Semiquantum approximation for the Reggeon field theory on a lattice

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The Reggeon field theory on a lattice is studied for values of the intercept α_0 near the critical point by using a semiquantum approximation to the single site and the linear chain Hamiltonian. Results are in quantitative agreement with those obtained by other methods and lend support to previous extrapolation procedures.

I. INTRODUCTION

One of the most convenient ways to study the properties of the Reggeon field theory (RFT) has been to introduce a lattice in impact-parameter space,[1] in which the nonvanishing intersite spacing b acts as an ultraviolet cutoff. A further approximation has been to truncate the single-site energy spectrum keeping only the two lowest levels, transforming the theory into a spin model. This spin approximation[2] is justified either for very high values of the bare Pomeron intercept α_0, or for arbitrarily small triple coupling r_0 and slope α_0', since in these cases there is a gap between the two lower energy levels and the higher excitations and the matrix elements of the Pomeron field take a particularly simple form which allows the study of the universal features of the critical structure of the RFT.[3] Unfortunately, for realistic values of b, r_0, and α_0' the single-site spectrum does not show a quasidegenerate ground-state doublet. Therefore, magnitudes which depend on the above parameters, as for example the critical intercept α_0^c, cannot be evaluated in this way. Using the expressions for the energy gap and matrix elements valid for $\alpha_0 \lesssim 1$ the not well justified truncation approximation has been used[4] to calculate the intercept renormalization, even though the method cannot be applied to values of α_0 as large as α_0^c for realistic values of the other parameters.

Since the phenomenological study of high-energy hadronic cross sections suggests a bare Pomeron intercept slightly above 1, it is indeed important to try to find approximate solutions to the Reggeon field theory for values of α_0 near the critical value. Besides, in order to justify the spin model, it is important to be able to study the effect of higher excitations and to include them if necessary.

A method that, in principle, would allow us to perform both calculations is the semiquantum approximation developed by Sachrajda, Weldon, and Blankenbecler[5] and Blankenbecler and Fulco.[6] However, the method has been only applied to quantum-field-theory Hamiltonians that are the sum of single-site kinetic and potential energy and intersite interaction terms, that is, of the Klein-Gordon type. The Reggeon-field-theory Hamiltonian is clearly not of this form. However, by using a nonunitary transformation[7] the RFT Hamiltonian can be cast into the required form and the method can be applied straightforwardly for most values of the parameters.

Unfortunately, the transformation has the undesired feature of excluding the ground state (vacuum) from the Hilbert space at each site. Therefore, although the semiquantum method allows a fast and fairly accurate evaluation of the single-site energy levels for a broad range of the α_0 values, the complete field-theory Hamiltonian can be studied in its lowest levels only by modifying the single-site potential to include the vacuum at each site. An additional technical difficulty is presented in the fact that the intersite interaction is so complicated as to preclude its complete diagonalization, which, for some values of the parameters would have improved the accuracy of the approximation. Even with these limitations, the single-site energy levels and the intercept renormalization turn out to be in reasonable agreement with previous treatments.[4,8,9]

In Sec. II the semiquantum approximation will be applied to the single-site Hamiltonian which defines what is known as Reggeon quantum mechanics (RQM). Section III will be devoted to the study of the complete Hamiltonian (RFT) in $D=1$, and Sec. IV will contain comments and conclusions.

II. REGGEON QUANTUM MECHANICS

We begin with the RFT Hamiltonian in a transverse lattice,

$$H=\sum_j\left[\Delta_0\bar{\psi}_j\psi_j+i\frac{\tilde{r}}{2}\bar{\psi}_j(\psi_j+\bar{\psi}_j)\psi_j+\tilde{\alpha}'\sum_{\hat{i}}(\bar{\psi}_{j+\hat{i}}-\bar{\psi}_j)(\psi_{j+\hat{i}}-\psi_j)\right], \tag{1}$$

where $\alpha_0=1-\Delta_0$ is the bare intercept, $\tilde{r}=r_0/b^{D/2}$, $\tilde{\alpha}'=\alpha_0'/b^2$, D is the dimensionality of the lattice, j indicates the site, and $\hat{i}$ the nearest neighbors. Furthermore $[\psi_i,\bar{\psi}_j]=\delta_{ij}$.

In the limit $\alpha_0'\to 0$ and using the Schrödinger picture, Eq. (1) reduces to the sum of the single-site forms

$$H_s=\Delta_0 a^\dagger a+i\frac{\tilde{r}}{2}a^\dagger(a+a^\dagger)a, \tag{2}$$

where a and $a^\dagger$ are annihilation and creation operators. To apply the semiquantum method[5] one must transform Eq. (2) into the sum of kinetic and potential terms. For this purpose it is convenient to use the Bargmann representation

$$\langle 0|e^{az}|n\rangle=\varphi_n(z),\qquad a=\frac{z}{dz},\quad a^\dagger=z \tag{3}$$

so that the eigenvalue equation becomes[7]

$$z\left[\Delta_0+i\frac{\tilde{r}}{2}\left(z+\frac{d}{dz}\right)\right]\frac{d}{dz}\varphi_n(z)=E_n\varphi_n(z). \tag{4}$$

For $\varphi_n(z)$ to be holomorphic and normalizable for $\Delta_0>0$, according to $\int(dzdz^*/2\pi i)e^{-zz^*}|\varphi_n|^2<\infty$, they must satisfy the following conditions:

$$\varphi_n(z)\underset{z\to-i\infty}{\sim}\text{constant},\quad \varphi_n(z)\underset{|z|\to 0}{\sim}z, \tag{5}$$

The ground state $\varphi_0(z)=1$, which trivially satisfies Eq. (4) for $E_0=0$, is therefore excluded from the set of eigenfunctions.

Restricting to the negative imaginary axis $z=-ix^2$ and performing the nonunitary transformation

$$\varphi=(x^2)^{1/4}\exp[\tfrac{1}{4}(x^2+\rho)^2]\,\hat{\varphi}=F\hat{\varphi}, \tag{6}$$

where $\rho=2\Delta_0/\tilde{r}$, one obtains the Schrödinger-type equation

$$\left\{-\frac{1}{2}\frac{d^2}{dx^2}+\frac{x^2}{2}[(x^2+\rho)^2-2]+\frac{3}{8}\frac{1}{x^2}\right\}\hat{\varphi}_n(x)=\frac{E_n}{\tilde{r}/4}\hat{\varphi}_n(x), \tag{7}$$

where all $\hat{\varphi}_n\in L^2(0,\infty)$ and vanish at the origin.

By analytic continuation of the S matrix from $\alpha_0<1$ and $\Delta_0>0$ to $\Delta_0<0$ it has been shown in Ref. 7 that the eigenvalue problem [Eq. (7)], with the same boundary conditions [Eq. (5)] is valid for $-\infty<\Delta_0<\infty$. Of course, even for $\Delta_0<0$ the vacuum state is outside the Hilbert space defined by the eigenvectors in Eq. (7). Consequently, we will refer to the eigenvalues of Eq. (7) as first excited, second excited, etc. levels.

The potential of Eq. (7), displayed in Fig. (1), develops a second well for $-\rho>3.3$. Its eigenvalues can be analytically calculated for

(i) $\rho\gg 1$ giving $E_n=n\Delta_0$, $n=1,2,\ldots$;

(ii) $\rho\ll -1$ with $E_1\sim 0$, E_2 and $E_3\sim|\Delta_0|$, etc. (8)

However, for realistic values of the parameters $\alpha_0'\approx 0.3$ GeV^{-2}, $r_0\approx 0.5$ GeV^{-1}; $b\approx 4$ GeV^{-1} (related to a cutoff in transverse momentum $-t\approx\pi^2/b^2\approx 1$ GeV^2) and $\alpha_0\approx 1$, the region of interest is $|\rho|\approx 0$.

The semiquantum method (SQ) (Ref. 5) allows us to calculate lower bounds to the eigenvalues of Eq. (7) for a wide range of ρ. It is based on the following Schwarz inequality for the kinetic energy part of the Hamiltonian:

$$E=\frac{1}{2}\left\langle\frac{d\varphi}{dx},\frac{d\varphi}{dx}\right\rangle+\langle\varphi,V\varphi\rangle \geq\frac{1}{2}\frac{\mathrm{Re}\langle g\varphi,d\varphi/dx\rangle^2}{\langle g\varphi,g\varphi\rangle}+\langle\varphi,V\varphi\rangle =\frac{1}{8}\frac{\langle\varphi,g'\varphi\rangle^2}{\langle\varphi,g^2\varphi\rangle}+\langle\varphi,V\varphi\rangle$$

for real wave functions $\varphi(x)$ and arbitrary $g(x)$. Considering localized states, lower bounds to E

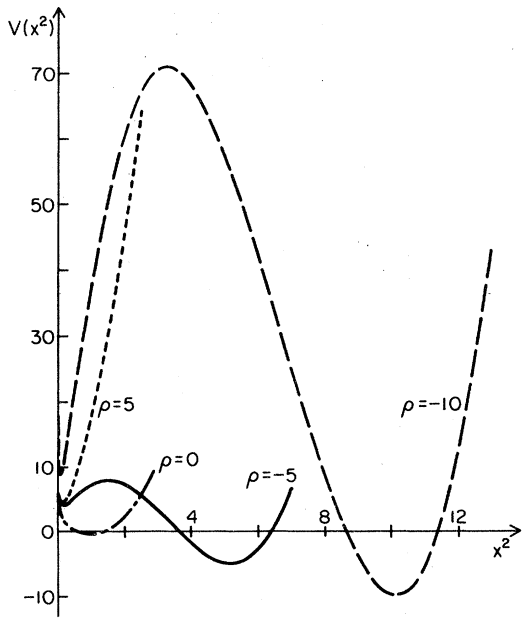

FIG. 1. Potential of single-site Hamiltonian for several values of ρ,

$V(x)=(x^2/2)\,[(x^2+\rho)^2-2]+\frac{3}{8}\,\frac{1}{x^2}$.

are obtained by minimizing

$$E(x)=\frac{1}{8}\frac{\langle g'(x)\rangle^2}{\langle g^2(x)\rangle}+\langle V(x)\rangle \tag{9}$$

with respect to x. If the wave function were exactly known the Schwarz inequality is an equality for $g(x)=\varphi'(x)/\varphi(x)$. Therefore the lower bound will be closest to the true eigenvalue provided a reasonable guess of the wave function can be made. This has been shown to be true for several cases.[5] For a wave function having m maxima M_i and $n+1$ zeroes N_i (in our problem one is always at $x=0$), one can write

$$g(x)=\frac{\prod_{i=1}^{m}(x^2-M_i)}{x\prod_{i=1}^{n}(x^2-N_i)}\,. \tag{10}$$

The classical expression $E(x)=\Pi^2(x)/2+\langle V(x)\rangle$ must be minimized in x and maximized with respect to variations of the parameters M_i and N_i.

For most of the interesting values of ρ, the potential has a single well and it is straightforward to guess the form of the wave functions. These are sketched in Fig. 2. In Table I we summarize the results for $\rho=5$. It is seen that the ratio of E_n is quite different from the one corresponding to $\rho\gg1$ and the values of E_1 and E_2 are in reasonable agreement (within 5%) with those obtained with a variational method[4] using five eigenfunctions for the $\rho\gg1$ case.[10]

TABLE I. One-well potential ($\rho=5$): E_i are eigenvalues from SQ method, M_i and N_i the values of the maxima and the nodes of the wave function which maximize the lower bound for the energy.

$\frac{E_1}{\tilde{r}/4}=11$	$M_1=0.3$				
$\frac{E_2}{\tilde{r}/4}=28$	$M_1=0.1$	$M_2=0.9$	$N_1=0.4$		
$\frac{E_3}{\tilde{r}/4}=50$	$M_1=0.05$	$M_2=0.4$	$M_3=1.3$	$N_1=0.2$	$N_2=0.8$

For the two-well configuration ($-\rho$ very large) guessing the form of the wave function is more difficult. If one starts from the analysis of two separated wells ($\alpha_0\to\infty$), and noticing that the eigenvalues in both wells coincide,[7] the suggested form of the wave functions will be as shown in Fig. 3(a). The use of the semiquantum method

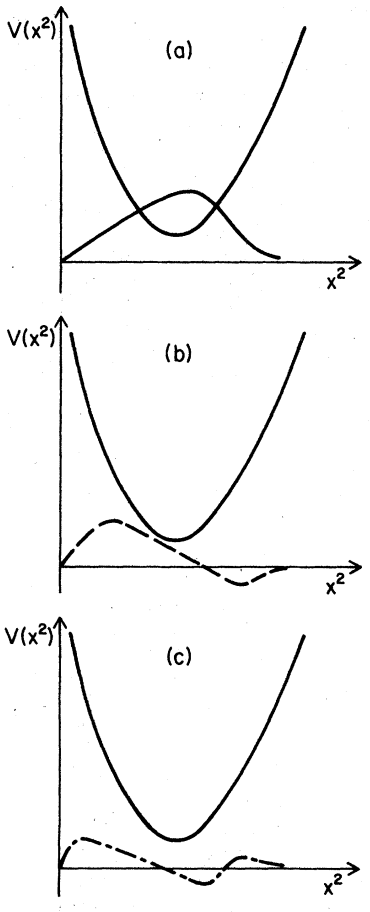

FIG. 2. Form of the approximate wave functions for the one-well potential for (a) first excited state (φ_1); (b) second excited state (φ_2); and (c) third excited state (φ_3).

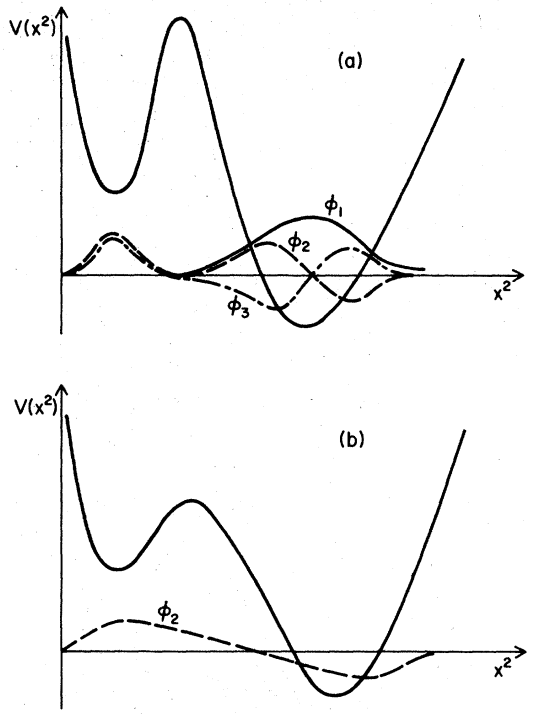

FIG. 3. Form of the approximate wave functions for the two-well potential: (a) φ_1, φ_2, and φ_3 for a very high barrier; (b) φ_2 for a low barrier.

gives, apart from E_1, pairs of almost degenerate levels (E_2, E_3), etc.

On the other hand, for small height of the barrier, it seems more reasonable to consider wave functions which are straightforward modifications of the ones for the single-well potential. Comparing with Fig. 3(a) it is clear that the first and third excited states will be the same whereas the second excited state will be as shown in Fig. 3(b). Therefore E_2 will be lower as a consequence of a large tunnel effect. For $\rho=-10$ which is an intermediate situation, the use of the semiquantum method with the wave functions of Fig. 3(b) gives the results reported in Table II. Even though E_2 may be underestimated, these results show that for not extremely high α_0, the energy spectrum of RQM corresponds to two almost degenerate levels E_0 and E_1, and higher excited levels E_2, E_3, etc. with large separations because of tunneling effects.

The SQ method allows to estimate the diagonal matrix elements of the field since it gives approximate values for the expectation values of the square of the field $\langle x_i^2\rangle$ corresponding to the lower bound in the energy of the corresponding state.[6] Table III shows $\langle n|\bar{\psi}|n\rangle$ for $\rho=-10$ and 5. The values of $\langle 1|\bar{\psi}|1\rangle$ are in agreement with both the asymptotic limit[3] and the variational technique.[4]

The computation of the nondiagonal matrix elements is not straightforward in the SQ approximation. One may be tempted to take the approximate wave functions with the values of the parameters determined by the minimization procedure and use them to obtain the overlap by direct integration. However, all matrix elements involving the vacuum require the knowledge of the derivative of the wave function at $x^2=0$ and this cannot be realistically obtained in our approximation where the vacuum is introduced by force in the Hamiltonian by modifying the potential. Matrix elements such as $\langle 0|\psi|1\rangle=\varphi_1'(0)$ may be estimated with a higher degree of confidence because the wave function for the first excited state could be reasonably approximated by a first degree polynomial times an exponential. The normalization condition, together with the value of the maximum obtained from the minimization procedure are enough to fix the two parameters required. Since, we are not interested in actually solving the spin model we have not evaluated these matrix elements. Of course, having an estimate of the value of $\langle 0|\psi|n\rangle$ for $n\geq 2$ is of interest to determine the validity of the spin-model calculations. However, the approximation for the higher-order wave functions may not warrant a reliable estimate.

TABLE III. Diagonal matrix elements of Pomeron field.

i	$\langle i\|\bar{\psi}\|i\rangle$ $\rho=-10$	$\rho=5$
1	10	0.35
2	7.4	0.54
3	6.5	0.62
4	6.1	...

III. REGGEON FIELD THEORY

We consider now the complete Hamiltonian Eq. (1) for a linear lattice ($D=1$). The use of the Bargmann representation for each site together with the transformation

$$\hat{H}=F^{-1}HF, \qquad F=\prod_j (x_j^2)^{1/4}\exp[\tfrac{1}{4}(x_j^2+\rho)^2] \tag{11}$$

gives the result

TABLE II. Two-well potential ($\rho=-10$): E_i are eigenvalues from SQ method, M_i and N_i the values of the maxima and the nodes of the wave function which maximize the lower bound for the energy.

$\frac{E_1}{\tilde{r}/4}\approx 0$	$M_1=10$					
$\frac{E_2}{\tilde{r}/4}=12$	$M_1=0.4$	$M_2=10.8$	$N_1=8.9$			
$\frac{E_3}{\tilde{r}/4}=24$	$M_1=0.15$	$M_2=8.1$	$M_3=11.3$	$N_1=3$	$N_2=9.7$	
$\frac{E_4}{\tilde{r}/4}=35$	$M_1=0.1$	$M_2=3$	$M_3=9$	$M_4=11.7$	$N_1=0.4$	$N_2=7.5$ $N_3=10.3$

$$\frac{\hat{H}}{\tilde{r}/4} = \sum_j \Big(-\frac{1}{2}\frac{d^2}{dx_j^2} + \frac{x_j^2}{2}\left[(x_j^2+\rho)^2 - 2\right] + \frac{3}{8}\frac{1}{x_j^2} + c(1 + 2x_j^4 - 4cx_j^2)$$
$$- c\Big\{ 2x_j^2 x_{j+1}^2 + \tfrac{1}{2}x_j^2\Big(\frac{1}{x_{j+1}^2} + \frac{1}{x_{j-1}^2}\Big) - cx_j^2\Big(\frac{x_j^2}{x_{j-1}^2} + \frac{x_{j-2}^2}{x_{j-1}^2} + \frac{x_{j+2}^2}{x_{j+1}^2} + \frac{x_j^2}{x_{j+1}^2}\Big)$$
$$- i\Big[\frac{d}{dt}(x_j^2) - \Big(\frac{x_{j+1}^2}{x_j}\dot{x}_j + \frac{x_j^2}{x_{j+1}}\dot{x}_{j+1}\Big)\Big]\Big\}\Big), \tag{12}$$

where $c = 2\tilde{\alpha}'/\tilde{r}$ and the commutator $i\dot{x}_j = [x_j, \hat{H}/(\tilde{r}/4)]$ has been introduced. The last line of Eq. (12) is non-Hermitian. Its first term is a total derivative so that the expectation value in a stationary state vanishes. The last term would also be a total derivative if $x_{j+1} \approx x_j$, i.e., for $k \approx 0$. Wishing to explore the spectrum near $t = 0$ we shall neglect the non-Hermitian terms altogether.

Comparing the Hamiltonian (12) with the original one (1), the problem is that through the transformation Eq. (2) one loses the vacuum state for every site. Therefore the solutions of Eq. (12) will correspond to states where all the sites are at least in the first excited level.

We will see first that these states give rise to an energy gap even though the single-site energy E_1 may be close to zero. A classical solution of the Hermitian part of Eq. (12) is $x_j^2 = \text{constant} = x_{c1}^2$ corresponding to the minimum of the potential. Expanding around this minimum $x_j^2 = x_{c1}^2 + \delta x_j^2$ and keeping only the quadratic terms, one gets,

$$\frac{\hat{H}}{\tilde{r}/4} \approx \sum_j \Big\{ \frac{1}{2}\Pi_j^2 + V(x_{c1}^2) + (\delta x_j)^2\Big[\frac{1}{2}\left[(x_{c1}^2+\rho)^2 - 2\right] + 5x_{c1}^2(x_{c1}^2+\rho) + 2(x_{c1}^2)^2 + \frac{1}{2}\times\frac{3}{4}\frac{3}{(x_{c1}^2)^2} + 2c\Big(4x_{c1}^2 - \frac{2}{x_{c1}^2} + 12c\Big)\Big]$$
$$+ \delta x_j \delta x_{j+1} 4c\Big(-2x_{c1}^2 + \frac{1}{x_{c1}^2} - 8c\Big) + \delta x_j \delta x_{j+2} 8c^2\Big\}, \tag{13}$$

The quadratic part of Eq. (13) may be diagonalized in terms of eigenmodes obtaining

$$\frac{\hat{H}}{\tilde{r}/4} \simeq \sum_j V(x_{c1}^2) + \sum_n \Big[\frac{1}{2}\Pi_n^2 + \tfrac{1}{2}D_n(\delta r_n)^2\Big], \tag{14}$$

whose solution for the ground-state energy is obtained[6] with the replacement $\Pi_n^2 = 1/4(\delta r_n)^2$, which gives through minimization

$$\frac{E}{\tilde{r}/4} \leq \sum_j V(x_{c1}^2) + \tfrac{1}{2}\sum_n D_n^{1/2}. \tag{15}$$

For the values of the parameters already quoted $c \simeq 0.1$. For $\rho = 0$ where $V(x_{c1}^2) = -0.13$, a four-site chain with periodic boundary conditions gives $D_1 = 7.63$, $D_2 = 8.06$, $D_3 = 8.16$, $D_4 = 9.03$, and a value of the energy $E/(\tilde{r}/4) = 5.24$. Since all D_n are larger than the corresponding one-site frequencies and since for $c \neq 0$ one would add a positive contribution to the energy, it is clear than an N-site chain will have $E_{\text{chain}} > NE_1$. This may be seen analytically when terms $O(c^2)$ are neglected, since the c-dependent terms are exactly the same as those analyzed in Ref. 6 [case (b)] giving therefore positive corrections to all the eigenvalues obtained for $c = 0$. This feature persists for larger c and $-\rho$ so that a nonzero gap always appears.

The more interesting states in which some sites are in the vacuum level can be simulated by replacing the one-site potential V by $V' = V(x)\theta(x - \delta) - h\theta(\delta - x)$. δ must be taken small enough to ensure that the wave function for the first few excited levels are not appreciably changed and h must be such that the first level in the narrow well is exactly at $E = 0$ and corresponds to a wave function concentrated near the origin. In this way it is clear that $\langle 0|\psi|n\rangle \approx \varphi_n'(0)$ which is the correct behavior.

In the following we shall analyze the case $\rho = 0$ for a two-site cell, a four-site cell, and a linear chain with periodic boundary conditions comparing the results with the spin approximation with realistic parameters.[4] We stress that the comparison must be done extrapolating the results of Ref. 4 to $\rho \approx 0$, since this region is in principle not attainable in that approach.

The two-site cell Hamiltonian is

$$\frac{\hat{H}_{12}}{\tilde{r}/4} = \tfrac{1}{2}\Pi_1^2 + V_1' + \tfrac{1}{2}\Pi_2^2 + V_2' + c(1 + 2x_1^4 - 4cx_1^2)$$
$$+ c(1 + 2x_2^4 - 4cx_2^2)$$
$$- c\Big[2x_1^2x_2^2 + \frac{1}{2}\Big(\frac{x_1^2}{x_2^2} + \frac{x_2^2}{x_1^2}\Big) - c\Big(\frac{x_1^4}{x_2^2} + \frac{x_2^4}{x_1^2}\Big)\Big]. \tag{16}$$

For the ground state both particles must be in the narrow well so that $\frac{1}{2}\Pi_i^2 + V_i' = 0$. Since the details of this well cannot be important we take $x_1 \approx x_2 \approx \delta \ll 1$ to give $E_0/(\tilde{r}/4) \approx c$, so that the position of the vacuum is almost unchanged.

For the first excited state our SQ approximation requires us to put one of the particles in the narrow well and the other in the broad one. Using for the particle in the broad well the form of Π_2 which corresponds to the first excited single-

site level we obtain

$$\frac{E_1(x_2)}{\tilde{r}/4} = \frac{1}{8x_2^2}\left(\frac{x_2^2+M}{x_2^2-M}\right)^2 + V_2' + 2c + 2c(x_2^4 - 2cx_2^2) - c\left(\frac{1}{2}\frac{x_2^2}{\delta^2} - c\frac{x_2^4}{\delta^2}\right). \tag{17}$$

There are two conditions that limit the choice of δ^2. First, it should be small enough as to ensure that the energy and wave functions of the excited states are not changed. Second, it should be large enough so that the semiquantum approximation for the first excited state is valid. It can be seen from the numerical calculation, that the maximization of Eq. (17) with respect to M gives increasing values of M for decreasing δ^2. Even though V_2' compensates the negative contribution coming from the interaction term, for M much larger than the value which maximized the one-site energy, the approximation of Π_2 in Eq. (17) is no longer valid. This phenomenon has been considered in Ref. 6. For a Hamiltonian with intersite interactions, the approximation of replacing each site kinetic term Π_i^2 by a function of the site coordinate x_i is accurate only when the intersite term is small. When the latter is large one should diagonalize the intersite term and express the kinetic terms as a sum over the normal modes n replacing each Π_n^2 by a function of the normal coordinate n. Since in our case we have not attempted to diagonalize the intersite term we are obliged to take δ^2 large enough to avoid that the optimum value of M be displaced to a point where the semiquantum oscillations would no longer correspond to the first excited state but to a more excited one. In that case the approximation for Π_2^2 for the one-site first excited state would be no longer appropriate and the collective coordinate description would be in order. Therefore we take $\delta^2 = 0.05$ which ensures that the displacement of M in this and in the subsequent steps is not larger than 0.1. Then $E_1/(\tilde{r}/4) \approx 2.09$ for $M = 1.1$ while the single-site result is $E_1/(\tilde{r}/4) \approx 2.35$ for $M = 1.05$.

The second excited level requires that both particles be in the broad well. Here we replace both Π_1 and Π_2 by the form corresponding to the first-excited single-site state. In this way $E_2/(\tilde{r}/4) \approx 5.4$ which is slightly larger than twice E_1 for one-site.

The four-site cell is calculated in a similar way. The ground state has again $E_0 \approx c$ and the first excited level must be averaged according to which particle is in the broad well, giving $E_1/(\tilde{r}/4) \approx 1.75$.

Finally, a linear chain with periodic boundary conditions gives for the first excited state a result which does not depend on which particle is placed in the broad well. This value is $E_1/(\tilde{r}/4) \approx 1.30$ for four sites, while for larger number of sites there is no variation provided $\delta^2 \ll 1$. The vacuum state is again at $E_0 \approx c$.

In Table IV we compare the one-site, two-site, four-site, and linear chain first-excited levels with the corresponding block-spin calculation for $D = 2$ extrapolated to $\alpha_0 \approx 1$. The results show a similar trend. Here the SQ approximation, despite giving lower bounds, shows results corresponding to energies higher than the ones obtained in the block-spin method. This is due to the fact that the latter results are for $D = 2$ where the intersite terms, which give negative contributions to the energy are effectively multiplied by a factor of $\sqrt{2}$.[3] We are interested here in showing only the qualitative features of both approximations. It is therefore reasonable that the intercept renormalization in the block-spin case (~0.05) is smaller than in the SQ approximation (~0.08).

IV. CONCLUSIONS

We have applied the semiquantum approximation to study the Reggeon field theory on a transverse lattice obtaining reliable results for realistic va-values of the bare parameters including values of the bare intercept near the critical point.

In the case of $\alpha_0' = 0$ the approximation reproduces quantitatively results obtained by other methods most of which involved extrapolations from values of $\alpha_0 \gg 1$. The main advantage here resides on the fact that all calculations where performed with a nonprogrammable pocket calculator in contrast with the sophisticated computer techniques of the variational methods.

For $\alpha_0' \neq 0$ we have modified the single-site potential in order to be able to include the vacuum into the energy levels of the Hamiltonian. We have tested this approximation by comparing our results for the two-site problem with previous calculations. We have then computed the values of the energy for higher excited states corroboraring previous estimates and therefore giving support to the spin model for values of α_0 above

TABLE IV. Comparison between the energy of the first excited state from the SQ method and the block-spin approximation ($\alpha_0 = 1$): E_1 single-site, E_1' two-site, E_1'' four-site, $E_{1\,\text{final}}$ fixed-point value. SQ is calculated for $D = 1$ and block-spin for $D = 2$ (extrapolated from $\alpha_0 = 0.9$).

	E_1	E_1'	E_1''	$E_{1\,\text{final}}$
SQ approx.	0.15	0.13	0.11	0.08
Block-spin approx.	0.15	0.12	0.10	0.05

the critical values. In principle, the semiquantum approximation allows us to keep more than two levels per site, however, since the method involves only classical quantities one loses all those states obtained by symmetrizing or antisymmetrizing the wave functions. Moreover, the complicated structure of the intersite interactions prevents its diagonalization and therefore reduces the validity of approximations that include higher excitations. Nevertheless, the simple many-site calculation shows remarkable agreement with previous results obtained by using the spin-block formalism. This enhances our confidence in the method and justifies further studies to include kink states, detailed features of the phase transition, and critical exponents.

As a further remark we note that the original Hamiltonian of the RFT is not of the Klein-Gordon form but involves pseudoparticle fields resembling fermion fields. The existence of a transformation that allows the application of the semiquantum method in this case may indicate the possibility of using this method for true fermionic systems. Further work on this subject is in progress.

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