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N/D analysis of $\rho\pi$ scattering in the 1^+ channel in Bardakci-Halpern–type models

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Amplitudes for $\rho\pi$ scattering obtained from lowest order graphs in two gauge models are unitarized in the $J^P = 1^+$, isospin-1 channel using the N/D formalism. There is no evidence for resonant behavior near the A_1 mass region.

I. INTRODUCTION

During the recent past, the status of A_1 as a resonance has become more and more open to doubt. It was first observed as a broad enhancement in the $J^P(I) = 1^+(1)$ channel of the diffractive reaction $\pi N \to 3\pi N$ at a $\rho\pi$ invariant mass about 1.1 GeV. A number of analyses of the experimental data have been made,[1] and all agree on the absence of a resonant A_1 state. The A_1 phase variation is quite flat, and the evidence for A_1 in nondiffractive processes is slender and insufficient.[2] On the other hand, theoretically there are reasons to expect an axial-vector meson. Weinberg sum rules, if vector and axial-vector functions are saturated with ρ and A_1, require the mass of A_1 to be $\sqrt{2}$ times the mass of ρ; just about the place where the $\rho\pi$ mass enhancement in $\pi N \to (3\pi)N$ is seen. Similarly, in the meson-mass spectrum generated by the quark model, it is natural to expect A_1.

Because of its ambiguous character, theoretical attempts have been made to understand A_1 in terms of a purely kinematic enhancement. The Deck mechanism,[3] and its later refined versions[4]—the so-called Reggeized Deck models—have been particularly successful in this direction. But, if A_1 is purely a kinematic enhancement, then one has to look seriously into the implications it has for chiral symmetry and the quark model.

It becomes of interest, therefore, to study $\rho\pi$ scattering in the 1^+ channel and see, even if qualitatively, whether a resonance can be generated. We consider the interaction of ρ and π as given by a gauge model having ρ as a gauge vector meson. Spontaneously broken gauge models are of special interest because they are renormalizable.

We take the lowest-order $\rho\pi$ scattering amplitudes from a gauge model introduced briefly in Sec. II and, using the discontinuities across the left-hand cuts given by them, unitarize them in the 1^+ channel making use of the N/D formalism. This is given in Sec. III. The same calculation has been done in Sec. IV using a chiral-SU(2) × SU(2) gauge model due to Bardakci[5] in which A_1 is assumed to exist. One would like to check whether the inclusion of amplitudes corresponding to the A_1 exchange in the u channel of $\rho\pi$ scattering produces a resonant behavior in the s channel.

The outcome, presented in Sec. V, is that in both the calculations there is no resonant behavior around the A_1 mass region. This means that the forces given by the gauge models do not seem to be attractive enough in the 1^+ channel, if indeed one can talk in terms of such nonrelativistic terminology.

Our calculations are certainly not a conclusive proof against A_1—though they are an argument against it. If we take this result seriously, then it appears that for the A_1 enhancement, one should either look for a kinematic explanation and try to use a nonlinear realization of chiral symmetry, or look for some deeper dynamical reasons to explain the poor phase-shift variation of A_1 in diffractive processes on the one hand, and the apparent absence of a 1^+ resonance in nondiffractive reactions on the other.[6]

II. A GAUGE MODEL FOR ρ AND π

Consider a group $L \times G$ where L is a local SU(2) group and G is a global SU(2) group. The pion is a triplet under L and ρ is taken to be the gauge vector meson of L. In other words, under an infinitesimal transformation of L,

$$\begin{aligned} &L(x) = 1 + i\vec{\theta}(x)\cdot\vec{\tau}\,, \\ &\vec{\pi} \to \vec{\pi} - 2\vec{\theta}(x)\times\vec{\pi}\,, \\ &\vec{\rho}_\mu \to \vec{\rho}_\mu - 2\vec{\theta}(x)\times\vec{\rho}_\mu + \frac{1}{g}\partial_\mu\vec{\theta}(x)\,. \end{aligned} \tag{2.1}$$

Under G, both ρ and π are invariant. We also consider Higgs scalars $M_0, N_0, \vec{M}, \vec{N}$ which transform under $L\times G$ as follows:

$$\begin{aligned} &\phi = M_0 + iN_0 + (\vec{M} + i\vec{N})\cdot\vec{\tau}\,, \\ &\phi \to L\phi G^{-1}. \end{aligned} \tag{2.2}$$

An invariant Lagrangian can now be written as

$$\mathcal{L} = -\tfrac{1}{8}\mathrm{tr}\{(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu - ig[\rho_\mu,\rho_\nu])^2\} + \tfrac{1}{4}\mathrm{tr}\{(\partial_\mu\pi - ig[\rho_\mu,\pi])^2\} + \tfrac{1}{4}\mathrm{tr}\{(\partial_\mu\phi - ig\rho_\mu\phi)(\partial^\mu\phi^\dagger + ig\phi^\dagger\rho^\mu)\}$$
$$+\tfrac{1}{4}\alpha^2\,\mathrm{tr}(\phi^\dagger\phi) - \tfrac{1}{4}\beta[\mathrm{tr}(\phi^\dagger\phi)]^2 + \tfrac{1}{2}\gamma\,\mathrm{tr}[(\phi^\dagger\phi)^2] - \tfrac{1}{2}\delta\,\mathrm{tr}(\pi^2\phi^\dagger\phi) - \tfrac{1}{4}\lambda[\mathrm{tr}(\pi^2)]^2\,, \tag{2.3}$$

where

$$\pi \equiv \vec{\pi}\cdot\vec{\tau}\,, \quad \text{and } \rho_\mu \equiv \vec{\rho}_\mu\cdot\vec{\tau}\,.$$

The local SU(2) symmetry can be broken spontaneously by requiring the vacuum expectation value of ϕ, $\langle\phi\rangle_0$, to be nonzero:

$$\langle\phi\rangle_0 = \eta\,. \tag{2.4}$$

Shifting the fields ϕ to $\phi' = \phi - \eta$, one finds in the usual manner that the ρ acquires a mass $g\eta$, the pion a mass $\sqrt{\delta}\eta$, and $M_0' \equiv M_0 - \eta$ and M have masses $\sqrt{2}\alpha$ and $2\sqrt{2}\gamma\eta$, respectively. N and N_0 remain massless.

We notice that under transformations for which $\vec{\theta}(x)$ is independent of x [see (2.1)] and the parameters of transformation of the two groups L and G are equal, the Lagrangian so obtained is still invariant. This can be identified with the isospin group.

A remark must be made concerning the gauge conditions and the presence of massless particles N_0 and $\vec{N}$. We can, for example, choose the gauge $\vec{N} = 0$. This corresponds to choosing the unitary gauge. It still leaves N_0 massless. We could have avoided N_0 by either choosing a smaller number of Higgs scalars, for example, breaking the symmetry by giving a nonzero vacuum-expectation value to some component of just one complex doublet instead of two; or, we could have retained similarity with the Bardakci-Halpern model[5] and extended the group to U(2). In the latter case, one has to introduce another vector meson corresponding to the U(1) part of U(2) = U(1) × SU(2). Then we can choose another gauge condition and eliminate N_0. However, in this work we do not require the extra couplings introduced in this way.

III. HELICITY AMPLITUDES FOR $\rho\pi$ SCATTERING AND N/D EQUATIONS

Writing the S matrix as $S = 1 + i(2\pi)^4 T$, the matrix element of T, $T_{\lambda_3\lambda_1}^{i_3i_4,i_1i_2}(s,t)$ can be decomposed generally as

$$T_{\lambda_3\lambda_1}(s,t) = (2\pi)^{-6}\epsilon^{\nu}_{\lambda_3}(p_3)\epsilon^{\mu}_{\lambda_1}(p_1)$$
$$\times[Ag_{\nu\mu} + Bp_{2\nu}p_{2\mu} + Cp_{1\nu}p_{2\mu} + Dp_{2\nu}p_{3\mu} + Ep_{1\nu}p_{3\mu}]\,, \tag{3.1}$$

where indices 1, 2, 3, 4 refer to $\rho(1) + \pi(2) \to \rho(3) + \pi(4)$ and we have omitted the isospin indices for convenience of writing. We note that there are actually only four independent amplitudes because time-reversal invariance requires that

$$C = B + D\,. \tag{3.2}$$

In the center-of-mass system, the "parity-conserving" amplitudes, i.e., amplitudes with definite total angular momentum and parity, can be written for the 1^+ channel in terms of the above invariant amplitudes as follows:

$$T_{11} = \tfrac{1}{4}\pi(2\pi)^{-6}(p/\sqrt{s})\times\int_{-1}^{1}dx[-A(1+x^2) + (D-E)p^2x(1-x^2)]\,,$$
$$T_{10} = \frac{1}{2\sqrt{2}}\pi(2\pi)^{-6}(p\Omega/M\sqrt{s})\times\int_{-1}^{1}dx(1-x^2)[-A + (B-C)p^2(\sqrt{s}/\Omega) + (D-E)p^2(1-x)]\,, \tag{3.3}$$
$$T_{01} = T_{10}$$
$$T_{00} = \tfrac{1}{2}\pi(2\pi)^{-6}(p\Omega^2/M^2\sqrt{s})\times\int_{-1}^{1}dx\left[A\left(\frac{p^2}{\Omega^2} - x\right) + \frac{p^2\sqrt{s}}{\Omega}\left(\frac{B\omega + C\Omega}{\Omega} + x(B-C)\right) + p^2(1-x)\left(\frac{D\omega + E\Omega}{\Omega} + x(D-E)\right)\right]\,,$$

where p is the magnitude of the center-of-mass momentum, Ω and ω are the energies of the ρ and π, respectively, and M is the mass of ρ. Also, the amplitudes A to E are supposed to be projected to the states with total isospin 1. We note that we have four amplitudes $T_{\alpha\beta}$, $\alpha,\beta = 1, 0$, because out of $\rho\pi$ helicity states one can construct two independent positive-parity states with angular momentum 1, namely, the one corresponding to ρ helicity zero, and the other corresponding to a linear combination of helicity +1 and −1. In $T_{\alpha\beta}$, α or $\beta = 1$ corresponds to the helicity ±1 combination. We also note that $T_{\alpha\beta}$ is symmetric as required by time-reversal invariance.

The amplitudes (3.3) must satisfy the unitarity condition

$$i(T^\dagger - T) = (2\pi)^4 TT^\dagger\,. \tag{3.4}$$

To write N/D equations, it is more convenient to define

$$\tilde{T}_{11} = \pi(2\pi)^4(\sqrt{s}/p)T_{11}\,,$$
$$\tilde{T}_{10} = \pi(2\pi)^4(m/p)T_{10}\,, \tag{3.5}$$
$$\tilde{T}_{00} = \pi(2\pi)^4(m^2/p\sqrt{s})T_{00}\,,$$

where m is the pion mass. Equation (3.4) now becomes

$$i(\tilde{T}-\tilde{T}^{\dagger})=-\tilde{T}\rho\tilde{T}^{\dagger}, \tag{3.6}$$

where ρ is the diagonal matrix

$$\begin{aligned}&\rho_{00}=p\sqrt{s}(\pi m^2)\,,\\&\rho_{11}=p/(\pi\sqrt{s})\,,\\&\rho_{10}=\rho_{01}=0\,.\end{aligned} \tag{3.7}$$

Let us write

$$\tilde{T}=ND^{-1}\,, \tag{3.8}$$

where N and D are 2×2 matrices. We assume that D is analytic in the s plane except for the physical cut [from $(m+M)^2$ to ∞], whereas N is analytic except for the so-called unphysical cuts—the latter being the singularities in the s plane arising from expressions (3.3) when we substitute for A, B, etc., the expressions obtained from lowest-order diagrams of field theory. Normalizing D at a point s_0 and taking the unitarity condition into account,

$$D(s)=1-\frac{s-s_0}{2\pi}\int_P ds'\,\frac{\rho(s')N(s')}{(s'-s_0)(s'-s)}\,, \tag{3.9}$$

$$N(s)=\int_U ds''\,\frac{\Delta(s'')D(s'')}{(s''-s)}\,. \tag{3.10}$$

The integrals in (3.9) and (3.10) go over the physical (P) and unphysical (U) cuts, respectively, and $\Delta(s'')$ is the discontinuity in $T(s'')$ across the unphysical cut divided by $2\pi i$.

By substituting the second of these equations in the first we get the following integral equation for D:

$$D(s)=1-\frac{s-s_0}{2\pi}\int_U ds''\,R(s,s'')\Delta(s'')D(s'')\,, \tag{3.11}$$

where

$$R(s,s'')=\int_P ds'\,\frac{\rho(s')}{(s'-s_0)(s'-s)(s''-s')}. \tag{3.12}$$

Equation (3.11) can be inverted to solve for D on the unphysical cut if we use for Δ the discontinuities of $\tilde{T}$ as obtained from the gauge model. These values can then be used to obtain N on the physical cut by (3.10). This, in turn, determines D on the physical cut as

$$\begin{aligned}\mathrm{Re}D(s)&=1-\frac{s-s_0}{2\pi}\,\mathrm{P}\int_P ds'\,\frac{\rho(s')N(s')}{(s'-s_0)(s'-s)}\,,\\ \mathrm{Im}D(s)&=-\tfrac{1}{2}\rho(s)N(s)\,.\end{aligned} \tag{3.13}$$

The lowest-order $\rho\pi$ diagrams in the gauge model discussed in Sec. II are shown in Fig. 1. They give rise to left-hand or unphysical cuts shown in Fig. 2. The discontinuities of $T_{\alpha\beta}$ across these cuts are given below.

ρ exchange in the t channel [Fig. 1(a)]:

$$\begin{aligned}\Delta_{11}^{\rho}(s)&=\tfrac{1}{8}g^2[(s-u')(1+y_\rho^{\,2})/p^2+4y_\rho(1-y_\rho^{\,2})]\\ \Delta_{10}^{\rho}(s)&=\frac{1}{4\sqrt{2}}(m\Omega/M\sqrt{s})[(s-u')/p^2\\&\qquad-4\omega/\Omega-4y_\rho](1-y_\rho^{\,2})\\&=\Delta_{01}(s)\\ \Delta_{00}^{\rho}(s)&=\tfrac{1}{4}g^2y_\rho(m^2\Omega^2/M^2s)\\&\qquad\times\left[-\frac{s-u'}{p^2}\left(\frac{p^2}{\Omega^2}-y_\rho\right)+\left(y_\rho+\frac{2\omega}{\Omega}+1\right)(1-y_\rho)\right],\end{aligned} \tag{3.14}$$

where

$$y_\rho=1+\frac{M^2}{2p^2} \tag{3.15}$$

and

$$u'=2M^2+2m^2-s+2p^2(1-y_\rho)\,. \tag{3.16}$$

π exchange in the u channel [Fig. 1(b)]:

$$\begin{aligned}\Delta_{11}^{\pi}(s)&=-g^2y_\pi(1-y_\pi^{\,2})\\ \Delta_{10}^{\pi}(s)&=\frac{1}{\sqrt{2}}g^2(m\Omega/M\sqrt{s})(1-y_\pi^{\,2})(y_\pi+\omega/\Omega)\\&=\Delta_{01}^{\pi}(s)\\ \Delta_{00}^{\pi}(s)&=g^2(m^2\Omega^2/M^2s)y_\pi(y_\pi+\omega/\Omega)^2\,,\end{aligned} \tag{3.17}$$

with

$$y_\pi=1+\frac{2M^2+2m^2-s}{2p^2}. \tag{3.18}$$

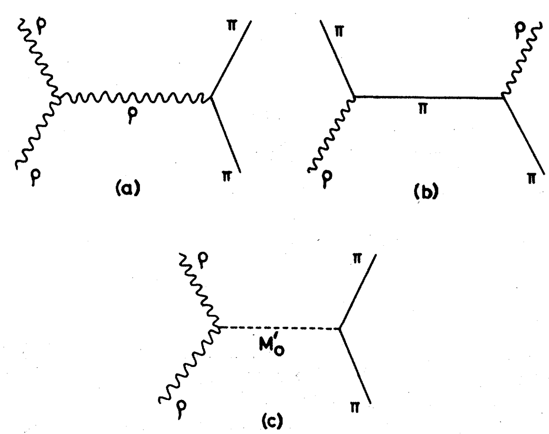

FIG. 1. Lowest-order diagrams giving left-hand discontinuities in the 1^+ channel.

Scalar exchange in the t channel [Fig. 1(c)]:

$$\begin{aligned}\Delta_{11}^{M}(s) &= \tfrac{1}{8} g^2(\eta^2\delta/p^2)(1+y_M{}^2)\\ \Delta_{10}^{M}(s) &= \frac{1}{4\sqrt{2}}\, g^2(\eta^2\delta/p^2)(m\Omega/M\sqrt{s})(1-y_M{}^2)\\ &= \Delta_{01}^{M}(s)\\ \Delta_{00}^{M}(s) &= -\tfrac{1}{4} g^2(\eta^2\delta/p^2)(m^2\Omega^2/M^2 s)(p^2/\Omega^2 - y_M)y_M\,,\end{aligned} \tag{3.19}$$

with

$$y_M = 1 + \frac{\mu^2}{2p^2}\,, \tag{3.20}$$

where μ is the mass of the scalar particle M_0'.

It must be noted that, because of the light mass of the pion, a part of the unphysical cut for the diagram in Fig. 1(b) lies entirely within the physical cut. This happens because $\rho\pi \to 3\pi$ is kinematically possible, so that s- and u-channel regions overlap. Such a situation can occur if we have unstable particles and needs special attention in a purely dispersion-theoretic approach. In our case we shall unitarize the amplitude on the physical cut, and it is this total unitarized amplitude which matters. Therefore, the cut due to pion exchange lying within the unitarity cut will not be included in the left-hand discontinuity.

For the calculation of discontinuities we take the following values for g, η, and δ:

$$g^2/4\pi = 0.4\,,$$

$$\eta = 330 \text{ MeV}\,,$$

$$\delta = 0.18\,,$$

which are fitted with the ρ and π masses and the $\rho \to 2\pi$ width of about 100 MeV.[7]

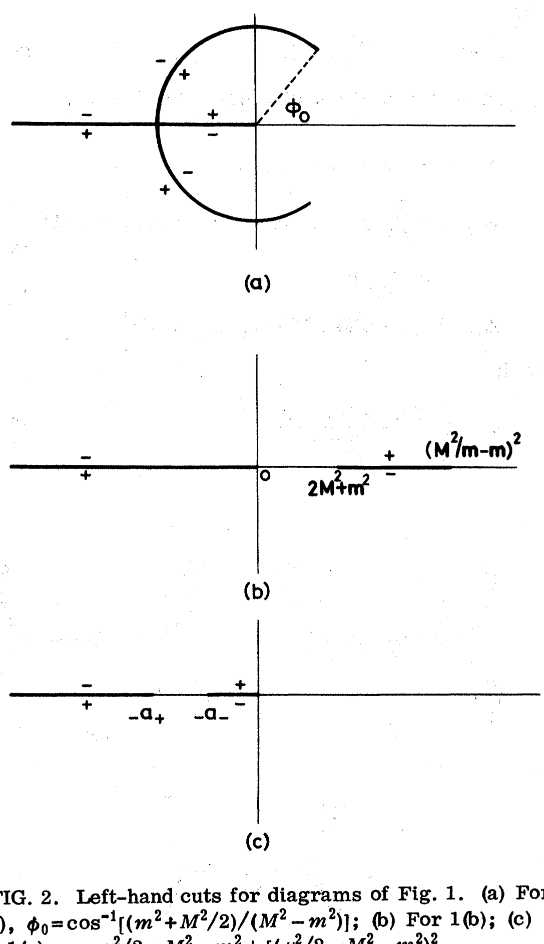

FIG. 2. Left-hand cuts for diagrams of Fig. 1. (a) For 1(a), $\phi_0 = \cos^{-1}[(m^2+M^2/2)/(M^2-m^2)]$; (b) For 1(b); (c) for 1(c), $a_\pm = \mu^2/2 - M^2 - m^2 \pm [(\mu^2/2 - M^2 - m^2)^2 - (M^2-m^2)^2]^{1/2}$. μ is the mass of M_0' and assumed to be $\geq 2M$. The + and − signs mean the following: $\tilde{T}$ on the + side of the cut minus, $\tilde{T}$ on the − side of the cut is equal to $2\pi i\Delta$.

IV. INCLUSION OF A_1 EXCHANGE IN u CHANNEL

In the gauge model of Sec. II, we had only ρ and π apart from Higgs scalars. A more interesting situation occurs in the Bardakci model where the symmetry group is the broken chiral SU(2) ×SU(2). A_1 itself is one of the gauge vector mesons, and in our N/D calculation we can include the discontinuities due to the diagram shown in Fig. 3(a). The

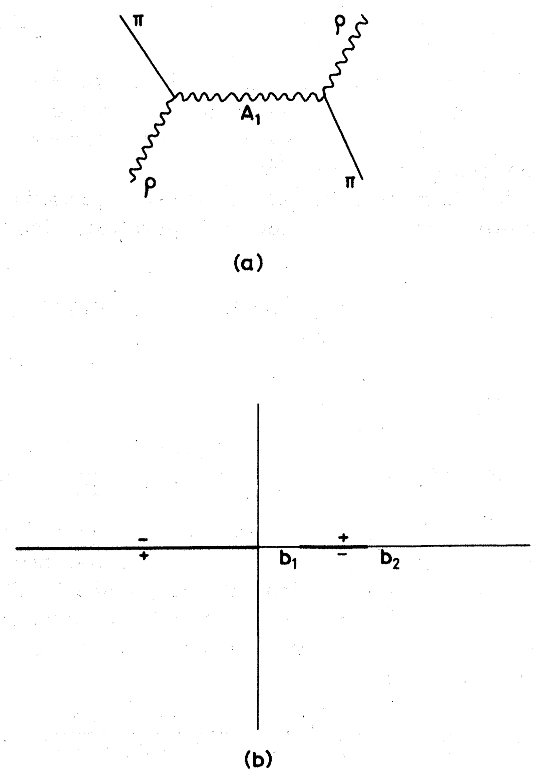

FIG. 3. (a) Diagram for the A_1 exchange in the u channel; (b) left-hand discontinuities caused by it, where $b_1 = 2M^2 + 2m^2 - m_A{}^2$, $b_2 = (M^2-m^2)^2/m_A{}^2$.

discontinuities for the case of the Bardakci model are written with a prime, and are given by

$$\Delta^{\rho\prime}=\gamma\Delta^{\rho}, \tag{4.1}$$

$$\Delta^{\pi\prime}=\gamma^2\Delta^{\pi}, \tag{4.2}$$

where Δ^{ρ} and Δ^{π} are discontinuities for the model of Sec. II, given by (3.15) and (3.17). γ is a dimensionless constant given in terms of the Bardakci-model parameters α and β by

$$\gamma=\frac{\beta^2+2\alpha^2}{\beta^2+4\alpha^2}\,.$$

The A_1 exchange gives rise to discontinuities given by

$$\Delta_{11}^{A}(s)=\frac{g^4\alpha^2\beta^2}{2p^2(\beta^2+4\alpha^2)}\,(1+y_A{}^2)\,,$$

$$\Delta_{10}^{A}(s)=\frac{g^4\alpha^2\beta^2 m\Omega}{2\sqrt{2}(\beta^2+4\alpha^2)M\sqrt{s}p^2}\,(1-y_A{}^2)\,, \tag{4.3}$$

$$\Delta_{00}^{A}(s)=\frac{g^4\alpha^2\beta^2m^2\Omega^2}{(\beta^2+4\alpha^2)M^2sp^2}\,y_A\left(\frac{p^2}{2\Omega^2}-y_A\right),$$

where

$$y_A=1+\frac{2M^2+2m^2-m_A{}^2-s}{2p^2}. \tag{4.4}$$

The unphysical cuts due to A_1 exchange are shown in Fig. 3(b). For discontinuities in this case we take

$g=4.7$,

$\alpha^2=7.1\times10^3\ \mathrm{MeV}^2$,

$\beta^2=2.7\times10^4\ \mathrm{MeV}^2$,

obtained by fitting the ρ and A_1 masses to be 765 MeV and 1.1 GeV, respectively, and the $\rho\to2\pi$ width to be 135 MeV.

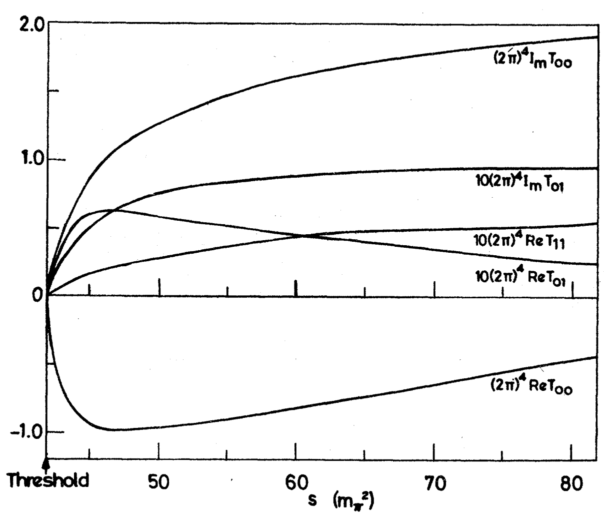

FIG. 4. $(2\pi)^4T_{\alpha\beta}$ obtained by solving N/D equations. M_0' mass is taken to be infinity. s is in units of pion-mass squared. Im T_{11} is too small to be shown. T_{01} is actually the symmetric part $(T_{01}+T_{10})/2$.

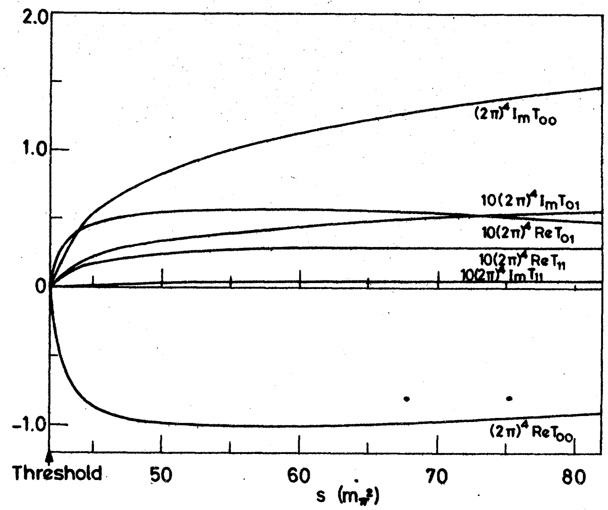

FIG. 5. $(2\pi)^4T_{\alpha\beta}$ obtained by solving N/D equations in the Bardakci Model. s is in units of pion mass squared. T_{01} is actually the symmetric part: $(T_{01}+T_{10})/2$.

V. RESULTS AND DISCUSSION

The solution of N/D equations with left-hand discontinuities as given in Secs. III and IV is shown in Figs. 4 and 5.

Fig. 4 gives the unitarized amplitudes for the model of Sec. II with the M_0' mass taken to be very large. There is almost no dependence of the amplitudes on the M_0' mass, varied from twice the ρ mass to infinity.

Fig. 5 shows the amplitudes calculated in the Bardakci model. The total cross sections in the 1^+ channel corresponding to the amplitudes in Figs. 4 and 5 are shown in Fig. 6. It is clear that

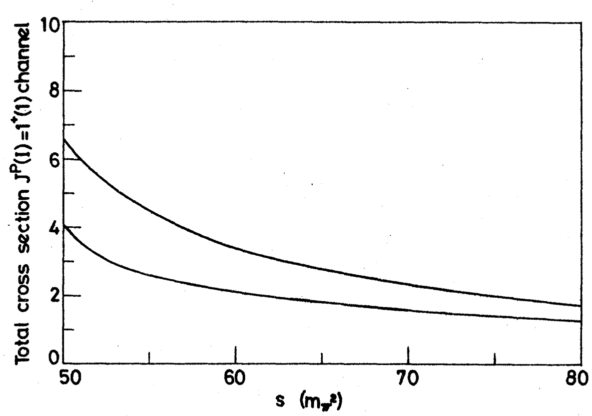

FIG. 6. Total cross sections (in units of m_π^{-2}) as a function of s (in units of m_π^2), corresponding to amplitudes in Figs. 4 and 5. The upper curve corresponds to the cross section for Fig. 4, and the lower one to Fig. 5. The A_1 mass region is about $s\approx62$.

there is not the slightest hint of a resonance.

In the following we make some remarks concerning the calculation and results:

(i) Looking at the expressions (3.14), (3.17), (3.19), and (4.3) for discontinuities Δ, we observe that Δ_{00} has a double pole and Δ_{01} has a single pole at $s=0$. This is of a purely kinematic origin, and depends on the particular choice (3.7) for the matrix ρ. In defining $\tilde{T}$ we could have multiplied T_{00}, T_{10}, and T_{11}, respectively, by $s\sqrt{s}/p$, s/p, and $\sqrt{s}/p$, which is as good a choice for removing the kinematical singularities due to $\sqrt{s}$ factors. In that case we would get

$$\rho_{00}=p/s\sqrt{s}\pi\;,\quad \rho_{11}=p/\sqrt{s}\pi\;,\quad \rho_{10}=\rho_{01}=0\;.$$

This choice avoids $1/s^2$ in $\tilde{T}_{\alpha\beta}$ at the cost of making its asymptotic behavior worse—the expression (3.10) for N becomes badly behaved for $s''\to\infty$. We are then at liberty to choose from the following two alternatives: "subtract" N at an arbitrary point with undetermined residue, or cut off the integral in (3.10) and treat the cutoff point as a parameter. We resort to neither of these, and, instead, shift the pole at $s=0$ by multiplying T_{00} by $[s/(s-a)]^2$ and T_{10} by $s/(s-a)$. Accordingly, ρ_{00} changes to $\rho_{00}\,[(s-a)/s]^2$. This does not change the asymptotic behavior of $\rho_{\alpha\beta}$. The results given are for $a=1$, in pion-mass units. Also, we have chosen s_0, the normalization point [see Eq. (3.9)], to be equal to 1.

(ii) The mass μ of the scalar M_0' has almost no influence on the amplitude. For this reason we have omitted the contribution of scalars in calculating with the Bardakci model.

(iii) We have checked that in our numerical calculation the matrices D and N are conjugate-symmetric. In particular, D is real on the negative real axis and takes complex-conjugate values on conjugate points. Similarly, N is real on the physical cut.

(iv) $T_{\alpha\beta}(s)$ is symmetric to a great extent. In the calculation with the model of Sec. II, the maximum value of $|T_{01}-T_{10}|/|T_{10}+T_{01}|$ is less than 0.05 for s ranging from the $\rho\pi$ threshold to $s=80$ in pion-mass units. Symmetry of the amplitude obtained from N/D equations when symmetric left-hand cut discontinuities are given as input is an analytic property. We consider it as a good feature of our calculation to be able to obtain this symmetry in a numerical calculation. We can improve it by choosing the pole at a, mentioned in (i), properly. However, for calculation with the Bardakci model, the symmetry is not so good, the asymmetry ratio being 0.15. This is due to the fact that with the introduction of the unphysical cut due to A_1 exchange (Fig. 3), the effect of the pole at $a=1$ is felt on discontinuities due to the A_1 exchange as well, the point b_1 in Fig. 3(b) being at about $s=2$. In this case we are unable to improve the symmetry of the amplitudes, and the results given in Fig. 5 are the best ones obtained.

(v) The unitarity condition for the amplitudes is very well satisfied. We have calculated the two eigenvalues x_1 and x_2 of the S matrix $S_{\alpha\beta}=\delta_{\alpha\beta}+i(2\pi)^4T_{\alpha\beta}$ as a function of s, and found that their modulus is very near 1 over the whole range. For example, for amplitudes shown in Fig. 4, the maximum difference of $|x_1|$ or $|x_2|$ from 1 is less than 0.06. For amplitudes in Fig. 5 it is less than 0.02.

In conclusion we would like to emphasize that our aim in the above calculation has been to see whether the lowest-order $\rho\pi$ amplitudes in gauge theories, when unitarized by the N/D method, produce a resonant behavior for A_1. We find that they do not. The parameters at our disposal g, μ, s_0, a, etc., show very little influence over the typical amplitudes shown in Figs. 4 and 5. It is possible to do this calculation at a much more sophisticated level, for example, by (a) continuing in the mass variable of ρ to take into account the fact that ρ is unstable and (b) by including more channels. In this connection it may be observed that the inclusion of other channels, for example the $K^*\bar{K}$ channel, whose importance has been emphasized by the calculation of Longacre and Aaron recently,[8] cannot be incorporated in our approach without extending the model itself to include K mesons. In our approach we have tried to see what implications gauge-field-theoretic couplings have for A_1. We are not trying to fit the A_1 parameters as done in a phenomenological approach. For this reason we are also unable to utilize the experimental information[9] on phase variation, because for us the phase variation (contained in the amplitudes $T_{\alpha\beta}$) is entirely fixed by the left-hand discontinuities obtained from the model.

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