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Quark forces and SU(6) configuration mixing

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Symmetry breaking through SU(6) configuration mixing is investigated, considering low-energy weak and electromagnetic processes and the relations to one-gluon-exchange potentials. We find evidence against d waves in nucleons and the possibility of an antisymmetric p-wave component.

I. INTRODUCTION

The classical quark shell model has been extended recently in two directions. On the one hand, the intermultiplet splittings (such as the Σ-Λ mass difference) have been explained with spin-dependent correction terms of the Breit-Fermi type, as suggested by one-gluon exchange.[1] On the other hand, the deviations from SU(6) of the current matrix elements (such as G_A/G_V) have been corrected in a scheme which extends the static SU(6) to the momentum dependence of the small components in the quark spinors.[2] Besides its simplicity, the first idea has the advantages of being related to quantum chromodynamics and of representing a well defined dynamical mechanism. This is not the case in the second approach, based on Dirac four-spinors, which can, moreover, become rather involved when considering configuration mixing or higher waves, as has been proposed lately in various contexts.[3] This possibility of configuration mixing is what we are going to investigate here. On the one hand, it can be obviously related to the simplified dynamics of one-gluon exchange, and on the other, it could even be an effective way of parametrizing complicated effects such as the relativistic corrections.

Therefore both in order to gain insights into the quark forces and to have a simpler formulation of SU(6) breaking, we investigate theoretically and phenomenologically possibilities of configuration mixing within the nonrelativistic quark model. Needless to say, if substantial relativistic corrections are shown to be required for the quark shell model, our considerations will be modified.

In the next section we consider the most general mixing for the $\underline{56}$ from double excitations, as required by parity. To be definite, we restrict ourselves to up and down quarks (nucleons and Δ) and perform in Sec. II a phenomenological analysis of the consistency of the higher-representation mixing. In Sec. III we consider the theoretical aspects of the mixing, relating the phenomenological mixings to the gluon-exchange operators and previous chiral-configuration-mixing formulations. In the last section we present the conclusions, among which are the following: (1) There is evidence against d waves; and (2) there is the possibility of p waves in the antisymmetric $(\underline{20}, 1^+)$. We also propose new tests for such admixtures.

II. PHENOMENOLOGY OF SU(6) MIXING

The lowest representations that could mix in principle with the ground state $(\underline{56}, 0^+)$ are, as required by parity conservation, the doubly excited $(\underline{70}, 0^+)$, $(\underline{70}, 2^+)$, and $(\underline{20}, 1^+)$ for the octet and $(\underline{56}, 2^+)$ and $(\underline{70}, 2^+)$ for the decuplet. The corresponding wave functions are given in Table I, with the following notation: subindex α stands for symmetry and β stands for antisymmetry in the first two quarks, N and Δ denote the SU(3) u and d quark contents, χ denotes the corresponding up and down spin states, and S and A mean total symmetry and antisymmetry, respectively. The space wave functions are $\varphi(l_\rho, l_\lambda; L)$, where the arguments refer to the angular momenta in the relative variables

$$\rho = \frac{1}{\sqrt{2}}\,(r_1 - r_2)\,,\; \lambda = \frac{1}{\sqrt{6}}\,(r_1 + r_2 - 2r_3) \tag{1}$$

$$l_1 + l_2 = \tfrac{1}{3}\,l_\lambda + l_\rho\,,\quad l_3 = \tfrac{2}{3}\,l_\lambda$$
$$L = l_1 + l_2 + l_3 = l_\rho + l_\lambda\,. \tag{2}$$

The main idea of our phenomenological analysis can be summarized as follows: We first work out the mixing required from the various representations to account by themselves for the reduction of the semileptonic form-factor ratio G_A/G_V from $\frac{5}{3}$ [of exact SU(6) with a pure $(\underline{56}, 0^+)$] to the experimental 1.25. These results for the mixings are then tested with the electromagnetic interactions: They have to preserve the successful SU(6) predictions of the magnetic-moment ratio $\mu_p/\mu_n = -\frac{3}{2}$ and the selection rule on the absence of an $E2$ transition amplitude for $\Delta^+ \to p\gamma$. These predictions are both theoretically and experimentally the least ambiguous, as compared, e.g., to the value of the $M1$ transition moment for the former reaction, the D/F ratio or the Δ^{++} magnetic moment, which are also considered.

TABLE I. SU(6) wave functions for N and Δ. The arguments in the space wave functions refer to $l_\rho, l\lambda; L$.

$\underline{56}, 0^+$	$\frac{1}{\sqrt{2}}(N_\alpha \chi_\alpha{}^{1/2} + N_\beta \chi_\beta{}^{1/2})\varphi_S(0,0;0)$
$\underline{70}, 0^+$	$\frac{1}{2}\{N_\alpha[\chi_\beta{}^{1/2}\varphi_\beta(1,1;0) - \chi_\alpha{}^{1/2}\varphi_\alpha(0,0;0)] + N_\beta[\chi_\alpha{}^{1/2}\varphi_\beta(1,1;0) + \chi_\beta{}^{1/2}\varphi_\alpha(0,0;0)]\}$
$\underline{70}, 2^+$	$\frac{1}{\sqrt{2}}\{N_\alpha[\chi_S{}^{3/2}\varphi_\alpha(2,0;2)]_{J=1/2} + N_\beta[\chi_S{}^{3/2}\varphi_\beta(1,1;2)]_{J=1/2}\}$
$\underline{20}, 1^+$	$\frac{1}{\sqrt{2}}\{N_\alpha[\chi_\beta{}^{1/2}\varphi_A(1,1;1)]_{J=1/2} + N_\beta[\chi_\alpha{}^{1/2}\varphi_A(1,1;1)]_{J=1/2}\}$
$\underline{56}, 0^+$	$\Delta_S \chi_S{}^{3/2}\varphi_S(0,0;0)$
$\underline{56}, 2^+$	$\Delta_S[\chi_S{}^{3/2}\varphi_S(2,0;2)]_{J=3/2}$
$\underline{70}, 2^+$	$\Delta_S\{[\chi_\alpha{}^{1/2}\varphi_\alpha(2,0;2)]_{J=3/2} + [\chi_\beta{}^{1/2}\varphi_\beta(1,1;2)]_{J=3/2}\}$

The matrix elements of additive single-particle operators, like all the currents mentioned above, are just three times their values for any one of the three quarks, for which we shall choose the one with coordinate λ relative to the center of mass of the other two. Denoting the mixing coefficients of the nucleon N by $\nu_{\dim,L}$ and the quark charges by Q_i, one has

$$|G_A/G_V| = 3\langle N|\sigma_{\lambda_z}\tau_\lambda^+|N\rangle = \tfrac{5}{3}\nu_{56,0}{}^2 - \tfrac{4}{3}\nu_{70,0}{}^2 - \tfrac{4}{3}\nu_{20,1}{}^2 - 2\nu_{70,2}{}^2 . \quad (3)$$

We see that all representations contribute, similarly lowering the SU(6) prediction of $\frac{5}{3}$. It is difficult to imagine theoretically how all three representations could mix in a similar amount with the nucleon, the wave functions, and the energy denominators being so different. In that case, anyway, the coefficients should be very small, because one would also have the corresponding reduction of all SU(6) predictions through normalization, even in processes, such as the nonleptonic, where only the $\underline{56}$ contributes; the effect of the individual mixing would be then very difficult to detect in a clear way. We concentrate therefore on the possibility of one particular representation mixing substantially with the nucleon. The coefficient required for that mixing by the experimental $|G_A/G_V| = 1.25$ would be 30% for the $(\underline{70}, 0^+)$ and $(\underline{20}, 1^+)$ and 20% for the $(\underline{70}, 2^+)$.

We proceed further to see how the predictions for the electromagnetic interactions are met by the various representations, of which only the $(\underline{70}, 0^+)$ modifies the $-\frac{3}{2}$ ground-state prediction for μ_p/μ_n, which becomes

$$\mu_p/\mu_n = -\tfrac{3}{2}\left[1 + \tfrac{1}{3}\left(\frac{\nu_{70,2}}{\nu_{56,0}}\right)^2\right]. \quad (4)$$

The $(\underline{70}, 0^+)$ (Ref. 4) can therefore mix only to a few percent and is not eligible for the substantial mixing required by $|G_A/G_V|$. We are then left to decide between the d waves in the $(\underline{70}, 2^+)$ or the p waves in the $(\underline{20}, 1^+)$, for which we compute the electric quadrupole $E2$ transition moment for the decay $\Delta^+ \to p\gamma$ which is zero (up to 4%) in agreement with the exact SU(6) selection rule.[5] Denoting by $\delta_{\dim,L}$ the corresponding coefficients for the Δ, one has

$$E_2 = 3\langle p|Q_\lambda Y_{20}(\Omega_\lambda)|\Delta\rangle \sim (\nu_{56,0}\delta_{70,2} - \delta_{56,0}\nu_{70,2}) . \quad (5)$$

The $(\underline{70}, 2^+)$ contributes with a term linear in the mixing coefficients of the octet, $\nu_{70,2}$, and decuplet $\delta_{70,2}$. Moreover, as we show in the next section, the standard spin-tensor term that could mix the $\underline{56}$ with the $(\underline{70}, 2^+)$ gives the opposite sign for the Δ and N coefficients. But we cannot even impose by hand $\delta_{70,2} \sim \nu_{70,2}$, to make a cancellation in Eq. (5), because then the $(\underline{70}, 2^+)$ would contribute with an incurably negative term [the second of Eq. (6)] to the $M1$ transition moment,

$$\mu_{\Delta p} = 3\langle N|(\tfrac{2}{3}Q_\lambda l_{\lambda_z} + Q_\lambda\sigma_{\lambda_z})|\Delta\rangle = \frac{2\sqrt{2}}{3}\nu_{56,0}\delta_{56,0} - \frac{\sqrt{2}}{3}\nu_{70,2}\delta_{70,2} + \tfrac{1}{3}\nu_{70,2}\delta_{56,2} - \frac{\sqrt{5}}{9}\delta_{70,2}\nu_{20,1} , \quad (6)$$

that would reduce the SU(6) value of $2\sqrt{2}/3$ by 30%, just the opposite of the experimental value.[4] To quantify the above qualitative disagreement with the experimental 4% upper limit for E_2/M_1, one has to compute the ratio E_2/M_1 in a model. In the oscillator, the result is propor-

TABLE II. Formal chiral SU(3) ×SU(3) content of the various SU(6) representations.

SU(6)	$(6,3)_8$	$(3,\bar{3})_8$	$(\bar{3},3)_8$	(3, 6)	(1, 8)	(8, 1)
$\underline{56}, 0^+$	1	⋯	⋯	⋯	⋯	⋯
$\underline{20}, 1^+$	⋯	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	⋯	⋯	⋯
$\underline{70}, 2^+$	$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{3}{20}}$	$\sqrt{\frac{1}{10}}$	$-\sqrt{\frac{3}{20}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{1}{10}}$

tional to the fourth power of the hadron radius, R. For $R\sim 1$ fm, the ratio is just around 4%, but for the standard oscillator radii, $R\sim 0.5$ fm or smaller,[6] the result would be far below the upper limit. On the other hand, if there is no appreciable mixing with $(70, 2^+)$, there is nothing from these decays against mixing of the nucleon with the $(\underline{20}, 1^+)$ nor against $(\underline{56}, 2^+)$ in the Δ, though this last possibility would reduce the magnetic moment of Δ^{++} from the SU(6) prediction

$$\mu(\Delta^{++})=2(1-\tfrac{2}{5}\delta_{56,2}{}^2-\tfrac{3}{5}\delta_{70,2}{}^2)\,. \tag{7}$$

Phenomenological analysis from proton Bremsstrahlung[7] gives $2\times(1\pm 40\%)$, but in order to decrease the error one has to go to higher energies and the analysis (based on Low's theorem) is no longer clear.

From the $\Sigma\to\Lambda$ and $n\to p$ semileptonic decay one obtains the D/F ratio, which in the mixed case would be

$$F/D=\tfrac{2}{3}(1-\nu_{20,1}{}^2/1-\tfrac{2}{3}\nu_{20,1}{}^2)\,, \tag{8}$$

in agreement with the experimental $F/D=0,58$ for the 30% mixing of $(\underline{20}, 1^+)$ that $|G_A/G_V|$ would require.

In summary, we find evidence against substantial mixing with d waves in the $(\underline{70}, 2^+)$ representation for the nucleons and Δ but no evidence against a mixing of the order of 30% for the nucleon with p-wave quarks in the $(\underline{20}, 1^+)$ nor against d waves in $(\underline{56}, 2^+)$ for Δ. A definite test of those representations should be possible in Δ^{++} neutrino production, for which new data will be soon available from Gargamelle.

For completeness, we end the discussion of the phenomenological implications of SU(6) mixing by recalling the chiral SU(3) ⊗ SU(3) representations, given in Table II. From Table II one can compare our results with those obtained from a chiral-representation-mixing framework,[8] even though the connection between its infinite-momentum frame and our static-quark model is not clear.

III. THEORETICAL IMPLICATIONS OF CONFIGURATION MIXING

From the success of the one-gluon perturbation Hamiltonian in explaining the multiplet splitting, the question arises if such terms can generate the various configuration mixings.

The Pauli reduction of the one-gluon exchange reads

$$\begin{aligned} H&=H_0+H_{ss}+H_T+H_{so}\,,\\ H_0&=-\tfrac{2}{3}\alpha_s\sum_{i<j}\frac{1}{r_{ij}}-\frac{1}{2m^2}\left(\frac{\vec{p}_i\cdot\vec{p}_j}{r_{ij}}+\frac{\vec{r}_{ij}\cdot(\vec{r}_{ij}\cdot\vec{p}_i)\vec{p}_j}{r_{ij}^3}\right),\\ H_{ss}&=\frac{16\pi}{9}\frac{\alpha_s}{m^2}\sum_{i<j}\delta(\vec{r}_{ij})\vec{s}_i\cdot\vec{s}_j\,,\\ H_T&=-\frac{2\alpha_s}{3m^2}\sum_{i<j}\frac{1}{r_{ij}^3}\left(\vec{s}_i\cdot\vec{s}_j-\frac{3(\vec{s}_i\cdot\vec{r}_{ij})(\vec{s}_j\cdot\vec{r}_{ij})}{r_{ij}^2}\right),\\ H_{so}&=\frac{\alpha_s}{3m^2}\sum_{i<j}\frac{1}{r_{ij}^3}[(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i-(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j\\ &\qquad+2(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j-2(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i] \end{aligned} \tag{9}$$

where α_s is the gluon coupling constant, $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$ and m the quark mass.

The spin-independent term H_0 can mix the ground state $(\underline{56}, 0^+)$ with radial excitations, in which we are not interested, since they will not change the current matrix elements.

H_{ss} mixes the ground state with $(\underline{70}, 0^+)$ and, using oscillator wave functions and level spacing, one obtains for the mixing coefficient

$$\frac{\nu_{70,0}}{\nu_{56,0}}=\frac{\langle N,(\underline{70},0^+)|H_{ss}|N,(\underline{56},0^+)\rangle}{M_{N=0}-M_{N=2}}=\frac{\alpha_s}{\sqrt{3\pi}}\left(\frac{\omega}{m}\right)^{1/2}, \tag{10}$$

where ω is the oscillator constant and $(\omega/m)^{1/2}$ is the mean quark velocity. The absolute value, $\nu_{70,0^+}{}^2\simeq 0.04$ is rather small, in agreement with the constraint from μ_p/μ_n mentioned after Eq. (4). The mixing angle being positive, it is different from the phenomenological mixing proposed by the Orsay group,[3] which should have a different origin.

The mixing generated by H_T for the nucleon is

$$\frac{\nu_{70,2}}{\nu_{56,0}}=\frac{\langle N,(\underline{70},2^+)|H_T|N,(\underline{56},0^+)\rangle}{M_0-M_2}=-\frac{\alpha_s}{4\pi}\left(\frac{\omega}{m}\right)^{1/2}, \tag{11}$$

while for Δ one has two contributions

$$\frac{\delta_{70,2}}{\delta_{56,0}}=\frac{\langle\Delta,(\underline{70},2)|H_T|\Delta,(\underline{56},0)\rangle}{M_0-M_2}=\frac{\alpha_s}{4\pi}\left(\frac{\omega}{m}\right)^{1/2}, \tag{12}$$

$$\frac{\delta_{56,2}}{\delta_{56,0}} = \frac{\langle \Delta, (\underline{56}, 2)|H_T|\Delta, (\underline{56}, 0)\rangle}{M_0 - M_2} = -\sqrt{2}\, \frac{\alpha_s}{4\pi}\left(\frac{\omega}{m}\right)^{1/2}. \quad (12)$$

As anticipated in the previous section, the main result is that the coefficients for N and Δ are opposite, giving a nonvanishing contribution to the $E2$ transition moment. The absolute value for $\nu_{\underline{70},2^+}$ and $\delta_{\underline{70},2^+}$ obtained from the parameter values of the fit to the spectrum, on the other hand, is very small (from 0.04 to 0.06) so that there is no inconsistency with the matrix elements in the one-gluon perturbed nonrelativistic approach.

Finally, the spin-orbit term could in principle mix the ground state with p waves, but one has

$$\langle N, (\underline{20}, 1)|H_{so}|N, (\underline{56}, 0)\rangle = 0\,. \quad (13)$$

for any one-body SU(3)-invariant spin-orbit term, such as the one given by one-gluon exchange.

IV. CONCLUSIONS

We have seen that the only possibility to explain the SU(6) breaking required by current matrix elements through mixing of the ground state with the lowest configurations in the ordinary quark shell model, would be a substantial mixing with the $(\underline{20}, 1^+)$. We have found the following evidence against d waves: with the mixing generated by an ordinary SU(3)-invariant-tensor term, the absence of $E2$ transitions, a well satisfied selection rule of the quark model, would be violated,[9] and if one chooses by hand the signs of the mixing coefficients to avoid this difficulty, one is in conflict with the magnitude of the $M1$ transition moment for $\Delta^+ \to p\gamma$.

As for the $(\underline{20}, 1^+)$, even though the phenomenology considered here would allow for as much as 30% mixing, in view of the simple analysis without any corrections, we should conclude just that there is a possibility of $(\underline{20}, 1^+)$ mixing, but obviously not that good a one; otherwise one should perhaps have seen $(\underline{20}, 1^+)$ resonances in formation experiments above 4.5 GeV, as 20 is contained in $\underline{35} \otimes \underline{20}$.

From a constituent point of view the quarks in those states would have a high kinetic energy and their spin-orbit splitting should also be larger, against which there is no evidence for the moment.

The operator responsible for the hypothetical mixing cannot be of the SU(3)-invariant type, and there seems to be evidence in the oscillator fit to the baryon spectrum by Jones, Dalitz, and Horgan,[10] for such anomalous spin-orbit terms.

In our opinion, the final test of these mixing possibilities might come from improved data on the neutrino production of Δ^{++}, looking for the $MA2$ selection rule of Andreadis *et al.*[11] Another possibility would be to look for implications in quark antisymmetry in the nucleon distribution functions.

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