General-relativistic treatment of the gravitational coupling between laser beams

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The present paper is an extension of the earlier work of Tolman, Ehrenfest, and Podolsky who investigated the gravitational interaction between "thin pencils of light." We calculate the gravitational field produced by a laser pulse traveling with a velocity $v < c$, and the trajectory of a probe pulse propagating through this field. The amplitude and phase variations of the probe pulse due to the presence of the laser pulse are calculated via the Einstein-Maxwell equations.

I. INTRODUCTION

Some years ago Tolman, Ehrenfest, and Podolsky[1] (TEP) investigated the gravitational interaction between "thin pencils of light." The present paper is an extension of their investigations motivated in part by the modern-day realization of high-intensity "thin pencils of light," i.e., the high-power laser. In view of precision modern optics and high-energy laser sources we are naturally lead to investigate the physical effects resulting from the gravitational interaction between intense laser beams.

In their original paper TEP show that "test rays of light" in the neighborhood of an intense pulse are not deflected when the test ray (probe pulse) is moving parallel with the intense pulse. However, an examination of their calculation shows that this is due to the fact that both pulses are moving on the light cone, i.e., are moving *in vacuo*. In the present calculation we show that if our probe and high-power pulses are propagating with velocities less than the speed of light, interesting interactions between the pulses can occur. For example, as indicated in Fig. 1, when an intense pulse propagates down a guided wave structure (which is essentially a hole bored out of a dielectric) and the probe pulse propagates in the dielectric parallel to the optical wave guide, a finite deflection δy occurs when the two pulses propagate in the same direction. Furthermore, we show that more subtle interactions can occur requiring the coupled Einstein-Maxwell equations for their description.

The gravitational field $h_{\mu\nu}$ produced by our high-power laser pulse is presented in Sec. II. In Sec. III we investigate the deflection of an optical pulse traveling through the gravitational field thus produced. This is essentially a ray-optical problem. However, in Sec. IV we proceed to calculate associated wave-optical phenomena. Using the coupled Einstein-Maxwell equations we find amplitude and phase variations of the probe pulse due to the presence of the high-intensity pulse. Discussion and numerical estimates are included in Sec. V.

The main thrust of this paper is theoretical. However, as argued in Sec. V, while the effects of gravitational coupling between laser pulses are very small, they are not small beyond the point of conceivable measurement. In fact, if the next 50 years sees a fraction of the technological development that has taken place since the TEP paper 50 years ago, we may well see experiments along these lines one day.

It is our intention to make this paper as self-contained as possible since it is hoped that some members of the quantum optics community may be amused by these calculations. To this end we have attempted to carefully define our terms and include some calculational details in appendices.

II. GRAVITATIONAL FIELD PRODUCED BY HIGH-POWER LASER PULSE

In this section, we consider the influence of our intense pulse on the curvature of local spacetime. That is, we calculate

$$g_{\mu\nu}(\vec{r},t)=\eta_{\mu\nu}+h_{\mu\nu}(\vec{r},t)\,, \tag{2.1}$$

where, in our convention, the flat spacetime metric is given by

$$\eta_{\mu\nu}=\begin{bmatrix} c^2 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix} \tag{2.2}$$

and $h_{\mu\nu}$ characterizes the contribution to the metric produced by the high-power laser pulse. Equation (2.2), of course, implies that

$$(x^0,x^1,x^2,x^3)=(t,x,y,z)\,.$$

We choose to display the speed of light *in vacuo* explicitly, because the velocities of our high-power laser pulse and probe pulse are given by v and u, respectively. With these three velocities entering our problem we find it useful to keep them all in full view rather than setting $c=1$.

The small correction $h_{\mu\nu}$ due to the presence of the laser pulse obeys the linearized Einstein equation[2]

$$\Box^2 h_{\mu\nu} = \kappa(T_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}T) \tag{2.3a}$$

in which the constant κ is given by

$$\kappa = 16\pi G/c^2 . \tag{2.3b}$$

In Eqs. (2.3a) and (2.3b), G is the gravitational constant, $T = T^{\alpha}{}_{\alpha}$, and the stress-energy tensor $T_{\mu\nu}$ is defined by[3]

$$T_{\mu\nu} = \epsilon_0(F_{\mu}{}^{\lambda}F_{\lambda\nu} + \tfrac{1}{4}\eta_{\mu\nu}F^{\sigma\rho}F_{\sigma\rho}) . \tag{2.4}$$

The covariant electromagnetic field tensor in our notation reads

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{bmatrix} \tag{2.5}$$

and the contravariant counterpart of Eq. (2.5) is given by the usual expression

$$F^{\mu\nu} = g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} . \tag{2.6}$$

We now proceed to calculate $h_{\mu\nu}$ as produced by the intense pulse. We assume the pulse propagates in the x direction with a velocity $v < c$. We could accomplish this by imagining our pulse to be traveling through a material medium having an index of refraction n or, perhaps more realistically, traveling down a "wave-guide" structure. For the purposes of this paper let us consider the pulse as propagating down such a "wave guide" in a transverse electric (TE) configuration. The essential features of the $\vec{E}$ and $\vec{B}$ fields are then summarized by the expressions[4]

$$E_2(x,t) = \mathcal{E}(\vec{r},t)\sin(\nu t - kx) , \tag{2.7a}$$

$$B_3(x,t) = \left(\frac{v}{c}\right)\frac{\mathcal{E}(\vec{r},t)}{c}\sin(\nu t - kx) , \tag{2.7b}$$

$$B_1(x,t) = \left[1 - \left(\frac{v}{c}\right)^2\right]^{1/2}\frac{\mathcal{E}(\vec{r},t)}{c}\cos(\nu t - kx) , \tag{2.7c}$$

where $\mathcal{E}(x,t)$ denotes the envelope of our pulse moving with velocity v, and we have ignored spatial variation in the y and z directions. Now, for a "thin" tightly focused pulse of duration T, we may write

$$\mathcal{E}^2(\vec{r},t) = E_0{}^2 A[\theta(v(t+T) - x) - \theta(vt - x)]\,\delta(y)\delta(z) , \tag{2.8}$$

where E_0 is the pulse amplitude, A is the effective cross-sectional area, and the step function $\theta(x)$ is defined by

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases} .$$

As shown in Appendix A, we then find the "pulse-induced" contribution to the metric to be given by

$$h_{\mu\nu} = h(\vec{r},t)M_{\mu\nu} , \tag{2.9}$$

where

$$h(\vec{r},t) = -(4G\rho A/c^2)\ln\left(\frac{v(t+T) - x + [(v(t+T) - x)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2}}{vt - x + [(vt - x)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2}}\right) \tag{2.10}$$

and

$$M_{\mu\nu} = \begin{bmatrix} 1 & -\frac{v}{c^2} & 0 & 0 \\ -\frac{v}{c^2} & \frac{v^2}{c^4} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2}\left[1 - \left(\frac{v}{c}\right)^2\right] \end{bmatrix} . \tag{2.11}$$

The radiation energy density ρ appearing in Eq. (2.10) is defined as

$$\rho = \tfrac{1}{2}\epsilon_0 E_0{}^2 . \tag{2.11a}$$

Finally, we note that for a short pulse such that

$$vT/[(x - vt)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2} \ll 1 , \tag{2.11b}$$

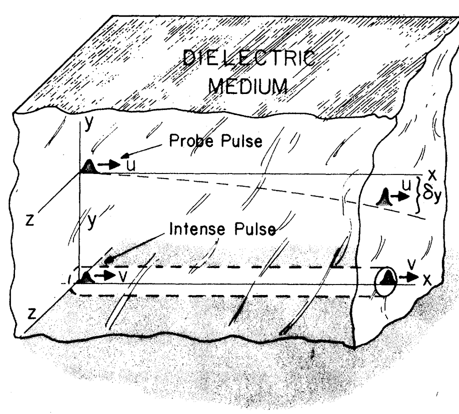

FIG. 1. Figure illustrating intense laser pulse propagating with velocity v through dielectric wave guide (i.e., hole bored in dielectric medium) and parallel-propagating probe pulse. Probe pulse is deflected by an amount δy as it moves through the medium with velocity u.

Eq. (2.10) becomes

$$h(r,t)=\frac{-(4G\rho V/c^2)}{[(x-vt)^2+(1-v^2/c^2)(y^2+z^2)]^{1/2}}\,, \quad (2.12)$$

where the "volume" of our pulse is given by

$$V=AvT\,.$$

For future reference we note that on the x axis Eq. (2.12) reduces to

$$h(x,t)=\frac{-(4G\rho V/c^2)}{|x-vt|}\,. \quad (2.13)$$

III. DEFLECTION OF PROBE PULSE

Consider now a weak probe pulse traveling in the x direction with a velocity u and in the vicinity of our high-power pulse (see Fig. 1). The probe pulse follows a trajectory determined by the gravitational potential due to the intense pulse, i.e., $h_{\mu\nu}(\vec{r},t)$ of Eq. (2.9). More explicitly our probe pulse obeys the general-relativistic analogy of Newton's second law, namely,

$$\frac{d^2x^\mu}{d\tau^2}+\begin{Bmatrix}\mu\\ \sigma\ \ \rho\end{Bmatrix}\frac{dx^\sigma}{d\tau}\frac{dx^\rho}{d\tau}=0\,, \quad (3.1)$$

where τ is a time parameter and $\{{}^{\ \mu}_{\sigma\ \rho}\}$, the Christoffel symbol of the second kind, is defined by

$$\begin{Bmatrix}\mu\\ \sigma\ \ \rho\end{Bmatrix}=\frac{1}{2}g^{\mu\nu}\left(\frac{\partial g_{\sigma\nu}}{\partial x^\rho}+\frac{\partial g_{\rho\nu}}{\partial x^\sigma}-\frac{\partial g_{\sigma\rho}}{\partial x^\nu}\right). \quad (3.2)$$

If we consider deflection in the y direction, then Eq. (1) implies

$$\frac{d^2x^2}{d\tau^2}+\begin{Bmatrix}2\\ \sigma\ \ \rho\end{Bmatrix}\frac{dx^\sigma}{d\tau}\frac{dx^\rho}{d\tau}=0\,. \quad (3.3)$$

The Christoffel symbol appearing in Eq. (3.3) can be calculated from Eqs. (3.2), (2.1), and (2.9) as follows: To the lowest order in $h_{\mu\nu}$ we may write

$$\begin{Bmatrix}2\\ \sigma\ \ \rho\end{Bmatrix}=\frac{1}{2}\eta^{22}\left(\frac{\partial h_{\sigma 2}}{\partial x^\rho}+\frac{\partial h_{\rho 2}}{\partial x^\sigma}-\frac{\partial h_{\sigma\rho}}{\partial x^2}\right), \quad (3.4)$$

where we have used the fact that $\eta^{\mu\nu}$ is diagonal. We may further simplify (3.4) by explicitly taking $h_{\mu\nu}$ from (2.9), obtaining

$$\begin{Bmatrix}2\\ \sigma\ \ \rho\end{Bmatrix}=\frac{1}{2}\frac{\partial h(\vec{r},t)}{\partial x^2}M_{\sigma\rho}\,, \quad (3.5)$$

where we have made use of the fact that

$$h_{\sigma 2}=0 \text{ for } \sigma=0,1,2,3 \quad (3.6)$$

in our problem.

Relations (3.3) and (3.5) then yield

$$\frac{d^2x^2}{d\tau^2}+\frac{1}{2}\frac{\partial h(\vec{r},t)}{\partial x^2}M_{\sigma\rho}\frac{dx^\sigma}{d\tau}\frac{dx^\rho}{d\tau}=0\,. \quad (3.7)$$

Writing this out explicitly, using (2.11), we have

$$\frac{d^2x^2}{d\tau^2}+\frac{1}{2}\frac{\partial h}{\partial x^2}\left[\left(\frac{dx^0}{d\tau}\right)^2-2\frac{\beta}{c}\frac{dx^1}{d\tau}+\frac{\beta^2}{c^2}\left(\frac{dx^1}{d\tau}\right)^2+\frac{1-\beta^2}{c^2}\left(\frac{dx^3}{d\tau}\right)^2\right]=0\,, \quad (3.8)$$

where $\beta=v/c$. We now choose the temporal parameter τ to be the time t, as measured in the laboratory frame. Then the time derivatives of x^μ as they appear in (3.8) take on their apparent physical meaning, and we write Eq. (3.8) as

$$\frac{d^2x^2}{dt^2}+\frac{1}{2}\frac{\partial h}{\partial x^2}\left(1-2\frac{\beta u}{c}+\frac{\beta^2u^2}{c^2}+\frac{1-\beta^2}{c^2}w^2\right)=0\,, \quad (3.9)$$

where u and w are the velocities of the probe pulse in the x and z directions. For the present problem we may choose w arbitrarily small and neglect the last term in parentheses in (3.9) and write

$$\frac{d^2x^2}{dt^2}+\frac{1}{2}\frac{\partial h}{\partial x^2}(1-\beta\beta')^2=0 \quad (3.10)$$

where $\beta'=u/c$. From (2.12) and (3.10) we find

$$\frac{d^2y}{dt^2}+\frac{(2G\rho V/c^2)(1-\beta^2)y}{[(x-vt)^2+(1-\beta^2)(y^2+z^2)]^{3/2}}(1-\beta\beta')^2=0\,, \quad (3.11)$$

where, for typographic convenience, we have reverted to an (x,y,z) notation.

Note that when $\beta=1$ the deflection vanishes,[6] that is, it is essential that the intense laser pulse propagate with a velocity $v<c$ in order for our probe pulse to "feel" its presence. Under these conditions an acceleration (deflection) of the probe pulse is apparent. The magnitude and conditions for possible observation of this result are discussed in Sec. V.

IV. AMPLITUDE AND PHASE VARIATIONS CALCULATED VIA EINSTEIN-MAXWELL EQUATIONS

The coupled Einstein-Maxwell equations are used in this section to calculate the amplitude and phase variations of our probe pulse due to the intense laser pulse. Here we have in mind a physical "setup" such that the high-intensity pulse (wavelength λ_h) propagates down a "wave guide" and the probe pulse (wavelength λ_p) follows. Note, however, that the velocity of the probe pulse is not the same as that of the high-power pulse. In particular, the probe pulse may be moving with a speed c. Although the calculation becomes a bit messy, the strategy is straightforward and proceeds as follows:

(1) The first (high-power) pulse contributes a tiny correction to the metric of flat spacetime $h_{\mu\nu}$.

(2) The second (probe) pulse is influenced by $h_{\mu\nu}(\vec{r},t)$ as produced by our first pulse. It is well (if not widely) known that an electromagnetic pulse

moving through curved spacetime reacts just as if it were propagating through ordinary "matter" having finite electric and magnetic susceptibility.[7]

(3) The results of propagating through the "pseudomatter" of point (2) are calculated following the usual approach of quantum optics.

We proceed with the calculation as outlined above. Chore (1) has already been carried out in Sec. II. Proceeding with (2), Maxwell equations for our probe pulse are

$$\frac{\partial\sqrt{|g|}F^{\mu\nu}}{\partial x^{\nu}}=0 \tag{4.1a}$$

and

$$\frac{\partial F_{\mu\nu}}{\partial x^{\lambda}}+\frac{\partial F_{\lambda\mu}}{\partial x^{\nu}}+\frac{\partial F_{\nu\lambda}}{\partial x^{\mu}}=0\,, \tag{4.1b}$$

where $|g|=|\det g|$. The metric, and therefore the influence of the high-power pulse, appear in the Maxwell equations through $|g|$ and $g^{\mu\nu}$ in Eq. (4.1a), since

$$F^{\mu\nu}=(\eta^{\mu\sigma}+h^{\mu\sigma})(\eta^{\nu\rho}+h^{\nu\rho})F_{\sigma\rho}\,. \tag{4.2}$$

We assume our probe pulse to be TEM and to propagate in the x direction, i.e.,

$$\vec{E}=\begin{bmatrix}0\\ E_2(\vec{r},t)\\ 0\end{bmatrix},\quad \vec{B}=\begin{bmatrix}0\\ 0\\ B_3(\vec{r},t)\end{bmatrix}. \tag{4.3}$$

Then from (2.2), (2.9), (2.5), (4.2), and (4.3) we find, as shown in Appendix B,

$$F^{\mu\nu}=\begin{bmatrix}0 & 0 & E_2/c^2-\gamma E_2 & 0\\ 0 & 0 & B_3-\gamma vE_2 & 0\\ -E_2/c^2+\gamma E_2 & -B_3+\gamma vE_2 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{4.4}$$

where $\gamma=h(1-\beta)/c^4$.

Substituting (4.4) in (4.1a) and noting that

$$\sqrt{|g|}=c+O(h^2)\,,$$

we have with $\mu=2$

$$\frac{\partial(cF^{2\nu})}{\partial x^{\nu}}=0\,, \tag{4.5}$$

which, in view of (4.4), leads to the simple relation

$$\frac{\partial F^{20}}{\partial x^0}+\frac{\partial F^{21}}{\partial x^1}=0\,. \tag{4.6a}$$

Writing (4.6a) in terms of $\vec{E}$ and $\vec{B}$ from (4.4) we find, after some simple algebra,

$$\frac{1}{c^2}\frac{\partial E_2}{\partial t}+\frac{\partial B_3}{\partial x}=\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)\left(\frac{h}{c^4}(1-\beta)E_2\right). \tag{4.6b}$$

The other Maxwell equation (4.1b) implies

$$\frac{\partial B_3}{\partial t}+\frac{\partial E_2}{\partial x}=0\,. \tag{4.7}$$

Finally, by use of (4.6b) and (4.7) we obtain

$$\frac{1}{c^2}\frac{\partial^2E_2}{\partial t^2}-\frac{\partial^2E_2}{\partial x^2}=\frac{\partial}{\partial t}\left(\frac{1}{v}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\left(\frac{h(\vec{r},t)v}{c^4}\right)(1-\beta)E_2\,. \tag{4.8}$$

Since the probe pulse is well focused and traveling down the x axis, we may take $y=z=0$ so that $h(x,t)$ is given by Eq. (2.13), and (4.8) becomes

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)E_2(x,t)=\alpha\frac{\partial}{\partial t}\left(\frac{1}{v}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\frac{E_2(x,t)}{|vt-x|}\,, \tag{4.9}$$

where

$$\alpha=-\frac{v}{c^4}(1-\beta)(4G\rho V/c^2)\,. \tag{4.10}$$

Thus we see that our probe pulse obeys a "driven wave equation" much as if it were propagating through an ordinary material medium.

We note that (4.9) describes a probe pulse propagating in free space while experiencing a gravitational interaction with the high-power pulse. If the probe were propagating in a dielectric medium with velocity u, then one can show that (4.9) and (4.10) become

$$\left(\frac{1}{u^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)E_2(x,t)=\alpha'\frac{\partial}{\partial t}\left(\frac{1}{v}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\frac{E_2}{|vt-x|}\,, \tag{4.9$'$}$$

where

$$\alpha'=-\frac{v}{c^2u^2}\left(1-\frac{uv}{c^2}\right)(4G\rho V/c^2)\,. \tag{4.10$'$}$$

However, for the physical "setup" as described in the beginning of this section, Eq. (4.9) is appropriate.

The physics in (4.9) is made clear by considering the special case of a plane-wave probe pulse of the form

$$E_2(x,t)=E(x,t)\cos[kx-\omega t+\phi(x,t)]\,, \tag{4.11}$$

where $E(x,t)$ and $\phi(x,t)$ are the envelope and phase, and

$$B_3(x,t)=\frac{E(x,t)}{c}\cos[kx-\omega t+\phi(x,t)] \tag{4.12}$$

Following the usual quantum-optical approach[8] (as mentioned in point 3 at the beginning of this section), we invoke the slowly varying amplitude and phase approximation as indicated below

$$\frac{\partial E(x,t)}{\partial x} \ll kE(x,t)\,,\qquad \frac{\partial E(x,t)}{\partial t} \ll \omega E(x,t)\,, \tag{4.13a}$$

$$\frac{\partial \phi(x,t)}{\partial x} \ll k\phi(x,t)\,,\qquad \frac{\partial \phi(x,t)}{\partial t} \ll \omega \phi(x,t)\,. \tag{4.13b}$$

Using (4.13a) and (4.13b), as shown in Appendix B, we obtain the following equations for $E(x,t)$ and $\phi(x,t)$:

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)E(x,t)=\left[\frac{\alpha\omega}{2k}(1-\beta)\frac{1}{(vt-x)^2}\right]E(x,t)\,, \tag{4.14a}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\phi(x,t)=\frac{\omega}{c}-k-\left[\frac{\alpha\omega^2}{2kv}(1-\beta)\frac{1}{vt-x}\right]. \tag{4.14b}$$

From (4.14a) we see that the amplitude of our probe pulse changes due to the gravitational coupling with the high-intensity pulse. Likewise the phase of the probe pulse as determined by (4.14b) will differ from its free-space (no high-power pulse) value $(\omega/c-k)x$. These results are further discussed in Sec. V, but it should be noted that such effects vanish when $v=c$, just as was the case in the deflection of the probe pulse.

V. DISCUSSION AND NUMERICAL ESTIMATES

A. Deflection of probe pulse

Consider the acceleration of our probe pulse when it is traveling in the xy plane with $\beta=\beta'$ and at $x=vt$ (see Fig. 1). In that case the acceleration of the probe toward the high-power pulse, as given by (3.11), becomes

$$\frac{d^2y}{dt^2}=-\frac{2Gm_{\text{eff}}}{y^2}(1-\beta^2)^{3/2}\,, \tag{5.1}$$

where m_{eff} is the energy in the high-power pulse divided by c^2. This is to be compared with the similar result calculated via simple "Newtonian" theory

$$\frac{d^2y}{dt^2}=-\frac{Gm_{\text{eff}}}{y^2}\,. \tag{5.2}$$

It is interesting to note that the general-relativistic result is roughly twice that obtained from Newtonian theory. The appearance of this factor of 2 is closely related to the fact that general relativity predicts a deflection of light in a gravitational field which is twice what a naive Newtonian calculation yields. Furthermore the acceleration d^2y/dt^2 vanishes when $\beta=1$.

Let us consider the order of magnitude of the deflection determined by (5.1). We are interested not in the precise deflection, but rather in ascertaining whether such a deflection could ever be observed or whether we would have to monitor the experiment for cosmic distances to see an effect. With this object in mind, let us further consider the *Gedanken* experiment sketched above. The typical dimensions of our probe and high-power pulses are taken as λ_p and λ_h, i.e., the wavelengths of the probe and high-power pulses, respectively.[9] Let us assume that we must deflect our probe pulse by say $\simeq 10^{-2}\lambda_p$ in order to detect the deflection. How far must we propagate in order to observe this deflection? If we assume $\lambda_p\sim\lambda_h=\lambda$, that typical distances y are likewise of order λ, and that $\beta=\beta'\sim 0.9$, then we have an acceleration

$$\frac{d^2y}{dt^2}\sim\frac{(G\epsilon_0 E_0{}^2 V/c^2)10^{-3/2}}{\lambda^2}\,. \tag{5.3}$$

In a time δt a deflection δy of magnitude

$$\delta y\sim[10^{-2}(G\epsilon_0 E_0{}^2V/c^2)\lambda^{-2}](\Delta t)^2 \tag{5.4}$$

would be observed, and a deflection $\delta y\sim 10^{-2}\lambda$ would be observed in a propagation distance $L=c\delta t$ given by[10]

$$L\sim[\lambda^3c^4/(G\epsilon_0E_0{}^2V)]^{1/2}\,. \tag{5.5}$$

The pulsed lasers presently under development for fusion applications involve megajoule energies, with $\lambda\sim 1\mu$, on a nanosecond time scale. If we consider our high-energy pulse to be a "mode-locked" picosecond pulse, then the pulse length could be of order $(10 \text{ to } 10^2)\lambda$, and our short-pulse limit is not wildly unreasonable. In such a case Eq. (5.5) yields a propagation estimate of

$$L(\lambda=1\mu)\sim 10^6\ \text{km}\,. \tag{5.6a}$$

Although this is a large distance, it is not cosmic and might be realized by bouncing a pulse between mirrors spaced ~1 km apart.

Finally, let us imagine that we have a high-power UV laser with a photon flux similar to that of the IR laser described above. Carrying out a deflection estimate as in the IR case, but now assuming $\lambda\sim 10^3$ Å, we find a hundredth wave deflection in a distance

$$L(\lambda\sim 10^3\ \text{Å})\sim 10^4\ \text{km}\,. \tag{5.6b}$$

We conclude that based on conceivable, 1μ and imaginable, 10^3Å, laser parameters experimentally interesting deflections are potentially attainable in "laboratory" distances.[11]

B. *Gedanken* experiment based on Einstein-Maxwell phase-shift effect

In order to establish order-of-magnitude estimates for the amplitude and phase variations of Sec. IV, we introduce the variables

$$\xi = x \tag{5.7a}$$

and

$$\tau = t - x/c \,. \tag{5.7b}$$

Substituting (5.7a) and (5.7b) into (4.14a) and (4.14b) we find

$$\frac{\partial E}{\partial \xi}(\xi,\mu) = \left(\frac{\alpha\omega}{2k}(1-\beta)\frac{1}{\mu^2}\right)E(\xi,\mu)\,, \tag{5.8}$$

$$\frac{\partial \phi}{\partial \xi}(\xi,\mu) = \frac{\omega}{c} - k - \left(\frac{\alpha\omega^2}{2kv}(1-\beta)\frac{1}{\mu}\right), \tag{5.9}$$

where we have denoted the separation between high intensity and probe pulse by

$$v\tau = vt - x \equiv \mu \,. \tag{5.10}$$

Thus the pulse amplitude varies exponentially as

$$E(x) = E(0)\exp\left(\int_0^x \Gamma(\mu)dx'\right), \tag{5.11}$$

where

$$\Gamma(\mu) = \frac{\alpha\omega}{2k}\frac{1-\beta}{\mu^2}\,. \tag{5.12}$$

Likewise the phase shift obtained is given by

$$\delta\phi(x) \equiv \phi(x) - \left(\frac{\omega}{c} - k\right)x$$

$$= \int_0^x \frac{-\alpha\omega^2}{2kv}\frac{1-\beta}{\mu}\,dx\,. \tag{5.13}$$

Small variations in amplitude, such as implied by Eq. (5.11), are essentially unmeasurable. It is much more reasonable to consider measuring small phase shifts; therefore, let us consider $\delta\phi$ in more detail. Taking α from Eq. (4.10), the phase-shift expression (5.13) may be estimated as

$$\delta\phi \sim \int_0^x \left(\frac{1}{c^5}(G\epsilon_0 E_0{}^2 V)(1-\beta)^2\frac{\omega}{\mu}\right)dx'\,. \tag{5.14a}$$

Using the UV laser parameters as discussed above in Eq. (5.14a), we estimate[12] the phase shift to go roughly as

$$\delta\phi \sim (10^{-20}\ \text{m}^{-1})x\,, \tag{5.14b}$$

a very small change. In fact, if one asks the man on the street "what is the smallest phase shift that could be measured," the usual answer is, "I can see a 'hundredth-wave' change in length so $\delta\phi = 2\pi\delta L/\lambda \sim 10^{-2}$." However, we can gain some encouragement by considering the precision measurements made possible by modern stable laser techniques. In recent experiments Jacobs and coworkers have made measurements of length changes δL to incredible precision.[13] They make use of the fact that the eigenfrequencies for a Fabry-Perot cavity are given by

$$\nu = n\pi c/L\,, \tag{5.15}$$

where n is the number of half-waves and L is the cavity length, to write

$$\delta\nu/\nu = \delta L/L\,. \tag{5.16}$$

Using heterodyne techniques they can measure $\delta\nu$ to a few hertz, and since, for their IR laser, $\nu \sim 10^{15}$ Hz, they have a sensitivity

$$\delta L/L \sim 10^{-15}\,. \tag{5.17}$$

Hence, for a 1-m cavity they "see" average displacements of a small fraction of a nuclear diameter. For a UV laser ($\lambda \sim 10^3$ Å) $\nu \sim 10^{16}$ Hz, and recent work has yielded frequency difference measurements of order[14] $\delta\nu \sim 10^{-8}$ Hz, so that we might reasonably have for a sensitivity

$$\delta L/L \sim 10^{-24}\,. \tag{5.18}$$

Let us denote the phase shift as

$$\delta\phi = \delta kL\,, \tag{5.19}$$

and further write this in terms of an effective displacement δL times the wave vector k, that is

$$\delta kL = k\delta L\,. \tag{5.20}$$

Therefore, we may write

$$\delta k = k\delta L/L\,,$$

then for $k \sim 10^8\ \text{m}^{-1}$ and taking $\delta L/L$ from (5.18), we have $\delta k \sim 10^{-16}\ \text{m}^{-1}$, and from (5.19) this implies a phase shift

$$\delta\phi \sim (10^{-16}\ \text{m}^{-1})L\,. \tag{5.21}$$

Comparing the "measurable" phase shift (5.21) with the estimated effect (5.14a), we see that a long interaction length "x" ($\sim 10^2$ m) and a short cavity length "L" (~1 cm) are suggested. This *Gedanken* experiment is clearly not an experimental call to arms, but rather an argument that such experiments are "thinkable."

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APPENDIX A: $h_{\mu\nu}$ DUE TO TE GUIDED WAVE

We first calculate the stress energy tensor for an arbitrary electromagnetic field from (2.4), (2.5), and (2.6) as well as the expression for $F_\mu{}^\lambda$,

$$F_\mu{}^\lambda = \eta^{\lambda\sigma} F_{\mu\sigma} . \tag{A1}$$

We find the stress energy tensor

$$\frac{T_{\mu\nu}}{\epsilon_0} = \begin{bmatrix} \frac{1}{2}(E^2+c^2B^2) & -(E_2B_3-E_3B_2) & -(E_3B_1-E_1B_3) & -(E_1B_2-E_2B_1) \\ -(E_2B_3-E_3B_2) & [\frac{1}{2}(E^2+c^2B^2)-(E_1{}^2+c^2B_1{}^2)]\frac{1}{c^2} & -\left(\frac{E_1E_2}{c^2}+B_1B_2\right) & -\left(\frac{E_1E_3}{c^2}+B_1B_3\right) \\ -(E_3B_1-E_1B_3) & -\left(\frac{E_1E_2}{c^2}+B_1B_2\right) & [\frac{1}{2}(E^2+c^2B^2)-(E_2{}^2+c^2B_2{}^2)]\frac{1}{c^2} & -\left(\frac{E_2E_3}{c^2}+B_2B_3\right) \\ -(E_1B_2-E_2B_1) & -\left(\frac{E_1E_3}{c^2}+B_1B_3\right) & -\left(\frac{E_2E_3}{c^2}+B_1B_3\right) & [\frac{1}{2}(E^2+c^2B^2)-(E_3{}^2+c^2B_3{}^2)]\frac{1}{c^2} \end{bmatrix} . \tag{A2}$$

Inserting Eqs. (2.7a), (2.7b), (2.7c) into (A2) we find

$$\frac{T_{\mu\nu}}{\epsilon_0} = \begin{bmatrix} \frac{1}{2}\mathcal{E}^2(\vec{r},t) & -\frac{1}{2}\frac{v}{c^2}\mathcal{E}^2(\vec{r},t) & 0 & 0 \\ -\frac{1}{2}\frac{v}{c^2}\mathcal{E}^2(\vec{r},t) & \frac{1}{2}\frac{v^2}{c^4}\mathcal{E}^2(\vec{r},t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\beta^2}{2c^2}\mathcal{E}^2(\vec{r},t) \end{bmatrix} \tag{A3}$$

where we have replaced high-frequency terms such as $\sin^2\omega t$, etc. by $\frac{1}{2}$: By so doing we are neglecting terms which go as $\frac{1}{2}\sin\omega t$, which average to zero in a short time. We may then write Eq. (A3) as

$$T_{\mu\nu} = \tfrac{1}{2}\epsilon_0 \mathcal{E}^2(\vec{r},t) M_{\mu\nu} , \tag{A4}$$

where

$$M_{\mu\nu} = \begin{bmatrix} 1 & \frac{-\beta}{c} & 0 & 0 \\ \frac{-\beta}{c} & \frac{\beta^2}{c^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2}(1-\beta^2) \end{bmatrix} . \tag{A5}$$

Next we note that the trace of $T_{\mu\nu}$ vanishes, that is,

$$T^\alpha{}_\nu = \eta^{\alpha\mu} T_{\mu\nu}$$

$$= \tfrac{1}{2}\epsilon_0 \mathcal{E}^2(\vec{r},t) \begin{bmatrix} \frac{1}{c^2} & \frac{-\beta}{c^3} & 0 & 0 \\ \frac{-\beta}{c^3} & \frac{-\beta^2}{c^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{c^2}(1-\beta^2) \end{bmatrix} ,$$

and therefore $T^\alpha{}_\alpha = 0$.

Hence, Eq. (2.3) becomes

$$\Box^2 h_{\mu\nu} = \kappa T_{\mu\nu} . \tag{A6}$$

From Eqs. (A4), (A5), and (A6) we see that

$$\Box^2 h_{00} = \kappa[\tfrac{1}{2}\epsilon_0 \mathcal{E}^2(x,t)] \tag{A7}$$

and

$$h_{01} = h_{10} = \frac{-\beta}{c} h_{00} , \tag{A8}$$

while

$$h_{33} = \frac{-1}{c}(1-\beta^2) h_{00} . \tag{A9}$$

We then have only to solve one differential equation in order to obtain h_{00}, h_{01}, and h_{33}. That equation is

$$\Box^2 h(\vec{r},t) = \kappa(\tfrac{1}{2}\epsilon_0 \mathcal{E}^2(\vec{r},t)) , \tag{A10}$$

where we have defined

$$h(\vec{r},t) \equiv h_{00}(r,t) . \tag{A11}$$

For a pulse moving with a speed v in the x direction we may write $h(r,t) = h(\vec{x}-vt,y,z)$. For this function the substitutions of $\bar{x} = x - vt$ changes the d'Alembertian of Eq. (A10) to

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)=(1-\beta^2)\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\,; \tag{A12}$$

now if we define

$$x=(1-\beta^2)^{1/2}\bar{x}\,, \tag{A13a}$$

$$y=\bar{y}\,, \tag{A13b}$$

$$z=\bar{z}\,, \tag{A13c}$$

we see that (A12) becomes

$$\frac{\partial^2}{\partial\bar{x}^2}+\frac{\partial^2}{\partial\bar{y}^2}+\frac{\partial^2}{\partial\bar{z}^2}\equiv\bar{\nabla}^2\,. \tag{A14}$$

This trick, which effectively removes time from the problem, is well known in electromagnetism.

In view of the above, Eq. (A10) now reads

$$\bar{\nabla}^2 h(\bar{x},\bar{y},\bar{z},\bar{t})=(\kappa\rho A)[\theta(v(t+T)-(1-\beta^2)^{1/2}\bar{x}) - \theta(vt-(1-\beta^2)^{1/2}\bar{x})]\delta(\bar{y})\delta(\bar{z})\,, \tag{A15}$$

where we have taken $\mathcal{E}(\vec{r},t)$ from (2.8) used (A13a), (A13b), (A13c) and the radiation energy density $\frac{1}{2}\epsilon_0 E_0$ is denoted by ρ. Since (A15) involves only $\bar{\nabla}^2$ (instead of $\Box^2$), we are relieved of worrying about time retardation, etc., and the solution is

$$\begin{aligned} h &= -\frac{1}{4\pi}(\kappa\rho A)\int_{-\infty}^{\infty}d\bar{x}'\int_{-\infty}^{\infty}d\bar{y}'\int_{-\infty}^{\infty}d\bar{z}'\,\frac{[\theta(v(t+T)-(1-\beta^2)^{1/2}\bar{x}')-\theta(vt-(1-\beta^2)^{1/2}\bar{x}')]\delta(\bar{y}')\delta(\bar{z}')}{[(\bar{x}-\bar{x}')^2+(\bar{y}-\bar{y}')^2+(\bar{z}-\bar{z}')^2]^{1/2}} \\ &= -\frac{1}{4\pi}(\kappa\rho A)\ln\left(\frac{[v(t+T)-x]+\{[v(t+T)-x]+(1-\beta^2)(y^2+z^2)\}^{1/2}}{(vt-x)+[(vt-x)^2+(1-\beta^2)(y^2+z^2)]^{1/2}}\right). \end{aligned} \tag{A16}$$

Equation (A16) together with (A7), (A8), and (A9) are concisely summarized in Eq. (2.9).

APPENDIX B: DERIVATION OF EQS. (4.4) AND (4.14a) and (4.14b)

From Eq. (4.2) we have

$$F^{\mu\nu}=\eta^{\mu\sigma}\eta^{\nu\rho}F_{\sigma\rho}+\eta^{\mu\sigma}h^{\nu\rho}F_{\sigma\rho}+h^{\mu\sigma}\eta^{\nu\rho}F_{\sigma\rho}+O(h^2)\,. \tag{B1}$$

To lowest order in h we may write

$$g^{\mu\nu}=\eta^{\mu\nu}-\eta^{\mu\rho}h_{\rho\sigma}\eta^{\sigma\nu}\,, \tag{B2}$$

so that

$$h^{\mu\nu}=-\eta^{\mu\rho}h_{\rho\sigma}\eta^{\sigma\nu}\,. \tag{B3}$$

Hence the last two terms in (B1) are

$$\eta^{\mu\sigma}h^{\nu\rho}F_{\sigma\rho}=-\eta^{\nu\alpha}h_{\alpha\beta}(\eta^{\mu\sigma}F_{\sigma\rho}\eta^{\rho\beta})\,, \tag{B4}$$

while

$$h^{\mu\sigma}\eta^{\nu\rho}F_{\sigma\rho}=-\eta^{\mu\alpha}h_{\alpha\beta}(\eta^{\beta\sigma}F_{\sigma\rho}\eta^{\rho\nu})\,. \tag{B5}$$

At this point it is convenient to define the "flat" space tensor field strength

$$\mathfrak{F}^{\mu\nu}=\eta^{\mu\sigma}F_{\sigma\rho}\eta^{\rho\nu}\,. \tag{B6}$$

From Eqs. (B1), (B4), (B5), and (B6) we may write $F^{\mu\nu}$ as

$$F^{\mu\nu}=\mathfrak{F}^{\mu\nu}-\eta^{\nu\alpha}h_{\alpha\beta}\mathfrak{F}^{\mu\beta}-\eta^{\mu\alpha}h_{\alpha\beta}\mathfrak{F}^{\beta\nu}\,. \tag{B6$'$}$$

Inserting $F_{\sigma\rho}$ from Eq. (2.5) into (B6) we obtain the usual flat-space contravariant field tensor

$$\mathfrak{F}^{\mu\nu}=\begin{bmatrix} 0 & E_1/c^2 & E_2/c^2 & E_3/c^2 \\ -E_1/c^2 & 0 & B_3 & -B_2 \\ -E_2/c^2 & -B_3 & 0 & B_1 \\ -E_3/c^2 & B_2 & -B_1 & 0 \end{bmatrix}. \tag{B7}$$

Taking $h_{\alpha\beta}$ from (2.9), carrying out the matrix multiplication in Eq. (B6), and taking $\vec{E}$ and $\vec{B}$ from (4.3), we find

$$F^{\mu\nu}=\mathfrak{F}^{\mu\nu}+\begin{bmatrix} 0 & 0 & \left(\frac{-E_2h_{00}}{c^4}\,\frac{-B_3h_{10}}{c^2}\right) & 0 \\ 0 & 0 & \left(\frac{E_2h_{01}}{c^2}+B_3h_{11}\right) & 0 \\ \left(\frac{E_2h_{00}}{c^4}+\frac{B_3h_{10}}{c^2}\right) & \left(\frac{-E_2h_{01}}{c^2}-B_3h_{11}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{B8}$$

where we have used the fact that for our probe pulse $\vec{E}=(0,E_2,0)$ and $\vec{B}=(0,0,B_3)$. Finally we recall from Sec. II:

$$h_{00}=h, \quad h_{01}=-\frac{v}{c^2}h, \quad h_{11}=\frac{v^2}{c^4}h. \tag{B9}$$

Inserting (B9) into (B8) and using $B_3=E_2/c$ yields (4.4).

For the convenience of the reader who is not familiar with this type of calculation we now indicate a few steps in the passage from (4.9) to (4.14). Consider the left-hand side of (4.9). Rewriting we have

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)E(x,t)\cos[\omega t-hx+\phi(x,t)]$$
$$=\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial t}\right)\left\{E(x,t)\left[\frac{\omega}{c}+k+\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)\phi(x,t)\right]\sin(\omega t-kx+\phi)+\left[\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)E(x,t)\right]\cos(\omega t-kx+\phi)\right\}. \tag{B10}$$

Using the slowly varying phase and amplitude approximation (4.14a), (4.14b) this becomes the left-hand side of (4.9):

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\left\{E(x,t)\left[2k+\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)\phi\right]\sin(\omega t-kx+\phi)\right\}. \tag{B11}$$

Likewise

$$2k\gg\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)\phi,$$

and so we are left with the final form

$$2k\left\{\left[\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)E(x,t)\right]\sin(\omega t-kx+\phi)+E(x,t)\left[\frac{\omega}{c}-k+\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\phi(x,t)\cos(\omega t-kx+\phi)\right]\right\}. \tag{B12}$$

Consider next the right-hand side of Eq. (4.9),

$$\alpha\frac{\partial}{\partial t}\left(\frac{1}{v}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\left(\frac{1}{|vt-x|}E(x,t)\cos[\omega t-kx+\phi(x,t)]\right),$$

noting that

$$\left(\frac{1}{v}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\frac{1}{|vt-x|}=0. \tag{B13}$$

We have

$$\alpha\frac{\partial}{\partial t}\left[\left(\frac{E(x,t)}{|vt-x|}\right)\left(k-\frac{\omega}{v}\right)\sin[\omega t-kx+\phi(x,t)]\right], \tag{B14}$$

where we have invoked the slowly varying phase and amplitude approximation. Continuing to neglect derivatives of $E(xt)$ and $\phi(xt)$, (B14) may be written

$$\alpha E(x,t)\frac{\omega}{v}\left(1-\frac{v}{c}\right)\left(\frac{v}{(vt-x)^2}\sin[\omega t-kz+\phi(x,t)]-\frac{\omega}{|vt-x|}\cos[\omega t-kz+\phi(x,t)]\right), \tag{B15}$$

which is, of course, equal to (B12). Equating the coefficients of the sine and cosine terms of (B12) and (B15) yields

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)E(x,t)=\left[\frac{\alpha\omega}{2k}\left(1-\frac{v}{c}\right)\frac{1}{(vt-x)^2}\right]E(x,t), \tag{B16a}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)\phi(x,t)=k-\frac{\omega}{c}-\left[\frac{\alpha\omega^2}{2kv}\left(1-\frac{v}{c}\right)\frac{1}{|vt-x|}\right]. \tag{B16b}$$

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[1] R. Tolman, P. Ehrenfest, and B. Podolsky, Phys. Rev. 37, 602 (1931).

[2] The linearized Einstein equations are clearly developed in C. Misner, K. Thorne, and J. Wheeler, *Gravitation* (Freeman, San Francisco, 1970), Chap. 18; and R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965), Chap. 8.

[3]We use MKS units throughout.

[4]See, for example, J. Slater, *Microwave Transmission* (McGraw-Hill, New York, 1942); or N. Kapany and J. Burke, *Optical Waveguides* (Academic, New York, 1972).

[5]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Sec. 5.1. We note that the parameter τ in Eq. (3.1) may be taken as the proper time since our probe photons are "off the light cone," i.e., move with velocity $v < c$.

[6]The fact that there is no interaction between coparallel light rays moving through the vacuum was first pointed out by Tolman, Ehrenfest, and Podolsky (Ref. 1). See also A. Lightman, W. Press, R. Price, and S. Teukolsky, *Problem Book in Relativity and Gravitation* (Princeton Univ. Press, Princeton, N.J., 1974), Problem 13.17.

[7]C. Moller, *The Theory of Relativity* (Oxford Univ. Press, London, 1972), 2nd edition, Section 10.9.

[8]See, for example, M. Sargent, M. Scully, and W. Lamb, *Laser Physics* (Addison-Wesley, Reading, Mass., 1974).

[9]Of course, a "thin pencil of light" such as this would experience a substantial spread due to diffraction in a short distance if the pulses were in a free space. Hence, we envision containing our intense pulse in a multiple mirror configuration or a guided wave structure. In such a case we might like to have $\lambda_p \ll \lambda_h$.

[10]In some such experiments one might wish to ensure a "perfect" vacuum within our guided wave structure. This could be accomplished by using the technique of W. Boreham and J. Hughes as discussed in their paper contained in the Digest of Technical Papers presented at the Tenth International Quantum Electronics Conference, IEEE, Cat. No. 78CH1301, 1QES, 1978 (unpublished). Thus we might envision a "precursor" pulse sweeping out the very dilute gas in our high (but not absolute) vacuum.

[11]Photon-photon scattering via material or vacuum polarization would not be a problem in the present experiment if we keep the pulses physically separated.

[12]In this estimate we have taken the group velocity to be $\sim v$, while the phase velocity $\sim c$. The appearance of the factor $(1/v)\partial/\partial t + \partial/\partial x$ in (4.9) requires us to keep these possible distinctions in mind lest this factor, and thus the phase shift, vanish.

[13]S. F. Jacobs, private communication. See, for example, J. Berthold, S. Jacobs, and M. Norton, Appl. Opt. 15, 1898 (1976).

[14]S. Ezekiel, private communication.

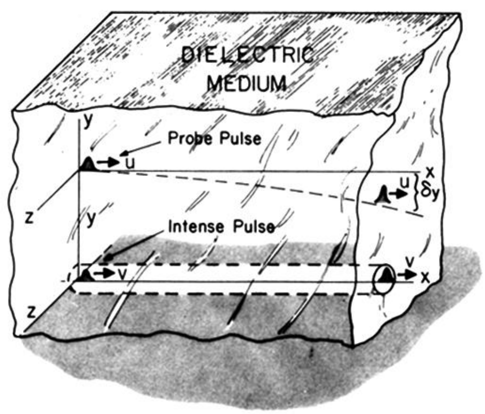

FIG. 1. Figure illustrating intense laser pulse propagating with velocity v through dielectric wave guide (i.e., hole bored in dielectric medium) and parallel-propagating probe pulse. Probe pulse is deflected by an amount δy as it moves through the medium with velocity u.