Comments and Addenda

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Soft-pion production in e^+e^- annihilations

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We show that we can apply the soft-pion theorem to the high-energy e^+e^- process, assuming the new extrapolation method, and point out that the nucleon-pole term plays an important role at PEP and PETRA energy.

It has been pointed out by Pais and Treiman[1] that spectral-function information on the weak axial-vector current can be extracted from soft-pion production in e^+e^- annihilations. In general, the soft-pion theorem is considered to be applicable only if $q\cdot p_i$ (q denotes the pion's momentum) is small for every momentum p_i in the process, since for large $q\cdot p_i$ the inclusive yields of pions become occupied mainly by those indirectly produced by the decays of heavier particles. Recently we have applied the soft-pion theorem[2] to deep-inelastic pion electroproduction in order to investigate the possibility of its applications at high energy.[3] Then our postulate in the previous paper that soft pions at high energy should be regarded as direct pions in the central region is consistent with the experiments.[4] In this note, we show that the transverse part of the spectral function of the weak axial-vector current can be determined with the use of the light-cone current algebra,[5] and that, using the picture based on the previous paper, we can apply the soft-pion theorem to the high-energy e^+e^- process with even large $q\cdot k$, where k is the photon momentum. Further, we point out that the nucleon-pole term, neglected in Ref. 1, becomes meaningful at PEP and PETRA energy.

Now we calculate σ^α defined as

$$\sigma^\alpha \equiv q^0 \frac{d\sigma(e^+e^- \to \pi^\alpha X)}{d^3q}, \tag{1}$$

where α means the charge of the soft pion. According to the usual technique,[2] the hadronic tensor $T^{\mu\nu}$ is given by

$$T^{\mu\nu} = \frac{(m_\pi^2 - q^2)^2}{m_\pi^4 F_c^2} \int d^4x\, d^4y\, d^4z \exp[-iq(x-z)+iky]\langle 0|T^\dagger(\partial_\lambda J_{a'}^{5\lambda}(x), J_b^\mu(y))\, T(\partial_\rho J_c^{5\rho}(z), J_d^\nu(0))\,|0\rangle, \tag{2}$$

where $a'=c$ specifies the pion charge, $b=d$ the electromagnetic currents, and the pion field is defined as

$$\phi_{\pi^a}(x) = \frac{1}{m_\pi^2 F_a}\, \partial_\lambda J_a^{5\lambda}(x). \tag{3}$$

F_a is the appropriate normalization factor for the pion field and is expressed by the pion decay constant, $f_\pi(=0.61 m_\pi)$, as $\sqrt{2}f_\pi$ in the case of the charged pion, and f_π in the case of the neutral pion.

Then in Eq. (2) we take the derivative out of the time-ordering product, integrate by parts, ignoring surface terms, and take the limit $q^+=0$ and $\vec{q}^\perp=0$.[6] In this limit, q^2 and $q\cdot k$ becomes zero and q^-k^+, respectively. Now the lightlike version of the soft-pion theorem is obtained by taking $q^-\to 0$ as

$$T^{\mu\nu}|_{q^\mu\to 0} \equiv W^{\mu\nu} = \frac{1}{F_c^2}\,(A^{\mu\nu}+B^{\mu\nu}+C^{\mu\nu}). \tag{4}$$

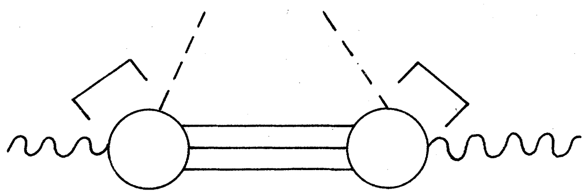

FIG. 1. The contribution from the equal–null-plane commutator, where the dashed line denotes the pion.

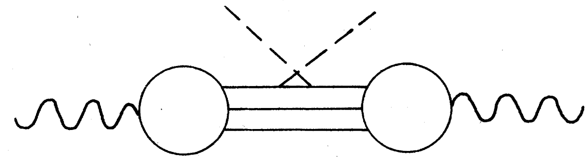

FIG. 2. The contribution from the nucleon- (antinucleon-) pole term, where the proper part of the axial-vector current attaches to the final nucleon (antinucleon).

The term $A^{\mu\nu}$ is the contribution from the equal–null-plane commutator term as shown in Fig. 1 and is given by

$$A^{\mu\nu}=f_{abe}f_{cdf}\int d^4x\exp(ik\cdot x)\times\langle 0|[J_e^{5\mu}(x),J_f^{5\nu}(0)]|0\rangle\,, \tag{5}$$

where a is the complex conjugate of a', and the algebra of currents based on lightlike quantization is used.[7] The term $B^{\mu\nu}$ is the contribution from the nucleon-pole term and the antinucleon-pole term[8] as shown in Fig. 2 and is given by

$$B^{\mu\nu}=\sum_{XN}\int d^4x\exp(ik\cdot x)\langle 0|J_b^{\mu}(x)|XN\rangle\langle N|J_a^{5+}(0)|N'\rangle\langle N'|J_c^{5+}(0)|N\rangle\langle NX|J_d^{\nu}(0)|0\rangle\,, \tag{6}$$

where $\sum_{XN}$ denotes the sum over the intermediate state, and N or N' denotes a suitable nucleon or antinucleon. The term $C^{\mu\nu}$ is the contribution from those as shown in Fig. 3 and is negligible.[9]

Since the dominant contribution to Eqs. (5) and (6) as $s\equiv k^2\to\infty$ comes from the $x^2\simeq 0$ region, we obtain, using the light-cone current algebra,[5]

$$\sigma^+=\frac{\alpha^2}{24\pi^2f_\pi^{\,2}s}\,[3+g_A^{\,2}(0)(\langle n\rangle_p+\langle n\rangle_{\bar n})\mathrm{Tr}(Q^2)]\,, \tag{7}$$

$$\sigma^-=\frac{\alpha^2}{24\pi^2f_\pi^{\,2}s}\,[3+g_A^{\,2}(0)(\langle n\rangle_n+\langle n\rangle_{\bar p})\mathrm{Tr}(Q^2)]\,, \tag{8}$$

$$\sigma^0=\frac{\alpha^2}{48\pi^2f_\pi^{\,2}s}\,[g_A^{\,2}(0)(\langle n\rangle_N+\langle n\rangle_{\bar N})\mathrm{Tr}(Q^2)]\,, \tag{9}$$

where the color degree of freedom is taken into account, α is the fine-structure constant, and $g_A(0)=1.22$ is the nucleon axial-vector coupling constant. In Eqs. (7)–(9), the term which depends on $\langle n\rangle_x$ is the nucleon-pole term, where $\langle n\rangle_p$ ($\langle n\rangle_n$) denotes the average multiplicity of the proton (the neutron), and $\langle n\rangle_N\equiv\langle n\rangle_p+\langle n\rangle_n$.

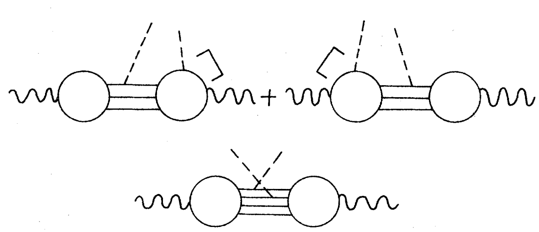

FIG. 3. The expression of the interference between the nucleon- (antinucleon-) pole term and the null-plane commutator.

In the e^+e^- process with large $q\cdot k$, as $q\cdot k>E_{\mathrm{c.m.}}m_\pi$, it seems that there is no room for application of the soft-pion theorem. But it would be premature to deny the possibility of the application of the soft-pion theorem to the high-energy process. We start with the following two assumptions. First, soft pions are the pions on which the momentum cutoff $|\vec{q}|<m_\pi$ is imposed in the photon rest frame, and which are not produced through the resonance decay. This assumption is based on our picture in the previous paper. Second, by use of the definition $z\equiv q\cdot k/s$ at fixed s, σ^α is smooth in small z, where the pion's momentum $\vec{q}$ is $|\vec{q}|<m_\pi$. These two assumptions give an effective guarantee of the application of the soft-pion theorem to the high-energy e^+e^- process with even large $q\cdot k$. Further, using these assumptions, we can extrapolate the theoretical value of σ^α at $z=0$ to the experimental value of σ^α at $z_{\min}$. Thus, these assumptions, in principle, can be compared with the experiments.

The theoretical values of the commutator term are compared with the experimental data[10] in Table I, when the nucleon-pole term is expected to be negligible. The three experimental values, σ_E, are taken from the region where R_E^{11} is regarded as constant, $R\equiv\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$. R_E denotes the experimental value, and R_T the theo-

TABLE I. E and T denote experimental and theoretical values, respectively. σ_E is half of the charged-pion cross section. Our estimate of the direct-pion ratio, $R_E\sigma_T/R_T\sigma_E$, is given at three energies.

$E_{\mathrm{c.m.}}$ (GeV)	σ_E (mb)	σ_T (mb)	$\frac{\sigma_T}{\sigma_E}$ (%)	R_E	R_T	$\frac{R_E\sigma_T}{R_T\sigma_E}$ (%)
3.0	35.9	4.0	11	2.6	2	14
6.2	28.0	0.93	3.3	5.5	$\frac{10}{3}$	5.4
7.4	24.4	0.66	2.7	5.5	$\frac{10}{3}$	4.5

retical one expected by the three-color quark-parton model. There is some difference between R_E and R_T. Then σ_T/σ_E is slightly small to represent the ratio of direct pions, since σ_T should be magnified to $R_E\sigma_T/R_T\sigma_E$ when we compare with the experimental value σ_E. Further, above $E_{\mathrm{c.m.}}=5$ GeV, the ratio R_E/R_T contains the effect of the heavy lepton. The contribution due to the commutator term, being flavor independent, decreases above charm threshold. Therefore, as shown in Eqs. (7)–(9), the nucleon-pole term, being flavor dependent, plays an important role at high energy. Practically, in the PEP and PETRA energy region, $E_{\mathrm{c.m.}}=30$ GeV, charged hadron multiplicity is expected to be $\langle n\rangle_{\mathrm{ch}}\simeq 7$, and even if the ratio $\langle n\rangle_p/\langle n\rangle_{\mathrm{ch}}\simeq 10^{-2}$, we obtain $\langle n\rangle_p+\langle n\rangle_{\bar{p}}\simeq 0.14$, which is not a negligible value in Eqs. (7)–(9). Thus, we find

$$\frac{\sigma^0}{\sigma^+}=\frac{g_A{}^2(0)(\langle n\rangle_N+\langle n\rangle_{\bar{N}})\mathrm{Tr}(Q^2)}{2[3+g_A{}^2(0)(\langle n\rangle_p+\langle n\rangle_{\bar{p}})\mathrm{Tr}(Q^2)]}, \tag{10}$$

which is meaningful at PEP and PETRA energy.

Finally, one of our theoretical results, $\sigma_T=\sigma_L$, is consistent with the experimental data.[11] Further, from the definition of Eq. (1), we integrate over the solid angle, using $d^3q=|\vec{q}|^2 d|\vec{q}|\sin\theta d\theta d\phi$ and the fact that the theoretical value is isotropic. The result is

$$s\frac{d\sigma^+}{dx}=s\frac{d\sigma^-}{dx}=\frac{\alpha^2x^2s}{8\pi f_\pi^2}(x^2+4m_\pi^2/s)^{-0.5}, \tag{11}$$

by neglecting the nucleon-pole term.

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