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Remarks on the parity-violating effects in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$

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The correlation expected between the Λ spin and the positron momentum in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ is presented for three distinct gauge models. We show that a measurement of this correlation effect may help to distinguish between the standard Weinberg-Salam model and models in which the electron's neutral-current coupling is pure vector or in which parity is conserved by all neutral currents in lowest order.

I. INTRODUCTION

Much attention has been recently devoted to the study of the parity-violating electron-hadron neutral-current interaction. The parity-violating effects induced by this interaction have been measured in transitions between atomic levels of bismuth[1] and more recently in the scattering of longitudinally polarized electrons by nucleons.[2] While the essentially null results of the atomic-physics experiments seem to disagree[1] with the prediction of the standard Weinberg-Salam (WS) model,[3] the electron-nucleon scattering experiment is in excellent agreement with this model.[2] In view of the contradictory nature of these results, it seems necessary to study the effect of the parity-violating electron-hadron interaction in alternative processes.

In this paper we calculate the correlation expected between the Λ spin and the positron momentum in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ for three distinct gauge models. The parity-violating effects induced by the electron-hadron weak-neutral-current interaction in this decay have been studied by Abers and Sharif[4] in a model-independent way. They obtained a general expression for the correlation between the Λ spin and the e^+ momentum when the Σ^0 particle is unpolarized. In the present paper we elaborate further on this correlation effect for three specific gauge models: (a) the WS model,[3] (b) the $SU(2) \times U(1)$ (vectorlike) model[5] in which the electron's neutral current is purely vector in lowest order, and (c) the $SU_L(2) \times SU_R(2) \times U(1)$ (left-right-symmetric) model[6] in which all neutral currents naturally conserve parity in lowest order.

In order to calculate the electron-hadron parity-violating effective interaction for these three models, we shall find useful the general approach introduced in Refs. 5 and 7 for the study of the electron-quark parity-violating effective Hamiltonian. In general, we have found that models (b) and (c) predict sizeable correlation effects in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ as a result of radiative corrections.

II. THE CORRELATION EFFECT

We shall use the Abers-Sharif[4] notation when possible. The four-momenta are denoted by $\Sigma^0(p')$, $\Lambda(p)$, $e^+(p_+)$, $e^-(p_-)$, and the decay kinematics depends on two variables

$$x = (q^2)^{1/2} \tag{2.1}$$

and

$$y = (E_+ - E_-)/p_\Lambda , \tag{2.2}$$

where $q = p_+ + p_-$ is the four-momentum transferred to the electron-positron pair, $E_\pm = p_\pm^0$, and p_Λ is the magnitude of the three-momentum of the Λ particle.

The invariant matrix element for the decay $\Sigma^0 \to \Lambda + e^+ + e^-$, including the weak-neutral-current contribution, is taken to be[4]

$$\mathfrak{M} = -\frac{e^2}{x^2} T_\mu L^\mu + \frac{G_F}{\sqrt{2}} W_\mu L_5^\mu , \tag{2.3}$$

where e and G_F are the electric and Fermi coupling constants. The hadronic and leptonic matrix elements of the electromagnetic and weak nuetral currents are given by

$$\begin{aligned} L_\mu &= \bar{v}(p_+)\gamma_\mu u(p_-) , \\ L_\mu{}^5 &= \bar{v}(p_+)\gamma_\mu(g_V + g_A\gamma_5)u(p_-) , \\ T_\mu &= f_2\bar{u}(p)i\sigma_{\mu\nu}(q^\nu/2M)u(p') , \\ W_\mu &= \bar{u}(p)[f_2' i\sigma_{\mu\nu}(q^\nu/2M) + C\gamma_\mu\gamma_5]u(p') , \end{aligned} \tag{2.4}$$

where $M = (M_\Sigma + M_\Lambda)/2$, and all form factors are approximated by constants. The interference between the electromagnetic and weak-neutral-current contributions induces a correlation between the Λ spin and the direction of the e^+ momentum given by[4]

$$P(x, y) = \frac{\partial^2\Gamma_+/\partial x\partial y - \partial^2\Gamma_-/\partial x\partial y}{\partial^2\Gamma_+/\partial x\partial y + \partial^2\Gamma_-/\partial x\partial y} , \tag{2.5}$$

with $\partial^2\Gamma_\pm/\partial x\partial y$ the differential decay rate for the Λ spin to be in the positive (negative) positron direction. The correlation function $P(x,y)$ is largest for small values of the Λ three-momentum P_Λ, or x near $\Delta \equiv M_\Sigma - M_\Lambda$. In this kinematic region, $P(x,y)$ can be approximated by[4]

$$P(\Delta, y) \cong \left(\frac{G_F M\Delta}{\sqrt{2}\alpha}\right)\left(\frac{\Delta}{p_\Lambda}\right)\left(\frac{y}{1+y^2}\right) \times\left[\frac{C_{2\Sigma\Lambda}}{f_2} - \frac{C_{1\Sigma\Lambda}}{f_2}\left(\frac{P_\Lambda}{M}\right)y\right], \quad (2.6)$$

where $C_{1\Sigma\Lambda} \equiv f_2' g_A$, $C_{2\Sigma\Lambda} \equiv C g_V$, and the constant $(G_F M\Delta/\sqrt{2}\alpha) \simeq 10^{-4}$.

On the other hand, the parity-violating neutral-current interaction between the electron and a quark is given by the effective Hamiltonian[5,7]

$$H_{\mathrm{PV}} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma_\mu\gamma_5 e(C_{1u}\bar{u}\gamma^\mu u + C_{1d}\bar{d}\gamma^\mu d + C_{1s}\bar{s}\gamma^\mu s) + \bar{e}\gamma_\mu e(C_{2u}\bar{u}\gamma^\mu\gamma_5 u + C_{2d}\bar{d}\gamma^\mu\gamma_5 d + C_{2s}\bar{s}\gamma^\mu\gamma_5 s)], \quad (2.7)$$

where the C's are constants that depend on the gauge model, and e, u, d, s are the field operators for the electron and the quarks of the type up, down, and strange, respectively. We shall use the values for the C's computed by Marciano and Sanda[5] for the three different models (a), (b), and (c). The expressions for the C's are given in the Appendix, including the dominant radiative corrections.

We shall make two reasonable approximations in order to express the parameters $C_{1\Sigma\Lambda}$ and $C_{2\Sigma\Lambda}$, which determine the parity-violating function (2.6), in terms of the C's defined in Eq. (2.7). In the first place we shall neglect strong-interaction corrections[8] to the effective Hamiltonian (2.7); and in the second, we shall use the vector and axial-vector charges associated with the various quarks as obtained through Bjorken's sum rule[9] for the decays $n \to pe^-\bar{\nu}_e$ and $\Xi^- \to \Xi^0 e^-\bar{\nu}_e$, in the limit of SU(3) symmetry.[7]

Under these conditions, the vector charges are simply given by the naive quark content of the hadrons, and the axial-vector charges are determined by the relations[7]

$$N_u^A - N_d^A = F + D, \quad N_d^A - N_s^A = F - D, \quad (2.8)$$

where F and D are the usual SU(3) coupling parameters, $F = 0.425$ and $D = 0.825$. From (2.8) it follows[7] that for the proton $N_u^A = 2F$, $N_d^A = F - D$, and $N_s^A = 0$; while for the Ξ^- particle we have $N_u^A = 0$, $N_d^A = -(F+D)$, and $N_s^A = -2F$. In the limit of exact SU(3) symmetry, it follows that for the Σ^- particle we have $N_u^A = 0$, $N_d^A = -2F$, and $N_s^A = -(F+D)$; while for the Σ^+ particle, $N_u^A = -2F$, $N_d^A = 0$, and $N_s^A = -(F+D)$. Therefore, for the Σ^0 and Λ particles we have finally, in the SU(3) limit, $N_u^A = N_d^A = -F$ and $N_s^A = -(F+D)$. From these results and Eqs. (2.3) and (2.7), we get

$$C_{1\Sigma\Lambda} \equiv f_2' g_A = C_{1u} + C_{1d} + C_{1s}, \quad (2.9)$$

$$C_{2\Sigma\Lambda} \equiv C g_V = -FC_{2u} - FC_{2d} - (F+D)C_{2s}. \quad (2.10)$$

We have listed the values for $C_{1\Sigma\Lambda}$ and $C_{2\Sigma\Lambda}$ in Table I for the three models (a), (b), and (c). These numerical results were obtained using the expressions for the C_1's and C_2's given in the Appendix. For the WS and vectorlike models we have used the specific values $\ln(M_Z/m) \approx \ln(M_L/m) \approx 4$, and three different values for the weak mixing angle θ_W, $\sin^2\theta_W = 0.21$, 0.24, and 0.33. For the left-right-symmetric model we have used $\sin^2\xi = 0.5$, and two extreme values for the mass ratio $M_R/M_L = 2.7$ and 200. We have also listed in Table I the value obtained for $C_{2\Sigma\Lambda} \equiv C g_V$ in Ref. 4, with g_V given by the WS model and $C \simeq 0.2$. This value was obtained[4] by relating C to the coupling of the axial-vector current which mediates the semileptonic decay $\Sigma^- \to \Lambda e^- \bar{\nu}_e$.

III. CONCLUSION

From the values listed in Table I and the expression (2.6) for the correlation effect, we see that it will be possible in principle to distinguish among the models discussed in this paper as far as their predictions for parity violation in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ are concerned. In particular, both models

TABLE I. A comparison of parity-violation parameters for the Weinberg-Salam model, vectorlike SU(2) ×U(1) model, and $SU_L(2) \times SU_R(2) \times U(1)$ model. Numerical results were obtained from the formulas in the text using the values quoted for $\delta \equiv \sin^2\theta_W$, $\epsilon \equiv M_R/M_L$, and $\sin^2\xi = 0.5$, $\ln M_Z/m \approx \ln M_2/m \approx 4$.

	(a) Weinberg-Salam model			(b) Vectorlike			(c) Left-right		Reference 4		
Quantity	$\delta = 0.21$	$\delta = 0.24$	$\delta = 0.33$	$\delta = 0.21$	$\delta = 0.24$	$\delta = 0.33$	$\epsilon = 2.7$	$\epsilon = 200$	$\delta = 0.21$	$\delta = 0.24$	$\delta = 0.33$
$C_{1\Sigma\Lambda}$	−0.493	−0.495	−0.503	0.035	0.031	0.02	0.002	−0.002	···	···	···
$C_{2\Sigma\Lambda}$	0.099	0.025	−0.2	0.73	0.66	0.43	0.026	0.011	0.016	0.004	−0.032

(b) and (c) predict sizeable correlation effects as a result of radiative corrections. Note that if the weak mixing angle θ_W is such that $\sin^2\theta_W \lesssim 0.25$, then one can distinguish between the vectorlike and the other two models by the order of magnitude of the correlation effect[10]: $P(\Delta, y) \sim 3\times 10^{-5}(\Delta/p_\Lambda)$ for the vectorlike model, whereas $P(\Delta, y) \lesssim 3\times 10^{-6}(\Delta/p_\Lambda)$ for the WS and the left-right-symmetric models.

A prominent feature of the correlation effect is that the $C_{1\Sigma\Lambda}$ term in Eq. (2.6) is practically negligible for all three models in the kinematical region where $p_\Lambda \ll \Delta$. Therefore, the correlation between the Λ spin and the e^+ momentum, in this kinematical region, is determined by the $C_{2\Sigma\Lambda}$ term, and from Eq. (2.10), by the interference between the leptonic vector and the hadronic axial-vector neutral-current couplings. This has to be contrasted with the atomic-physics case, where the parity-violating effects are determined by the interference between the leptonic axial-vector and the hadronic vector neutral-current couplings. In this respect, a measurement of the correlation effect in the decay $\Sigma^0 \to \Lambda + e^+ + e^-$ will give us information on the parity-violating electron-hadron neutral-current interaction which is not available from the atomic-physics experiments.

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APPENDIX

In this Appendix we give the expression for the C_1's and C_2's which determine the effective electron-quark Hamiltonian (2.7) for the three different models. Details of the models and of the calculations involved can be found in Ref. 5. Note that, since in all three models considered the down and strange quarks play a symmetric role, we have $C_{1d} = C_{1s}$ and $C_{2d} = C_{2s}$.

WS model:

$$C_{1u} = \tfrac{1}{2}(1 - \tfrac{8}{3}\sin^2\theta_W) + \frac{\alpha}{2\pi}\left[\frac{1}{\sin^2\theta_W} + (1 - 4\sin^2\theta_W)\ln(M_Z{}^2/m^2) + \frac{3}{8\sin^2 2\theta_W}(1 - \tfrac{8}{3}\sin^2\theta_W)[1 + (1 - 4\sin^2\theta_W)^2]\right], \quad \text{(A1)}$$

$$C_{1d} = C_{1s} = -\tfrac{1}{2}(1 - \tfrac{4}{3}\sin^2\theta_W) - \frac{\alpha}{4\pi}\left[\frac{1}{2\sin^2\theta_W} - (1 - 4\sin^2\theta_W)\ln(M_Z{}^2/m^2) - \frac{3}{4\sin^2 2\theta_W}(1 - \tfrac{4}{3}\sin^2\theta_W)[1 + (1 - 4\sin^2\theta_W)^2]\right], \quad \text{(A2)}$$

$$C_{2u} = \tfrac{1}{2}(1 - 4\sin^2\theta_W) + \frac{\alpha}{2\pi}\left[\frac{1}{\sin^2\theta_W} + (1 - \tfrac{8}{3}\sin^2\theta_W)\ln(M_Z{}^2/m^2) + \frac{3}{8\sin^2 2\theta_W}(1 - 4\sin^2\theta_W)[1 + (1 - \tfrac{8}{3}\sin^2\theta_W)^2]\right], \quad \text{(A3)}$$

$$C_{2d} = C_{2s} = -\tfrac{1}{2}(1 - 4\sin^2\theta_W) - \frac{\alpha}{4\pi}\left[\frac{1}{2\sin^2\theta_W} - (1 - \tfrac{4}{3}\sin^2\theta_W)\ln(M_Z{}^2/m^2) - \frac{3}{4\sin^2 2\theta_W}(1 - 4\sin^2\theta_W)[1 + (1 - \tfrac{4}{3}\sin^2\theta_W)^2]\right], \quad \text{(A4)}$$

where m is a typical hadronic mass, M_Z is the mass of the neutral vector boson, and θ_W is the weak mixing angle.

Vectorlike model:

$$C_{1u} = \frac{\alpha}{\pi}\left[\frac{3}{8\sin^2\theta_W} + \cos 2\theta_W \ln(M_Z{}^2/m^2) + (\cot 2\theta_W)^2(\cos 2\theta_W - \tfrac{1}{4})\right], \quad \text{(A5)}$$

$$C_{1d} = C_{1s} = \frac{\alpha}{\pi}\left[\frac{3}{8\sin^2\theta_W} + \tfrac{1}{2}\cos 2\theta_W \ln(M_Z{}^2/m^2) + \tfrac{1}{2}(\cot 2\theta_W)^2(\cos 2\theta_W + \tfrac{1}{2})\right], \quad \text{(A6)}$$

$$C_{2u} = \cos 2\theta_W + \frac{5\alpha}{8\pi\sin^2\theta_W}, \quad \text{(A7)}$$

$$C_{2d} = C_{2s} = -\cos 2\theta_W - \frac{5\alpha}{8\pi\sin^2\theta_W}. \quad \text{(A8)}$$

Left-right-symmetric model:

$$C_{1u} = \frac{\alpha}{\pi}\left[\tfrac{14}{3}\ln(M_R/M_L) + \frac{1}{\sin^2\xi}\right], \quad \text{(A9)}$$

$$C_{1d} = C_{1s} = -\frac{\alpha}{\pi}\left[\tfrac{7}{3}\ln(M_R/M_L) + \frac{1}{4\sin^2\xi}\right], \quad \text{(A10)}$$

$$C_{2u}=\frac{\alpha}{\pi}\left[7\ln(M_R/M_L)+\frac{1}{\sin^2\xi}+\frac{2}{9}\ln(M_L/m)\right], \quad \text{(A11)}$$

$$C_{2d}=C_{2s}=-\frac{\alpha}{\pi}\left[7\ln(M_R/M_L)+\frac{1}{4\sin^2\xi}+\frac{4}{9}\ln(M_L/m)\right], \quad \text{(A12)}$$

where M_R and M_L are the masses of the right- and left-handed charged vector bosons, and ξ is a mixing angle.

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