relation (14), taking into account the experimental errors in $f_K/f_\pi f_+(0)$. For details see Ref. 13, Chap. VI, Sec. 4. (e) The fact that we explicitly avoided a Goldstone κ meson is clear; if we had indeed the κ meson as a Goldstone boson, i.e., $F_\kappa=0(1)$,[16] and correspondingly $m_\kappa{}^2=0(\lambda)$, then the graphs where a κ meson is exchanged would have been of the *same* order as the graphs we separated out in Fig. 1 and *not* of order $O(\lambda^2)$. Therefore, in the case that we have indeed a Goldstone κ meson, the results (5) and (6) would be changed by extra contributions where a κ meson is exchanged. The diagrams are given in Figs. 5 and 6.

Thus we are faced with κK and $\kappa\pi$ decay and annihilation processes, so the modified equations (5) and (6) are not very interesting. The main point is that in case of a κ Goldstone boson, the Ademollo-Gatto theorem is invalidated by these κ terms. Alternatively, one can derive in this case an Adler-Weisberger sum rule for $f_+{}^2(0)$ using the commutator

$$\langle\pi^+(p)|[F^{4+i5},F^{4-i5}]|\pi^+(p)\rangle=\langle\pi^+(p)|F^3|\pi^+(p)\rangle.$$

There, $f_+(0)$ is equal to 1 plus a correction of order 1, since $F_\kappa=O(1)$.

The only way to distinguish the two cases is to have a precise experimental value for $f_+(0)$, i.e., a precise value for the Cabibbo angle.

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[16] $\langle 0|V_\mu{}^{4+i5}(0)|\kappa^-(k)\rangle\equiv iF_\kappa k_\mu$.

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Dynamics of a Double-Peaked Resonance*

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We present evidence that a double-peaked resonance implies two nearby poles of the S matrix rather than the interference of the resonance with a background where singularities are relatively far away.

It was suggested by two of the authors[1] that it is possible for a single, inelastic resonance to exhibit a double-peaked structure. In the example given in Ref. 1, it was demonstrated that a non-Breit-Wigner form for the inelasticity η could generate a form for δ, via the Ball-Frazer mechanism, which results in a double-peaked resonance. Within the framework of the analysis presented there, it was impossible to say if the example actually corresponded to having only a single resonance interfering with a background whose singularities are relatively far away or whether there were really two nearby poles of the S matrix.[2,3] Since then we have attempted to generate double-peaked structures corresponding to a single resonance by using multichannel ND^{-1} models and by using the inelastic single-channel Frye-Warnock equations. We have not succeeded in our efforts to find a physically meaningful example of a double-peaked resonance which can be explained as a single inelastic resonance. We conclude that a double-peaked structure in a single partial wave must be due to the coherent interference of two resonances.

We first tried to produce a double-peak resonance within the multichannel ND^{-1} framework. The nondiagonal, unphysical cut terms were approximated by a single-pole term; for the diagonal interactions, we used four poles in the first channel and two poles in the other channels. We worked with models containing up to four channels where one of the channels was used to produce a bound state which gave rise to a resonance in the other channels when the interchannel coupling was turned on. Despite considerable effort in adjusting the input parameters, we were unable to produce a double-peaked structure in which there was only one resonance pole.

The reason for the failure of the ND^{-1} model to give the desired effect could be that the potential term is not sufficiently complicated. We decided to *assume* a double-peaked structure and then compute the form of the potential term. We present an example of such

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[1] P. Coulter and G. Shaw, Phys. Rev. Letters **21**, 634 (1968).

[2] By a "double-peaked structure" we mean that there is a dip in the middle of a peak in the cross section as in Fig. 2. We do not refer to the dips that frequently occur in nuclear-physics problems near a resonance due to coherent interference with a complex background.

[3] By resonance we mean a peak in the cross section which is associated with a pole of the amplitude on some unphysical sheet. For a discussion of enhancements not associated with poles of the S matrix see G. Calucci, L. Fonda, and G. C. Ghirardi, Phys. Rev. **166**, 1719 (1968).

a calculation for an S wave in which η has a Breit-Wigner shape and δ is modified from a Breit-Wigner form in Figs. 1 and 2, so that it barely exhibits a double peak. Since η and δ are assumed to be known in the physical region, we can compute the potential term $B(s)$ in the physical region by using

$$B(s)=\mathrm{Re}A(s)-\frac{1}{\pi}P\int_{s_e}^{\infty}\frac{\mathrm{Im}A(s')}{s'-s}ds', \qquad (1)$$

where P denotes a principal-value integral and s_E is the elastic threshold. The accuracy of the calculation can be checked by inserting $B(s)$ in the Frye-Warnock equations[4] to recompute A:

$$\frac{2\eta(s)}{1+\eta(s)}\mathrm{Re}N(s)=\bar{B}(s)+\frac{1}{\pi}P\int_{s_E}^{\infty}\frac{\bar{B}(s')-(s/s')\bar{B}(s)}{s'-s}\times\frac{2\rho(s')\,\mathrm{Re}N(s')}{1+\eta(s')}ds', \qquad (2)$$

$$D(s)=1-\frac{s}{\pi}\int_{s_e}^{\infty}\frac{2\rho(s')\,\mathrm{Re}N(s')}{s'(1+\eta(s'))}\frac{ds'}{s'-s}, \qquad (3)$$

$$\bar{B}(s)=B(s)+\frac{1}{\pi}P\int_{s_I}^{\infty}\frac{1-\eta(s')}{2\rho(s')}\frac{ds'}{s'-s}, \qquad (4)$$

$$A=N/D=(\eta e^{2i\delta}-1)/2i\rho\equiv T/\rho.$$

ρ is a kinematical factor and s_I is the inelastic threshold.

We assume a Breit-Wigner amplitude ($l=0$)

$$T=\Gamma_E/[m^2-s-i(\Gamma_E+\Gamma_I)], \qquad (5)$$

where Γ_E is the elastic width given by

$$\Gamma_E=\gamma_E(s-s_E)^{1/2}. \qquad (6)$$

If we also assume

$$\Gamma_I=\gamma_I(s-s_I)^{1/2}, \qquad (7)$$

where Γ_I is the inelastic width, then A has no left-hand cut and B is zero. Then we modify A as presented in Fig. 1 from a Breit-Wigner form. We obtain, using Eq. (1), the nonzero B given in Fig. 3, which is still small by comparison to $\bar{B}$. Using this B, we solve the Frye-Warnock equations (2)–(4) and obtain an output amplitude which agrees with the input amplitude to within 1% and reproduces the double peak.

Unfortunately, we only know $B(s)$ in the physical region. In order to continue B away from real s to examine its singularity structure, we fit B using a Padé approximant

$$B(s)=\sum^{m-1}a_ns^n/\sum^{m}b_ns^n. \qquad (8)$$

The computed form of $B(s)$ is shown in Fig. 3. Since B changes sign three times, we must at least consider

[4] G. Frye and R. Warnock, Phys. Rev. **130**, 478 (1963).

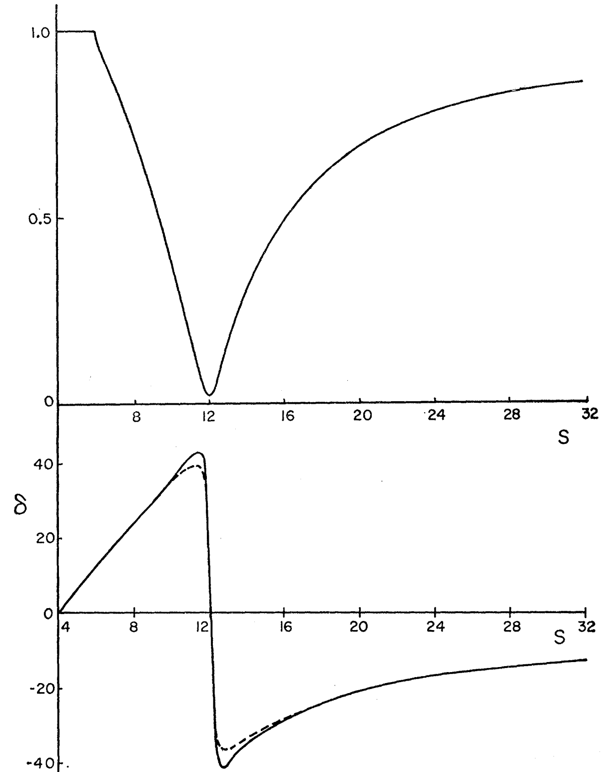

FIG. 1. (a) Inelastic factor resulting from the Breit-Wigner amplitude [Eq. (4)] versus s. We use units $\hbar=c=1$. The parameters are $s_E=4$, $s_I=6$, $\gamma_E=1$, $\gamma_I=1.189$, ‡$=(s-s_E)^{1/2}$, $m^2=12$. (b) Phase shift (solid line):

$$\delta=\delta_{BW}\{1+0.1/[(s-12)^2+0.5]\},$$

where δ_{BW} (dashed line) is the phase shift resulting from the Breit-Wigner amplitude, in degrees.

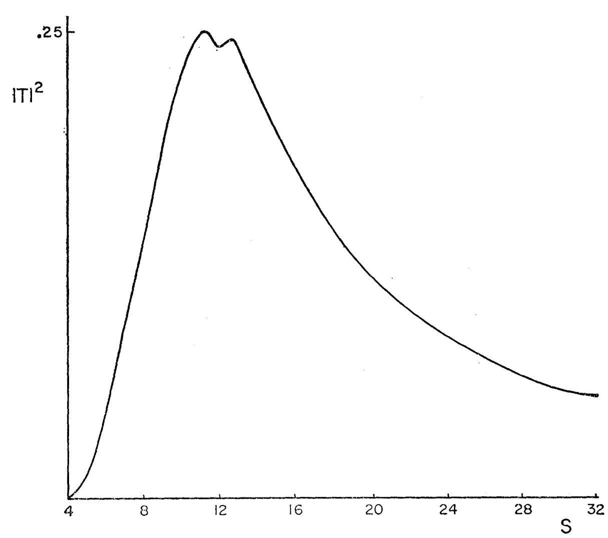

FIG. 2. Plot of $|T|^2$ versus s showing the doubled-peaked structure resulting from η and the solid line δ given in Fig. 1.

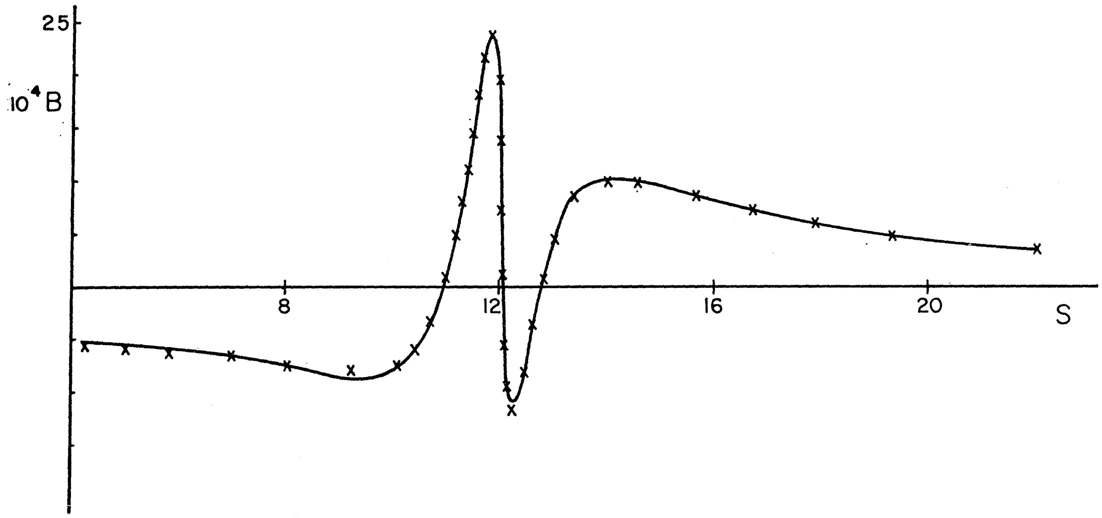

FIG. 3. The crosses show potential term computed from the solid curves in Fig. 1, using the dispersion relation, Eq. (1). Solid line shows the Padé-approximant fit [Eq. (9)] to this potential.

$m=4$. The numerical fit to Fig. 3 is

$$B(s)=0.0039\frac{(s-12.70)(s-12.01)(s-10.95)}{(s-a)(s-a^*)(s-b)(s-b^*)}, \quad (9)$$

where

$$a=11.87+1.59i, \quad b=12.01+0.29i. \quad (10)$$

B has two pair of complex-conjugate poles near the resonance. If we use Eq. (9) as input to the Frye-Warnock equations (2)–(4), we again obtain the graph shown in Fig. 2.

If the poles in B had been on the left, then this solution might have made sense physically. However, since the poles are so near the resonance, the example is entirely unrealistic. Using more parameters to fit B does not eliminate the nearby poles. If the dip in the resonance is made more pronounced, we obtain even wilder behavior in $B(s)$. Although we have not exhausted all possibilities for producing double-peaked structures in a single resonance, we regard it as highly unlikely that such structure can be due to anything but two coherent resonances.[2,3]