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Radiative corrections in photon-photon collisions induced by e^+e^- or $e^\pm e^\pm$ processes

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In this work we present an evaluation of radiative corrections to order α^5 pertaining to γ-γ collision experiments performed with an $e^\pm e^\pm$ storage ring in which the electrons scattered are detected at small angle. We check that the infrared divergences originating from elastic corrections are canceled due to opposite divergences from real-photon bremsstrahlung. Numerical results for given experimental conditions allow one to compare the various contributions to radiative corrections in processes such as $ee \to eeX$ with $X = e^+e^-, \mu^+\mu^-, \pi^+\pi^-$

I. INTRODUCTION

In this work, we present a study of radiative corrections in photon-photon collisions via e^+e^- or $e^\pm e^\pm$ colliding beams. The physical interest of the process $ee \to ee\gamma\gamma \to eeX$ $(C=+1)$, which will enable us to study the $C=+1$ hadronic states, without hadronic spectator, has been widely dealt with[1-5].

In order α^4 in quantum electrodynamics, the Feynman graph corresponding to this process is the graph (P) (Fig. 1). γ-γ collision experiments without detection of the scattered primary electrons meet some difficulties owing to the background (which actually comes from the other Feynman graphs of the same order in α). It has been shown[1,2,3,6] that one can practically eliminate this background by detecting the scattered primary electrons in the forward direction. In that case, the virtual photons are quasireal. This is the principle of the experiment started in 1978 at the electron storage ring DCI (Orsay).[7] Many of the aspects of this study are relative to the conditions of that experiment.

In order α^5, radiative corrections to the graph (P) are due, on one hand, to "elastic" corrections contained in the interference between the principal graph (P) and the graphs of Fig. 2. On the other hand, they come from "inelastic" corrections expressed by the graphs of Fig. 3. Indeed, as for the latter ones, the tagging system of the electrons has a finite resolution ΔE in energy which allows events of the types (4) or (5) to occur, i.e., emission of a photon k with small energy lying between 0 and ΔE. However, in order to avoid infrared divergences, we shall assign to this photon a mass λ which will tend to zero after eliminating the infinities, and we shall separate the emitted photons into two classes: "soft" photons $(\lambda \leq k_0 \leq \Lambda)$ and "hard" photons $(\Lambda \leq k_0 \leq \Delta E)$ by means of a cutoff Λ.

The corrected differential cross section will be written in a general way as follows: $d\sigma=(1+\delta)d\sigma_0 + dC$, where $d\sigma_0$ is the cross section relative to the graph (P), and where δ and dC are the corrections coming from graphs (1) to (5): $\delta=\delta(1)+\delta(2)+\delta(3)+\delta(4+5)$, $dC=dC(1)+dC(2)+dC(3)+dC(4+5)$. In the calculations we deal with here, we neglect the graph (3) and the interference between graphs (4) and (5). In fact, their contributions are probably unimportant.[8,9] One thus gets $\delta\simeq\delta(1)+\delta(2)+\delta(4)+\delta(5)$, $dC\simeq dC(1)+dC(2)+dC(4)+dC(5)$.

In Sec. II, we shall review briefly the general results on γ-γ collisions and the computation of $d\sigma_0$ in the approximation of quasireal photons, and we shall evaluate the elastic corrections. In Sec. III, we shall consider the inelastic corrections, for the emission of a soft photon, in order to elinimate the infrared divergences, and for the emission of a hard photon. The full results will be presented in Sec. IV together with their numerical outcome.

We emphasize the fact that our calculations are based on three main types of approximations: (i) extreme relativistic approximation, (ii) approximation of quasireal photons (or equivalent-photon approximation), and (iii) approximation of soft photons (for inelastic corrections). These three categories of approximations will be defined hereafter.

II. UNCORRECTED CROSS SECTION AND ELASTIC CROSS SECTION

In the beginning of this section, we shall be interested in the calculation of the cross section of the process represented by the graph (P). This calculation has already been made by many authors, and for all details we refer the reader to references.[1,2,3,10] Before giving general results, let us define the kinematical variables corresponding to the experimental situation (laboratory frame) in Fig. 4. Four-momenta of the particles are defined as follows $(+,-,-,-$ metric): incident electrons $p_1=(E,\vec{p})$, $p_2=(E,-\vec{p})$; scattered electrons $p_1'=(E_1', \vec{p}_1', \theta_1\varphi_1)$, $p_2'=(E_2', \vec{p}_2', \theta_2, \varphi_2)$

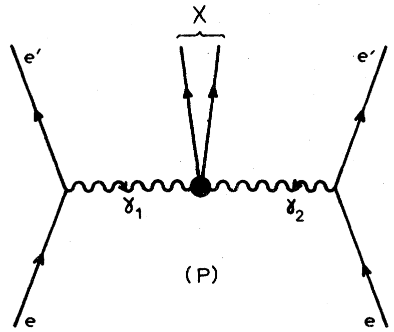

FIG. 1. Main graph (P) for the production of a system X collisions photon-photon induced by $e^\pm e^\pm$ scattering.

with $\theta_1, \theta_2 \leq \Theta_{\max}$; quasireal photons $q_1=(\omega_1, \vec{q}_1)$, $q_2=(\omega_2, \vec{q}_2)$; system X produced $X=q_1+q_2=p_1-p_1'+p_2-p_2'$. m is the electron mass, and we suppose $m/E, m/E_1', m/E_2' \ll 1$. We shall define $t_i=-q_i^2=-(p_i-p_i')^2$; $\rho_i=t_i/4m^2$ $(i=1,2)$; $W=(X^2)^{1/2}$.

The wholly differentiated cross section of the process studied is written in a usual way[3]

$$d\sigma_0=\frac{32\pi^2\alpha^4}{E^2t_1^2t_2^2}\; L_{\mu\rho}^{(1)}\left(\sum X^{\mu\nu}X^{\rho\sigma *}\right)L_{\nu\sigma}^{(2)} \times\delta^{(4)}(X-q_1-q_2)\,\frac{d^3p_1'}{2E_1'}\,\frac{d^3p_2'}{2E_2'}\,d\Gamma_X\,, \qquad (1)$$

where $d\Gamma_X$ is the phase space of the decay particles of the X system $(X\rightarrow\eta_1+\eta_2+\cdots+\eta_n)$:

$$d\Gamma_X=(2\pi)^{-3n}\prod_{\alpha=1}^{n}\zeta_\alpha d^3\eta_\alpha/2\eta_{0\alpha}\,,$$

with $\zeta_\alpha=2$ for a fermion and 1 for a boson; $L_{\mu\rho}^{(1)}$ and $L_{\nu\sigma}^{(2)}$ are the dynamical tensors of the left-hand and right-hand $ee\gamma$ vertices and $X^{\mu\nu}$ is the electromagnetic current associated with the partial process $q_1+q_2\rightarrow X$; $\sum$ means a sum over spin states. If we are not interested in studying individual particle variables of the system produced,

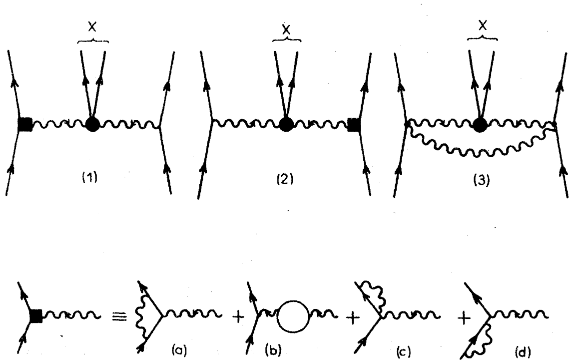

FIG. 2. Graphs (1), (2), (3) ocurring to order α^5 [when interfering with graph (P)] in elastic corrections. The squared box is defined as the sum of vertex correction (a), vacuum polarization (b), and mass renormalization (c) and (d) effects.

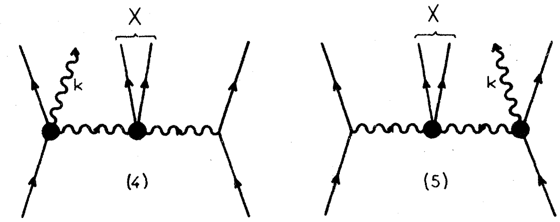

FIG. 3. Graphs (4) and (5) occuring to order α^5 in inelastic corrections. They are characterized by the emission of a real photon k.

we must integrate them over; then it is convenient to set

$$W^{\mu\nu\rho\sigma}=[(2\pi)^4/2]\int\left(\sum X^{\mu\nu}X^{\rho\sigma *}\right)\delta^{(4)}(X-q_1-q_2)d\Gamma_X\,.$$

It has been shown[3] that, in the approximation of quasireal photons, this quantity can be written as

$$W^{\mu\nu\rho\sigma}\simeq R^{\mu\rho}R^{\nu\sigma}W_{TT}(W) +\tfrac{1}{2}(R^{\mu\nu}R^{\rho\sigma}+R^{\mu\sigma}R^{\rho\nu}-R^{\mu\rho}R^{\nu\sigma})\tau_{TT}(W)\,, \quad (2)$$

where $R^{\mu\nu}=-g^{\mu\nu}+[q_1\cdot q_2(q_1^\mu q_2^\nu+q_1^\nu q_2^\mu)-q_1^2q_2^\mu q_2^\nu-q_2^2q_1^\mu q_1^\nu]/\mathfrak{X}$ and $\mathfrak{X}=(q_1\cdot q_2)^2-q_1^2q_2^2$. It has been shown also that $W_{TT}=W^2\sigma_{\gamma\gamma}(W)/16\pi^2\alpha^2$ and $\tau_{TT}=W^2\sigma_{TT}{}^2(W)/16\pi^2\alpha^2$, where $\sigma_{\gamma\gamma}(W)$ is the photoproduction cross section and $\sigma_{TT}{}^2(w)$ is the difference between cross sections for scattering transverse photons with the parallel and orthogonal linear polarizations. The validity of this approximation is submitted to the condition of quasireality: $t_1/W^2, t_2/W^2\ll 1$.

A simple calculation shows that the τ_{TT} term is proportional to $\cos 2\varphi^*$, where φ^* is the relative azimuthal angle of the outgoing electrons in the γ-γ center-of-mass frame. In the approximation of quasireal photons, it is shown[10] that $\varphi^*\simeq\varphi$, where φ is the laboratory relative azimuthal angle. Furthermore, for small angles θ_1 and θ_2, W is independent of φ $(W^2=4\omega_1\omega_2)$. After integration over the azimuthal angles of the outgoing electrons, one gets

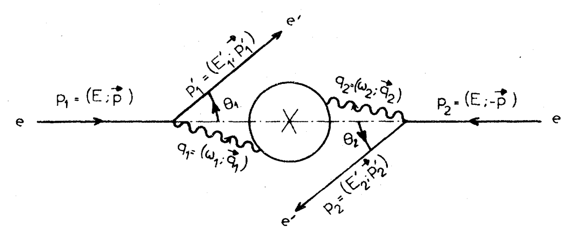

FIG. 4. Kinematical diagram in the lab frame for the process $ee\rightarrow ee\gamma\gamma\rightarrow eeX$. The central blob corresponds to the system X produced.

$$d^4\sigma_0/dE_1'dE_2'd\cos\theta_1\,d\cos\theta_2=\alpha^2W^6E_1'E_2'E^2\sigma_{\gamma\gamma}(W)\left[\left(1-\frac{\omega_1}{E}\right)\left(1-\frac{t_{1\min}}{t_1}\right)+\frac{\omega_1{}^2}{2E^2}\left[\left(1-\frac{\omega_2}{E}\right)\left(1-\frac{t_{2\min}}{t_1}\right)+\frac{\omega_2{}^2}{2E^2}\right]/16\pi^2t_1t_2\right.,$$

where $t_{i\,\min}=m^2\omega_i{}^2/EE_i'$ $(i=1,2)$.

Using $dE_1'dE_2'd\cos\theta_1\,d\cos\theta_2=d\omega_1d\omega_2dt_1dt_2/4E^2E_1'E_2'$, we get $d^2\sigma_0/d\omega_1d\omega_2=\sigma_{\gamma\gamma}(W)N(\omega_1)N(\omega_2)/\omega_1\omega_2$, where

$$N(\omega_i)=(\alpha/\pi)[(1-\omega_i/E+\omega_i{}^2/2E^2)\ln(t_{i\max}/t_{i\min})-(1-\omega_i/E)(1-t_{i\min}/t_{i\max})]\quad(i=1,2)\,.$$

$t_{i\max}$ is defined by the experimental choice of the angle $\theta_{\max}$. In order to obtain the invariant-mass distribution in W, let us set $\omega_1=\omega$ and $\omega_2=W^2/4\omega_1$. Thus,

$$\frac{d\sigma_0}{dw}\simeq\frac{2}{w}\,\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}}N(\omega)N\left(\frac{W^2}{4\omega}\right)\frac{d\omega}{\omega}\,. \tag{3}$$

$\omega_{\min}$ and $\omega_{\max}$ are defined as follows: Let us assume that, due to experimental conditions, ω_1 and ω_2 both lie between aE and bE $(0<a<b<1)$. Using $W^2=4\omega_1\omega_2$ we get the integration limits (Fig. 5):

$$\omega_{\min}=\max(aE,\,W^2/4bE)\text{ and }\omega_{\max}=\min(bE,\,W^2/4aE)\,.$$

Now, we consider the eleastic corrections described in Fig. 2. Because of the symmetry between graphs (1) and (2), it is sufficient to compute the contributions relative to the interference between graph (1) and the main graph (P). Let us call E_μ the electromagnetic current corresponding to the squared box of Fig. 2. Graphs (c) and (d) which contribute to E_μ, can be neglected because of the mass renormalization of the electron. Thus, we consider only graphs (a) and (b) which have been widely dealt with.[8,9,11] Let us give a short survey of the results.

To separate contributions from (a) and (b), let us set $E_\mu=E_\mu^{(a)}+E_\mu^{(b)}$. $E_\mu^{(a)}$ can be written in a general way: $E_\mu^{(a)}=L\gamma_\mu+\Lambda_\mu(p_1,p_1')$ where L is absorbed by the charge renormalization and, thus, can be neglected; the second term is defined by

$$\Lambda_\mu(p_1,p_1')=-\frac{\alpha}{2\pi}\left[(I_F+I_D)\gamma_\mu+\frac{J}{4m}[\gamma_\mu,\gamma_\nu]q_1^\nu\right],$$

where

$$I_F=2+\ln X_0\left[\frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\ln(\rho_1+1)^{1/2}\frac{2+3\rho_1}{2[\rho_1(\rho_1+1)]^{1/2}}\right]$$
$$+\frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\{\Phi(\alpha_1)-\Phi(-\beta_1)+\tfrac{1}{2}[\Phi(-2\beta_1)-\Phi(2\alpha_1)]\},$$

$$I_D=\left(-2+\frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\ln X_0\right)\ln\frac{m}{\lambda},\quad J=\frac{\ln X_0}{2[\rho_1(\rho_1+1)]^{1/2}},$$

$$\alpha_1=[\rho_1(\rho_1+1)]^{1/2}-\rho_1,\quad\beta_1=[\rho_1(\rho_1+1)]^{1/2}+\rho_1,\quad X_0=\frac{\beta_1}{\alpha_1},\quad\Phi(\alpha)=\int_0^\alpha\ln|1-x|\,\frac{dx}{x}\,.$$

Φ is a Spence function, variations of which were given in the nice work of Tsai.[9] Let us remark that I_D is a divergent term which becomes infinite when λ goes to zero.

The general expansion of $E_\mu^{(b)}$ is $E_\mu^{(b)}=(\not{q}_1q_{1\mu}-q_1{}^2\gamma_\mu)(C+\alpha k/\pi)/q_1{}^2$, where C is absorbed by charge renormalization and, thus, can be neglected. Using the Dirac equation, we can see that the $\not{q}_1q_{1\mu}$ term vanishes; thus, $E_\mu^{(b)}$ is reduced to $-\alpha k\gamma_\mu/\pi$ with

$$K=\{\tfrac{5}{3}-1/\rho_1-(1-1/2\rho_1)[(\rho_1+1)/\rho_1]^{1/2}\ln X_0\}/3\,.$$

Finally

$$(E_\mu)_{\text{eff}}=-(\alpha/2\pi)\{(I_F+I_D+2K)\gamma_\mu+J[\gamma_{\mu 1}\gamma_\nu]q_1^\nu/4m\}\,.$$

We then write the "elastic" cross section [for graph (1)] as

$$d\sigma(1)=\delta(1)d\sigma_0+dC(1)\,,$$

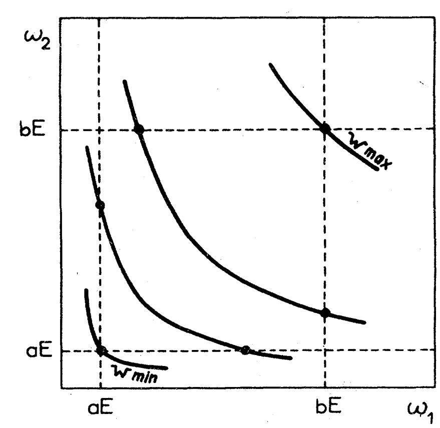

FIG. 5. Variations of ω_2 as a function of ω_1 at constant W. When ω_1 and ω_2 are lying between aE and bE, these curves allow us to determine the limits of variation of ω_1.

where

$$\delta(1) = -\frac{\alpha}{\pi}(I_F + I_D + 2K)\,,$$

$$dC(1) = \frac{16\pi\alpha^5}{E^2 t_1{}^2 t_2{}^2} J\left(\sum X^{\mu\nu} X^{\sigma *}_{\mu}\right) L^{(2)}_{\nu\sigma}\delta^{(4)}(X - q_1 - q_2) \times \frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'}\, d\Gamma_X\,.$$

$\delta(1)$ is a divergent term, and we will see below that this divergence is cancelled by an opposite divergence coming from the soft-photon contribution. However, we can obtain the invariant-mass distribution $(dC/dW)_I$ by multiplying $dC(1)$ by 2 to take into account the contribution of graph (2), and by integrating over phase space in the same way as in the calculation of the uncorrected cross section:

$$\left(\frac{dC}{dW}\right)_I = -\frac{2}{W}\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}} M(\omega) N\left(\frac{W^2}{4\omega}\right)\frac{d\omega}{\omega}\,, \qquad (4)$$

where

$$M(\omega) = \frac{\alpha}{\pi}\int_{t_{1\min}}^{t_{1\max}} \frac{\alpha J}{\pi m^2}\left(1 - \frac{\omega}{E}\right)\frac{t_{1\min}}{t_1}\, dt_1\,.$$

III. INELASTIC CROSS SECTION

Now we consider the contaminating process $ee \to eeX\gamma$ characterized by the emission of a real photon with its energy lying between 0 and ΔE, where ΔE is the finite resolution of the tagging system of the scattered electrons. This kind of process is described to order α^5 by the graphs (4) and (5) of Fig. 3. To avoid infrared divergences, we ascribe, in a first step, a mass λ to the radiated photon, where λ is as small as we like. Thus, for the convergent terms, we will take $\lambda = 0$; on the other hand, we conserve λ in the divergent terms.

Because of the symmetry of the graphs (4) and (5) and because we neglect their interference, it is sufficient to compute $d\sigma(4)$ given by

$$d\sigma(4) = \frac{16\alpha^5}{E^2 q_1{}^4 t_2{}^2}\mathfrak{L}^{(1)}_{\mu\rho}\left(\sum X^{\mu\nu} X^{\rho\sigma *}\right) L^{(2)}_{\nu\sigma} \times \delta^{(4)}(X - q_1 - q_2)\frac{d^3k_0}{2k_0}\frac{d^3p_1'}{2E_1'}\frac{d^3p_2'}{2E_2'}\, d\Gamma_X\,,$$

where $\mathfrak{L}^{(1)}_{\mu\rho}$ is the electromagnetic tensor described in Fig. 6 (semivirtual Compton effect) and expressed as $\mathfrak{L}^{(1)}_{\mu\rho} = (-g^{\alpha\beta} + k^\alpha k^\beta/\lambda^2)\sum R_{\alpha\mu} R^*_{\beta\rho}$, with

$$\begin{aligned} R_{\alpha\mu} &= \bar u(p_1')\left(\gamma_\alpha \frac{1}{\not p_1' + \not k - m}\gamma_\mu + \gamma_\mu \frac{1}{\not p_1 - \not k - m}\gamma_\alpha\right) u(p_1) \\ &= \bar u(p_1')\ \gamma_\alpha \frac{1}{\not p_1' + \not k - m}\gamma_\mu + \gamma_\mu \frac{1}{\not p_1 - \not k - m}\gamma_\alpha\ u(p_1) \\ &= \bar u(p_1')\left[2\left(\frac{p_{1\alpha}'}{D_1'} - \frac{p_{1\alpha}}{D_1}\right)\gamma_\mu + \frac{\gamma_\alpha \not k \gamma_\mu}{D_1'} + \frac{\gamma_\mu \not k \gamma_\alpha}{D_1}\right] u(p_1)\,, \end{aligned}$$

where $D_1 = 2kp_1 - \lambda^2$ and $D_1' = 2kp_1' + \lambda^2$.

A rather lengthy calculation, taking into account all the terms of $R_{\alpha\mu}$, then leads to following formula (neglecting λ, as we said, in the nondivergent terms):

$$\begin{aligned} \mathfrak{L}^{(1)}_{\mu\rho} = {} & \left[2\left(\frac{1}{D_1'} - \frac{1}{D_1}\right) + \frac{4(t_1 + 2m^2)}{D_1 D_1'} - 4m^2\left(\frac{1}{D_1{}^2} + \frac{1}{D_1'{}^2}\right)\right] L^{(1)}_{\mu\rho} \\ & + \left[(t_1 - 2m^2)\left(\frac{1}{D_1'} - \frac{1}{D_1}\right) - \left(\frac{D_1}{D_1'} + \frac{D_1'}{D_1}\right) - 2m^2\left(\frac{D_1'}{D_1{}^2} - \frac{D_1}{D_1'{}^2}\right)\right] g_{\mu\rho} - \frac{8m^2}{D_1 D_1'} k_\mu k_\rho - \frac{4}{D_1} p_{1\mu} p_{1\rho} + \frac{4}{D_1'} p_{1\mu}' p_{1\rho}' \\ & + 2\left(\frac{t_1 + 2m^2}{D_1 D_1'} - \frac{2m^2}{D_1'{}^2} + \frac{1}{D_1'}\right)(k_\mu p_{1\rho} + k_\rho p_{1\mu}) + 2\left(-\frac{t_1 + 2m^2}{D_1 D_1'} + \frac{2m^2}{D_1{}^2} + \frac{1}{D_1}\right)(k_\mu p_{1\rho}' + k_\rho p_{1\mu}')\,, \end{aligned} \qquad (5)$$

or, taking account of momentum conservation,

$$\begin{aligned} \mathfrak{L}^{(1)}_{\mu\rho} = {} & \frac{8m^2}{D_1 D_1'} L^{(1)}_{\mu\rho} + \left[-\frac{2t_1{}^2}{D_1 D_1'} + 2(m^2 - t_1)\left(\frac{1}{D_1'} - \frac{1}{D_1}\right) - \left(\frac{D_1}{D_1'} + \frac{D_1'}{D_1}\right) - 2m^2\left(\frac{D_1'}{D_1{}^2} - \frac{D_1}{D_1'{}^2}\right) + 2m^2 t_1\left(\frac{1}{D_1{}^2} + \frac{1}{D_1'{}^2}\right)\right] g_{\mu\rho} \\ & + 4\left(\frac{1}{D_1'} - \frac{1}{D_1} - \frac{2m^2}{D_1'{}^2} + \frac{t_1}{D_1 D_1'}\right) p_{1\mu} p_{1\rho} + 4\left(\frac{1}{D_1'} - \frac{1}{D_1} - \frac{2m^2}{D_1{}^2} + \frac{t_1}{D_1 D_1'}\right) p_{1\mu}' p_{1\rho}' \\ & + 2\left(\frac{2m^2 - t_1}{D_1 D_1'} + \frac{2m^2}{D_1'{}^2} - \frac{1}{D_1'}\right)(q_{1\mu} p_{1\rho} + q_{1\rho} p_{1\mu}) + 2\left(\frac{t_1 - 2m^2}{D_1 D_1'} - \frac{2m^2}{D_1{}^2} - \frac{1}{D_1}\right)(q_{1\mu} p_{1\rho}' + q_{1\rho} p_{1\mu}') - \frac{8m^2}{D_1 D_1'} q_{1\mu} q_{1\rho}\,. \end{aligned} \qquad (6)$$

It becomes necessary to be very careful with notations. We shall continue using notations like t_1 and ω_1, but with restricted meanings: $t_1 = -(p_1 - p_1')^2$ and $\omega_1 = E - E_1'$. In fact, we now have $p_1 - p_1' = q_1 + k$ instead of $p_1 - p_1' = q_1$. For graph (4), ω_2 and t_2 are quantities characterizing the exchanged photon q_2; the same is no longer true for ω_1 and t_1 with respect to q_1. Yet we can still write

$$d^3k\, d^3p_1' d^3p_2'/8k_0 E_1' E_2' = (k_0{}^2 - \lambda^2)^{1/2}\, dk_0\, d\cos\theta_\gamma\, d\varphi_\gamma\, d\omega_1\, d\omega_2\, dt_1\, dt_2\, d\varphi_1\, d\varphi_2/32E^2\,,$$

so that, after integration over φ_1 and the phase space of the central system,

$$d^8\sigma(4) \simeq (\alpha^5 |\vec{k}|/(2\pi)^3 q_1{}^4 t_2{}^2 E^4)\mathcal{L}^{(1)}_{\mu\rho} W^{\mu\nu\rho\sigma} L^{(2)}_{\nu\sigma} d\omega_1 d\omega_2 dt_1 dt_2 d\varphi dk_0 d\cos\theta_\gamma d\varphi_\gamma .$$

To be able to continue the calculations and eliminate divergent terms at the limit $\lambda \to 0$, we divide the energy range of the radiated photon into two parts: (1) the range $\lambda \leq k_0 \leq \Lambda$ corresponding to soft photons and which contains infrared divergences when λ goes to zero; (2) the range $\Lambda \leq k_0 \leq \Delta E$ corresponding to hard photons. Λ is a cutoff which separates the two ranges; its value will be determined later on. To begin, we consider the soft-photons case.

Assuming Λ to be small enough, we make the following approximations. We shall first neglect k_μ in the $\delta^{(4)}(X-q_1-q_2)$ distribution, which means that the tensor $W^{\mu\nu\rho\sigma}$ is identical to the one calculated for the uncorrected cross section; the terms in k_μ will also be neglected in (5) which implies, accounting for conservation of the electromagnetic current, that

$$\mathcal{L}^{\text{soft}}_{\mu\rho}(1) \simeq A L^{(1)}_{\mu\rho} + B g_{\mu\rho} ,$$

where

$$A = \frac{4}{D_1 D_1'}(t_1 + 2m^2) - 4m^2\left(\frac{1}{D_1^2} + \frac{1}{D_1'^2}\right) \text{ and } B = 2m^2\left(\frac{1}{D_1'} - \frac{1}{D_1}\right) - \left(\frac{D_1}{D_1'} + \frac{D_1'}{D_1}\right) - 2m^2\left(\frac{D_1'}{D_1^2} - \frac{D_1}{D_1'^2}\right).$$

We shall also replace $q_1{}^4$ by $t_1{}^2$ in the denominator of the propagator of the exchanged photon q_1. Setting

$$(\mathcal{A}, \mathcal{B}) = \tfrac{1}{2}\int_\lambda^\Lambda (k_0{}^2 - \lambda^2)^{1/2} dk_0 \int_{-1}^{+1} d\cos\theta_\gamma \int_0^{2\pi} d\varphi_\gamma (A, B) ,$$

we get

$$\delta(4) = \frac{\alpha}{2\pi^2}\mathcal{A} , \quad dC(4) = \frac{2\alpha^5}{(2\pi)^3 t_1{}^2 t_2{}^2 E^4}\mathcal{B} g_{\mu\rho} W^{\mu\nu\rho\sigma} L^{(2)}_{\nu\sigma} d\omega_1 d\omega_2 dt_1 dt_2 d\varphi .$$

The calculation of $\mathcal{A}$ and $\mathcal{B}$ is very tedious and is exposed in Appendix A. It results in the following expressions:

$$\mathcal{A} = \mathcal{A}_F + \mathcal{A}_D ,$$

where

$$\begin{aligned}\mathcal{A}_F = 2\pi \ln\frac{EE_1'}{m^2} - \pi\frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\Bigg\{&-2\ln\frac{E}{m}\ln X_0 + \ln X_0 \ln[4(\rho_1+1)]^{1/2} - \ln X_0 \ln\left(1 - \frac{\omega_1}{E}\right)\left(1 - \frac{t_{1\min}}{t_1}\right)\\ &+\Phi\left(\frac{X_0}{1+X_0}\right) - \Phi\left(\frac{1}{1+X_0}\right) - \Phi\left(\frac{\omega_1/E}{\omega_1/E - 2\alpha_1}\right)\\ &+\Phi\left(\frac{\omega_1/E}{X_0(\omega_1/E - 2\alpha_1)}\right) - \Phi\left(\frac{X_0\omega_1/E}{2\beta_1 + \omega_1/E}\right) + \Phi\left(\frac{\omega_1/E}{2\beta_1 + \omega_1/E}\right)\Bigg\},\end{aligned}$$

$$\mathcal{A}_D = 2\pi\left(-2 + \frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\ln X_0\right)\ln\frac{\Lambda}{\lambda} ,$$

and

$$\begin{aligned}\mathcal{B} = m^2\pi\Lambda\Bigg\{&\frac{2}{E_1'}\ln\frac{2E_1'}{m} - \frac{2}{E}\ln\frac{2E}{m} + (1+2\rho_1)\left(\frac{1}{E_1'} - \frac{1}{E}\right) + \left(\frac{E_1'}{E^2} - \frac{E}{E_1'^2}\right)\\ &+\left[1 - \frac{m^2}{EE_1'}(1+2\rho_1)\right]\left[\frac{E}{E_1'^2}\ln\frac{2E_1'}{m} - \frac{E_1'}{E^2}\ln\frac{2E}{m}\right]\Bigg\}\\ -\pi\Lambda^2\Bigg\{&m^2\left(\frac{E_1'}{E^3} + \frac{E}{E_1'^3}\right) + \left[1 - \frac{m^2}{EE_1'}(1+2\rho_1)\right]\left(\frac{E_1'}{E} + \frac{E}{E_1'}\right) + \frac{m^2}{E_1'^2}\left(1 + 2\rho_1 - \frac{E}{E_1'}\right)\ln\frac{2E_1'}{m}\\ &+\frac{m^2}{E^2}\left(1 + 2\rho_1 - \frac{E_1'}{E}\right)\ln\frac{2E}{m}\Bigg\}.\end{aligned}$$

The comparison of the expressions of $\delta(1)$ and $\delta(4)$ shows that the infrared divergences can be eliminated, so that we get

$$\delta(1) + \delta(4) = \frac{\alpha}{\pi}\left(-2 + \frac{1+2\rho_1}{[\rho_1(\rho_1+1)]^{1/2}}\ln X_0\right)\ln\frac{\Lambda}{m} + \frac{\alpha}{2\pi^2}\mathcal{A}_F - \frac{\alpha}{\pi}(I_F + 2K) ,$$

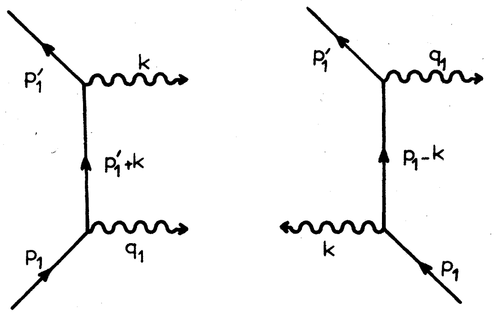

FIG. 6. Feynman graphs entering in the semivirtual Compton effect.

which is a finite quantity independent of λ.

Considering that the relation $W^2=4\omega_1\omega_2$ is still valid in the case of soft photons emitted, we easily get [after multiplication by 2 to take into account graph (5)] the contribution coming from $\delta(1)+\delta(2)+\delta(4)+\delta(5)$:

$$\left(\frac{dC}{dW}\right)_{\mathrm{II}}=\frac{2}{W}\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}}Q(\omega)N\left(\frac{W^2}{4\omega}\right)\frac{d\omega}{\omega}, \qquad (7)$$

with

$$Q(\omega)=\frac{2\alpha}{\pi}\int_{t_{\min}}^{t_{\max}}[\delta(1)+\delta(4)] \times\left[\left(1-\frac{\omega}{E}\right)\left(1-\frac{t_{1\min}}{t_1}\right)+\frac{\omega^2}{2E^2}\right]\frac{dt_1}{t_1},$$

and where $\omega_{\min}, \omega_{\max}, t_{1\min}, t_{1\max}$ have been defined in Sec. II.

We can also get the invariant mass distribution coming from $dC(4)$ and $dC(5)$. It is easy to show that

$$g_{\mu\rho}W^{\mu\nu\rho\sigma}L^{(2)}_{\nu\sigma}=-\frac{t_2}{\omega_2^2}E^2\left[\left(1-\frac{\omega_2}{E}\right)\left(1-\frac{t_{2\min}}{t_2}\right)+\frac{\omega_2^2}{2E^2}\right] \times\frac{W^2}{4\pi^2\alpha^2}\sigma_{\gamma\gamma}(W),$$

wherefrom we deduce:

$$\left(\frac{dC}{dW}\right)_{\mathrm{III}}=-\frac{2}{W}\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}}P(\omega)N\left(\frac{W^2}{4\omega}\right)\frac{d\omega}{\omega}, \qquad (8)$$

where

$$P(\omega)=\frac{\alpha}{\pi}\int_{t_{1\min}}^{t_{1\max}}\frac{\alpha\mathcal{B}}{\pi^2m^2}\left(1-\frac{\omega}{E}\right)\frac{t_{1\min}}{t_1^2}\,dt_1 .$$

Finally, the contribution of elastic plus inelastic corrections of soft photons is

$$\frac{d\sigma}{dW}(\text{elastic}+\text{soft})=-\frac{2}{W}\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}}[M(\omega)-Q(\omega)+P(\omega)]N\left(\frac{W^2}{4\omega}\right)\frac{d\omega}{\omega}. \qquad (9)$$

Let us compute now the contribution to radiative corrections due to the emission of a hard photon. We shall use the expression of $d\sigma(4)$, where we replace $\mathcal{L}^{(1)}_{\mu\rho}$ by (6) after neglecting terms in $q_{1\mu}$, $q_{1\rho}$ (because of current conservation). However, in contradistinction with the case of soft photons, here we are not allowed to neglect k_μ, so that $q_1^2\simeq-t_1-2k_0\omega_1(1-\cos\theta_\gamma)$ and $W^2\simeq4\omega_1\omega_2-2k_0[\omega_1+\omega_2+(\omega_2-\omega_1)\cos\theta_\gamma]$.

In $W^{\mu\nu\rho\sigma}$ as well, we replace q_1 by $p_1-p_1'-k$; our calculation thus includes more terms and becomes more complicated; using here again the quasireal-photon approximation, we are led to

$$\mathcal{L}^{(1)}_{\mu\rho}W^{\mu\nu\rho\sigma}L^{(2)}_{\nu\sigma}=F_1(t_1,t_2,\omega_1,\omega_2,\varphi,k_0,\cos\theta_\gamma,\varphi_\gamma)W_{TT}(W)+F_2(t_1,t_2,\omega_1,\omega_2,\varphi,k_0,\cos\theta_\gamma,\varphi_\gamma)\tau_{TT}(W),$$

with

$$F_i=\sum_{j=1}^{4}c_{ij}H_j,$$

$$H_1=\frac{8m^2}{D_1D_1'},\quad H_3=4\left(\frac{1}{D_1'}-\frac{1}{D_1}\right)-\frac{8m^2}{D_1'^2}+\frac{4t_1}{D_1D_1'},\quad H_4=4\left(\frac{1}{D_1'}-\frac{1}{D_1}\right)-\frac{8m^2}{D_1^2}+\frac{4t_1}{D_1D_1'},$$

$$H_2=-\frac{2t_1^2}{D_1D_1'}+2(m^2-t_1)\left(\frac{1}{D_1'}-\frac{1}{D_1}\right)-\left(\frac{D_1}{D_1'}+\frac{D_1'}{D_1}\right)-2m^2\left(\frac{D_1'}{D_1^2}-\frac{D_1}{D_1'^2}\right)+2m^2t_1\left(\frac{1}{D_1^2}+\frac{1}{D_1'^2}\right),$$

$$(c_{11},c_{12},c_{13},c_{14})=(L^{(1)}_{\mu\rho},g_{\mu\rho},p_{1\mu}p_{1\rho},p_{1\mu}'p_{1\rho}')R^{\mu\rho}R^{\nu\sigma}L^{(2)}_{\nu\sigma},$$

$$(c_{21},c_{22},c_{23},c_{24})=(L^{(1)}_{\mu\rho},g_{\mu\rho},p_{1\mu}p_{1\rho},p_{1\mu}'p_{1\rho}')R^{\mu\nu}R^{\rho\sigma}L^{(2)}_{\nu\sigma}-\tfrac{1}{2}(c_{11},c_{12},c_{13},c_{14}).$$

In order to compute the cross section more easily, we shall take, both in F_1 and F_2, $t_1=t_{1\min}$, $t_2=t_{2\min}$, thus suppressing the dependence of those factors on φ and φ_γ. Defining

$$\mathfrak{F}_i(W,\omega_1,k_0,\cos\theta_\gamma)=F_i(t_{1\min},t_{2\min},\omega_1,0,k_0,\cos\theta_\gamma,0)\,,$$

one thus obtains

$$\frac{d\sigma}{dW}(\text{hard})\simeq\frac{\alpha^5W}{\pi E^4}\int_\Lambda^{\Delta E}k_0dk_0\int_{-1}^{+1}d\cos\theta_\gamma\int_{\omega_{\min}}^{\omega_{\max}}d\omega_1\,\frac{\mathfrak{F}_1W_{TT}+\mathfrak{F}_2\tau_{TT}}{2\omega_1-k_0(1+\cos\theta_\gamma)}\int_{t_{1\min}}^{t_{1\max}}\frac{dt_1}{q_1^4}\int_{t_{2\min}}^{t_{2\max}}\frac{dt_2}{t_2^2}\,,$$

thus

$$\frac{d\sigma}{dW}(\text{hard})=\left(\frac{dC}{dW}\right)_{\text{IV}}+\left(\frac{dC}{dW}\right)_{\text{V}}\,.$$

with

$$\left(\frac{dC}{dW}\right)_{\text{IV}}=\frac{\alpha^3W^3}{4(2\pi)^3E^4}\int_\Lambda^{\Delta E}k_0dk_0\int_{-1}^{+1}d\cos\theta_\gamma$$
$$\times\int_{\omega_{\min}}^{\omega_{\max}}\frac{(t_{1\max}-t_{1\min})(t_{2\max}-t_{2\min})\mathfrak{F}_1\sigma_{\gamma\gamma}d\omega_1}{t_{2\min}t_{2\max}[2\omega_1-k_0(1+\cos\theta_\gamma)][t_{1\min}+2k_0\omega_1(1-\cos\theta_\gamma)][t_{1\max}+2k_0\omega_1(1-\cos\theta_\gamma)]}\,,$$

$$\left(\frac{dC}{dW}\right)_{\text{V}}=\text{same with }\mathfrak{F}_1\sigma_{\gamma\gamma}\to\mathfrak{F}_2\sigma^\tau_{TT}\,,$$

where the limits of integration over ω_1 are now

$$\omega_{\min}=\max\left(aE,\,\frac{W^2+2bEk_0(1+\cos\theta_\gamma)}{4bE-2k_0(1-\cos\theta_\gamma)}\right),\quad\omega_{\max}=\min\left(bE,\,\frac{W^2+2aEk_0(1+\cos\theta_\gamma)}{4aE-2k_0(1-\cos\theta_\gamma)}\right).$$

IV. NUMERICAL RESULTS AND CONCLUSIONS

In order to separate soft from hard photons, we took the cutoff Λ equal to the electron mass. This choice is convenient since, for $\Lambda<m$ the radiative corrections keep the same value. In addition, it has the advantage that the term proportional to $\ln(\Lambda/m)$ vanishes in $(dC/dW)_{\text{II}}$.

We evaluated the radiative corrections for various possible values of W, chosen according to the conditions of the experiment at DCI (Orsay)[7]: $a=0.2$, $b=0.5$, $E=0.8$ GeV, $\Delta E=14$ MeV, $\theta_{\max}=10$ mrad, for the processes $ee\to eee^+e^-$, $ee\mu^+\mu^-$, $ee\pi^+\pi^-$. The corresponding elementary cross sections $\sigma_{\gamma\gamma}(W)$ and $\sigma^\tau_{TT}(W)$ for these processes are given in Appendix B. Tables I–III show the numerical results obtained, and Fig. 7(a) and 7(b) the curves computed for $d\sigma_0/dW$ and $(d\sigma/dW)$.

We notice that, in all cases, $(dC/dW)_{\text{I}}$ and $(dC/dW)_{\text{III}}$ are negligible with respect to $(dC/dW)_{\text{II}}$. The latter quantity thus represents the whole of elastic corrections plus corrections due to soft photons. With the cutoff chosen, one has practically

$$\frac{d\sigma}{dW}(\text{soft}+\text{elastic})\simeq\frac{4\alpha^2}{\pi^2W}\sigma_{\gamma\gamma}(W)\int_{\omega_{\min}}^{\omega_{\max}}N\!\left(\frac{W^2}{4\omega}\right)\int_{t_{1\min}}^{t_{1\max}}\left(1-\frac{\omega}{E}\right)\left(1-\frac{t_{1\min}}{t_1}\right)+\frac{\omega^2}{2E^2}\left(\frac{\alpha_F}{2\pi}-I_F-2K\right)\frac{dt_1}{t_1}d\omega\,.$$

As for the inelastic corrections due to hard photon emission, we notice they are generally much smaller than those computed above. On the other hand, they strongly depend on the process considered, i.e., on

TABLE I. Numerical values of $d\sigma_0/dW$, $(dC/dW)_{\text{I}}$, $(dC/dW)_{\text{II}}$, $(dC/dW)_{\text{III}}$, $(dC/dW)_{\text{IV}}$, $(dC/dW)_{\text{V}}$, $(d\sigma/dW)_{\text{corr}}$ and $\%=100[(d\sigma/dW)_{\text{corr}}-d\sigma_0/dW]/(d\sigma_0/dW)$ as functions of W for $ee\to eee^+e^-$.

W (GeV)	$\frac{d\sigma_0}{dW}$	$\left(\frac{dC}{dW}\right)_{\text{I}}$	$\left(\frac{dC}{dW}\right)_{\text{II}}$	$\left(\frac{dC}{dW}\right)_{\text{III}}$ (10^{-36} cm^2/GeV)	$\left(\frac{dC}{dW}\right)_{\text{IV}}$	$\left(\frac{dC}{dW}\right)_{\text{V}}$	$\left(\frac{d\sigma}{dW}\right)_{\text{corr}}$	%
0.344	3.95×10^3	−0.37	−392.07	3.42	55.81	-0.33×10^{-4}	3.62×10^3	−8.43
0.440	6.31×10^3	−1.03	−650.50	4.95	105.08	-0.48×10^{-4}	5.76×10^3	−8.59
0.536	3.34×10^3	−0.86	−341.81	2.67	67.67	-0.22×10^{-4}	3.07×10^3	−8.15
0.632	0.95×10^3	−0.32	−96.60	0.78	23.15	-0.05×10^{-4}	0.88×10^3	−7.68
0.728	0.20×10^3	−0.09	−20.25	0.17	5.55	-0.01×10^{-4}	0.18×10^3	−7.46

TABLE II. Numerical values of $d\sigma_0/dW$, $(dC/dW)_{\rm I}$, $(dC/dW)_{\rm II}$, $(dC/dW)_{\rm III}$, $(dC/dW)_{\rm IV}$, $(dC/dW)_{\rm V}$, $(d\sigma/dW)_{\rm corr}$ and $\% = 100[(d\sigma/dW)_{\rm corr} - d\sigma_0/dW]/(d\sigma_0/dW)$ as functions of W for $ee \to ee\mu^+\mu^-$.

W (GeV)	$\frac{d\sigma_0}{dW}$	$\left(\frac{dC}{dW}\right)_{\rm I}$	$\left(\frac{dC}{dW}\right)_{\rm II}$	$\left(\frac{dC}{dW}\right)_{\rm III}$ $(10^{-36}$ cm^2/GeV)	$\left(\frac{dC}{dW}\right)_{\rm IV}$	$\left(\frac{dC}{dW}\right)_{\rm V}$	$\left(\frac{d\sigma}{dW}\right)_{\rm corr}$	%
0.344	0.56×10^3	−0.05	−55.38	0.48	7.88	−1.68	0.51×10^3	−8.73
0.440	1.11×10^3	−0.18	−114.19	0.87	18.45	−2.44	1.01×10^3	−8.81
0.536	0.66×10^3	−0.17	−67.55	0.53	13.37	−1.13	0.61×10^3	−8.32
0.632	0.20×10^3	−0.07	−20.70	0.17	4.96	−0.24	0.19×10^3	−7.79
0.728	0.04×10^3	−0.02	−4.61	0.04	1.26	−0.04	0.04×10^3	−7.55

the system X produced. That fact is due to the differences between the expressions of $\sigma^{\tau}_{TT}(W)$ obtained for various types of particle pairs: electrons, muons, or (pointlike) pions. In the case of lepton pair production, one is justified, as is shown by the numerical results of Tables I and II, in neglecting $(dC/dW)_{\rm V}$ with respect to $(dC/dW)_{\rm IV}$. On the contrary, for pion pair production (see Table III), one has to consider both contributions $(dC/dW)_{\rm IV}$ and $(dC/dW)_{\rm V}$.

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APPENDIX A

In order to calculate $\mathcal{A}$ and $\mathcal{B}$, one must evaluate the integrals

$$I_1=\tfrac{1}{2}\int_\lambda^\Lambda dk_0(k_0^2-\lambda^2)^{1/2}\int\int\frac{d\Omega_\gamma}{D_1^2}\,,\quad I_1'=\tfrac{1}{2}\int_\lambda^\Lambda dk_0(k_0^2-\lambda)^{1/2}\int\int\frac{d\Omega_\gamma}{D_1'^2}\,,$$

$$I_2=\tfrac{1}{2}\int_\lambda^\Lambda dk_0(k_0^2-\lambda^2)^{1/2}\int\int\frac{d\Omega_\gamma}{D_1D_1'}\,,\quad I_3=\tfrac{1}{2}\int_\lambda^\Lambda k_0dk_0\int\int\left(\frac{1}{D_1'}-\frac{1}{D_1}\right)d\Omega_\gamma\,,$$

$$I_4=\tfrac{1}{2}\int_\lambda^\Lambda k_0dk_0\int\int\frac{D_1}{D_1'}+\frac{D_1'}{D_1}\,d\Omega_\gamma\,,\quad I_5=\tfrac{1}{2}\int_\lambda^\Lambda k_0dk_0\int\int\left(\frac{D_1'}{D_1^2}-\frac{D_1}{D_1'^2}\right)d\Omega_\gamma\,.$$

I_3, I_4, I_5 are nondivergent integrals at the limit $\lambda\to0$. They can be calculated without any difficulty by taking $\vec{p}_1$ (resp. $\vec{p}_1'$) as the polar axis. They lead to following expression of $\mathcal{B}$:

$$\mathcal{B}=2m^2I_3-I_4-m^2I_5\,.$$

Choosing $\vec{p}_1$ ($\vec{p}_1'$) as the polar axis in the calculation of I_1 (I_1'), one gets

$$I_1=\tfrac{1}{2}\pi K_\lambda(E,|\vec{p}_1|,1),\quad I_1'=\tfrac{1}{2}\pi K_\lambda(E_1',|\vec{p}_1'|,-1)\,,$$

TABLE III. Numerical values of $d\sigma_0/dW$, $(dC/dW)_{\rm I}$, $(dC/dW)_{\rm II}$, $(dC/dW)_{\rm III}$, $(dC/dW)_{\rm IV}$, $(dC/dW)_{\rm V}$, $(d\sigma/dW)_{\rm corr}$ and $\% = 100[(d\sigma/dW)_{\rm corr} - d\sigma_0/dW]/(d\sigma_0/dW)$ as functions of W for $ee \to ee\pi^+\pi^-$.

W (GeV)	$\frac{d\sigma_0}{dW}$	$\left(\frac{dC}{dW}\right)_{\rm I}$	$\left(\frac{dC}{dW}\right)_{\rm II}$	$\left(\frac{dC}{dW}\right)_{\rm III}$ $(10^{-37}$ cm^2/GeV)	$\left(\frac{dC}{dW}\right)_{\rm IV}$	$\left(\frac{dC}{dW}\right)_{\rm V}$	$\left(\frac{d\sigma}{dW}\right)_{\rm corr}$	%
0.344	6.21×10^2	−0.05	−61.55	0.53	8.76	12.54	5.81×10^2	−6.41
0.440	10.64×10^2	−0.17	−109.79	0.84	17.74	21.15	9.94×10^2	−6.60
0.536	6.32×10^2	−0.16	−64.69	0.51	12.81	10.10	5.91×10^2	−6.56
0.632	2.00×10^2	−0.07	−20.34	0.16	4.87	2.20	1.87×10^2	−6.56
0.728	0.45×10^2	−0.02	−4.63	0.04	1.27	0.37	0.42×10^2	−6.64

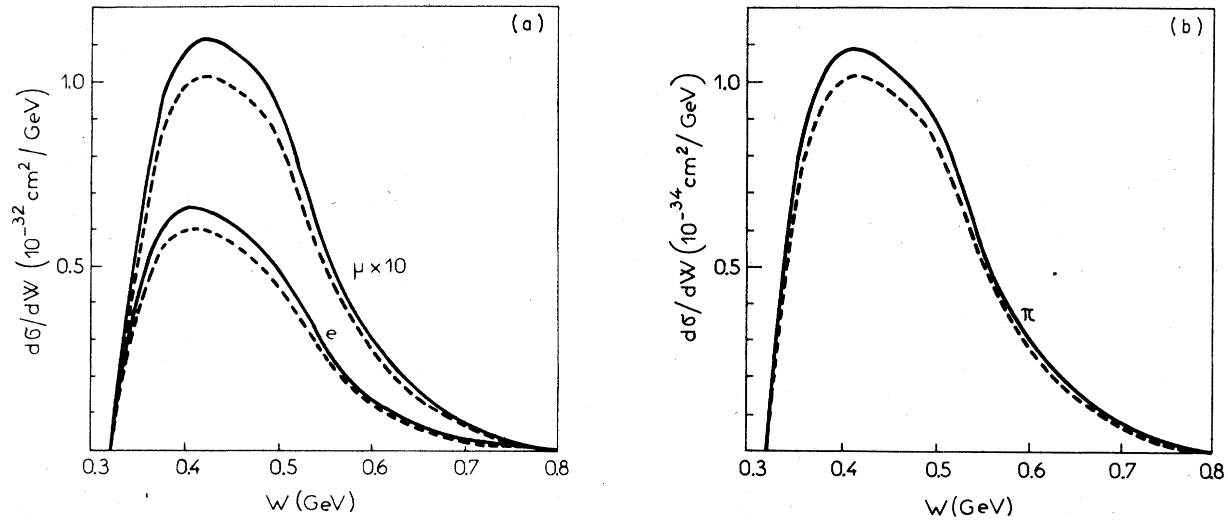

FIG. 7. Variations of $d\sigma_0/dW$ (solid curve) and $(d\sigma/dW)_{\rm corr}$ (dashed curve) as functions of the invariant mass W, expressed in 10^{-32} cm^2/GeV for e^+e^- production and in 10^{-33} cm^2/GeV for $\mu^+\mu^-$ production (a) and in 10^{-34} cm^2/GeV for $\pi^+\pi^-$ production (b).

where

$$K_\lambda(\alpha,\beta,\gamma)=\int_\lambda^\Lambda \frac{(k_0^2-\lambda^2)^{1/2}dk_0}{(k_0\alpha-\frac{1}{2}\lambda^2\gamma)^2-\beta^2(k_0^2-\lambda^2)}\,.$$

Using the identity

$$\frac{1}{AB}=\int_0^1[Ax+B(1-x)]^{-2}\,dx\,,$$

and defining $p_x=xp_1'+(1-x)p_1$, I_2 can be expressed in a similar form:

$$I_2=\tfrac{1}{2}\pi\int_0^1 K_\lambda(E_x,|\vec{p}_x|,-2x+1)dx\,.$$

The calculation of $K_\lambda(\alpha,\beta,\gamma)$, although lengthy and tedious, does not involve any major difficulty and leads, at the limit $\lambda\to 0$, to

$$K(\alpha,\beta,\gamma)=\lim_{\lambda\to0}K_\lambda(\alpha,\beta,\gamma)=\frac{1}{\alpha^2-\beta^2}\left(\ln 2\frac{\Lambda}{\lambda}-\frac{\alpha}{2\beta}\ln\frac{(\alpha+\beta)^2}{\alpha^2-\beta^2}\right)\equiv K(\alpha,\beta,0)\,.$$

Using the extreme-relativistic approximation, one gets

$$I_1+I_1'=\frac{\pi}{2m^2}\ln\frac{\Lambda^2m^2}{\lambda^2EE_1'}\,,\qquad I_2=\frac{\pi}{2}\left(\ln\frac{\Lambda}{\lambda}\int_0^1\frac{dx}{p_x^2}-\frac{1}{2}\int_0^1\frac{1}{p_x^2}\ln\frac{E_x^2}{p_x^2}dx\right)\,.$$

One can show that

$$\int_0^1\frac{dx}{p_x^2}=\frac{1}{2m^2[\rho_1(\rho_1+1)]^{1/2}}\ln X_0\,,$$

$$\int_0^1\frac{1}{p_x^2}\ln\frac{E_x^2}{p_x^2}dx=\frac{1}{2[\rho_1(\rho_1+1)]^{1/2}}\left\{\left(2\ln\frac{E}{m}+\ln\left[\left(1-\frac{\omega_1}{E}\right)\left(1-\frac{t_{1\rm min}}{t_1}\right)\right]-\ln[4(\rho_1+1)]^{1/2}\right)\ln X_0\right.$$
$$+\Phi\left(\frac{\omega_1/E}{\omega_1/E-2\alpha_1}\right)-\Phi\left(\frac{\omega_1/E}{X_0(\omega_1/E-2\alpha_1)}\right)-\Phi\left(\frac{X_0\omega_1/E}{\omega_1/E+2\beta_1}\right)$$
$$\left.-\Phi\left(\frac{\omega_1/E}{\omega_1/E+2\beta_1}\right)-\Phi\left(\frac{X_0}{1+X_0}\right)+\Phi\left(\frac{1}{1+X_0}\right)\right\}\,.$$

One then gets $\mathfrak{a}$ in the form $\mathfrak{a}=-4m^2(I_1+I_1')+8m^2(1+2\rho_1)I_2$.

APPENDIX B

The elementary cross sections for various photon-photon collision processes are as follows.

(a) $\gamma\gamma \to e^+e^-$ or $\mu^+\mu^-$:

$$\sigma_{\gamma\gamma}(W) = \frac{4\pi\alpha^2}{W^2}\left[\left(1+\frac{4m_0^2}{W^2}-\frac{8m_0^4}{W^4}\right)L-\left(1+\frac{4m_0^2}{W^2}\right)\frac{\Delta t}{W^2}\right]$$

$$\sigma^{\tau}_{TT}(W) = -\frac{16\pi\alpha^2 m_0^2}{W^6}(\Delta t + 2m_0^2 L)\ .$$

(b) $\gamma\gamma \to \pi^+\pi^-$ (Born terms, pointlike pions):

$$\sigma_{\gamma\gamma}(W) = \frac{2\pi\alpha^2}{W^2}\left[\frac{\Delta t}{W^2}\left(1+\frac{4m_0^2}{W^2}\right)-\frac{2m_0^2}{W^2}\left(2-\frac{4m_0^2}{W^2}\right)L\right],$$

$$\sigma^{\tau}_{TT}(W) = \frac{8\pi\alpha^2 m_0^2}{W^6}(\Delta t + 2m_0^2 L)\ ,$$

where m_0 is the mass of the particles produced, and

$$\Delta t = W^2\left(1-\frac{4m_0^2}{W^2}\right)^{1/2},\quad L = 2\ln\frac{W}{2m_0}\left(1+\frac{\Delta t}{W^2}\right).$$

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