New theory of gravitation

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A new theory of gravity is proposed in which the geometry of space-time is determined by a nonsymmetric field structure. The theory satisfies the following requirements: (1) general covariance, (2) (weak) principle of equivalence, (3) the field equations are derivable from a Lagrangian action principle, (4) the theory agrees with all the classical (weak gravitational field) tests of Einstein's general relativity. The field equations for the nonsymmetric Hermitian $g_{\mu\nu}$ lead to a rigorous static spherically symmetric solution for the gravitational field in empty space that excludes the essential singularity at $r = 0$ from physical space-time. It is expected that the predictions of the theory will differ significantly from Einstein's theory of gravitation for compact sources or supermassive stars. Matter undergoing gravitational collapse is prevented from forming a black hole (in physical space-time) of the kind predicted in Einstein's theory, when a new gravitational parameter l that appears as a constant of integration in the solution satisfies $l > 2m$.

I. INTRODUCTION

Einstein's 1915 theory of gravitation,[1] based on a Riemannian metric tensor $g_{\mu\nu}$, although experimentally verified with a good degree of accuracy for comparatively weak gravitational fields, suffers from the consequence that the very notions of space-time become meaningless at the singularities of collapsed stars and cosmology.

Einstein devoted many years to searching for an extension of his gravitational theory that would possess rigorous solutions regular everywhere in space-time; one such attempt, which he claimed was the most natural extension of his theory, was based on a nonsymmetric tensor field $g_{\mu\nu}$.[2-4]

The present author, together with collaborators, has investigated[5-8] the nonsymmetric theory with the identification $g_{[\mu\nu]} \sim F_{\mu\nu}$. However, there is no compelling reason Einstein's original theory[3,4] should be interpreted as somehow including the electromagnetic field. Indeed, we show in the following that a purely gravitational interpretation leads to a rigorous spherically symmetric static solution that excludes the essential singularity at $r=0$ from physical space-time. Moreover, the theory can be made to agree with all the classical tests of general relativity.

We shall base the new theory on the following postulates:

(I) The laws of physics are invariant under general coordinate transformations.

(II) The (weak) principle of equivalence is valid for the total gravitational field structure.[9]

(III) The field equations are derivable from a Lagrangian action principle and they are second-order partial differential equations.

(IV) The geometry of space time, as determined by the gravitational sources, is non-Riemannian and described by a nonsymmetric Hermitian tensor $g_{\mu\nu}$ and a nonsymmetric Hermitian connection $\Gamma^{\lambda}_{\mu\nu}$.

(V) The world lines of material test particles follow the equations of paths[10]

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0 . \tag{1.1}$$

II. THE FIELD STRUCTURE AND THE FIELD EQUATIONS

The geometry of space-time will be determined by a nonsymmetric tensor $g_{\mu\nu}$ which decomposes into

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]} , \tag{2.1}$$

where $g_{[\mu\nu]}$ is a pure imaginary skew tensor. The $g_{\mu\nu}$ is Hermitian symmetric $g_{\mu\nu} = \bar{g}_{\nu\mu}$ and this replaces the symmetry property of $g_{\mu\nu}$ in Einstein's gravitational theory (Riemannian geometry). The contravariant tensor $g^{\mu\nu}$ is related to the covariant tensor $g_{\mu\nu}$ by the equation

$$g^{\mu\nu}g_{\sigma\nu} = g^{\nu\mu}g_{\nu\sigma} = \delta^\mu_\sigma , \tag{2.2}$$

where the order of the indices is important.

The concept of parallel displacement of a vector A^μ can be extended to the nonsymmetric field by

$$\delta A^\mu = -\Gamma^\mu_{\alpha\beta}A^\alpha dx^\beta , \tag{2.3}$$

$$\delta A_\mu = \Gamma^\alpha_{\mu\beta}A_\alpha dx^\beta , \tag{2.4}$$

where $\Gamma^\mu_{\alpha\beta}$ is a (nonsymmetric) Hermitian affine connection $\Gamma^\mu_{\alpha\beta} = \bar{\Gamma}^\mu_{\beta\alpha}$, which decomposes according to

$$\Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{(\alpha\beta)} + \Gamma^\mu_{[\alpha\beta]} . \tag{2.5}$$

As in the symmetrical theory, a curvature tensor may be derived by parallel displacement of a vector along a boundary of an infinitesimal surface element

$$R^{\sigma}_{\mu\nu\rho}(\Gamma)=(\Gamma^{\sigma}_{\mu\nu,\rho}-\Gamma^{\sigma}_{\alpha\nu}\Gamma^{\alpha}_{\mu\rho}) - (\Gamma^{\sigma}_{\mu\rho,\nu}-\Gamma^{\sigma}_{\alpha\rho}\Gamma^{\alpha}_{\mu\nu}). \tag{2.6}$$

The contracted curvature tensor is

$$R_{\mu\nu}(\Gamma)=(\Gamma^{\beta}_{\mu\nu,\beta}-\Gamma^{\beta}_{\alpha\nu}\Gamma^{\alpha}_{\mu\beta}) - (\Gamma^{\beta}_{\mu\beta,\nu}-\Gamma^{\beta}_{\alpha\beta}\Gamma^{\alpha}_{\mu\nu}). \tag{2.7}$$

The field equations are derived by using the Palatini method.[6] We define a (non-Hermitian) connection $W^{\lambda}_{\mu\nu}$ in terms of $\Gamma^{\lambda}_{\mu\nu}$ by the equation

$$W^{\lambda}_{\mu\nu}=\Gamma^{\lambda}_{\mu\nu}-\tfrac{2}{3}\delta^{\lambda}_{\mu}W_{\nu}, \tag{2.8}$$

where

$$W_{\nu}=\tfrac{1}{2}(W^{\sigma}_{\nu\sigma}-W^{\sigma}_{\sigma\nu}). \tag{2.9}$$

It follows that

$$\Gamma_{\mu}\equiv\Gamma^{\sigma}_{[\mu\sigma]}=0. \tag{2.10}$$

We shall adopt the Einstein notation for covariant differentiation[3]

$$A^{\mu+}{}_{;\sigma}=A^{\mu}{}_{,\sigma}+A^{\rho}\Gamma^{\mu}_{\rho\sigma}, \tag{2.11}$$

$$A^{\mu-}{}_{;\sigma}=A^{\mu}{}_{,\sigma}+A^{\rho}\Gamma^{\mu}_{\sigma\rho} \tag{2.12}$$

and

$$A_{\mu+;\sigma}=A_{\mu,\sigma}-A_{\rho}\Gamma^{\rho}_{\mu\sigma}, \tag{2.13}$$

$$A_{\mu-;\sigma}=A_{\mu,\sigma}-A_{\rho}\Gamma^{\rho}_{\sigma\mu}. \tag{2.14}$$

The Lagrangian density of the theory is chosen to be

$$\mathfrak{L}=\mathfrak{g}^{\mu\nu}[R_{\mu\nu}(W)-8\pi T_{\mu\nu}], \tag{2.15}$$

where $\mathfrak{g}^{\mu\nu}=\sqrt{-g}\,g^{\mu\nu}$ is the fundamental tensor density and $T_{\mu\nu}$ is the (nonsymmetric) Hermitian energy-momentum tensor. Moreover, it can be shown that

$$R_{\mu\nu}(W)=R_{\mu\nu}(\Gamma)+\tfrac{2}{3}(W_{\mu,\nu}-W_{\nu,\mu}) \tag{2.16}$$

and $R_{\mu\nu}(W)$ is a Hermitian contracted curvature tensor, since $R_{\mu\nu}(\Gamma)$ is Hermitian and W_{μ} is a purely imaginary vector field.

From the variational principle

$$\delta L=\delta\int\mathfrak{L}\,d^4x=0 \tag{2.17}$$

we get the field equations

$$g_{\mu+\nu-;\sigma}=0, \tag{2.18}$$

$$\mathfrak{g}^{[\mu\nu]}{}_{,\nu}=0, \tag{2.19}$$

$$R_{\mu\nu}(W)-\tfrac{1}{2}g_{\mu\nu}R(W)=8\pi T_{\mu\nu}, \tag{2.20}$$

where $R(W)=g^{\mu\nu}R_{\mu\nu}(W)$.

These are the equations that we shall use to determine the structure of the *pure gravitational field*. Equation (2.19) can be shown to be equivalent to (2.10). In empty space ($T_{\mu\nu}=0$) the field equation (2.20) becomes[4]

$$R_{(\mu\nu)}(\Gamma)=0, \tag{2.21}$$

$$R_{[\mu\nu],\sigma}(\Gamma)+R_{[\nu\sigma],\mu}(\Gamma)+R_{[\sigma\mu],\nu}(\Gamma)=0. \tag{2.22}$$

III. CONSERVATION LAWS

Let us write the Lagrangian density as (we have set $T_{\mu\nu}=0$)

$$\mathfrak{L}=\mathfrak{L}_E+\mathfrak{L}^*,\quad \mathfrak{L}^*=\tfrac{2}{3}\mathfrak{g}^{\mu\nu}(W_{\mu,\nu}-W_{\nu,\mu}), \tag{3.1}$$

where $\mathfrak{L}_E=\mathfrak{g}^{\mu\nu}R_{\mu\nu}(\Gamma)$ is Einstein's Lagrangian density.[3] We define the (Hermitian) tensor density $\mathcal{T}^*_{\mu\nu}$ by

$$\mathcal{T}^*_{\mu\nu}=\sqrt{-g}\,T^*_{\mu\nu}=-\frac{1}{8\pi}\frac{\partial\mathfrak{L}^*}{\partial g^{\mu\nu}}. \tag{3.2}$$

If we write

$$G_{\mu\nu}(\Gamma)=8\pi T^*_{\mu\nu}, \tag{3.3}$$

where

$$G_{\mu\nu}(\Gamma)=R_{\mu\nu}(\Gamma)-\tfrac{1}{2}g_{\mu\nu}R(\Gamma), \tag{3.4}$$

then it can be shown that[8] the Bianchi identities give

$$\mathrm{Re}(\mathfrak{g}^{\alpha\nu}T^*_{\rho-\nu-;\alpha})=0. \tag{3.5}$$

We have that

$$\mathfrak{g}^{\mu\nu}R_{\mu\nu}(\Gamma)=(\mathfrak{g}^{\lambda\mu}\Gamma^{\alpha}_{\lambda\mu}-\delta^{\alpha}_{\mu}\mathfrak{g}^{\lambda\mu}\Gamma^{\beta}_{\lambda\beta})_{,\alpha}+\mathfrak{L}', \tag{3.6}$$

where

$$\mathfrak{L}'=\mathfrak{g}^{\lambda\mu}(\Gamma^{\alpha}_{\lambda\beta}\Gamma^{\beta}_{\alpha\mu}-\Gamma^{\beta}_{\lambda\mu}\Gamma^{\alpha}_{\beta\alpha}), \tag{3.7}$$

from which it follows that

$$R_{\mu\nu}(\Gamma)=\frac{\partial\mathfrak{L}'}{\partial\mathfrak{g}^{\mu\nu}}-\frac{\partial}{\partial x^{\rho}}\left(\frac{\partial\mathfrak{L}'}{\partial\mathfrak{g}^{\mu\nu}_{,\rho}}\right). \tag{3.8}$$

If we require that the action

$$L'=\int\mathfrak{L}'\,d^4x$$

remain invariant under linear transformations of the form

$$x^{\mu}\to\tilde{x}^{\mu}=x^{\mu}+\epsilon\xi^{\mu}(x) \tag{3.9}$$

for arbitrary constant ξ^{ρ} and an infinitesimal ϵ, we find that

$$\delta L'=8\pi\epsilon\int 2\xi^{\rho}\mathfrak{t}^{\alpha}_{\rho,\alpha}d^4x=0, \tag{3.10}$$

where $\mathfrak{t}^{\alpha}_{\rho}$, a tensor density only under linear coordinate transformations, is given by

$$8\pi\mathfrak{t}^{\alpha}_{\rho}=\tfrac{1}{2}(G_{\rho\nu}\mathfrak{g}^{\alpha\nu}+G_{\nu\rho}\mathfrak{g}^{\nu\alpha}) -\frac{1}{2}\frac{\partial\mathfrak{L}'}{\partial\mathfrak{g}^{\mu\nu}_{,\alpha}}\mathfrak{g}^{\mu\nu}_{,\rho}-\delta^{\alpha}_{\rho}\mathfrak{L}'. \tag{3.11}$$

Thus we have an ordinary divergence conservation law

$$t^{\alpha}_{\rho,\alpha}=0\,. \tag{3.12}$$

Callaway[11] showed that the equations of motion of a particle derived from the field equations (2.19), (2.21), and (2.22) do not contain any direct Lorentz-force contributions $F_{\mu\nu}v^{\nu}$, where v^{ν} is the velocity four-vector: *a* particle moves as if it were uncharged. This was a serious criticism of Einstein's theory when interpreted as a unification of gravitation and electromagnetism. However, in our case, *it works to our advantage.* Since there is no direct gravitational Lorentz-force term coupling to moving particles, the principle of equivalence is preserved.

IV. RIGOROUS SOLUTION OF THE FIELD EQUATIONS

The field equation (2.18) may be solved explicitly for the $\Gamma^{\lambda}_{\mu\nu}$ in terms of the $g_{\mu\nu}$ and its derivatives,[12] provided that $g=\det(g_{\mu\nu})\neq 0$. The general form of the $g_{\mu\nu}$ for a static spherically symmetric field was given by Papapetrou[12] and a general solution derived by Vanstone,[13] but we shall restrict ourselves to the solution with the line element[12]

$$ds^2=\left(1-\frac{2m}{r}\right)\left(1-\frac{l^4}{r^4}\right)dt^2 - \left(1-\frac{2m}{r}\right)^{-1}dr^2-r^2d\Omega^2 \tag{4.1}$$

and

$$g_{[14]}=\frac{il^2}{r^2}\,. \tag{4.2}$$

The metric (4.1) tends to the flat space-time metric as $r\to\infty$ and when $l=0$ reduces to the Schwarzschild metric of general relativity. The tensor component $g_{[14]}$ describes a new long-range gravitational field that does not affect a material test particle directly through equations of motion.

The boundary conditions $g_{[14]}\to 0$ and $g_{[23]}\to 0$ as $r\to\infty$ can be shown[13] to lead to $g_{[23]}\equiv 0$. If we also choose $g_{22}=-r^2$ in order to obtain the flat-space coordinates at $r=\infty$, then the solution (4.1) and (4.2) is the unique spherically symmetric solution in the theory.

The metric (4.1) does not have any essential singularity in *physical* space-time, because for $r<l$ the metric does not have the signature of a physical space-time. When $r=l$ a "surface of concealment" forms around the origin. No timelike or null world lines can penetrate this surface and, therefore, *physical space-time in the theory is free of the essential singularity at* $r=0$.

From the field euqation $R_{(14)}=0$, we find that $\dot\alpha/\alpha=0$ where $\alpha=-g_{11}$ and $\dot\alpha=d\alpha/dt$. The nonstatic solution is then

$$g_{22}=-r^2,\quad \alpha=\left(1-\frac{2m}{r}\right)^{-1},$$
$$g_{44}=h(t)\left(1-\frac{2m}{r}\right)\left(1-\frac{l^4}{r^4}\right), \tag{4.3}$$
$$g_{[23]}=0,\quad g_{[14]}=h^{1/2}(t)\frac{il^2}{r^2},$$

where $h(t)$ is an arbitrary function of t. By choosing a new time coordinate

$$t'=\int h^{1/2}(t)dt\,, \tag{4.4}$$

we prove the Birkhoff[14] theorem for our gravitational theory.

V. PROPERTIES OF THE METRIC AND THE EQUATIONS OF MOTION

There are three distinct solutions of the line element (4.1) : (1) $l<2m$, (2) $l=2m$, and (3) $l>2m$. We can envisage three possible changes of signature depending on the value of l. Denote by Σ the signature of space-time. As r decreases to zero, g_{44} can change sign twice, while g_{11} changes sign only once at $r=2m$. For $l<2m$ the signature changes from $\Sigma=-2$ to $\Sigma=0$ at $r=l$. For $l=2m$ the signature changes from $\Sigma=-2$ to $\Sigma=0$. Finally, for $l>2m$ the signature changes from $\Sigma=-2$ to $\Sigma=-4$, and $r=l$ is a null surface inside which space is Euclidean four-dimensional, excluding the region $r<l$ from physical space-time. In particular, the essential singularity at $r=0$ is no longer part of physical space.

If we demand that the surface $r=l$ should be below the Schwarzschild event horizon at $r=2m$, then $l<2m$. On the other hand, if $l>2m$, a collapsing star would never form a black-hole event horizon of the kind predicted by general relativity; such an event horizon would be concealed from view by the null surface $r=l$. In all cases a star undergoing gravitational collapse would be prevented from reaching the singular point $r=0$.

Since we have adopted the Hermitian affine connection $\Gamma^{\lambda}_{\mu\nu}$ as the affine connection by means of which we define the parallel transfer of a vector, it seems natural to require that (1.1) be the equation of motion in the theory. Since $\Gamma^{\lambda}_{\mu\nu}$ is Hermitian, $\Gamma^{\lambda}_{\mu\nu}=\overline{\Gamma}^{\lambda}_{\nu\mu}$, it can be shown using Eq. (2.18) that the length of an arbitrary complex vector is preserved under parallel transfer.[6] The quadratic

$$g_{(\mu\nu)}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}=\text{const} \tag{5.1}$$

is an integral of (1.1). This can be proved by multiplying (1.1) by $g_{(\mu\nu)}dx^{\nu}/ds$ and using (2.18).

VI. BEHAVIOR OF TEST PARTICLES IN THE METRIC WITH $l > 2m$

We study the motion of test particles in the metric (4.1) with the condition that $l > 2m$ with the notation and evaluation of $\Gamma^{\lambda}_{\mu\nu}$ of Ref. 6. The orbits will be chosen to lie in the equatorial plane, whereby $\theta = \frac{1}{2}\pi$ and $\dot{\theta} \equiv d\theta/d\tau = 0$. We also choose $ds^2 = E d\tau^2$, where E is a constant.

Two of the equations (1.1) are, for $x^3 = \phi$ and $x^4 = t$,

$$\ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} = 0\,, \tag{6.1}$$

$$\ddot{t} - \frac{\alpha'}{\alpha}\dot{r}\dot{t} = 0\,. \tag{6.2}$$

If we rewrite the constant of motion (5.1) as

$$g_{(\mu\nu)}\dot{x}^{\mu}\dot{x}^{\nu} = E\,, \tag{6.3}$$

then we get

$$\gamma\dot{t}^2 - \alpha\dot{r}^2 - r^2\dot{\phi}^2 = E\,, \tag{6.4}$$

$$r^2\dot{\phi} = J\,, \tag{6.5}$$

$$\dot{t} = \alpha\tilde{E}\,, \tag{6.6}$$

where J is the (constant) angular momentum and $\tilde{E}$ is a constant which we choose to set equal to unity. It follows that

$$\dot{r} = \left[1 - \frac{l^4}{r^4} - \left(1 - \frac{2m}{r}\right)\left(E + \frac{J^2}{r^2}\right)\right]^{1/2} \tag{6.7}$$

and

$$\tau - \tau_0 = \int_{r_0}^{r} \frac{dr'}{[1 - l^4/r'^4 - (1 - 2m/r')(E + J^2/r'^2)]^{1/2}}\,, \tag{6.8}$$

where τ_0 and r_0 are some initial proper time and position, respectively.

Consider now the radial motion of a freely falling material test particle ($E = 1$) with $J = 0$:

$$\dot{r} = \frac{1}{r^{1/2}}\left(2m - \frac{l^4}{r^3}\right)^{1/2}. \tag{6.9}$$

Then $\dot{r} = 0$ when

$$r = r_l = (l^4/2m)^{1/3}\,.$$

But since $l > 2m$, it follows that $r_l > l$ *and the falling body comes to rest before it reaches* $r = l$. A radially moving material test particle is repelled by the sphere defined by the radius $r = l$ and is prevented from reaching either $r = 2m$ or the essential singularity at $r = 0$. Thus the world lines of radially moving material test particles are timelike complete. For radial motion we also get from (6.8)

$$\tau - \tau_0 = \frac{1}{3m}[(2mr^3 - l^4)^{1/2} - (2mr_0^3 - l^4)^{1/2}]\,, \tag{6.10}$$

which becomes *complex* for

$$r < r_l = (l^4/2m)^{1/3}\,,$$

corresponding to the repulsion from $r = r_l$. These results can be extended without difficulty to the case of nonradial motion with $J \neq 0$.

For $l > 2m$ the metric contains a "hard sphere" that deflects all material moving test particles. The Schwarzschild event horizon at $r = 2m$ and the essential singularity at $r = 0$ are excluded from physical space-time.

There is a branch point in dr/dt at $r = l$ and at this point $dr/dt = 0$. A radial null world line that approaches $r = l$ can be continued smoothly round the branch point onto the second Riemann sheet of dr/dt, where t goes into $-t$. This second sheet represents a time-reversed submanifold isometric with our own. In this way it may be possible to continue analytically radial null world lines.

VII. THE PHYSICAL INTERPRETATION OF THE PARAMETER l AND SOME ASTROPHYSICAL PREDICTIONS

The gravitational red-shift of atomic spectral lines measured by Pound, Rebka, and Snider[15,16] provides an upper limit for l. The present theory predicts for this experiment

$$C \equiv \frac{\Delta\nu}{\nu} = \frac{\left(1 - \frac{2M_\oplus}{R_\oplus}\right)^{1/2}\left(1 - \frac{l^4}{R_\oplus^4}\right)^{1/2}}{\left(1 - \frac{2M_\oplus}{R}\right)^{1/2}\left(1 - \frac{l^4}{R^4}\right)^{1/2}} - 1$$

$$\cong -\frac{M_\oplus h}{R_\oplus^2} - \frac{2l^4}{R_\oplus^4}\left(\frac{h}{R_\oplus}\right), \tag{7.1}$$

where $R_\oplus = 6.38 \times 10^8$ cm, $R = R_\oplus + h$ with $h = 2.26 \times 10^3$ cm, and $M_\oplus = 5.98 \times 10^{27}$ g. The Einstein prediction is

$$C_E = -\frac{M_\oplus h}{R_\oplus^2} = -2.46 \times 10^{-15}\,. \tag{7.2}$$

The experimental value obtained by Pound and Snider is[16]

$$|C_{\exp}| = (2.45 \pm 0.019) \times 10^{-15}. \tag{7.3}$$

Denoting by $\Delta C_{\exp}$ the experimental error, we get

$$l_\oplus < \frac{R_\oplus}{2^{1/4}}\left(\frac{R_\oplus}{h}\right)^{1/4}(\pm|\Delta C_{\exp}| + |C_E| - |C_{\exp}|)^{1/4}$$

$$= 4.61 \times 10^{-5} R_\oplus\left(\frac{R_\oplus}{h}\right)^{1/4}, \tag{7.4}$$

which yields $l_\oplus < 6.8$ km. A similar calculation for the gravitational red-shift of spectral lines emitted from the surface of the sun yields the upper bound

$$l_{\odot} \lesssim 2\times 10^4 \text{ km}.$$

The fractional red-shift of the spectral lines emitted from the surface of any star is predicted in the new theory to be

$$z \equiv \frac{\Delta\lambda}{\lambda} = \left(1-\frac{2m}{r}\right)^{-1/2}\left(1-\frac{l^4}{r^4}\right)^{-1/2} - 1 . \qquad (7.5)$$

In general relativity, $z < 0.615$,[17] preventing the gravitational red-shift from being an explanation of the large red-shifts observed for quasistellar radio sources[18] whose z's can be as large as 2–3. We can obtain red-shifts 2–3 for reasonable values of l and thus the quasistellar sources could be massive objects lying outside our galaxy but not at large cosmological distances.

VIII. CONCLUSIONS

We have demonstrated that an exact solution of the theory leads to a space-time in which matter undergoing gravitational collapse is prevented from reaching $r = 0$, thus avoiding the breakdown of physics encountered in Einstein's theory of gravity.

Another important feature of the theory is that a black-hole event horizon and an essential singularity at $r = 0$ can be excluded from the physical manifold, without contradicting the classical predictions of Einstein's theory, provided that we have $l > 2m$ and that we limit l to a calculable maximum value. For strong gravitational fields, the experimental predictions of the new theory may be significantly different from Einstein's theory.

The problem of the predictability of the big-bang cosmological model has been studied using the new theory.[19] It was found that an exact homogeneous anisotropic solution of the field equations has analytic properties that exclude the singularity at the origin of time $t = 0$ from physical space-time.

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