally, it should now be possible to include a reasonably honest calculation of the Σ-Λ conversion process (which for no good reason we distinguish from the other final-state scattering effects) since there is now experimental information available concerning low-energy Y-N scattering. Not only will this supply an important missing ingredient to the theoretical $\Lambda\pi^-He^3$ spectrum, but also it will play an important role in understanding the corresponding process where the final-state nucleus is unbound.[31]

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[31] A recent calculation of this process using a cluster expansion of the T matrix has been done by B. R. Wienke, Phys. Rev. D 1, 2514 (1970).