Bethe-Salpeter wave functions. Our discussion has also shown how a superconvergent dispersion relation could come about without the need to introduce spurious mesons.

Clearly we cannot expect perfect agreement with the data yet because we have oversimplified. There is a spurious essential singularity at $t=4M_p^2$, due to our Gaussian approximation, which is not valid for large x. This point in the timelike region is, of course, in the physical region for $\bar{p}p\rightarrow$ leptons. This and the small discrepancies with the data for small q^2 can be remedied by the selection of a more realistic wave function, perhaps with an exponential tail. In the following paper we give a formal treatment of the spin problem and present the definition of the electromagnetic form factor of the nucleons in a quark model. In a future paper we hope to present detailed calculations which include all the known vector mesons ρ, ω, and ϕ.

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Wave Functions and Form Factors for Relativistic Composite Particles. II

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We extend our relativistic description of composite particles to the case of clusters of spin-$\frac{1}{2}$ particles. From this, explicit expressions for the nucleon vector current form factors are derived. We find that the nucleon's electric form factor has the form

$$G^E(t)=\alpha^{-1}S_0(-t/\alpha)\sum_{i=1}^{3}q^iG_{ib}{}^1(t),$$

where S_0 is the nonrelativistic form factor, $\alpha=1-t/4M^2$, and q^i is the charge and $G_{ib}{}^1(t)$ the electric form factor for the ith quark. A similar expression is obtained for the magnetic form factor.

I. INTRODUCTION

In the preceding paper,[1] we discussed spinless composite particles which behave as nonrelativistic clusters in their rest frames. Here we generalize these results to include spin.[2] In particular, we derive explicit expressions for the nucleon form factors.

We derive a substitution law for the relativistic form factors which permits us to obtain them from the nonrelativistic form factors. We apply these results to the quark model of the nucleons and find for the proton electric form factor,

$$G_p{}^E(t)=\alpha^{-1}S_p\left(\frac{-t}{\alpha}\right)\sum_{i=1}^{3}q^iG_{ib}{}^1(t)\,,$$

where S_p is the nonrelativistic strong-interaction form factor of the proton, $G_{ib}{}^1$ is the electric form factor of the ith quark, q^i is the charge of the ith quark, and

$$\alpha=1-t/4M_p{}^2.$$

A similar expression holds for the magnetic form factor.

In Sec. II, to define our notation, we review briefly the description of elementary relativistic particles with spin. This is generalized in Sec. III to the case of relativistic particles with an arbitrary complex internal structure. In Sec. IV we consider particles whose rest-frame structure is described by a wave function. In Sec. V we relate the matrix elements of transition operators to the form factors. We derive our formulas for the form factors in Sec. VI. The proof of two mathematical relations is delegated to Appendices A and B.

II. RELATIVISTIC SPINNING PARTICLES

Let the ket $|pjma\rangle$ denote an elementary-particle state with four-momentum p, spin j with z component m in the rest frame, and other discrete quantum numbers a.[3,4] These states have the inner product

$$\langle p'j'm'a'|pjma\rangle=2p^0\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{jj'}\delta_{mm'}\delta_{aa'}\,. \quad (1)$$

We denote an inhomogeneous Lorentz transformation by (Λ,h). Let L_p denote a pure Lorentz transformation that takes the rest-frame vector $\hat{p}=((p^2)^{1/2},\mathbf{0})$ into p.

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Then

$$\Lambda(\hat{p})=(L_{\Lambda p})^{-1}\Lambda L_p \tag{2}$$

is an element of the $\hat{p}$ little group corresponding to Λ.

Let $D_{\alpha\alpha'}{}^j(R)$ denote the spin-j representation of the rotation group. Then, as is well known, there is a unitary representation $U(\Lambda,h)$ of the inhomogeneous Lorentz group defined by

$$U(\Lambda,h)|p,j,m,a\rangle=e^{-i\Lambda p\cdot h}|\Lambda p,j,m',a\rangle D_{m'm}{}^j(\Lambda(p)), \tag{3}$$

where summation over all allowed m' is understood.

III. COMPOSITE PARTICLES

The kets $|pjma\rangle$ can be written as tensor products

$$|pjma\rangle=|p\rangle\otimes|jma\rangle. \tag{4}$$

The kets $|p\rangle$ describe the c.m. motion; the nets $|jma\rangle$ describe the rest-frame structure of the particle. In the simplest case, this structure consists of only spin and unitary spin. In the more general case of a bound cluster, it may be more complicated. We describe it by a ket $|X\rangle$.

The particle is then described by the tensor product

$$|p,X\rangle=|p\rangle\otimes|X\rangle, \tag{5}$$

with the inner product

$$\langle p',X'|p,X\rangle=\langle p'|p\rangle\langle X'|X\rangle. \tag{6}$$

In the rest frame the rotations are the only allowed Lorentz transformations. We assume, therefore, that there is a unitary representation of the rotation group $V(R)$ which acts on $|X\rangle$. Now we can define a representation of the inhomogeneous Lorentz group by

$$U(\Lambda,h)|p,X\rangle=\exp[-i\Lambda p\cdot h]|\Lambda p\rangle\otimes V(\Lambda(p))|X\rangle. \tag{7}$$

IV. WAVE-FUNCTION CLUSTERS

Of special interest are composite particles which can be described in the rest frame as a bound cluster of n subparticles. We assume the subparticles have spin $\frac{1}{2}$. Let the ith subparticles be located in the rest frame at the vector position $\mathbf{x}_i$. Let the z component of its spin be α_i and its unitary spin a_i. We fix the c.m. to be at the origin by

$$\sum_{i=1}^{n}\mathbf{x}_i=0. \tag{8}$$

The total ket is

$$|p,\{\mathbf{x}_i,\ \alpha_i,\ a_i;\ i=1,\ldots,n\}\rangle, \tag{9}$$

with the inner product

$$\langle p',\{\mathbf{x}_i',\alpha_i',a_i'\}|p,\{\mathbf{x}_i,\alpha_i,a_i\}\rangle$$
$$=2p^0\delta^{(3)}(\mathbf{p}-\mathbf{p}')\sum_{\mathrm{perm}\,\pi}\epsilon_\pi\prod_{j=1}^{n}\delta_{\alpha_j,\alpha_{\pi j}'}\delta_{\alpha_j,\alpha_{\pi j}'}$$
$$\times\prod_{i=1}^{n-1}\delta^{(3)}(\mathbf{x}_i-\mathbf{x}_{\pi i}'). \tag{10}$$

Here πj is a permutation of the subparticles; $\epsilon_\pi=(N!)^{-1}$ for Bose statistics, $\mathrm{sgn}\pi(N!)^{-1}$ for Fermi statistics, and whatever is appropriate if some of the subparticles are distinguishable.

We define $U(\Lambda,h)$ on these kets by

$$U(\Lambda,h)|p,\{\mathbf{x}_i,\alpha_i,a_i\}\rangle=\exp[-i\Lambda p\cdot h]$$
$$\times|\Lambda p,\{\Lambda(p)\mathbf{x}_i,\alpha_i',a_i'\}\rangle\prod_{i=1}^{n}D_{\alpha_i'\alpha_i}{}^{1/2}(\Lambda(p)). \tag{11}$$

Not all combinations of the kets (9) will necessarily correspond to physical states. We construct the physical states as follows: Let $\{\Psi_A(l,m;\mathbf{x}_1,\ldots,\mathbf{x}_n)\}$ be a set of wave functions carrying angular momentum l, m. Let $\langle\chi_1\cdots\chi_n|ss_z\rangle$ be a Clebsch-Gordon coefficient for coupling the n spins together to give total spin s, s_z. Let $\langle l\tilde{m},ss_z|jm\rangle$ be the coefficient for coupling l and s together and let $\langle a_1\cdots a_n|a\rangle$ be the coefficient for coupling together all the unitary spins. Then we can interpret

$$|p,A,JMa\rangle=\sum_l\prod_{i=1}^{n-1}\int d^3x_i\sum_{\alpha_1\cdots\alpha_n}\sum_{ms_z}\Psi_A(lm,\mathbf{x}_1\cdots\mathbf{x}_n)$$
$$\times\langle\alpha_1\cdots\alpha_n|ss_z\rangle\langle lmss_z|JM\rangle$$
$$\times\langle a_1\cdots a_n|a\rangle|p,\{\mathbf{x}_j,\alpha_j,a_j\}\rangle \tag{12}$$

as a physical state provided that we take the mass square p^2 equal to the one appropriate for a bound state of spin J, z component M, unitary spin a, and internal state A.

V. TRANSITION OPERATORS

To calculate scattering, we will need to know matrix elements of the form

$$\langle p',B,j'm',a'|j_b{}^\mu(0)|p,A,jm,a\rangle. \tag{13}$$

Here $j_b{}^\mu(x)$ is some field operator carrying a tensor index μ and an $SU(3)$ index b. This operator can be written quite generally in terms of cluster kets as

$$j_b{}^\mu(x)=\sum\prod_{i=1}^{n}\prod_{j=1}^{n'}\int d^3x_i\int d^3y_j\int d^4p$$
$$\times K_b{}^\mu(p,\{\mathbf{x}_i,\alpha_i,a_i\},\{\mathbf{y}_j,\beta_i,b_j\})\int d^4p'|p',\{y,\beta,b\}\rangle$$
$$\times\langle p,\{\mathbf{x},\alpha,a\}|e^{-i(p'-p)x}, \tag{14}$$

where K is an unknown function with the right support and transformation properties. The sum in (14) goes over all the discrete quantum numbers.

We will derive here a simple approximation for $j_b{}^\mu(x)$ guided by the Breit-frame impulse approximation of I. We make the additivity assumption

$$j_b{}^\mu(x)=\sum_{i=1}^{n}j_{bi}{}^\mu(x), \tag{15}$$

where $j_{bi}{}^{\mu}$ acts only on the ith subparticle. Each such $j_{bi}{}^{\mu}$ can in turn be written as a sum of operators $\Gamma_{bi}{}^{\mu}$ which change kets of momentum p and mass m into kets of momentum p' and mass m':

$$j_{bi}{}^{\mu}(x)=\sum_{mm'}\int d^4p'\,\theta(p'^0)\delta(p'^2-m^2)\times\int d^4p\,\theta(p^0)\delta(p^2-m^2)\Gamma_{b,i}{}^{\mu}(p',p)e^{-i(p'-p)x}. \quad (16)$$

As in I, we assume that the transition $p\to p'$ takes place instantaneously in the p, p' Breit frame. Then

$$\Gamma_{bi}{}^{\mu}(p',p)=\prod_{j=1}^{n-1}\int d^3x_j\,E_{bi}{}^{\mu}(\{\mathbf{x}_i\},p',p), \quad (17)$$

where the integration is over the spatial coordinates of the subparticles in the Breit frame. We assume that $E_{bi}{}^{\mu}$ changes only the $SU(6)$ coordinates of the ith subparticle. As in I, there will also be a factor $\exp[-i(p'-p)\xi_i]$ in E, where $\xi_i=(0,\mathbf{x}_i)$.

Following the argument in I, a particle located at in the Breit frame has a rest-frame coordinate

$$\mathbf{y}_i=\mathrm{VP}\{L_p{}^{-1}(\xi_i-p(p\cdot\xi_i)/p^2)\} \quad (18)$$

$$=\mathrm{VP}\{L_p{}^{-1}\xi_i\}=\mathbf{H}_p\xi_i. \quad (19)$$

Collecting all this, we assume that

$$j_b{}^{\mu}(x)=\sum_i\sum_{mm'}\int d^4p'\,\theta(p'^0)\delta(p'^2-m'^2)\int d^4p\,\theta(p^0)\times\delta(p^2-m^2)\sum_{\alpha'\alpha a a'}\prod_{k=1}^{n-1}\int d^3x_k\exp[-i(p'-p)(x+\xi_i)]\times|p',\{\mathbf{X}_{p'}\xi,\alpha',a'\}\rangle\prod_{j\neq i}\delta_{\alpha_j'\alpha_j}\delta_{a_j'a_j}\times(J_b{}^{\mu}(p',p)_{\alpha'\alpha a'a})_i\langle p,\{\mathbf{H}_p\xi,\alpha,a\}|, \quad (20)$$

where

$$(J_b{}^{\mu}(p',p)_{\alpha'\alpha a'a})_i=\langle p'\alpha_i'a_i'|j_{bi}{}^{\mu}(0)|p\alpha_ia_i\rangle \quad (21)$$

is the matrix element of the tensor operator $j_{bi}{}^{\mu}$ between single subparticle states extrapolated off the i subparticle mass shell. The indices α_i and a_i always refer to the ith subparticle, but in the following we will drop the subindex i as was done on the right-hand side of (20).

We show now that the expression (20) can be written in manifestly covariant form. We note that[1]

$$[(p+p')^2]^{(n-1)/2}\prod_{i=1}^{n-1}d^4\xi_i\delta((p+p')\cdot\xi_i)=\prod_{i=1}^{n-1}d^3x_i \quad (22)$$

when evaluated in the Breit frame.

Let $u^{\alpha}(p)$ be a Dirac spinor with four-momentum p, z component of spin in the rest frame α, and normalized to

$$\bar{u}^{\alpha}(p)u^{\beta}(p)=\delta_{\alpha\beta}. \quad (23)$$

In the p, p' Breit frame

$$\bar{u}^{\alpha}(p')u^{\alpha}(p)=G(t,m',m)\delta_{\alpha\beta}, \quad (24)$$

where G is a complicated function of $t=(p'-p)^2$ and the masses. In the particular case when $p'^2=p^2$,

$$G(t)=(2m)^{-1}[(p'+p)^2]^{1/2}=(1-t/4m^2)^{1/2}. \quad (25)$$

In general, we can write

$$J_{bi}{}^{\mu}(p',p)_{\alpha'\alpha a'a}=\bar{u}^{\alpha}(p')A_{bia'a}{}^{\mu}(p',p)u^{\beta}(p), \quad (26)$$

where $A_{bi}{}^{\mu}$ is a tensor built out of the p', p, and the γ matrices. We claim that

$$j_{bj}{}^{\mu}(x)=\sum_{mm'}\int d^4p\,\theta(p'^0)\delta(p'^2-m'^2)\int d^4p\,\theta(p^0)\times\delta(p^2-m^2)\{[(p'+p)^2]^{1/2}[G(t,m',m)]^{-1}\}^{n-1}\times\sum_{\alpha'\alpha a'a}\prod_{k=1}^{n-1}\int d\xi_k\,\delta(\xi_k\cdot(p'+p))\times\exp[-i(p'-p)(x+\xi_j)]|p',\{\mathbf{H}_{p'}\xi,\alpha',a'\}\rangle\times\prod_{j\neq i}\bar{u}^{\alpha'}(p')u^{\beta}(p)\delta_{\alpha_j'\alpha_j}(\bar{u}^{\alpha'}(p')A_{ba'a}{}^{\mu}(p',p)u^{\alpha}(p))_i\times\langle p,\{\mathbf{H}_p\xi,\alpha,a\}|, \quad (27)$$

is Lorentz covariant.

To prove it we must show that

$$U(\Lambda,h)j_b{}^{\mu}(x)U(\Lambda,h)^{-1}=[C^{\mu\nu}(\Lambda)]^{-1}j_b{}^{\nu}(\Lambda x+h), \quad (28)$$

where C is the appropriate representation of the Lorentz group.

Using Eqs. (27), (11), and (A1), we find that $U(\Lambda,h)j_b{}^{\mu}(x)U(\Lambda,h)^{-1}$ is of the form

$$\sum_j\int d\mu(p',p,\xi)\exp[-i(p'-p)(x+\xi_j)]\times\exp[-i\Lambda(p'-p)h]|\Lambda p',\{\mathbf{H}_{\Lambda p'}\Lambda\xi,\beta'\}\rangle\times\langle\Lambda_p,\{\mathbf{H}_{\Lambda p}\Lambda\xi,\gamma\}|\prod D_{\beta'\alpha}{}^{1/2}(\Lambda p')\bar{u}^{\alpha}(p')u^{\beta}(p)\times D_{\beta\gamma}{}^{1/2*}(\Lambda p)D_{\beta'\alpha}{}^{1/2}(\Lambda p')\bar{u}^{\alpha}(p')A^{\mu}(p',p)u^{\beta}(p)\times D_{\beta\gamma}{}^{1/2*}(\Lambda p), \quad (29)$$

where $d\mu$ is a manifestly invariant volume element, and where we have suppressed unnecessary indices.

Using (B1), we see that

$$D_{\beta'\alpha}{}^{1/2}(\Lambda p')\bar{u}^{\alpha}(p')\begin{pmatrix}1\\A^{\mu}\end{pmatrix}u^{\beta}(p)D_{\beta\gamma}{}^{1/2*}(\Lambda p)$$

$$=\bar{u}^{\beta'}(\Lambda p')S(\Lambda)\begin{pmatrix}1\\A^{\mu}\end{pmatrix}S^{-1}(\Lambda)u^{\gamma}(\Lambda p)$$

$$=\bar{u}^{\beta'}(\Lambda p')\begin{pmatrix}1\\ [C^{\mu\nu}(\Lambda)]^{-1}A^{\nu}(\Lambda p,\Lambda p',\Lambda\xi)\end{pmatrix}u^{\gamma}(\Lambda p), \quad (30)$$

since, by construction, and by the properties of the γ matrices,

$$S(\Lambda)A^\mu(p',p,\xi)S(\Lambda)^{-1}=[C^{\mu\nu}(\Lambda)]^{-1}A^\nu(\Lambda p',\Lambda p,\Lambda\xi). \quad (31)$$

The assertion (28) now follows upon making the change of variable p, p', $\xi \to \Lambda^{-1}p$, $\Lambda^{-1}p'$, $\Lambda^{-1}\xi$.

VI. MATRIX ELEMENTS

In this section we express the matrix elements of the transition operators between the states of Eq. (12) in terms of the nonrelativistic form factors.

Let ϕ^α be the two-component spinor which generates u^α by

$$u^\alpha(\hat{p})=\begin{pmatrix}\varphi^\alpha\\ 0\end{pmatrix}. \quad (32)$$

Then we can write

$$(\bar{u}^{\alpha'}(p')A_{ba'a}{}^\mu(p',p)u^\alpha(p))_i=(\phi^{\alpha'\dagger}B_{ba'a}{}^\mu\phi^\alpha)_i, \quad (33)$$

where $B_{bi}{}^\mu$ is a matrix operator acting on the spin and $SU(3)$ indices of the ith subparticle.

Let $|ss_za\rangle$ be that part of the rest-frame ket of the cluster which carries just spin and $SU(3)$ indices. The operator B can be considered as acting on these kets. We also introduce the form factors

$$\langle pB'l'm'|pAlm\rangle$$
$$\prod_{i=1}^{n=1}\int d^3x_i\,\psi_{Bm'}{}^{l'*}(\mathbf{H}_{p'}\xi_i)\psi_{Am}{}^l(\mathbf{H}_p\xi_i)e^{-i(p'-p)\xi_k}, \quad (34)$$

where the integral is to be evaluated in the $p+p'$ Breit frame. This is the spatial wave-function part of the matrix element. Then

$$\langle p'Bj'm'a'|j_{bk}{}^\mu(0)|pAjma\rangle = \sum_{ll'mm's_zs_z'}\langle p'Bl'm'|pAlm\rangle_k\langle j'm'|l'm's's_z'\rangle \times\langle lmss_z|jm\rangle\langle s's_z'a'|B_{bk}{}^\mu|ss_za\rangle. \quad (35)$$

When $p^2=p'^2=M_A{}^2$, the spatial wave-function part simplifies to

$$\langle p'Bl'm'|pAlm\rangle_k =\alpha^{(1-n)/2}S_{BA}{}^k((\mathbf{p}'-\mathbf{p})\alpha^{-1/2},l'm',lm), \quad (36)$$

where

$$\alpha=1-t/4M_A{}^2 \quad (37)$$

and $S_{BA}{}^k$ is the nonrelativistic form factor

$$S_{BA}{}^k(\mathbf{q},l'm'lm) =\prod_{k=1}^{n-1}\int d^3x_i\,\psi_{Bm'}{}^{l'}(\{\mathbf{x}_i\})\psi_{Am}{}^l(\{\mathbf{x}_i'\})e^{-i\mathbf{q}\cdot\mathbf{x}_k}. \quad (38)$$

The proof of this is the same as for the corresponding result in I.

VII. NUCLEON VECTOR FORM FACTORS

As an example, we work out the vector form factors for the nucleon. We use the quark model. In this model we have

$$l=0,\quad B=A,\quad j=\tfrac{1}{2},\quad \text{and}\quad n=3. \quad (39)$$

For simplicity, we assume the spatial wave function to be symmetric under all permutations of the quark position vectors.

The wave-function part of the form factor is

$$\langle p'A00|pA00\rangle_i=\alpha^{-1}S_A(-t/\alpha) \quad (40)$$

and is independent of the quark index i. Here S is the nonrelativistic form factor

$$S_A(\mathbf{q}^2)=\prod_{i=1}^{n-1}\int d^3x_i|\psi_A(\{\mathbf{x}_i\})|^2e^{-i\mathbf{q}\cdot\mathbf{x}_i}, \quad (41)$$

which is a function of q^2 only, because of the rotational invariance of the wave functions.

We assume that the current operator $j_b{}^\mu$ is diagonal with respect to isospin and hypercharge. The single-quark matrix element of Eq. (20) is then

$$\langle p'\alpha_i'a_i'|j_{bi}{}^\mu(0)|p\alpha_ia_i\rangle=q_b{}^i\delta_{a_ia_i'}\bar{u}^{\alpha_i}(p')[F_i{}^1(t)\gamma^\mu +iF_i{}^2(t)\sigma^{\mu\nu}q_\nu M_A{}^{-1}]u^{\alpha_i}(p). \quad (42)$$

Here $q_b{}^i$ is the b charge carried by the ith quark; t is $(p'-p)^2=q^2$. $F_i{}^1$ and $F_i{}^2$ are scalar functions of t. Expressed in terms of spinors $\phi^{\alpha i}$ in the rest frame of A,

$$\langle p'\alpha_i'a_i'|j_{bi}{}^\mu(0)|p\alpha_ia_i\rangle =q_b{}^i\delta_{aa'}\phi^{\alpha'\dagger}[G_i{}^1(t),-i\mathbf{q}\times\boldsymbol{\sigma}(2M)^{-1}G_i{}^2(t)]\phi^\alpha, \quad (43)$$

where

$$\begin{aligned}G_i{}^1(t)&=F_i{}^1-(\mathbf{q}^2/4m)F_i{}^2,\\ G_i{}^2(t)&=F_i{}^1+F_i{}^2.\end{aligned} \quad (44)$$

The b-vector form factor can now be written in the Breit frame

$$\langle p'A|j_b{}^\mu(0)|pA\rangle=\alpha^{-1}S_A(-t/\alpha)\langle X|B^\mu|X\rangle, \quad (45)$$

where $|X\rangle$ is the rest-frame $SU(6)$ ket and the components of B^μ are

$$\begin{aligned}B^0&=\sum_{i=1}^3 q_b{}^iG_{bi}{}^1(t),\\ \mathbf{B}&=-i(2M)^{-1}\sum_{i=1}^3 q_b{}^i\mathbf{q}\times\boldsymbol{\sigma}^iG_{bi}{}^2(t).\end{aligned} \quad (46)$$

Here $\mathbf{q}$ is the momentum transfer evaluated in the Breit frame. The Pauli matrix $\boldsymbol{\sigma}^i$ acts only on the spin coordinates of the ith quark.

The vector operator

$$\mathbf{C}=\sum_{i=1}^3 q_b{}^i\boldsymbol{\sigma}^iG_{bi}{}^2(t) \quad (47)$$

can, in principle, connect states with different spins.

That part of $\mathbf{C}$, C_A, which connects only nucleon states of spin $\frac{1}{2}$, must be proportional to the total spin operator

$$\boldsymbol{\sigma}_T=\sum_{i=1}^{3}\boldsymbol{\sigma}_i \tag{48}$$

or

$$\mathbf{C}_A=D_A(t)\boldsymbol{\sigma}_T. \tag{49}$$

We obtain the proportionality factor D_A by taking the expectation value of the third component C_3 in a spin-up nucleon state

$$D_A(t)=\langle\tfrac{1}{2}\,\tfrac{1}{2}A\,|\,C_3\,|\,\tfrac{1}{2}\,\tfrac{1}{2}A\rangle. \tag{50}$$

The expectation value of the z spin of the ith quark in A is

$$\langle\tfrac{1}{2}\,\tfrac{1}{2}A\,|\,\sigma_{i3}\,|\,\tfrac{1}{2}\,\tfrac{1}{2}A\rangle=\epsilon_i \tag{51}$$

and this yields

$$D_A(t)=\sum_{i=1}^{3}q_b{}^i\epsilon_iG_{bi}{}^2(t). \tag{52}$$

It is customary to define the b "electric" and "magnetic" form factors for A by

$$\delta_{ss'}G_{bA}{}^E(t)=\langle p'\tfrac{1}{2}s'A\,|\,j_b{}^0(0)\,|\,p\tfrac{1}{2}sA\rangle, \tag{53}$$

$$-(i/2M)\mathbf{q}\times\langle\tfrac{1}{2}s'A\,|\,\boldsymbol{\sigma}\,|\,\tfrac{1}{2}sA\rangle G_{bA}{}^M(t)=\langle p'\tfrac{1}{2}s'A\,|\,\mathbf{j}_b(0)\,|\,p\tfrac{1}{2}sA\rangle, \tag{54}$$

where all quantities are to be evaluated in the Breit frame. From this we get

$$G_{bA}{}^E(t)=\alpha^{-1}S_A\left(\frac{-t}{\alpha}\right)\sum_{i=1}^{3}q_b{}^iG_{bi}{}^1(t), \tag{55}$$

$$G_{bA}{}^M(t)=\alpha^{-1}S_A\left(\frac{-t}{\alpha}\right)\sum_{i=1}^{3}q_b{}^i\epsilon_iG_{bi}{}^2(t). \tag{56}$$

VIII. DISCUSSION

We have developed a general method for writing form factors for bound clusters of spin-$\frac{1}{2}$ particles. Here we have applied the procedure to the quark model of the nucleon to derive the vector current form factors for the nucleons.

It should be borne in mind that these results rest on the validity of a particular kind of impulse approximation. Namely, we have assumed that the transition from the initial to the final cluster state takes place instantaneously in the Breit frame. This seems reasonable, but we have at present no way of testing this assumption other than by comparing the results with experiment. This requires as additional input some information about the wave function.

In a future article we shall present the results of a realistic calculation of the different electromagnetic form factors of the nucleons. From our derivation it is evident that our method is relevant in the spacelike region and will give the most significant corrections for large negative q^2. We shall show that the above procedure provides a significant improvement over the dipole fit.

APPENDIX A

Here we prove the relation

$$\Lambda(p')\mathbf{H}_{p'}\xi=\mathbf{H}_{\Lambda p'}\Lambda\xi. \tag{A1}$$

We have, by definition,

$$\Lambda(p')\mathbf{H}_{p'}\xi=\Lambda(p')\ \mathrm{VP}\{L_{p'}{}^{-1}\Delta(p')\xi\} \tag{A2}$$

$$=\mathrm{VP}\{\Lambda(p')L_{p'}{}^{-1}\Delta(p')\xi\}, \tag{A3}$$

since a pure rotation commutes with VP (taking the three-vector part). The right-hand side of (A3) can be simplified step by step since

$$\Lambda(p')L_{p'}{}^{-1}\Delta(p')\xi=L_{\Lambda p'}{}^{-1}\Lambda L_{p'}{}^{-1}\Delta_{p'}\xi \tag{A4}$$

$$\begin{aligned}&=L_{\Lambda p'}{}^{-1}\Lambda\Delta_{p'}\xi\\&=L_{\Lambda p'}{}^{-1}\Lambda[\xi-p(p\cdot\xi)/p^2]\\&=L_{\Lambda p'}{}^{-1}[\Lambda\xi-\Lambda p(p\cdot\xi/p^2)]\end{aligned}$$

$$=L_{\Lambda p'}{}^{-1}\Delta_{\Lambda p'}\Lambda\xi, \tag{A5}$$

which is all that is needed to prove (A1).

APPENDIX B

Here we prove the transformation law for spinors

$$D_{\beta\alpha}{}^{1/2}(\Lambda_p)\bar{u}^\alpha(p)=\bar{u}^\beta(\Lambda p)S(\Lambda) \tag{B1}$$

and

$$u^\alpha(p)D_{\alpha\beta}{}^{1/2*}(\Lambda_p)=S^{-1}(\Lambda)u^\beta(\Lambda p), \tag{B2}$$

which were used in our proof of covariance.

We define the $u^\alpha(p)$ in terms of boosted rest-frame spinors by

$$u^\alpha(p)=S(L_p)u^\alpha(\hat{p}). \tag{B3}$$

The spinor $S(\Lambda)u^\alpha(p)$ is then some linear combination of the $u^\alpha(\Lambda p)$,

$$S(\Lambda)u^\alpha(p)=u^\beta(\Lambda_p)C_{\beta\alpha}, \tag{B4}$$

with some coefficients $C_{\beta\alpha}$. By (B3) this is

$$S(\Lambda)S(L_p)u^\alpha(\hat{p})=S(L_{\Lambda p})u^\beta(\hat{p})C_{\beta\alpha}. \tag{B5}$$

Thus,

$$S(L_{\Lambda p})^{-1}S(\Lambda)S(L_p)u^\alpha(\hat{p})=u^\beta(\hat{p})C_{\beta\alpha}, \tag{B6}$$

or, by (2),

$$S(\Lambda_p)u^\alpha(\hat{p})=u^\beta(\hat{p})C_{\beta\alpha}. \tag{B7}$$

Now, for $p^2>0$, $S(\Lambda_p)$ is a representative of a rotation and we choose the basis spinors $u^\alpha(\hat{p})$ so that we get exactly

$$S(\Lambda_p)u^\alpha(\hat{p})=u^\beta(\hat{p})D_{\beta\alpha}{}^{1/2}(\Lambda_p), \tag{B8}$$

which implies $C_{\beta\alpha}=D_{\beta\alpha}{}^{1/2}(\Lambda_p)$. Equation (B4) is then

$$S(\Lambda)u^\alpha(p)=u^\beta(\Lambda_p)D_{\beta\alpha}{}^{1/2}(\Lambda_p). \tag{B9}$$

Equation (B1) then follows by the unitarity of the $D_{\beta\alpha}{}^{1/2}$ and the relation

$$\gamma^0S(\Lambda)^\dagger\gamma^0=S(\Lambda)^{-1}. \tag{B10}$$