PHYSICAL REVIEW D VOLUME 2, NUMBER 6 15 SEPTEMBER 1970

Finite Theory of Quantum Electrodynamics*

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(Received 15 May 1970)

A new theory of quantum electrodynamics is presented, which is relativistically invariant, gauge invariant, unitary, and free of divergences. In this theory, mass renormalization, charge renormalization, and wave-function renormalization are all finite. Experimental consequences are discussed, and theoretical implications, especially those related to causality, are analyzed.

I. INTRODUCTION

In spite of the spectacular success of the renormalized theory of quantum electrodynamics,[1] there remain unsatisfactory aspects concerning the inherent mathematical ambiguities in manipulating divergent expressions. Even apart from questions of mathematical rigor, there are serious difficulties when one tries to extend the renormalization process to hadronic systems. Consider, for example, the problem of the mass difference between $\pi^{\pm}$ and π^0. It seems reasonable to assume that this mass difference is due purely to the electromagnetic interaction. Yet, according to the usual theory, this mass difference is calculated to be infinite; furthermore, it can be shown that under rather general assumptions such an infinity, being closely connected with the equal-time commutator between the current operator and its derivatives, cannot be removed through strong interactions.[2] The same divergence difficulties exist for all observed (therefore finite) mass differences between different hadrons in the same isospin multiplet; all such mass differences are found to be infinite according to the usual theory of quantum electrodynamics. Similarly, by making the universality assumption about the weak interaction, one expects that at least the ratio between the radiative correction to the Fermi constant G_V in a β decay and that to the μ-decay constant G_μ should be finite. Again, it is infinite in the conventional theory.[3]

Now, the various fractional mass differences between different hadrons in the same isospin multiplet are certainly finite, and are all of the order of the fine structure constant α. By using the Cabibbo theory, the amount of the radiative correction to G_V/G_μ can be deduced from the observed rates of β decay and μ decay; it is, of course, finite, and also of the order of α. The fact that all these quantities are of the right order of magnitude strongly indicates that they are indeed due to second-order electromagnetic processes. It appears then that there must be fundamental changes in our basic formulation of quantum field theory, so that unrenormalized masses and unrenormalized coupling constants can become finite.

Recently, it has been found[4–6] that there exists a general class of field theories in which the S matrix is fully unitary, but the Lagrangian is not Hermitian. This makes it possible to construct relativistic local field theories which satisfy the unitarity requirement and are free from divergences. In particular, by replacing in the electromagnetic interaction the usual zero-mass photon field A_μ by a complex field

$$\phi_\mu = A_\mu + iB_\mu , \tag{1.1}$$

where B_μ is a massive boson field associated with a negative metric, it is possible to remove all infinities from the electromagnetic mass differences between hadrons, as well as those associated with radiative corrections to weak decays.

The purpose of the present paper is, on the one hand, to supply some further details of this new form of quantum electrodynamics, and on the other hand to extend a similar modification also to the fermion fields, so that the unrenormalized electric charge is also finite. The resulting theory is then completely free from divergent expressions.

In order to gain proper perspective, we shall first review briefly in Sec. II some elementary properties of quantum theories with indefinite metric, and then proceed to analyze the relations between the usual commutation (or, anticommutation) relations and the metric of the system. It is shown that, under rather general conditions, in the case of Fermi statistics the positive definiteness, or indefiniteness, of the metric is *uniquely* determined by the equal-time anticommutation relations. Thus, once the Lagrangian for a fermion system is given, the metric is completely specified (of course, up to a canonical transformation, connected

* Research supported in part by the U. S. Atomic Energy Commission.

[1] At present, quantum electrodynamics agrees with experiment better than ever: J. Aldins, S. J. Brodsky, A. J. Dufner, and T. Kinoshita, Phys. Rev. Letters **23**, 441 (1969); Phys. Rev. D **1**, 2378 (1970); T. Appelquist and S. J. Brodsky, Phys. Rev. Letters **24**, 562 (1970).

[2] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); G. C. Wick and B. Zumino, Phys. Letters **25B**, 479 (1967); I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, Phys. Rev. Letters **19**, 1064 (1967); D. G. Boulware and S. Deser, *ibid.* **20**, 1399 (1968).

[3] T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1958); E. S. Abers, R. E. Norton, and D. A. Dicus, Phys. Rev. Letters **18**, 676 (1967); E. S. Abers, D. A. Dicus, R. E. Norton, and H. R. Quinn, Phys. Rev. **167**, 1461 (1968). For a general discussion and more complete literature, see A. Sirlin, *ibid.* **176**, 1871 (1968).

[4] T. D. Lee and G. C. Wick, Nucl. Phys. **B9**, 209 (1969); **B10**, 1 (1969).

[5] T. D. Lee, in *Quanta*, edited by P. G. O. Freund, C. J. Goebel, and Y. Nambu (Chicago U. P., Chicago, 1970), p. 260.

[6] T. D. Lee, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969*, edited by J. S. Bell (CERN, Geneva, 1969), p. 427.

with possible changes of basis vectors). For the case of Bose statistics, the usual commutation relations do not uniquely determine the metric; there remains a free choice whether the metric is positive definite, or indefinite.

In Sec. III, we give the details for the modified photon field $A_\mu+iB_\mu$. The interaction between the negative-metric B_μ field and the charged lepton and hadron pair states requires that the modified photon propagator have, besides the usual photon pole at

$$-(\text{four-momentum})^2=0, \tag{1.2}$$

also complex poles at

$$-(\text{four-momentum})^2=(m_B\pm\tfrac{1}{2}i\gamma_B)^2, \tag{1.3}$$

where, to $O(\alpha)$, the partial width $(\gamma_B)_{\text{lepton}}$ due to charged lepton pairs can be calculated, and is given by

$$(\gamma_B)_{\text{lepton}}=\tfrac{2}{3}\alpha m_B. \tag{1.4}$$

As will be discussed, in order to maintain unitarity, it is necessary that the poles due to the negative-metric B_μ field are, indeed, *off* the real axis.

It is well known[7] that in order to regularize the usual charge renormalization, one cannot, because of gauge invariance, simply modify, say, the electron propagator by introducing convergence factors. This difficulty is resolved in Sec. IV. We introduce two new Dirac spinor fields; their charges are imaginary $\pm ie$, and their masses are complex

$$m_F\pm\tfrac{1}{2}i\gamma_F, \tag{1.5}$$

where m_F and γ_F are both real and not zero. Both fields obey Fermi statistics. As will be shown, in this theory, the metric is uniquely determined by the canonical anticommutation relations and is indefinite; the gauge invariance and, therefore, current conservation are fully satisfied by introducing the usual minimal electromagnetic interaction; the unitarity is maintained because of the complex masses; and the charge renormalization becomes finite since, because of their $\pm ie$ charges, the loop diagram due to these new fermion fields is of the opposite sign from the usual one.

The modified Feynman rule is given in Sec. V. Explicit evaluations of charge renormalization and photon propagator are carried out in Sec. VI. Experimental consequences of this new finite theory of quantum electrodynamics are discussed in Sec. VII, and the causality problem is analyzed in Sec. VIII.

II. INDEFINITE METRIC

In this section we collect, for the convenience of the reader, some well-known facts about vector spaces with an indefinite metric and self-adjoint operators,[8] and we discuss the relations between the usual canonical commutation (or anticommutation) rules and the general structure of metric that can be used in a quantum theory.

Let us consider a complex vector space $\mathcal{H}$; let $|x\rangle$, $|y\rangle, \ldots$ be vectors in $\mathcal{H}$, and let $|1\rangle$, $|2\rangle, \ldots$ be a basis, so that

$$|x\rangle=\sum_i x_i|i\rangle, \tag{2.1}$$

where x_1, $x_2, \ldots$ are complex numbers. We define the scalar product $\langle x|y\rangle$ to be the Hermitian form

$$\langle x|y\rangle=\sum_{i,j}\eta_{ij}x_i^*y_j, \tag{2.2}$$

where an asterisk denotes complex conjugation, and the matrix

$$\eta=(\eta_{ij}) \tag{2.3}$$

is assumed to be a *nonsingular Hermitian matrix.* One calls the metric indefinite if η is not a positive definite matrix. From (2.2), one has, of course,

$$\langle i|j\rangle=\eta_{ij}. \tag{2.4}$$

A linear operator A_{op} can be specified by the equations

$$A_{\text{op}}|i\rangle=\sum_j A_{ji}|j\rangle, \tag{2.5}$$

where the A_{ij}'s are complex numbers; the adjoint operator $\bar{A}_{\text{op}}$ is defined by the equation

$$\langle x|\bar{A}_{\text{op}}|y\rangle=\langle y|A_{\text{op}}|x\rangle^* \tag{2.6}$$

for all vectors $|x\rangle$ and $|y\rangle$, where $\langle x|A_{\text{op}}|y\rangle$ denotes the scalar product between $|x\rangle$ and $A_{\text{op}}|y\rangle$. According to (2.2) and (2.4), we have

$$\langle x|A_{\text{op}}|y\rangle=\sum_{i,j,k}x_i^*\eta_{ij}A_{jk}y_k. \tag{2.7}$$

The expectation value $\langle x|A_{\text{op}}|x\rangle$ of a self-adjoint operator, defined by

$$A_{\text{op}}=\bar{A}_{\text{op}}, \tag{2.8}$$

is, therefore, real for all vectors $|x\rangle$.

All these equations can, of course, be expressed in terms of the appropriate matrix notations. For example, Eqs. (2.2) and (2.7) can be simply written as

$$\langle x|y\rangle=\bar{x}y \tag{2.9}$$

and

$$\langle x|A_{\text{op}}|y\rangle=\bar{x}Ay, \tag{2.10}$$

where y is a column matrix whose ith matrix element is y_i, $\bar{x}$ is a row matrix whose ith matrix element is

$$\bar{x}_i=\sum_j x_j^*\eta_{ji}, \tag{2.11}$$

and A is the square matrix

$$A=(A_{ij}). \tag{2.12}$$

[7] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949); see also R. P. Feynman, Phys. Rev. **76**, 749 (1949).

[8] As is well known, an indefinite metric in the space of state vectors was first used by P. A. M. Dirac, Proc. Roy. Soc. (London) **A180**, 1 (1942).

Note that the matrix element A_{ij} should not be confused with the scalar product between the basis vector $|i\rangle$ and the vector $A_{\text{op}}|j\rangle$, which is given by

$$\langle i|A_{\text{op}}|j\rangle=(\eta A)_{ij}. \tag{2.13}$$

It is worthwhile to point out that the well-known transformation law of the matrix η implies that η does not represent a *bona fide* operator. In fact, if one chooses new basis vectors $|1'\rangle$, $|2'\rangle$, ...,

$$|i'\rangle=\sum_j T_{ji}|j\rangle, \tag{2.14}$$

then, in terms of the new basis, (2.1) becomes

$$|x\rangle=\sum_i x_i|i\rangle=\sum_i x_i'|i'\rangle. \tag{2.15}$$

Thus $x_i=\sum_j T_{ij}x_j'$, or, symbolically,

$$x'=T^{-1}x, \tag{2.16}$$

where, again, x and x' are column matrices, and T^{-1} is the inverse of the matrix $T=(T_{ij})$. The transformed matrix A' for the operator A_{op}, defined by

$$A_{\text{op}}|i'\rangle=\sum_j A_{ji}'|j'\rangle, \tag{2.17}$$

is given by

$$A'=(A_{ij}')=T^{-1}AT. \tag{2.18}$$

On the other hand, relative to the new basis, the metric is obviously given by the matrix $\eta'=(\eta_{ij}')$, where

$$\eta_{ij}'=\langle i'|j'\rangle, \tag{2.19}$$

so that

$$\eta'=T^\dagger\eta T. \tag{2.20}$$

In view of the difference between (2.18) and (2.20) it is worthwhile to make a clear distinction between η and an *operator*. For example, algebraic equations involving η are not in general independent of the choice of basis. *Bona fide* operators, of course, do not have this defect.[9] Similarly, the eigenvalues of an operator are basis independent. Those of η are not. This has its good side: One can not only transform η to a diagonal form, but also assume that the diagonal matrix elements are equal to ±1 (zero being excluded by the assumption that η is nonsingular). At any rate, the eigenvalues of η have no invariant meaning. The only invariant property of η is the "signature" which (in a finite dimensional space) is defined to be the number of its positive eigenvalues minus that of its negative eigenvalues.

We now turn to the general question of different classes of the metric η allowed in a quantum theory. [All η's related by the transformation (2.20) are said to belong to the same class.] Through the usual quantization rules, one has, to begin with, the appropriate commutation, or anticommutation, relations for a set of operators a and their adjoints $\bar{a}$; furthermore, one is often only interested in the case in which the matrix representation of the entire set of these operators a and $\bar{a}$ is irreducible. As we shall see, in such a case, these commutation, or anticommutation, relations then completely determine the possible classes of the metric η.

We note that in any basis, because of (2.6) and $\eta=\eta^\dagger$, the matrix representations of a and $\bar{a}$ are related by

$$\bar{a}=\eta^{-1}a^\dagger\eta, \tag{2.21}$$

and therefore the over-all sign of η cannot be determined by any set of algebraic equations between a and $\bar{a}$. Of course, such an over-all sign has *no* physical meaning, since it merely changes the entire S matrix by a minus sign; in any quantum theory, only the relative phase has no physical significance. Thus, it is convenient to take care of this trivial over-all sign choice by generalizing the definition of the same class of η to include also

$$\eta\rightarrow-\eta, \tag{2.22}$$

in addition to those related by (2.20); i.e.,

$$\eta\rightarrow T^\dagger\eta T. \tag{2.20'}$$

The mathematical details for the determination of different classes of η are given in Appendices A and B. Here, we shall discuss some of the main conclusions. Consider, for example, the case of a single harmonic oscillator obeying Fermi-Dirac statistics. We have

$$a^2=0 \quad\text{and}\quad \bar{a}^2=0. \tag{2.23}$$

There are two different possibilities for the anticommutation relation between a and $\bar{a}$:

$$\text{(i)}\quad a\bar{a}+\bar{a}a=1; \tag{2.24}$$

$$\text{(ii)}\quad a\bar{a}+\bar{a}a=-1. \tag{2.25}$$

As will be shown in Appendix A, in case (i), by using (2.22), the metric η can always be set to be positive definite; this corresponds to the usual positive-metric case, in which

$$\langle x|x\rangle \text{ is positive} \tag{2.26}$$

for all vectors $|x\rangle$, and therefore one can always choose

[9] For this reason, we have chosen the notation in which η is omitted in the scalar-product symbol, given by the left-hand side of (2.2) (being "absorbed" in the definition of $\langle x|$). Such a notation has, of course, already been used occasionally, for example, by W. Heisenberg [Nucl. Phys. **4**, 532 (1957)]. It differs from the more commonly used notation in the literature, for example, in Pauli [Rev. Mod. Phys. **15**, 175 (1943)], and also in our earlier papers on the subject (Refs. 4–6). The present notation of not explicitly displaying η has the advantage of avoiding unnecessary noncovariant equations.

Similarly, one notes that while the matrix representation $\bar{A}$ of the adjoint operator $\bar{A}_{\text{op}}$ transforms in the same way as the matrix representation A of any operator A_{op}, the Hermitian conjugate $A^\dagger$ does not. Thus, it is always of considerable convenience to use only A and $\bar{A}$, but not $A^\dagger$.

In this connection, it is important to dispel the notion that certain noncovariant relations involving η, which occur in almost every treatment of the subject, are really indispensable. For example, as will be shown in the following, noncovariant equations such as $\eta a=-a\eta$ and $\eta=(-1)^{\bar{a}a}$, that are usually assumed in the theory of a harmonic oscillator with indefinite metric, can be completely replaced by the more general covariant condition (2.27).

the basis so that

$$\eta=1. \tag{2.26'}$$

In case (ii), there is also only a *single* class for the metric η; by using (2.22), one can always require, instead of (2.26), that

$$\langle x|(-1)^{\bar{a}a}|x\rangle \text{ be positive} \tag{2.27}$$

for all vectors $|x\rangle$. The metric η is therefore indefinite. If one wishes, one may choose a specific basis so that

$$\eta=(-1)^{\bar{a}a}. \tag{2.27'}$$

We note that (2.26) and (2.27) are valid for all bases, while (2.26′) and (2.27′) are, of course, basis dependent. In either case, the quantum of this oscillator is said to be of positive or negative metric, depending on whether case (i) or case (ii) holds.

In the case of a single harmonic oscillator obeying Bose-Einstein statistics, the commutation relation

$$a\bar{a}-\bar{a}a=-1 \tag{2.28}$$

can be reduced to

$$a\bar{a}-\bar{a}a=1 \tag{2.29}$$

by interchanging the roles of a and $\bar{a}$. As will be shown in Appendix B, this commutation relation limits the metric η to one of the following three classes.

(i) *Definite metric case*. In this case (2.26) holds and therefore (2.26′) is always a possible choice.

(ii) *Indefinite metric case (normal)*. In this case (2.27) holds, and therefore (2.27′) is always a possible choice.

(iii) *Abnormal case*. In this case the metric η is indefinite; unlike the situation in cases (i) and (ii), the eigenvalues of $\bar{a}a$ are not integers, and they have neither upper nor lower bound.

In the following, we shall impose the condition that the operator $\bar{a}a$ should be bound either from above, or from below; this would ensure, at least for the free oscillator, a lower bound to the energy spectrum. This additional condition then rules out the abnormal case (iii). The class of metric is then uniquely determined by specifying whether η is definite or indefinite. The quantum of this oscillator is said to be of positive or negative metric, depending on whether η is definite or indefinite.

For physical applications of a quantum theory with indefinite metric, we choose the Hamiltonian H to be a self-adjoint operator. Let $|r\rangle$ denote any eigenvector of H with a real eigenvalue. We recall the elementary, but important, property[4] that the S matrix is unitary if

$$\langle r|r\rangle \text{ is positive} \tag{2.30}$$

for *all* such eigenvectors $|r\rangle$ with real eigenvalues. In such a theory, each physical observable is represented by a self-adjoint operator A, and each physical state $|\ \rangle$ must be a linear superposition of only eigenstates $|r\rangle$ belonging to the real eigenvalues. The expectation value of A is then given by the usual formula,

$$\langle A\rangle=\langle|A|\rangle/\langle\ |\ \rangle. \tag{2.31}$$

III. PHOTON FIELD

In this section we shall review briefly the basic formalism of the modified photon field

$$\phi_\mu=A_\mu+iB_\mu. \tag{3.1}$$

For clarity, we consider first a simple system consisting of three fields: A_μ, B_μ, and a charged lepton field ψ_l which can be either the usual electron field or the usual muon field. (The inclusion of other fermion fields of an unusual type will be discussed in Sec. IV.)

The Lagrangian density of this simple system is given by

$$\mathcal{L}_{\text{free}}(\phi)+\mathcal{L}(l,\phi), \tag{3.2}$$

where

$$\mathcal{L}_{\text{free}}(\phi)=-\tfrac{1}{4}(G_{\mu\nu}{}^2+F_{\mu\nu}{}^2)-\tfrac{1}{2}(m_B{}^0B_\mu)^2, \tag{3.3}$$

$$\mathcal{L}(l,\phi)=-\bar{\psi}_l\gamma_4\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu}-ie_0\phi_\mu\right)+m_l{}^0\right]\psi_l, \tag{3.4}$$

$$F_{\mu\nu}=\frac{\partial}{\partial x_\mu}A_\nu-\frac{\partial}{\partial x_\nu}A_\mu, \tag{3.5}$$

$$G_{\mu\nu}=\frac{\partial}{\partial x_\mu}B_\nu-\frac{\partial}{\partial x_\nu}B_\mu, \tag{3.6}$$

e_0 is the unrenormalized charge, and $m_B{}^0$ and $m_l{}^0$ are the unrenormalized masses of the B_μ and ψ_l fields. In the above expressions, $\bar{\psi}_l$ denotes the adjoint operator of ψ_l, and ϕ_μ is a self-adjoint vector field[10]; i.e.,

$$\bar{\phi}_\mu=+\phi_\mu \quad \text{for} \quad \mu\neq 4$$

and

$$\bar{\phi}_4=-\phi_4.$$

Consequently, A_μ is a self-adjoint field,

$$\bar{A}_\mu=+A_\mu \quad \text{for} \quad \mu\neq 4 \tag{3.7}$$

and

$$\bar{A}_4=-A_4, \tag{3.8}$$

but B_μ is not,[11]

$$\bar{B}_\mu=-B_\mu \quad \text{for} \quad \mu\neq 4 \tag{3.9}$$

and

$$\bar{B}_4=+B_4. \tag{3.10}$$

The Hamiltonian H and its quantization can be carried out by following the usual canonical treatment. We may choose as generalized coordinates the lepton field ψ_l, the transverse electromagnetic vector potential

[10] Throughout the paper, the subscripts μ and ν denote the space-time indices; $\mu=4$ is the time component with $x_4=it$. All boldface letters denote three-dimensional vectors.

[11] If one wishes, one may replace iB_μ by B_μ', which is a self-adjoint vector field. Then, of course, the free Lagrangian for B_μ' appears in (3.3) with an unusual sign.

A_j^{tr} (in the Coulomb gauge[12]), and the spatial component B_j of the massive Boson field; their conjugate momenta are, respectively, $i\bar{\psi}_l$, $-E_j^{tr}$, and

$$\Pi_j = iG_{4j}. \quad (3.11)$$

At equal time, one has

$$\{\psi_l(\mathbf{r},t),\bar{\psi}_l(\mathbf{r}',t)\} = \delta^3(\mathbf{r}-\mathbf{r}'), \quad (3.12)$$

$$[-E_j^{tr}(\mathbf{r},t), A_k^{tr}(\mathbf{r}',t)] = -i(\delta_{jk}-\nabla^{-2}\nabla_j\nabla_k)\delta^3(\mathbf{r}-\mathbf{r}'), \quad (3.13)$$

and

$$[\Pi_j(\mathbf{r},t), B_k(\mathbf{r}',t)] = -i\delta_{jk}\delta^3(\mathbf{r}-\mathbf{r}'). \quad (3.14)$$

It is convenient to introduce the usual Fourier components of these fields. For example, we may write

$$\mathbf{B} = \sum_{\mathbf{k},t}(2\Omega\omega_k)^{-1/2}(a_{\mathbf{k}}{}^t e^{i\mathbf{k}\cdot\mathbf{r}} - \bar{a}_{\mathbf{k}}{}^t e^{-i\mathbf{k}\cdot\mathbf{r}})\hat{e}_{\mathbf{k}}{}^t + \sum_k (2\Omega\omega_k)^{-1/2}(a_{\mathbf{k}}{}^l e^{i\mathbf{k}\cdot\mathbf{r}} - \bar{a}_{\mathbf{k}}{}^l e^{-i\mathbf{k}\cdot\mathbf{r}})(\omega_k\hat{k}/m_B{}^0) \quad (3.15)$$

and

$$\mathbf{\Pi} = \sum_{\mathbf{k},t}(2\Omega\omega_k)^{-1/2}(a_{\mathbf{k}}{}^t e^{i\mathbf{k}\cdot\mathbf{r}} + \bar{a}_{\mathbf{k}}{}^t e^{-i\mathbf{k}\cdot\mathbf{r}})(-i\omega_k\hat{e}_{\mathbf{k}}{}^t) + \sum_{\mathbf{k}}(2\Omega\omega_k)^{-1/2}(a_{\mathbf{k}}{}^l e^{i\mathbf{k}\cdot\mathbf{r}} + \bar{a}_{\mathbf{k}}{}^l e^{-i\mathbf{k}\cdot\mathbf{r}})(-im_B{}^0\hat{k}), \quad (3.16)$$

where $\hat{e}_k{}^1$, $\hat{e}_k{}^2$, and $\hat{k} = |\mathbf{k}|^{-1}\mathbf{k}$ form a right-handed orthonormal set of unit vectors,

$$\omega_k = [\mathbf{k}^2 + (m_B{}^0)^2]^{1/2}, \quad (3.17)$$

and Ω is the volume of the system. The commutation relation (3.14) becomes then

$$[a_{\mathbf{k}}{}^\alpha, \bar{a}_{\mathbf{q}}{}^\beta] = -\delta_{\mathbf{kq}}\delta_{\alpha\beta}, \quad (3.18)$$

where α, β denote either $t=1$, 2, or l. As already discussed in the previous section, the anticommutation relation determines the metric of ψ_l to be positive; the commutation relations (3.13) and (3.14) limit the metric of these Boson fields to two possibilities: either positive or negative. We will now specify the metric of the system by requiring[13]

$$\langle\ |(-1)^{N_B}|\ \rangle \text{ to be positive} \quad (3.19)$$

for all vectors $|\ \rangle$, where

$$N_B \equiv -\sum_{\mathbf{k},t}\bar{a}_{\mathbf{k}}{}^t a_{\mathbf{k}}{}^t - \sum_{\mathbf{k}}\bar{a}_{\mathbf{k}}{}^l a_{\mathbf{k}}{}^l. \quad (3.20)$$

That is, A_μ is of positive metric and B_μ is of negative metric.

It is easy to verify that the total Hamiltonian H is self-adjoint. The equations of motion can be derived by using the usual Heisenberg equations, and the compatibility of the equal-time commutation relations (3.12)–(3.14) with relativistic invariance can be established following the usual arguments. For the unitarity property, we note that for the free field ($e_0=0$), our basic condition (2.30) is not satisfied since there exist negative-metric eigenstates of the free Hamiltonian with real eigenvalues, e.g., states with $N_B=$ odd integers. [For the free system, the condition (2.30) is, of course, irrelevant since the S matrix is the unit matrix.] In order to satisfy our basic condition (2.30), it is necessary to have $e_0\neq 0$. Consider, for example, the negative-metric state $|B\rangle$ of a single free B_μ quantum, i.e., $N_B=1$. For $m_B{}^0 > 2m_l{}^0$, such a state, in the limit $e_0=0$, is degenerate[14] with the positive-metric lepton pair states $|l^+l^-\rangle$. It can be readily verified that this degeneracy is removed in the presence of the interaction. Furthermore, the resulting eigenvalues are[15]

$$m_B \pm \tfrac{1}{2}i\gamma_B, \quad (3.21)$$

where γ_B and m_B are both real, denoting the width and the renormalized mass of the B_μ quantum. The corresponding eigenvectors $|\pm\rangle$ of these two complex eigenvalues consist of a coherent mixture of $|B\rangle$ and $|l^+l^-\rangle$; these two eigenvectors both have zero norm,

$$\langle +|+\rangle = \langle -|-\rangle = 0, \quad (3.22)$$

and can be normalized so that

$$\langle +|-\rangle = 1. \quad (3.23)$$

The remaining eigenstates $|r\rangle$ with real eigenvalues can then be shown to have positive norm, and therefore (2.30) is satisfied. Further details will be given in Secs. V and VI.

Since the propagator of the modified photon field $\phi_\mu = A_\mu + iB_\mu$ can be easily seen to be proportional to k^{-4} in the high-momentum limit as $k\to\infty$, one finds that, through a straightforward power counting, except for charge renormalization all higher-order processes of the Lagrangian (3.2) are finite. In order to render charge renormalization finite, additional new fermion fields have to be introduced; these will be discussed in Sec. IV.

[12] The Coulomb gauge is chosen here purely for convenience. In the Coulomb gauge, A_μ is given by $A_j = A_j^{tr}$ and $A_4 = i\phi$, where A_j^{tr} satisfies $(\partial A_j^{tr}/\partial r_j)=0$ and ϕ is the solution of $\Delta\phi = -e_0\bar{\psi}_l\psi_l$. Correspondingly, the electric field is given by $E_j = E_j^{tr} + E_j^{long}$, where $E_j^{tr} = -(\partial A_j^{tr}/\partial t)$ and $E_j^{long} = -(\partial\phi/\partial r_j)$. In deriving the canonical momentum of A_j^{tr} as $-E_j^{tr}$, we have, as usual, first set in the Lagrangian the spatial integral $\int E_j^{tr}E_j^{long}d^3r = 0$, and then taken its derivative with respect to $(\partial A_j^{tr}/\partial t)$.

[13] We emphasize that (3.19), as well as other equations in this section, are all basis independent. Of course, it follows from (3.19) that, if one wishes, one can always choose a specific basis so that $\eta = (-1)^{N_B}$.

[14] If $m_B{}^0 < 2m_l{}^0$, then, in the limit $e_0=0$, this negative-metric state is degenerate with the positive-metric three-photon state. The introduction of $e_0\neq 0$ would also remove this degeneracy and make it possible to satisfy (2.30).

[15] After the completion of this paper, Norman Kroll kindly drew our attention to the fact that in connection with his investigation on *ad hoc* modifications of quantum electrodynamics [Nuovo Cimento **45**, 65 (1966)], he has also noted the possibility of such complex poles and some of their consequences. The so-called "causality difficulty" mentioned in Kroll's paper, however, has been resolved in Ref. 4. (See also Sec. VIII of the present paper.)

IV. FERMION FIELDS

Let

$$\psi_F \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{4.1}$$

denote a new fermion field, where ψ_1 and ψ_2 are both four-component Dirac fields, and

$$\bar{\psi}_F = (\bar{\psi}_1 \, \bar{\psi}_2)$$

denotes the adjoint field, where the bar is used in the sense of Eq. (2.6). The Lagrangian density for a system consisting of A_μ, B_μ, the usual charged lepton fields ψ_e, ψ_μ, and this new fermion field ψ_F is given by

$$\mathcal{L} = \mathcal{L}_{\rm free}(\phi) + \mathcal{L}(F,\phi) + \sum_{l=e,\mu} \mathcal{L}(l,\phi), \tag{4.2}$$

where $\mathcal{L}_{\rm free}(\phi)$ and $\mathcal{L}(l,\phi)$ are given, respectively, by (3.3) and (3.4),

$$\mathcal{L}(F,\phi) \equiv -\bar{\psi}_F \tau_a \gamma_4 \left[\gamma_\lambda \left(\frac{\partial}{\partial x_\lambda} + e_0 \tau_b \phi_\lambda \right) + M_F{}^0 \right] \psi_F, \tag{4.3}$$

and τ_a, τ_b are two of the usual (2×2) Pauli matrices which commute with all Dirac matrices γ_μ and satisfy

$$\tau_a{}^2 = \tau_b{}^2 = 1, \tag{4.4}$$

$$\tau_a{}^\dagger = \tau_a, \quad \tau_b{}^\dagger = \tau_b, \tag{4.5}$$

and

$$\tau_a \tau_b + \tau_b \tau_a = 0. \tag{4.6}$$

In (4.3), $M_F{}^0$ is also a (2×2) matrix, given by

$$M_F{}^0 = m_F{}^0 + i\tfrac{1}{2}\gamma_F{}^0 \tau_b, \tag{4.7}$$

where $m_F{}^0$ and $\gamma_F{}^0$ are real numbers. The total Lagrangian density is therefore self-adjoint, because of (4.5) and (4.6). From (4.3), one finds that the conjugate momentum of ψ_F is $i\bar{\psi}_F \tau_a$. The canonical quantization rule gives then, in addition to (3.12)–(3.14),

$$\{\psi_F(\mathbf{r}',t), \bar{\psi}_F(\mathbf{r},t)\} = \tau_a \delta^3(\mathbf{r}-\mathbf{r}'). \tag{4.8}$$

By following the standard procedure of passing from Lagrangian to Hamiltonian, one finds the Heisenberg equation for the new fermion field,

$$\gamma_\lambda \left(\frac{\partial}{\partial x_\lambda} + e_0 \tau_b \phi_\lambda \right) \psi_F + M_F{}^0 \psi_F = 0, \tag{4.9}$$

which implies

$$\frac{\partial \bar{\psi}_F}{\partial x_\lambda} \tau_a \gamma_4 \gamma_\lambda - \bar{\psi}_F \tau_a \gamma_4 (e_0 \tau_b \gamma_\lambda \phi_\lambda + M_F{}^0) = 0.$$

We have, therefore, two conservation laws:

$$\frac{\partial}{\partial x_\lambda} (\bar{\psi}_F \tau_a \tau_b \gamma_4 \gamma_\lambda \psi_F) = 0 \tag{4.10}$$

and

$$\frac{\partial}{\partial x_\lambda} (i\bar{\psi}_F \tau_a \gamma_4 \gamma_\lambda \psi_F) = 0, \tag{4.10$'$}$$

which correspond to the two similar conservation laws for the usual charged lepton fields,

$$\sum_{l=e,\mu} \frac{\partial}{\partial x_\lambda} (i\bar{\psi}_l \gamma_4 \gamma_\lambda \psi_l) = 0 \tag{4.11}$$

and

$$\frac{\partial}{\partial x_\lambda} (i\bar{\psi}_e \gamma_4 \gamma_\lambda \psi_e - i\bar{\psi}_\mu \gamma_4 \gamma_\lambda \psi_\mu) = 0. \tag{4.11$'$}$$

The total electromagnetic current density j_μ is given by

$$j_\lambda \equiv e_0 \bar{\psi}_F \tau_a \tau_b \gamma_4 \gamma_\lambda \psi_F - ie_0 \sum_{l=e,\mu} \bar{\psi}_l \gamma_4 \gamma_\lambda \psi_l, \tag{4.12}$$

and it is clearly conserved:

$$\partial j_\mu / \partial x_\mu = 0. \tag{4.13}$$

Under the gauge transformation

$$A_\mu \to A_\mu + \partial\Lambda/\partial x_\mu \tag{4.14}$$

and

$$B_\mu \to B_\mu, \tag{4.15}$$

the charged lepton field ψ_l ($l = e, \mu$) has the usual transformation

$$\psi_l \to e^{ie_0\Lambda} \psi_l, \tag{4.16}$$

and therefore

$$\bar{\psi}_l \to \bar{\psi}_l e^{-ie_0\Lambda};$$

the new fermion field ψ_F transforms according to

$$\psi_F \to e^{-e_0 \tau_b \Lambda} \psi_F, \tag{4.17}$$

and therefore

$$\bar{\psi}_F \to \bar{\psi}_F e^{-e_0 \tau_b \Lambda}.$$

By using (4.2), it can be readily verified that the theory is gauge invariant.

Just as in (3.19), we will now specify the metric of the system by requiring

$$\langle \,|(-1)^{N_B+N_F}|\, \rangle \text{ to be positive} \tag{4.18}$$

for all vectors $|\,\rangle$, where N_B is given by (3.20) and N_F is given by Eq. (C16) of Appendix C. By following the general discussions given in Sec. II, it is easy to see that the factor $(-1)^{N_F}$ is completely determined by the anticommutation relation (4.8), just as is the factor $(-1)^{\bar{a}a}$ in (2.27). The details are given in Appendix C. In the same appendix, the free Hamiltonian of the system is explicitly diagonalized. As will be shown there, the frequency spectrum of the free ψ_F field is given by

$$[\mathbf{k}^2 + (m_F{}^0 \pm \tfrac{1}{2} i\gamma_F{}^0)^2]^{1/2}. \tag{4.19}$$

We note that in the boson case, the bare mass $m_B{}^0$ of B_μ is assumed to be real; the width γ_B is acquired through the electromagnetic interaction between B_μ and the charged lepton pairs, and therefore it is not an independent parameter, as will be calculated in Sec. VI. On the other hand, in the fermion case, because of the conservation laws (4.10) and (4.10′), in order to ensure unitarity we have to assume a nonzero width $\gamma_F{}^0$ for the free ψ_F field.

V. FEYNMAN DIAGRAMS

In the present theory, because of complex singularities such as (3.21) and (4.19), the Green's function $U(t, -t)$ of the time-dependent Schrödinger equation in, say, the interaction representation diverges exponentially in the limit $t \to \infty$. On the other hand, the S matrix is, of course, well defined in terms of the eigenvectors of the total Hamiltonian with real eigenvalues; its matrix elements are given by

$$S_{r'r} = \langle r'^{\,\mathrm{in}} | r^{\mathrm{out}} \rangle , \tag{5.1}$$

where the superscript "out" (or "in") denotes states consisting of only plane waves and outgoing (or incoming) waves. Thus, one does not have the usual relation between the S matrix and the limit $U(t, -t)$ at $t = \infty$. Nevertheless, as pointed out in Ref. 4, it is possible to separate $U(t, -t)$ into a sum

$$U(t, -t) = U^{\mathrm{reg}}(t, -t) + U^{\mathrm{exp}}(t, -t) , \tag{5.2}$$

where as $t \to \infty$, $U^{\mathrm{exp}}(t, -t)$ diverges exponentially, but

$$\lim_{t\to\infty} U^{\mathrm{reg}}(t, -t) = S . \tag{5.3}$$

From (5.1) and (5.3), one obtains a set of modified Feynman rules which will be briefly reviewed in this section.

In general, in a quantum field theory with an indefinite metric, any S-matrix element can always be given by a sum over an appropriate set of Feynman graphs, just as in the usual theory with a definite metric. Each graph still stands for a multidimensional integral, integrated over a domain of some virtual four-momenta, which will be labeled collectively as k_μ; the integration over the space component $\mathbf{k}$ remains, as usual, over the entire real region, but that over the time component k_0 is now along a complex path. We note that, unlike the case in the usual theory, the integrand now has complex singularities. To obtain the correct integration path in the complex k_0 plane, it is convenient to first regard all imaginary parts γ_i of these complex singularities as independent parameters. For $\gamma_i = 0$, we have the usual Feynman rule: At fixed $\mathbf{k}$, the integration over k_0 is along the real axis, and the positions of the singularities of the integrand are determined by the usual $i\epsilon$ rule. Alternatively, one may regard (still for $\gamma_i = 0$) all singularities to be on the real axis, but the Feynman path C is slightly detoured so as to go either above or below the appropriate singularities. Now as γ_i increases from zero to its final value, these singularities will move continuously. We shall require the integration path C for dk_0 to be continuously

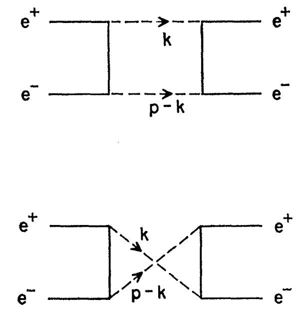

FIG. 1. Two examples of fourth-order e^+e^- scattering diagrams.

deformed in such a way that none of the singularities ever crosses C. It is clear that C is unambiguously defined only if there is no pinching along the path; i.e., if complex singularities on different sides of the path do not coalesce. As we shall see, such pinching occurs only over an integration domain of "zero" measure, and therefore does not contribute to the Feynman integral.

For clarity, let us consider some specific diagrams for, say, the elastic scattering of e^+e^- given in Fig. 1. The dashed line refers to the propagator of

$$\phi_\mu = A_\mu + iB_\mu ,$$

which, as will be explicitly calculated in Sec. VI, contains complex poles at

$$-(\text{four-momentum})^2 = M_B{}^2 \quad \text{and} \quad (M_B{}^*)^2 , \tag{5.4}$$

where

$$M_B = m_B + \tfrac{1}{2} i \gamma_B . \tag{5.5}$$

Thus, the integrand for either diagram in Fig. 1 has complex poles at, among others,

$$k_0 = (\mathbf{k}^2 + M_B{}^2)^{1/2} \tag{5.6}$$

and

$$k_0 = p_0 - [(\mathbf{p} - \mathbf{k})^2 + (M_B{}^*)^2]^{1/2} , \tag{5.7}$$

where p_μ denotes the total momentum of e^+ and e^-. According to the above "modified" Feynman rule, the pole (5.6) should be below the integration path C and the pole (5.7) should be above C. This is always possible except when these two poles coalesce, which occurs when

$$p_0 = (\mathbf{k}^2 + M_B{}^2)^{1/2} + [(\mathbf{p} - \mathbf{k})^2 + (M_B{}^*)^2]^{1/2} . \tag{5.8}$$

Except for the special center-of-mass reference frame, in any reference frame the total momentum $\mathbf{p}$ is not zero, and therefore Eq. (5.8) represents two independent conditions in order to satisfy both its imaginary and real parts:

$$|\mathbf{k}| = |\mathbf{p} - \mathbf{k}| \tag{5.9}$$

and

$$p_0 = \tfrac{1}{2}\, \mathrm{Re}(\mathbf{k}^2 + M_B{}^2)^{1/2} , \tag{5.10}$$

since p_0 denotes the total energy of e^+ and e^-, which must be real.

In $\mathbf{k}$ space, the points that satisfy these two conditions lie on a circle whose center is at $\tfrac{1}{2}|\mathbf{p}|$. For all other points, there is no pinching and the Feynman integral over k_0 is well defined; the result of this k_0 integration is a function of $\mathbf{k}$ which is singular at the above-mentioned circle, but with singularities no worse than

$$\{p_0-(\mathbf{k}^2+M_B{}^2)^{1/2}-[(\mathbf{p}-\mathbf{k})^2+(M_B{}^*)^2]^{1/2}\}^{-1}. \quad (5.11)$$

The subsequent integration over $\mathbf{k}$ is therefore absolutely convergent and well defined[16]; the ambiguity of the integrand over such a set of points of zero measure (a circle) in $\mathbf{k}$ space does not lead to any ambiguity in the final result. Therefore, in any system (except the center-of-mass system), the Feynman integral can be obtained by continuous deformation from the usual Feynman path. For the particular center-of-mass frame $\mathbf{p}=0$, we require the corresponding Feynman integral to be obtained as the limit $\mathbf{p}\rightarrow 0$.

The actual Feynman integration for the above diagrams can be carried out in exactly the same way as done in Sec. 5 of Ref. 5. The result is that, apart from the usual cut corresponding to the normal two-zero-mass-photon exchange, there is no additional imaginary part in the scattering amplitude due to the virtual two complex-mass heavy boson states. Consequently, unitarity holds. The usual condition of analyticity is, of course, violated.

The above discussions can be readily carried out for more complicated diagrams involving multiphoton exchanges. If the intermediate state consists of three or more particles, the equation for pinching always gives two independent conditions in any reference frame, including the center-of-mass frame. Take, for example, the case of three particles of energies, say,

$$\omega_k=(\mathbf{k}^2+M_B{}^2)^{1/2}, \quad \omega_q{}^*=[\mathbf{q}^2+(M_B{}^*)^2]^{1/2},$$

and $\nu_{p-k-q}=|\mathbf{p}-\mathbf{k}-\mathbf{q}|$. Just as in Eq. (5.8), the condition for pinching occurs when

$$p_0=\omega_k+\omega_q{}^*+\nu_{p-k-q}, \quad (5.12)$$

where $\mathbf{p}$ and p_0 denote the total momentum and energy of the system. Equation (5.12) implies two separate conditions,

$$|\mathbf{k}|=|\mathbf{q}| \quad (5.13)$$

and

$$\tfrac{1}{2}(p_0-\nu_{p-k-q})=\mathrm{Re}\,\omega_k, \quad (5.14)$$

independently of whether $\mathbf{p}=0$ or not. Just as in the above case of two-photon exchange, the points that satisfy these two conditions (5.13) and (5.14) give zero contribution to the Feynman integral.

In these Feynman integrals, since the domain of all three-momenta is kept real, but that of the fourth components is complex, the question of relativistic invariance naturally arises, especially for more complicated diagrams involving several complex masses. In this connection, there exists an alternative, but manifestly covariant, prescription given by Cutkosky, Landshoff, Olive, and Polkinghorne[17]; in their prescription, whenever there are a pair of regularized photon lines, one always first sets the complex masses in one line to be M_B and $M_B{}^*$, and in the other line $M_B{}'$ and $M_B{}'^*$. The physical case $M_B=M_B{}'$ is obtained as the limit $M_B{}'\rightarrow M_B$. Relativistic invariance then becomes obvious; for example, in the two-photon-exchange diagrams discussed, for arbitrary $M_B\neq M_B{}'$, the Feynman integral is well defined in any system $\mathbf{p}\neq 0$ or $\mathbf{p}=0$. For simple diagrams, these two prescriptions give identical results. For more complicated diagrams there may be some differences,[17] in which case one should adopt the prescription of Cutkosky *et al.* with (whenever necessary) some further specifications as to the correct order of limits. For example, Cutkosky *et al.* found that, for the so-called double ice cream cone diagram (which corresponds to a diagram of order at least e^{10}), depending on the order of limits, their prescrrption leads to two different expressions for the S matrix, each being fully relativistic and unitary. The mathematical complexities involved in these diagrams of rather high order have prevented us from appreciating fully their arguments. Nevertheless, it seems reasonable to expect that such an ambiguity can be resolved by noting the Bose statistics nature of photons. All limits must be taken symmetrically with respect to different internal photon lines. This would then lead to a unique answer for the double ice cream cone diagram.

VI. PHOTON PROPAGATOR

As an explicit example, we shall evaluate the propagator D of the modified photon field

$$\phi_\mu=A_\mu+iB_\mu. \quad (6.1)$$

It is convenient to represent all propagators between the two fields A_μ and B_μ by a (2×2) matrix

$$\mathfrak{D}\equiv\begin{pmatrix} D_{AA} & D_{AB} \\ D_{BA} & D_{BB}\end{pmatrix}, \quad (6.2)$$

where the two subscripts denote, respectively, the initial and final states to be either A_μ or B_μ. The propagator for the coherent mixture $A_\mu+iB_\mu$ is then given by

$$D=\tilde{\xi}\mathfrak{D}\xi, \quad (6.3)$$

[16] To show this, it is only necessary to study the $\mathbf{k}$ integration near the singularities of (5.11). Let us use the cylindrical coordinates $\mathbf{k}=(\rho\cos\phi, \rho\sin\phi, z)$, where the z axis is parallel to $\mathbf{p}$. The singularities of (5.11) lie on the circle $z=\frac{1}{2}|\mathbf{p}|$ and $\rho=\rho_0$, where ρ_0 is its radius, determined by (5.10), i.e., $\rho_0=\frac{1}{2}\,\mathrm{Re}\,\omega_0$ and $\omega_0=(\rho_0{}^2+\frac{1}{4}|\mathbf{p}|^2+M_B{}^2)^{1/2}$. In the neighborhood of this circle, it is convenient to write $\rho=\rho_0+r\cos\theta$ and $z=\frac{1}{2}|\mathbf{p}|+r\sin\theta$. The region $r\leqq\epsilon\ll 1$ denotes, then, a doughnut which contains all singularities of (5.11); its cross section is a small circle of radius ϵ. Neglecting $O(\epsilon^2)$, one finds, inside this doughnut region, $(5.11)=[r(a\cos\theta+ib\sin\theta)]^{-1}$, where a and b are two real constants, different from zero, and given explicitly by $a=-\rho_0[\omega_0{}^{-1}+(\omega_0{}^*)^{-1}]$ and $b=i\frac{1}{2}\times[\omega_0{}^{-1}-(\omega_0{}^*)^{-1}]\,|\mathbf{p}|$. Since $d^3k=r\rho_0 dr d\theta d\phi$, the $\mathbf{k}$ integration of (5.11) over this doughnut region is simply

$$2\pi\epsilon\rho_0\int_0^{2\pi} d\theta\,(a\cos\theta+ib\sin\theta)^{-1},$$

which goes to zero as $\epsilon\rightarrow 0$.

[17] R. E. Cutkosky, P. V. Landshoff, D. Olive, and J. C. Polkinghorne, Nucl. Phys. **B12**, 281 (1969). We wish to thank Dr. Cutkosky for a discussion on the double ice cream cone diagram.

where

$$\xi=\begin{pmatrix}1\\ i\end{pmatrix} \tag{6.4}$$

and $\tilde{\xi}$ is the transpose. For the free fields, we have

$$\mathfrak{D}=\mathfrak{D}_{\text{free}}=-i\begin{pmatrix} k^{-2}\delta_{\mu\nu}+(\cdots k_\mu k_\nu) & 0 \\ 0 & [k^2+(m_B{}^0)^2]^{-1}[\delta_{\mu\nu}+(m_B{}^0)^{-2}k_\mu k_\nu] \end{pmatrix}, \tag{6.5}$$

and therefore (6.3) becomes

$$D_{\text{free}}=\frac{-i}{k^2}\left(\frac{(m_B{}^0)^2}{k^2+(m_B{}^0)^2}\right)\delta_{\mu\nu}+(\cdots k_\mu k_\nu), \tag{6.6}$$

where $(\cdots k_\mu k_\nu)$ denotes gauge-dependent terms, which are always proportional to $k_\mu k_\nu$.

In Fig. 2, we introduce

$$i(k^2\delta_{\mu\nu}-k_\mu k_\nu)P \tag{6.7}$$

to represent the sum of all irreducible self-energy graphs in the propagator D_{AA}; because of the coherent mixture (6.1) that appears in all electromagnetic interactions, identical sets of irreducible self-energy graphs also appear in the other propagators D_{AB}, D_{BA}, and D_{BB}. Let Π be a (2×2) matrix, defined by

$$\Pi\equiv\xi\tilde{\xi}i(k^2\delta_{\mu\nu}-k_\mu k_\nu)P. \tag{6.8}$$

One, then, has

$$\mathfrak{D}=\mathfrak{D}_{\text{free}}+\mathfrak{D}_{\text{free}}\Pi\mathfrak{D}_{\text{free}}+\mathfrak{D}_{\text{free}}\Pi\mathfrak{D}_{\text{free}}\Pi\mathfrak{D}_{\text{free}}+\cdots, \tag{6.9}$$

which leads to

$$D=\frac{-i(m_B{}^0)^2\delta_{\mu\nu}}{k^2[k^2+(m_B{}^0)^2(1-P)]}+(\cdots k_\mu k_\nu). \tag{6.10}$$

In Fig. 2, the dashed line represents D itself. It is easy to see that the sum of the first four lowest-order graphs is finite, although each of them diverges logarithmically. With respect to the other graphs of higher order in α, since D is proportional to k^{-4} as $k\to\infty$, they are all finite, as may be shown by means of the "power counting theorem."[18] In this respect, the gauge invariance of our prescription is crucial, since it leads to the cancellation of terms that would otherwise lead to divergences.[19]

To calculate this sum, *we shall follow the modified Feynman rule given in Sec. V*. The result is

$$P(z)=-(\alpha/\pi)[\tfrac{2}{3}\ln(M_FM_F{}^*/m_em_\mu)+f(z,m_e)+f(z,m_\mu)-f(z,M_F)-f(z,M_F{}^*)]+O(\alpha^2), \tag{6.11}$$

where $\alpha\cong(137)^{-1}$, $M_F=m_F+\tfrac{1}{2}i\gamma_F$,

$$z=-k^2, \tag{6.12}$$

$$f(z,\lambda)=(3z)^{-1}(4\lambda^2+2z)[1+(\tfrac{1}{4}z-\lambda^2)J(z,\lambda)], \tag{6.13}$$

$$J(z,\lambda)=\int_0^1\frac{dx}{\lambda^2-x(1-x)z}=\frac{2}{F}\ln\frac{F-z}{F+z}, \tag{6.14}$$

and

$$F=(z^2-4\lambda^2z)^{1/2}. \tag{6.15}$$

We note that in (6.14) the integration path for x is along the real axis from 0 to 1; this follows from the modified Feynman rule, after some transformations and calculations. In (6.11), we have omitted the superscript 0 in all masses, since to $O(\alpha)$ it is immaterial whether one uses the bare masses or the observed masses.

For $\lambda=m_l$ ($l=e$ or μ), $J(z,m_l)$ has a branch point at $z=4m_l{}^2$, and we may choose the cut to be along the real axis from $z=4m_l{}^2$ to ∞. Outside the cut, the integral representation (6.14) is valid everywhere. The imaginary part of $f(z,m_l)$ along the cut is given by

$$\operatorname{Im}f(z,m_l)=\pm\pi\left(\frac{z+2m_l{}^2}{3z}\right)\left(\frac{z-4m_l{}^2}{z}\right)^{1/2}, \tag{6.16}$$

where the + or − sign depends on z being slightly above or below the cut. For $\lambda=M_F$, or $M_F{}^*$, $f(z,\lambda^2)$ is analytic everywhere except at the branch point $z=4M_F{}^2$, or $4(M_F{}^*)^2$. Furthermore, it follows from (6.14) that

$$f(z,\lambda)^*=f(z^*,\lambda^*) \tag{6.17}$$

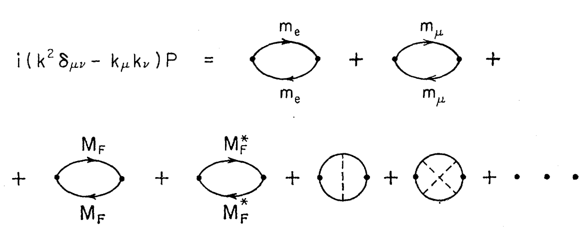

FIG. 2. Sum of irreducible photon self-energy graphs. The charges carried by the four fermion lines, which are labeled by their masses m_e, m_μ, M_F, and $M_F{}^*$, are, respectively, e_0, e_0, ie_0, and $-ie_0$.

[18] For a proof of this theorem, without postulating the validity of contour rotations in the energy variables, see W. Zimmermann, Commun. Math. Phys. **11**, 1 (1968).

[19] We note that each of the first four graphs in Fig. 2 becomes logarithmically divergent (and therefore the sum becomes finite) only if the integration is performed after the gauge-invariant term $(k^2\delta_{\mu\nu}-k_\mu k_\nu)$ is factored out. Quite often, it is much more convenient to do the integration before any factoring. In such a case, in order to avoid any mathematical ambiguity, one may define $\Pi(m_e{}^2,m_\mu{}^2,M_F{}^2,M_F{}^{*2})$ to be the formal sum of all irreducible self-energy graphs, as before, but replace (6.8) by

$$i\xi\tilde{\xi}(k^2\delta_{\mu\nu}-k_\mu k_\nu)P=\lim_{\Lambda^2\to\infty}[\Pi(m_e{}^2,m_\mu{}^2,M_F{}^2,M_F{}^{*2})-\Pi(\Lambda^2+m_e{}^2,\Lambda^2+m_\mu{}^2,\Lambda^2+M_F{}^2,\Lambda^2+M_F{}^{*2})].$$

By taking the limit $\Lambda\to\infty$ after the integration, one can then unambiguously arrive at (6.11).

and therefore $f(z,M_F)+f(z,M_F^*)$ is real along the entire real axis, in accordance with the unitarity condition. By using (6.10), (6.11), and (6.16), one finds that besides the usual photon pole $k^2=0$, D has poles at

$$-k^2=(m_B\pm\tfrac{1}{2}i\gamma_B)^2\,, \tag{6.18}$$

where to $O(\alpha)$ and neglecting $(m_l/m_B)^2$ as compared to unity,

$$\gamma_B=\tfrac{2}{3}\alpha m_B \tag{6.19}$$

and

$$m_B=(m_B{}^0)[1-\tfrac{1}{2}\,\mathrm{Re}P(m_B{}^2)]\,. \tag{6.20}$$

The ratio of the renormalized charge e and the unrenormalized charge e_0 is, by definition, given by the behavior of D as $z\to 0$, i.e.,

$$D\to(-i/k^2)(e/e_0)^2 \quad \text{as } k^2\to 0\,. \tag{6.21}$$

Since $f(z,\lambda^2)\to\frac{1}{9}$ as $z\to 0$, one finds for charge renormalization,

$$\left(\frac{e_0}{e}\right)^2=1+\frac{\alpha}{3\pi}\ln\left(\frac{M_F M_F{}^*}{m_e m_\mu}\right)^2+O(\alpha^2)\,, \tag{6.22}$$

which, of course, is finite. As $z\to\infty$ and slightly above the real axis, one finds

$$\mathrm{Re}P\to(2\alpha/\pi z)(M_F{}^2+M_F{}^{*2}-m_e{}^2-m_\mu{}^2)\,,$$
$$\mathrm{Im}P\to-\tfrac{2}{3}\alpha\,.$$

Therefore, one finds [after neglecting higher-order terms in both $O(\alpha/k^2)$ and $O(\alpha^2)$] that in the timelike region, as $-k^2\to\infty+i\epsilon$, where $\epsilon=0+$,

$$D\to-i(m_B{}^0)^2k^{-2}[k^2+(m_B{}^0)^2+\tfrac{2}{3}i\alpha(m_B{}^0)^2]^{-1}\,, \tag{6.23}$$

while in the spacelike region, as $k^2\to\infty$,

$$D\to-i(m_B{}^0)^2k^{-2}[k^2+(m_B{}^0)^2]^{-1}\,. \tag{6.24}$$

Except for the constant imaginary parts in (6.23), both limiting expressions are exactly the same as D_{free}.

VII. EXPERIMENTAL CONSEQUENCES

The following remarks concern some experimental consequences of the theory.

(1) The modification of the photon propagator from the usual $-ik^{-2}$ to $D(k)$, given by (6.10), in the present theory can be observed through any electromagnetic processes sensitive to the high k^2 behavior of the photon propagator, such as $e^\pm p$ and $\mu^\pm p$ scatterings for spacelike k^2, or pair productions of e^+e^- and $\mu^+\mu^-$ from any high-energy collision processes involving leptons and/or hadrons for timelike k^2. In the timelike region, for $-k^2$ near $m_B{}^2$, the transition probability, being proportional to $|D(k)|^2$, would exhibit a k^2 dependence identical to the standard Breit-Wigner resonance formula; on the other hand, the transition amplitude should have a phase that is of an opposite sign ($-90°$ instead of $+90°$). At $-k^2$ away from $m_B{}^2$, one may use the zeroth-order expression for $D(k)$:

$$D(k)=\frac{-im_B{}^2}{k^2(k^2+m_B{}^2)}\delta_{\mu\nu}+(\cdots k_\mu k_\nu)\,. \tag{7.1}$$

Consider, e.g., the differential cross section of

$$e^+e^-\to\mu^+\mu^- \tag{7.2}$$

in the present theory; it differs from that in the usual quantum electrodynamics by a simple multiplicative factor

$$[m_B{}^2/(k^2+m_B{}^2)]^2\,. \tag{7.3}$$

It is of interest to note that this factor can be rather substantial even if $-k^2$ is quite far from the resonance region. For example, for $m_B\sim 20$ GeV the width γ_B is only ~ 200 MeV; yet, at, say, $(-k^2)^{1/2}\sim 10$ GeV, the above factor is ~ 1.8.

(2) There should be a deviation in the gyromagnetic ratio g of the muon from the usual expression due to such a modification in the photon propagator:

$$\delta g=-(3\pi)^{-1}\alpha(m_\mu/m_B)^2\,. \tag{7.4}$$

From the present experimental result,[20] one concludes that

$$m_B>5\ \text{GeV}\,. \tag{7.5}$$

This limit is also consistent with the present high-energy experimental results[21] on

$$p+\text{uranium}\to p+\mu^++\mu^-+\cdots\,. \tag{7.6}$$

(3) In principle, m_B can also be determined from the finite value of mass differences between hadrons in the same isospin multiplet and from radiative corrections to weak decays. In practice this is difficult, since all these terms depend on m_B only logarithmically, and none of these terms can be calculated accurately. For example, the mass difference Δm_π between $\pi^\pm$ and π^0 in the usual chiral $SU_2\times SU_2$ phenomenological Lagrangian method is infinite, while in the present theory, it is of course finite. By using the same approximation for the strong-interaction vertex, one finds

$$\Delta m_\pi=\frac{3\alpha m_\rho{}^2}{8\pi m_\pi}\left[2\ln 2+\tfrac{1}{8}(1+\delta^2)\frac{m_\pi{}^2}{m_\rho{}^2}\ln\frac{m_B{}^2}{m_\rho{}^2}+O\left(\frac{m_\pi{}^2}{m_\rho{}^2}\right)\right]$$
$$\cong 5\left[1+0.003(1+\delta^2)\ln\left(\frac{m_B}{m_\rho}\right)^2+O\left(\frac{m_\pi{}^2}{m_\rho{}^2}\right)\right]\text{MeV}\,, \tag{7.7}$$

where the first term was first derived by Das *et al.*,[22] δ is the anomalous gyromagnetic ratio of the A_1 meson, and

[20] J. Bailey *et al.*, Phys. Letters **28B**, 287 (1968); see also E. Picasso, in *Proceedings of the Third International Conference on High-Energy Physics and Nuclear Structure* (Plenum, New York, 1970), p. 615.

[21] J. Christenson, G. Hicks, P. Limon, L. M. Lederman, B. Pope, and E. Zavattini, Phys. Rev. Letters (to be published).

[22] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).

$O(m_\pi^2/m_\rho^2)$ denotes terms proportional to (m_π^2/m_ρ^2) but remaining finite even in the limit $m_B \to \infty$. The entire expression (7.7) is, of course, identical to those obtained by Gerstein *et al.*,[23] except for replacing their *ad hoc* cutoff parameter Λ by m_B. It is clear that once Δm_π is made finite, it becomes rather insensitive to the precise value of m_B.

For the radiative correction to weak decays, the dependence on m_B is again only logarithmical. The result is similar to those calculated[24] by using the charged intermediate vector boson $W^\pm$, except for replacing m_W by m_B. If such calculations involving strong-interaction vertices could be made accurate, then it would be possible to determine m_B from the observed value of the Cabibbo angle θ. This is, of course, far from the actual case. Assuming that $\theta = 0.22$, the best estimate[25] for m_W in the intermediate boson theory is $\ln(m_W/m_N) = 2.8 \pm 0.8$ if quark algebra is applicable and $\ln(m_W/m_N) = 3.5 \pm 1.0$ if field algebra is applicable. This leaves a large admissible range for m_B (assuming $m_B \sim m_W$) from about 7 to 90 GeV.

Thus, the best way to determine m_B is through the direct observation of possible deviations from the conventional quantum electrodynamics predictions at high energy.

(4) By using (6.10) and (6.11), one sees that both m_F and γ_F, and therefore also the finite value of charge renormalization $(e_0/e)^2$, can be determined by accurately measuring the photon propagator $D(k)$ at high k^2. However, these measurements are more difficult since it is then necessary to measure $D(k)$ at least to an accuracy comparable to, or better than, $O(\alpha)$.

(5) We remark that an attractive, but highly speculative, idea is to regard B_μ as the neutral component of the hypothetical charged intermediate boson field $W_\mu^\pm$ for the weak interaction, in which case one would expect $m_B \sim m_W$. The further speculation that *the semi-weak interaction coupling constant* $g^2/4\pi$ *is*, in fact, *the same as the fine structure constant* α leads to

$$m_B \sim m_W \sim (4\pi\alpha/G_V)^{1/2} \sim 100 \text{ GeV}. \qquad (7.8)$$

VIII. CAUSALITY

The presence of complex singularities near the physical region, but on the physical sheet, has an unusual effect on the propagation of wave packets in a collision process. As has been discussed in Ref. 4, because of the uncertainty principle, such unusual effects disappear if one studies *only* the average position of the wave packet; this then automatically removes all of the so-called "causality difficulties" or, more precisely, those difficulties that could be directly related to a classical description. In quantum physics, as we shall see, in the first place, there is no general agreement as to the precise meaning of causality. While in the present theory, unusual effects do occur if one analyzes the shape of the wave packet, such effects, though unusual, are not in contradiction with anything known at present about the physical world. Furthermore, it should be clear that such effects *cannot* ever lead to logical difficulties (i.e., self-contradictory predictions), since they are the mathematical consequences of a set of well-defined self-consistent equations. In the following, we shall briefly review these unusual effects and their connection with the causality question.

In most papers on causality, attempts have been made to transform the somewhat ill-defined problem of causality to that of relativistic invariance, which can be stated with precision. In the classical derivation of the dispersion formula for light waves,[26] one begins by assuming a sharp wave front for a physical signal in the theory; e.g., the signal is zero for the space-time region specified by, say, $t<0$ and $x<0$. The subsequent requirement that this wave front should not travel faster than the velocity of light leads to the well known classical dispersion formula which in fact rules out complex singularities such as those that appear in the present theory. We recall that in quantum field theory, it is not possible to construct such a sharp wave front for the incoming wave for all $t<0$, since that would require the superposition of plane waves of all frequencies ν, negative as well as positive. Even for the zero-mass photon field, a coherent mixture of many photon states necessarily covers *only* the *positive* energy range, and therefore $\nu \geq 0$, by using the time-dependent Schrödinger equation; for a massive field, the range of physically allowable frequencies is even smaller, since $\nu \geq h^{-1} \times (\text{mass}) > 0$. The impossibility of constructing a wave packet with a sharply defined front makes it impossible to apply such a classical argument.[27]

Another approach is to use local field theory. One requires the commutators between any two field operators at points separated by a spacelike distance to be zero. This requirement is *satisfied* in the present theory. The derivation[28] of the usual analyticity condition is not applicable to our theory because it assumes the energy spectrum to be real (or, the underlying Lagrangian to be Hermitian), which is obviously not true in

[23] I. S. Gerstein, B. W. Lee, H. T. Nieh, and H. J. Schnitzer, Phys. Rev. Letters **19**, 1064 (1967); **20**, 825 (1968).

[24] For a more recent discussion, see A. Sirlin, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. C. Prentki and J. Steinberger (CERN, Geneva, 1968). The fact that, in the intermediate boson theory of the weak interaction, the $O(\alpha)$ radiative correction to the ratio (G_V/G_μ) is finite in the conventional form of quantum electrodynamics has, of course, been known for quite some time [T. D. Lee, Phys. Rev. **128**, 899 (1962)].

[25] The values quoted are based on the recent calculations by A. Sirlin (private communication). We thank Dr. Sirlin for making these values available to us.

[26] R. Kronig, J. Opt. Soc. Am. **12**, 547 (1926); H. A. Kramers, Atti. Congr. Intern. Fis. Como **2**, 545 (1927); See also A. Sommerfeld, Ann. Phys. (Paris) **44**, 177 (1914); L. Brillouin, *ibid.* **44**, 203 (1914).

[27] N. G. Van Kampen, Phys. Rev. **89**, 1072 (1953); **91**, 1267 (1953).

[28] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. **95**, 1612 (1954).

the present theory. The violation of the usual analyticity condition is, however, totally consistent with the requirement of relativistic invariance.

Still another description of causality that has been used in the literature[29] is one deriving from the study of the average motion of a wave packet. For example, in the case of a simple S-wave elastic scattering, if the incoming wave packet has an average relative position $\langle r_{\rm in}\rangle = -vt$ for time $t \ll 0$, then the outgoing wave packet, for $t \gg 0$, has an average relative position

$$\langle r_{\rm out}\rangle = vt + l\,, \tag{8.1}$$

where

$$l = -2\langle d\delta/dk\rangle\,, \tag{8.2}$$

δ denotes the phase shift, and k is the relative momentum. A $-90°$ resonance, such as the one required by the B_μ field in our theory, would contribute a positive value for l, and therefore give rise to an advancement of the outgoing wave packet. However, it can be shown,[30] under very general conditions, that one has the inequality

$$l < l_{\rm max} = O(\Delta^{-1})\,, \tag{8.3}$$

where Δ denotes the momentum width of the *incoming* wave packet. Therefore, it is not possible to draw any strong conclusion concerning causality by studying the average position of a wave packet.

Rather unusual behavior of wave packets can, nevertheless, be demonstrated to exist in the present theory. In general, these unusual properties concern the detailed shape of the wave packet. Consider, for example, the elastic collision of e^+ and e^- in the center-of-mass system at the resonance energy m_B. For clarity, let us assume the radial dependence of the incoming wave to be given by

$$\phi^{\rm in}(r,t) \propto r^{-1} e^{-\Delta|r+t|} \tag{8.4}$$

at large relative distances r. As shown in Ref. 4, the presence of a complex pole, such as $(m_B + \frac{1}{2}i\gamma_B)$ in the photon propagator, implies that the outgoing wave has a radial dependence given by

$$\phi^{\rm out}(r,t) \propto r^{-1} e^{-\Delta(t-r)} \quad \text{for } t > r\,, \tag{8.5}$$

but

$$\phi^{\rm out}(r,t) \propto r^{-1}\left[e^{-\Delta(t-r)} - \frac{8\gamma\Delta}{(2\Delta+\gamma)^2} e^{-\frac{1}{2}\gamma(t-r)} \right] \quad \text{for } t < r\,, \tag{8.6}$$

where $\gamma = \gamma_B$, the first term $e^{-\Delta(t-r)}$ merely reproduces the shape of the initial wave, but the second term $e^{-\frac{1}{2}\gamma(t-r)}$ is quite unusual. Nevertheless, the presence of such a term is, of course, perfectly compatible with the requirement of relativistic invariance as well as with all existing experiments.

We note that at present in any high-energy experiment almost nothing is known concerning the *shape* of wave packets. Without some detailed knowledge of the shape of wave packets, one can study only the average positions $\langle r\rangle$, which, as mentioned above, are insensitive to the complex singularities. In order to see the unusual tail $e^{-\frac{1}{2}\gamma(t-r)}$, we may consider a measurement which can differentiate the time-advanced region, say $(t-r) > \tau$, from the time-retarded region $(t-r) < \tau$, where τ represents the experimental space-time resolution. At present, the best value of the time resolution in any high-energy experiment is $\tau \sim 10^{-10}$ sec. Assuming, for example, $m_B \sim 20$ GeV and therefore $\gamma_B \sim 200$ MeV, the full intensity of the unusual tail, integrated over the entire time-advanced region from $(t-r) = \tau$ to $(t-r) = \infty$, comes out to be

$$\sim \exp(-\gamma_B\tau) \sim \exp[-3\times 10^{13}]\,. \tag{8.7}$$

The smallness of this probability[31] makes it unlikely that we can realistically detect such an unusual effect in any near future (assuming that we can reach a center-of-mass energy $\sim$ the resonance energy m_B). Of course, in principle, this effect should be measurable.

In any quantum theory, what one really studies are only correlations between various events occurring at different space-time regions. The impossibility of constructing during $t<0$ a sharp wave front for the incoming wave makes it also not possible to give a strict causal interpretation to such correlations. Thus, there does *not* exist a sharply defined causality principle. (We regard requirements such as the usual zero commutator of two local field operators separated by a spacelike distance as simply an expression of relativistic invariance and local canonical quantum field theory, but not of causality.[32]) The attribution as to which effect should be regarded as "noncausal," therefore, has a certain degree of arbitrariness, except in the classical limit.[33] It seems nevertheless appropriate to call the above described unusual tail $e^{-\frac{1}{2}\gamma(t-r)}$ in the outgoing wave packet "noncausal," although one must emphasize that there is *no* logical difficulty in having this particular

[29] E. P. Wigner, Phys. Rev. **98**, 145 (1955). Professor Wigner has kindly pointed out to us a different formulation of causality in his paper in *International School of Physics "Enrico Fermi," Course 29* (Academic, New York, 1964), p. 40. This formulation is not based on wave packets and is, therefore, not directly related to the point being made here. See also M. Froissart, M. L. Goldberger, and K. M. Watson, Phys. Rev. **131**, 2820 (1963); cf., however, H. M. Nussenzveig, Nuovo Cimento **20**, 694 (1961).

[30] See Ref. 4. The arguments used in Ref. 4 can be extended to the nonresonance region as well. The inequality (8.3) holds, provided the integral $\int (d\delta/dk)dk$, integrated over the entire momentum width of the incoming wave, is $\lesssim \pi$.

[31] The two-photon exchange processes can lead to a larger value, though still much too small to be detected at present. See T. D. Lee (Ref. 5) and Cutkosky *et al.* (Ref. 17).

[32] A similar opinion is also expressed by G. Wanders [Nuovo Cimento **14**, 168 (1959)], whose conclusions on causality seem to agree with ours also in other respects. General questions of causality have also been discussed by B. Ferretti, Nuovo Cimento **43**, 506 (1966); **43**, 516 (1966).

[33] Since a macroscopic body does not necessarily imply the validity of a classical limit (e.g., the superfluidity phenomenon), there also does not exist in quantum physics any general macrocausality principle that can be sharply defined without further qualifications.

kind of "noncausal" effect; the question of whether it indeed exists in nature can only be resolved by future experimentation.

ACKNOWLEDGMENTS

We wish to thank N. Christ, S. Coleman, R. Jackiw, and B. Zumino for helpful discussions.

APPENDIX A

Let us study anticommutation relations for one oscillator, assuming

$$a^2=\bar{a}^2=0\,,\quad a\bar{a}+\bar{a}a=\pm 1\,. \tag{A1}$$

The case with the + sign is, of course, well known, but we can study the two cases simultaneously if we define $N=\pm\bar{a}a$; then

$$N^2=N\,. \tag{A2}$$

As is well known, N satisfying (A2) can always be diagonalized. In fact, any vector ψ can always be decomposed into a sum of two vectors (one of which may be zero) $N\psi$ and $(1-N)\psi$ which are eigenvectors of N, corresponding to the eigenvalues 1 and 0, respectively. Both eigenvalues certainly exist; for if we assume a vector ψ with the eigenvalue 0, $N\psi=0$, then from $a\bar{a}\psi=\pm(\psi-N\psi)=\pm\psi$ we see that $\phi=\bar{a}\psi\neq 0$, since $a\phi=\pm\psi$; furthermore, $N\phi=\pm\bar{a}a\bar{a}\psi=\bar{a}\psi=\phi$, and therefore ϕ belongs to the eigenvalue 1. Conversely, if ϕ is an eigenvector belonging to the eigenvalue 1, $N\phi=\phi$, then $a\phi$ belongs to the eigenvalue 0, for $Na\phi=\pm\bar{a}a^2\phi=0$; notice again that $a\phi$ cannot be zero, since $N\phi=\pm\bar{a}a\phi=\phi\neq 0$. These calculations are well known from the ordinary case, where $\bar{a}=a^\dagger$ and the + sign holds in Eq. (A1). We have repeated them here, merely to remind the reader of the fact that the analysis makes no use whatsoever of the metric; it is based *entirely* on the algebraic structure of (A1). Pursuing the analysis further, one sees that the whole vector space $\mathcal{H}$ is the direct sum of two subspaces $\mathcal{H}=\mathcal{H}_0\oplus\mathcal{H}_1$, in one of which the operator $N=0$, while in the other it acts as the identity $N=1$. We have seen that if ψ belongs to the first subspace $\mathcal{H}_0$, then $\phi=\bar{a}\psi$ belongs to $\mathcal{H}_1$, and we have the relations

$$\begin{aligned}\bar{a}\psi&=\phi\,,\quad a\phi=\pm\psi\,,\\ a\psi&=0\,,\quad \bar{a}\phi=0\,.\end{aligned} \tag{A3}$$

The same relations are obtained if we start from any vector ϕ of $\mathcal{H}_1$, and define $\psi=\pm a\phi$. From this we see that $\bar{a}$ maps $\mathcal{H}_1$ into 0, and $\mathcal{H}_0$ into $\mathcal{H}_1$. Regarded as a mapping of $\mathcal{H}_0$ into $\mathcal{H}_1$, $\bar{a}$ has $\pm a$ as a left inverse. Likewise a maps $\mathcal{H}_0$ into 0, and $\mathcal{H}_1$ into $\mathcal{H}_0$; regarded as a mapping of $\mathcal{H}_1$ into $\mathcal{H}_0$, a has $\pm\bar{a}$ as a left inverse. It follows that $\mathcal{H}_0$ and $\mathcal{H}_1$ have the same dimensionality. Choosing bases $\psi_1,\ldots,\psi_n$ in $\mathcal{H}_0$ and $\phi_1,\ldots,\phi_n$ in $\mathcal{H}_1$ such that ψ_k and ϕ_k are related to each other in the same way as ψ and ϕ in (A3), a and $\bar{a}$ are reduced to the canonical form

$$a=\begin{pmatrix}0 & \pm I\\ 0 & 0\end{pmatrix},\quad \bar{a}=\begin{pmatrix}0 & 0\\ I & 0\end{pmatrix}, \tag{A4}$$

where I, of course, is the n-dimensional unit matrix. This may be obtained from the usual case, if in (A1) one identifies $\bar{a}$ with $\pm a^\dagger$. Notice again, however, that no use has been made of Hermiticity.

In order to determine uniquely the metric η, we shall impose the condition that *the set of matrices a and $\bar{a}$ should be irreducible*. It follows, then, from (A4) that both a and $\bar{a}$ are (2×2) matrices, i.e., there is only one ψ and one ϕ. To conform to the notations used in Sec. II, we shall denote ψ and ϕ by $|0\rangle$ and $|1\rangle$, respectively. Equation (A3) becomes

$$\begin{aligned}\bar{a}|0\rangle&\equiv|1\rangle\,,\quad a|1\rangle\equiv\pm|0\rangle\,,\\ a|0\rangle&=0\,,\qquad \bar{a}|1\rangle=0\,.\end{aligned} \tag{A5}$$

Thus, one finds

$$\begin{aligned}&\langle 1|0\rangle=\pm\langle 1|a|1\rangle=\pm\langle 1|\bar{a}|1\rangle^*=0\,,\\ &\langle 0|1\rangle=0\,,\end{aligned} \tag{A6}$$

and

$$\langle 1|1\rangle=\langle 0|a\bar{a}|0\rangle=\pm\langle 0|0\rangle\,.$$

By using transformation (2.22), we can always choose $\langle 0|0\rangle$ to be +1, and therefore there are only two classes specified by (2.26) and (2.27).

The preceding result can be applied immediately to the case of ν "oscillators" with anticommuting variables $a_1, a_2, \ldots, a_\nu$. We assume

$$\{a_r,a_s\}=\{\bar{a}_r,\bar{a}_s\}=0\,, \tag{A7}$$

while

$$\{\bar{a}_r,a_s\}=\epsilon_r\delta_{rs}\quad(\epsilon_r=\pm 1)\,. \tag{A8}$$

Then the above analysis can be applied immediately, say to the first pair of operators, a_1 and $\bar{a}_1$. We may in fact cut short all remaining calculations, if we notice that if we set $a_r^+=\epsilon_r\bar{a}_r$, the above equations have the usual form. The notation does not require that a_r^+ be Hermitian conjugate to a_r; it is simply another operator. The result is the customary one; if the set of matrices is assumed irreducible, the span has 2^ν dimensions. A basis may be chosen, in which the basis vectors are

$$|n_1,n_2,\ldots,n_\nu\rangle\,,$$

where each $n_k=0$ or 1. These vectors satisfy

$$a_k|0,0,\ldots,0\rangle=0\,, \tag{A9}$$

$$\epsilon_k\bar{a}_ka_k|n_1,n_2,\ldots,n_\nu\rangle=n_k|n_1,n_2,\ldots,n_\nu\rangle\,, \tag{A10}$$

and

$$|n_1,n_2,\ldots,n_\nu\rangle=(\bar{a}_1)^{n_1}(\bar{a}_2)^{n_2}\cdots(\bar{a}_\nu)^{n_\nu}|0,0,\ldots,0\rangle\,. \tag{A11}$$

We note in passing, that in a somewhat more general formulation (A8) could be replaced by

$$\{\bar{a}_r,a_s\}=K_{rs}\,, \tag{A12}$$

where the matrix $K=(K_{rs})$ is a *nonsingular Hermitian* matrix. This greater generality is only fictitious; a linear transformation $b_r=\sum_s T_{sr}a_s$, $\bar{b}_r=\sum_s T_{sr}^*\bar{a}_s\equiv\sum_s T_{rs}^\dagger\bar{a}_s$ will change the matrix K to $T^\dagger KT$. As is well known, one can always choose T so that (A12) reduces to (A8), with $\epsilon_r=\pm1$. Although trivial, this generalization is nevertheless useful in practice, as we shall see in Appendix C.

Just as in the case of a single oscillator, the metric η is completely determined, up to a real proportionality constant. By using (A9)–(A11), one easily sees that

$$\langle n_1',n_2',\ldots,n_\nu'|n_1,n_2,\ldots,n_\nu\rangle=\prod_{r=1}^{\nu}\epsilon_r^{n_r}\delta_{n_r'n_r}\times\langle 0,0,\ldots,0|0,0,\ldots,0\rangle. \quad \text{(A13)}$$

APPENDIX B

We examine here the consequences of the commutation relation

$$a\bar{a}-\bar{a}a=-1, \quad \text{(B1)}$$

which, by interchanging the roles of a and $\bar{a}$, is of course completely equivalent to the alternative form

$$a\bar{a}-\bar{a}a=+1.$$

Just as in the case for the anticommutation relation, we need the following assumption: (i) The algebra of matrices generated by a, $\bar{a}$ is irreducible, i.e., there is no invariant subspace. In addition, we assume (ii) there is at least one eigenvector $|\psi\rangle$ of the operator $\bar{a}a$:

$$\bar{a}a|\psi\rangle=\lambda|\psi\rangle. \quad \text{(B2)}$$

(For brevity, in this section, a vector is denoted by the symbols $|\psi\rangle$ or ψ as seems most convenient.)

The metric η, which is to be determined, is, as always, assumed to be Hermitian and nonsingular; therefore, there is no vector $\phi\neq0$ such that

$$\langle\ |\phi\rangle=0 \quad \text{(B3)}$$

for all vectors $|\ \rangle$.

The argument that follows is fairly trivial, as it relies on the customary construction of a string of eigenvectors of $\bar{a}a$,

$$\ldots,\ \bar{a}^2\psi,\ \bar{a}\psi,\ \psi,\ a\psi,\ a^2\psi,\ \ldots, \quad \text{(B4)}$$

corresponding to the eigenvalues

$$\ldots,\ \lambda-2,\ \lambda-1,\ \lambda,\ \lambda+1,\ \lambda+2,\ \ldots. \quad \text{(B5)}$$

The eigenvalue equations

$$\bar{a}a\phi_p=(\lambda+p)\phi_p\,,\quad \phi_p=a^p\psi$$

and

$$\bar{a}a\chi_p=(\lambda-p)\chi_p\,,\quad \chi_p=\bar{a}^p\psi \quad \text{(B6)}$$

follow, of course, from (B1) by recursion, with the qualification that the string may stop on the right at ϕ_p, if $a\phi_p=0$ (or on the left at χ_q, if $\bar{a}\chi_q=0$). In any event, by a well-known argument, the eigenvectors in (B4) form an independent system, their eigenvalues in (B5) being all different. In addition one easily sees that

$$a\phi_p=\phi_{p+1}\,,\quad \bar{a}\phi_p=\bar{a}a\phi_{p-1}=(\lambda+p-1)\phi_{p-1}\,,$$
$$\bar{a}\chi_p=\chi_{p+1}\,,\quad a\chi_p=a\bar{a}\chi_{p-1}=(\lambda-p)\chi_{p-1}. \quad \text{(B7)}$$

All these equations hold for $p=1,\ 2,\ \ldots$ provided one defines $\phi_0=\chi_0=\psi$. Thus the string of vectors (B4) spans our invariant subspace, which according to our assumption (i) must be the whole space. Clearly the string may not stop on both sides, since (B1) cannot hold in a finite dimensional space (by the usual trace argument). We now have three possibilities.

(1) The string stops at the left: $\chi_{p+1}=0$ but $\chi_p\neq0$. $0=a\chi_{p+1}=(\lambda-p-1)\chi_p$; hence $\lambda=p+1$.

(2) The string stops at the right: $\phi_{p+1}=0$ but $\phi_p\neq0$ (where p may also be $=0$). But then from (B7), $0=\bar{a}\phi_{p+1}=(\lambda+p)\phi_p$; hence $\lambda=-p$.

(3) The string extends to infinity in both directions: In this case, as we shall see, λ is an arbitrary *real* number but not an integer.

It is now easy to see that in all three cases the metric of the space is completely determined, apart from a real proportionality constant. Two distinct vectors in (B4) are mutually orthogonal, since they belong to different eigenvalues of a self-adjoint operator. None of them can have zero norm, or we would have degeneracy, Eq. (B3). From (B6) and (B7) we have finally

$$\langle\phi_{p+1}|\phi_{p+1}\rangle=\langle\phi_p|\bar{a}a|\phi_p\rangle=(\lambda+p)\langle\phi_p|\phi_p\rangle$$

and

$$\langle\chi_{p+1}|\chi_{p+1}\rangle=\langle\chi_p|a\bar{a}|\chi_p\rangle=(\lambda-p-1)\langle\chi_p|\chi_p\rangle. \quad \text{(B8)}$$

These equations determine the norm of all vectors up to a common proportionality constant. They show that λ must be real. Furthermore, if a vector in the string (B4) has a negative eigenvalue, then the vector to the right of it (if it exists) has a square norm of the opposite sign. This never occurs in case (1), which is therefore recognized as the ordinary oscillator with positive-definite metric by setting $\bar{a}=b$ and $a=b^\dagger$. Case (2) is likewise recognized as Dirac's or Pauli's oscillator with indefinite metric. Choosing suitable normalization factors, one can reduce Eqs. (B7) to one of the standard forms commonly employed.

We shall not mention case (1) any further. In case (2) or (3) we may (by a suitable selection of the vector called ψ in the string) assume that

$$0\leqslant\lambda<1.$$

Case (2) corresponds to $\lambda=0$, in this case the vectors $\phi_1,\ \phi_2,\ \ldots$ do not exist, and the eigenvalues are $\ldots,\ -2,\ -1,\ 0$. From (B8), and by choosing

$$\langle\chi_0|\chi_0\rangle \text{ positive},$$

one finds (2.27), i.e.,

$$\langle\ |(-1)^{\bar{a}a}|\ \rangle \text{ positive}$$

for all vectors $|\ \rangle$.

In case (3), $0<\lambda<1$, we may define new basis vectors, setting

$$|\lambda\rangle=\psi,$$
$$|\lambda+p\rangle=[\lambda(\lambda+1)\cdots(\lambda+p-1)]^{-1/2}a^p|\lambda\rangle \qquad \text{(B9)}$$

and

$$|\lambda-p\rangle=i^{-p}[(1-\lambda)(2-\lambda)\cdots(p-\lambda)]^{-1/2}\bar{a}^p|\lambda\rangle.$$

Then we get, denoting by $|\mu\rangle$ one of the new basis vectors ($\mu=\lambda$ or $\lambda\pm p$),

$$a|\mu\rangle=\mu^{1/2}|\mu+1\rangle$$
and
$$\bar{a}|\mu\rangle=(\mu-1)^{1/2}|\mu-1\rangle. \qquad \text{(B10)}$$

Notice again that, when $\mu<0$, $\mu^{1/2}=+i|\mu|^{1/2}$ by definition. Thus, one finds

$$\langle\mu+1|\mu+1\rangle=(\mu/|\mu|)\langle\mu|\mu\rangle, \qquad \text{(B11)}$$

i.e., all $\langle\mu|\mu\rangle$ have the same sign for $\mu>0$, but alternate signs for $\mu<0$.

APPENDIX C

In this appendix, we discuss the explicit diagonal form of the free Hamiltonian for the ψ_F field discussed in Sec. IV. According to (4.3), the free Lagrangian density is

$$\mathcal{L}_{\text{free}}=-\bar{\psi}_F\tau_z\gamma_4\left(\gamma_\lambda\frac{\partial}{\partial x_\lambda}+M_F{}^0\right)\psi_F, \qquad \text{(C1)}$$

where

$$M_F{}^0=m_F{}^0+\tfrac{1}{2}i\gamma_F{}^0\tau_x, \qquad \text{(C2)}$$

and for convenience, we have set

$$\tau_a=\tau_z \quad \text{and} \quad \tau_b=\tau_x. \qquad \text{(C3)}$$

The free Hamiltonian is then given by

$$H_{\text{free}}=\int d^3r\,\bar{\psi}_F\tau_z(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta M_F{}^0)\psi_F, \qquad \text{(C4)}$$

in which, as usual, $\gamma_4=\beta$ and $\gamma_j=-i\beta\alpha_j$ for $j=1, 2, 3$.

It is useful to introduce the c number 4-component spinor functions $u_\pm(\mathbf{p},s)$ and $v_\pm(\mathbf{p},s)$, defined by

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M)u_+(\mathbf{p},s)=E_\mathbf{p}u_+(\mathbf{p},s),$$
$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M^*)u_-(\mathbf{p},s)=E_\mathbf{p}{}^*u_-(\mathbf{p},s), \qquad \text{(C5)}$$
$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M)v_+(-\mathbf{p},s)=-E_\mathbf{p}v_+(-\mathbf{p},s),$$

and

$$(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta M^*)v_-(-\mathbf{p},s)=-E_\mathbf{p}{}^*v_-(-\mathbf{p},s),$$

where

$$E_\mathbf{p}=(\mathbf{p}^2+M^2)^{1/2}, \qquad \text{(C6)}$$

$$M=m_F{}^0+\tfrac{1}{2}i\gamma_F{}^0, \qquad \text{(C7)}$$

and $s=\pm1$ denotes the usual helicity. From (C5), it can be readily verified that the following orthogonality relations hold:

$$u_+{}^\dagger(\mathbf{p},s)v_-(-\mathbf{p},s')=v_+{}^\dagger(-\mathbf{p},s)u_-(\mathbf{p},s')=0. \qquad \text{(C8)}$$

By a suitable choice of the normalization factors, one can set

$$\sum_s[u_+(\mathbf{p},s)u_-{}^\dagger(\mathbf{p},s)+v_+(-\mathbf{p},s)v_-{}^\dagger(-\mathbf{p},s)]=I, \qquad \text{(C9)}$$

where I is the (4×4) unit matrix.

At any given time,

$$\psi_F=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

can be expanded in terms of these spinors

$$\psi_1=\sum_{\mathbf{p},s}(2\Omega)^{-1/2}[a_+(\mathbf{p},s)u_+(\mathbf{p},s)+a_-(\mathbf{p},s)u_-(\mathbf{p},s)$$
$$+\bar{b}_-(-\mathbf{p},s)v_+(-\mathbf{p},s)+\bar{b}_+(-\mathbf{p},s)v_-(-\mathbf{p},s)]e^{i\mathbf{p}\cdot\mathbf{r}} \qquad \text{(C10)}$$

and

$$\psi_2=\sum_{\mathbf{p},s}(2\Omega)^{-1/2}[a_+(\mathbf{p},s)u_+(\mathbf{p},s)-a_-(\mathbf{p},s)u_-(\mathbf{p},s)$$
$$+\bar{b}_-(-\mathbf{p},s)v_+(-\mathbf{p},s)-\bar{b}_+(-\mathbf{p},s)v_-(-\mathbf{p},s)]e^{i\mathbf{p}\cdot\mathbf{r}}, \qquad \text{(C11)}$$

where Ω is the volume of the system. Correspondingly, the adjoint operators are given by

$$\bar{\psi}_1=\sum_{\mathbf{p},s}(2\Omega)^{-1/2}[\bar{a}_+(\mathbf{p},s)u_+{}^\dagger(\mathbf{p},s)+\bar{a}_-(\mathbf{p},s)u_-{}^\dagger(\mathbf{p},s)$$
$$+b_-(-\mathbf{p},s)v_+{}^\dagger(-\mathbf{p},s)+b_+(-\mathbf{p},s)v_-{}^\dagger(-\mathbf{p},s)]e^{-i\mathbf{p}\cdot\mathbf{r}} \qquad \text{(C12)}$$

and

$$\bar{\psi}_2=\sum_{\mathbf{p},s}(2\Omega)^{-1/2}[\bar{a}_+(\mathbf{p},s)u_+{}^\dagger(\mathbf{p},s)-\bar{a}_-(\mathbf{p},s)u_-{}^\dagger(\mathbf{p},s)$$
$$+b_-(-\mathbf{p},s)v_+{}^\dagger(-\mathbf{p},s)-b_+(-\mathbf{p},s)v_-{}^\dagger(-\mathbf{p},s)]e^{-i\mathbf{p}\cdot\mathbf{r}}. \qquad \text{(C13)}$$

By using (C9) and the quantization rule (4.8) (setting $\tau_a=\tau_z$), one finds

$$\{a_+(\mathbf{p},s),\bar{a}_-(\mathbf{p}',s')\}=\{b_+(\mathbf{p},s),\bar{b}_-(\mathbf{p}',s')\}$$
$$=\{a_-(\mathbf{p},s),\bar{a}_+(\mathbf{p}',s')\}=\{b_-(\mathbf{p},s),\bar{b}_+(\mathbf{p}',s')\}$$
$$=\delta_{ss'}\delta_{\mathbf{pp}'} \qquad \text{(C14)}$$

and *all other equal-time anticommutators are zero*. These anticommutators are seen to correspond to the more general type of Eq. (A12), rather than the "diagonal" case of Eqs. (A7) and (A8) of Appendix A. The relations may be diagonalized by introducing new operators

$$a_1(\mathbf{p},s)=[a_+(\mathbf{p},s)+a_-(\mathbf{p},s)]/\sqrt{2},$$
$$a_2(\mathbf{p},s)=[a_+(\mathbf{p},s)-a_-(\mathbf{p},s)]/\sqrt{2}, \qquad \text{(C15)}$$

and similar expressions for the b operators. Then

$$\{a_\rho(\mathbf{p},s),\bar{a}_\sigma(\mathbf{p}',s')\}=\{b_\rho(\mathbf{p},s),\bar{b}_\sigma(\mathbf{p}',s')\}=\epsilon_\rho\delta_{ss'}\delta_{\mathbf{pp}'}\delta_{\rho\sigma}, \quad \text{(C14}')$$

where ρ, $\sigma=1$ or 2, and $\epsilon_1=1$, $\epsilon_2=-1$. All other equal-time anticommutators are zero. It then follows from the arguments given in Appendix A that the metric η is uniquely determined, up to the transformations (2.20′) and (2.22). We find, for a system consisting only of the fermion field ψ_F,

$$\langle\,|(-1)^{N_F}|\,\rangle \text{ to be positive}$$

for all vectors $|\,\rangle$, where

$$N_F=-\sum_{\mathbf{p},s}[\bar{a}_2(\mathbf{p},s)a_2(\mathbf{p},s)+\bar{b}_2(\mathbf{p},s)b_2(\mathbf{p},s)]. \quad \text{(C16)}$$

In terms of the Fourier components $a_\pm$ and $b_\pm$, the free Hamiltonian becomes

$$H_{\text{free}}=\sum_{\mathbf{p},s}\{E_{\mathbf{p}}[\bar{a}_-(\mathbf{p},s)a_+(\mathbf{p},s)-b_+(\mathbf{p},s)\bar{b}_-(\mathbf{p},s)]+E_{\mathbf{p}}^*[\bar{a}_+(\mathbf{p},s)a_-(\mathbf{p},s)-b_-(\mathbf{p},s)\bar{b}_+(\mathbf{p},s)]\}. \quad \text{(C17)}$$

We remark that, if we follow the prescriptions of Appendix A, we will naturally arrive at a basis in which the operators $\bar{a}_\rho a_\rho$, $\bar{b}_\rho b_\rho$ are diagonal; in this basis $\bar{a}_-a_+$, etc., are not diagonal. We notice, however, that the metric is positive definite with respect to the degrees of freedom with $\rho=1$ and indefinite for $\rho=2$ [see Eq. (C14′)]. Furthermore, in the basis we have described, because of (A13), the matrix representation η of the metric, defined by (2.3) and (2.4), is diagonal, and the matrices representing a_1, a_2, b_1, b_2 (we omit the indices $\mathbf{p}$, s for simplicity) obey the rule

$$\bar{a}_\rho=\epsilon_\rho a_\rho^\dagger, \quad \bar{b}_\rho=\epsilon_\rho b_\rho^\dagger. \quad \text{(C18)}$$

In this basis, therefore,

$$\bar{a}_-=a_+^\dagger, \quad \bar{b}_-=b_+^\dagger, \qquad \bar{a}_+=a_-^\dagger, \quad \bar{b}_+=b_-^\dagger, \quad \text{(C19)}$$

and Eqs. (C14) take the standard form for fermion operators,

$$\{a_\lambda(\mathbf{p},s),a_\lambda^\dagger(\mathbf{p}',s')\}=\{b_\lambda(\mathbf{p},s),b_\lambda^\dagger(\mathbf{p}',s')\}=\delta_{\mathbf{pp}'}\delta_{ss'}, \quad \text{(C20)}$$

where $\lambda=+$ or $-$ and all other equal-time anticommutators remain zero. Similarly, the free Hamiltonian becomes

$$H_{\text{free}}=\sum_{\mathbf{p},s}\{E_{\mathbf{p}}[a_+^\dagger(\mathbf{p},s)a_+(\mathbf{p},s)-b_+(\mathbf{p},s)b_+^\dagger(\mathbf{p},s)]+E_{\mathbf{p}}^*[a_-^\dagger(\mathbf{p},s)a_-(\mathbf{p},s)-b_-(\mathbf{p},s)b_-^\dagger(\mathbf{p},s)]\}. \quad \text{(C21)}$$

Now, technically we are still in the basis in which $\bar{a}_\rho a_\rho=\epsilon_\rho a_\rho^\dagger a_\rho$ and $\bar{b}_\rho b_\rho=\epsilon_\rho b_\rho^\dagger b_\rho$ are diagonal, and therefore H_{free} is not yet diagonal. However, through a familiar *unitary* transformation, one can easily transform the basis vectors into the eigenvectors of $a_+^\dagger a_+$, $a_-^\dagger a_-$, $b_+^\dagger b_+$, and $b_-^\dagger b_-$. Because this is a unitary transformation, the transformation law (2.20) governing the metric is the same as that for a *bona fide* operator. Therefore, in this new basis, Eqs. (C18)–(C21) remain valid and the free Hamiltonian (C21) is diagonal, though the matrix η is no longer diagonal.

It is useful, especially in the presence of interactions, to define

$$\psi_\pm=2^{-1/2}(\psi_1\pm\psi_2). \quad \text{(C22)}$$

The free Hamiltonian (C4) then becomes

$$H_{\text{free}}=\int[\bar{\psi}_-(-i\boldsymbol{\alpha}\cdot\nabla+\beta M)\psi_++\bar{\psi}_+(-i\boldsymbol{\alpha}\cdot\nabla+\beta M^*)\psi_-]d^3r. \quad \text{(C23)}$$

By using (4.3), one finds that the electromagnetic interaction is given by

$$H_{\text{int}}=\int e_0(\bar{\psi}_-\gamma_4\gamma_\mu\psi_+-\bar{\psi}_+\gamma_4\gamma_\mu\psi_-)\phi_\mu d^3r. \quad \text{(C24)}$$

Furthermore, according to (4.8),

$$\{\psi_+(\mathbf{r},t),\bar{\psi}_-(\mathbf{r}',t)\}=\{\psi_-(\mathbf{r},t),\bar{\psi}_+(\mathbf{r}',t)\}=\delta^3(\mathbf{r}-\mathbf{r}') \quad \text{(C25)}$$

and

$$\{\psi_+(\mathbf{r},t),\bar{\psi}_+(\mathbf{r}',t)\}=\{\psi_-(\mathbf{r},t),\bar{\psi}_-(\mathbf{r}',t)\}=0. \quad \text{(C26)}$$

From these relations and Eqs. (C10)–(C13), it is easy to see that ψ_+ annihilates "particles" of mass M and charge ie_0, and creates "antiparticles" of mass M and charge $-ie_0$, while $\bar{\psi}_-$ creates "particles" of mass M and charge ie_0, etc. By using (C8) and (C9), one can then proceed to derive the appropriate propagators and vertex functions for these fermion fields. The resulting expressions are just like the usual ones, except that the masses M and M^* are complex and the charges ie_0 and $-ie_0$ are imaginary.