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Elastic nucleon-nucleon scattering in a gauge field theory

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Nucleon-nucleon scattering is studied in a spontaneously broken gauge field theory with the gauge boson describing the ρ meson. Gauge invariance is demonstrated explicitly. An inversion of the $^3P_0$ phase shift (but no zero) is obtained in a [1/1] scalar Padé approximation.

I. INTRODUCTION

It is well known that vector-meson exchange (ρ and ω) plays an important role in the description of the nucleon-nucleon force. Field-theoretical models have been studied which demonstrate this fact. Pure Born-term approximations for ρ and ω exchange (see e.g., Ref. 1) have been performed, and so have calculations in the framework of the Bethe-Salpeter equation in ladder approximation.[2] In particular, in the latter case a cutoff is introduced in a phenomenological manner. Renormalizable field theories avoid such parameters but are less successful by now in describing the phase shifts. Here it is hoped that perturbation theory is summable by means of Padé approximants;[3] one reason for this is that no resonances appear in this process. The Yukawa model (with only pions exchanged) was investigated by various authors.[4] Beyond that, the ω meson was included[5] in an Abelian gauge theory with considerable improvement for the D waves but not for the P waves. Since the ρ meson is responsible for bending down the $^3P_0$ phase shift, e.g., its inclusion is of great importance.

Renormalizable field-theoretical models that include spin-1 and isospin-1 particles (ρ meson) must be non-Abelian gauge field theories according to our present knowledge.[6] In Sec. II a de Wit–type model[7] of a gauge field theory with spontaneous symmetry breaking is presented, which is applicable to the description of the nucleon-nucleon interaction. In Secs. III and IV we explicitly demonstrate how gauge invariance comes out and how renormalization is performed. Since the technical problems encountered are particularly difficult for the ρ meson, we consider only this one at present. The ω can also be treated in a model with spontaneous symmetry breaking but will be considered later. Section V contains the numerical results for nucleon-nucleon scattering.

The model of Sec. II describes the nucleons, the pions, and the triplet of ρ mesons (A_μ^i). Scalar fields χ_0, χ_i ($i=1,2,3$) serve as a Higgs multiplet. By the Higgs-Kibble mechanism, the formerly massless fields, A_μ^i become massive, the fields χ_i form a triplet of Goldstone bosons, and Λ is the Higgs particle. The Higgs multiplet is introduced such that isospin is a good quantum number. This leads in our case to an $SU(2)_L \times SU(2)_G$ (L and G for local and global, respectively) version of a de Wit–type model[7] with the Higgs meson having no direct coupling to the nucleons. Since the phase shifts are not very sensitive to the Higgs-meson mass, it may be chosen as large.

II. THE MODEL

We start[7] with an $SU(2)_L \times SU(2)_G$-invariant Lagrangian, where L and G stand for local and global respectively. The transformation properties of the fields with respect to the SU(2) groups are given in Table I. Later the scalar fields χ_0, χ_i ($i=1,2,3$) shall serve as a Higgs multiplet. Their commutation relations with the charges Q_L and Q_G are

$$
\begin{aligned}
[Q_L^i, \chi_j] &= \frac{i}{2}\epsilon_{ijk}\chi_k - \frac{i}{2}\delta_{ij}\chi_0 ,\\
[Q_L^i, \chi_0] &= \frac{i}{2}\chi_i ,\\
[Q_G^i, \chi_j] &= \frac{i}{2}\epsilon_{ijk}\chi_k + \frac{i}{2}\delta_{ij}\chi_0 ,\\
[Q_G^i, \chi_0] &= -\frac{i}{2}\chi_i .
\end{aligned} \tag{2.1}
$$

The $SU(2)_L \times SU(2)_G$-invariant Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\text{inv}} = {} & \tfrac{1}{2}D_\mu\phi_i D^\mu\phi_i - \tfrac{1}{2}m_\pi^2\phi_i^2 + \tfrac{1}{2}D_\mu\chi_i D^\mu\chi_i + \tfrac{1}{2}D_\mu\chi_0 D^\mu\chi_0 \\
& - \tfrac{1}{2}m_\chi^2(\chi_0^2+\chi_i^2) + \tfrac{i}{2}\bar\psi\gamma^\mu D_\mu\psi - \tfrac{i}{2}D_\mu\bar\psi\gamma^\mu\psi - M\bar\psi\psi \\
& - \tfrac{1}{4}\lambda_1(\phi_i^2)^2 - \tfrac{1}{4}\lambda_2(\chi_0^2+\chi_i^2)^2 - \lambda_3\phi_i^2(\chi_0^2+\chi_j^2) \\
& - ig_\pi\bar\psi\gamma_5\tau_i\psi\phi_i - \tfrac{1}{4}F_{\mu\nu}^i F_i^{\mu\nu} ,
\end{aligned} \tag{2.2}
$$

where the D_μ are the covariant derivatives and $F_{\mu\nu}^i$

TABLE I. Transformation properties of the fields with respect to $SU(2)_L$ and $SU(2)_G$.

Field	$SU(2)_L$	$SU(2)_G$
Pion: $\phi_i(x)$, $i=1,2,3$	1	0
Nucleon: $\psi(x)$	$\frac{1}{2}$	0
Scalar fields: $\chi_0(x), \chi_i(x)$, $i=1,2,3$	$\frac{1}{2}$	$\frac{1}{2}$

is the field-strength tensor:

$$\begin{aligned} D_\mu\phi_i &= \partial_\mu\phi_i + g_A\epsilon_{ijk}A_{\mu j}\phi_k\,, \\ D_\mu\chi_i &= \partial_\mu\chi_i + \frac{g_A}{2}\chi_0 A_{\mu i} + \frac{g_A}{2}\epsilon_{ijk}A_{\mu j}\chi_k\,, \\ D_\mu\chi_0 &= \partial_\mu\chi_0 - \frac{g_A}{2}A_{\mu i}\chi_i\,, \\ D_\mu\psi &= \partial_\mu\psi - ig_A\frac{\tau_i}{2}\psi A_{\mu i}\,, \\ D_\mu\bar\psi &= \partial_\mu\bar\psi + ig_A\bar\psi\frac{\tau_i}{2}A_{\mu i}\,, \\ F^i_{\mu\nu} &= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_A\epsilon_{ijk}A^j_\mu A^k_\nu\,. \end{aligned} \tag{2.3}$$

Assuming that $m_\chi^2<0$, the symmetry is spontaneously broken. We put

$$\begin{aligned} \chi_0(x) &= \Lambda(x) + c_0\,, \\ m_\chi^2 &= -c_0^2\lambda_2\,. \end{aligned} \tag{2.4}$$

By the Higgs-Kibble mechanism, the formerly massless fields A^i_μ become massive, the fields χ_i form a triplet of Goldstone bosons, and Λ is the Higgs particle. We find

$$\begin{aligned} m_A &= \tfrac{1}{2}g_A c_0\,, \\ m_\Lambda &= (2\lambda_2)^{1/2}c_0\,. \end{aligned} \tag{2.5}$$

We remark that both $SU(2)_L$ and $SU(2)_G$ are broken, but the group $SU(2)_I$, the generators of which are given by

$$Q^i_I = Q^i_L + Q^i_G\,, \tag{2.6}$$

remains an exact symmetry. It is interpreted as the physical isospin group of strong interactions. The $SU(2)_I$ quantum numbers of our fields are $I=1$ for ϕ_i, χ_i, A^i_μ, $I=\frac{1}{2}$ for ψ, and $I=0$ for Λ. The total Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + \mathcal{L}_B + \mathcal{L}_{\text{FP}}\,, \tag{2.7}$$

with the gauge-breaking term

$$\mathcal{L}_B = -\frac{1}{2\xi}(\partial^\mu A^i_\mu - \xi m_A\chi_i)^2 \tag{2.8}$$

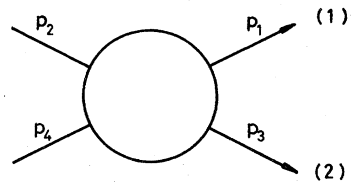

FIG. 1. Labeling of momenta for the nucleon-nucleon S-matrix element. (1) and (2) refer to particles 1 and 2, respectively.

of the 't Hooft gauge, and the corresponding Lagrangian of the Faddeev-Popov ghosts c_i

$$\begin{aligned} \mathcal{L}_{\text{FP}} = {} & \partial^\mu\bar c_i\partial_\mu c_i - \xi m_A^2\bar c_i c_i + g_A\epsilon_{ijk}\partial^\mu\bar c_i A^j_\mu c_k \\ & - \xi m_A^2 c_0^{-1}\bar c_i c_i\Lambda - \xi m_A^2 c_0^{-1}\epsilon_{ijk}\bar c_i c_j\chi_k\,. \end{aligned} \tag{2.9}$$

Working in the 't Hooft gauge has the advantage that there is no A^i_μ-χ_i propagator in lowest order and that all unphysical fields get equal, nonvanishing masses. We get

$$m_\chi^2 = m_{\text{FP}}^2 = \xi m_A^2\,. \tag{2.10}$$

The unphysical pole of the A^i_μ field is also located at ξm_A^2. Explicitly, we get the same propagator for the Faddeev-Popov and "would-be" Goldstone particles

$$\Delta^\chi_F(k) = \Delta^{\text{FP}}_F(k) = \frac{1}{k^2-\xi m_A^2}\,. \tag{2.11}$$

The propagator of the Yang-Mills boson A^i_μ is

$$\Delta_{F\mu\nu}(k) = -\frac{g_{\mu\nu}}{k^2-m_A^2} - \frac{k_\mu k_\nu}{m_A^2}\left(\frac{1}{k^2-m_A^2} - \frac{1}{k^2-\xi m_A^2}\right)\,. \tag{2.12}$$

For $\xi\to\infty$, the unphysical parts of the propagators vanish (unitary gauge).

III. GAUGE INVARIANCE

Apart from the mass terms, yielding the propagators (2.11) and (2.12), there are also coupling terms in the gauge-breaking and Faddeev-Popov parts of the Lagrangian [see (2.8) and (2.9)] depending explicitly on the gauge parameter ξ. Gauge invariance means that the sum of all contributing Feynman graphs yields an S-matrix element, which is independent of ξ order by order in perturbation theory. The S matrix can be written as

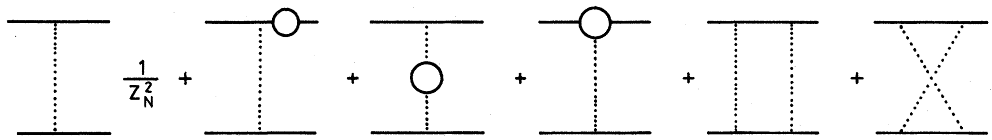

FIG. 2. Fourth-order contributions to the nucleon-nucleon scattering. Z_N is the renormalization constant of the nucleon. The dotted lines represent the exchanged particles (π and ρ meson, respectively). Circles in the pole terms stand for self-energy and vertex corrections. Insertions of these at all possible places are not shown.

$$(2\pi)^{6\,\text{out}}\langle p_1 p_3 | p_2 p_4\rangle^{\text{in}} = \int \prod_{i=1}^{4} dx_i \exp[i(p_1x_1 + p_3x_3 - p_2x_2 - p_4x_4)] Z_N^{-2} \times \bar{u}(p_1)(m - \not{p}_1)\bar{u}(p_3)(m-\not{p}_3)\langle 0 | T[\psi_U(x_1)\bar{\psi}_U(x_2)\psi_U(x_3)\bar{\psi}_U(x_4)] | 0\rangle (m - \not{p}_2)u(p_2)(m-\not{p}_4)u(p_4)\,, \quad (3.1)$$

where ψ_U are the unnormalized nucleon fields and Z_N is the renormalization constant of the nucleon field. The latter has explicitly to be taken into account because of its gauge dependence. The labeling of the momenta is shown in Fig. 1.

We calculate all fourth-order Feynman graphs contributing in our model as shown in Figs. 2–7, applying the dimensional regularization procedure of 't Hooft and Veltman.[8] Figure 2 shows the general form of the fourth-order graphs, and the following figures give the details of the self-energy and vertex corrections. The fourth-order contribution to the S matrix is ($g_\rho \equiv g_A$, $m_\rho \equiv m_A$):

$$\begin{aligned}(2\pi)^{6\,\text{out}}\langle p_1 p_3 | p_2 p_4\rangle^{\text{in}} = (2\pi)^4\delta(p_1 + p_3 - p_2 - p_3)(-i)g_\rho^2 \\ \times\Big[\bar{u}(p_1)\gamma_\mu \frac{\tau_i}{2} u(p_2)\bar{u}(p_3)\gamma_\rho\frac{\tau_i}{2} u(p_4)\Delta^{\mu\rho}(q)2(Z_N - 1) \\ + \bar{u}(p_1)\gamma_\mu\frac{\tau_i}{2} u(p_2)\bar{u}(p_3)\gamma_\rho\frac{\tau_i}{2} u(p_4)\Delta^{(2)\mu\rho}(q) \\ + \bar{u}(p_1)\gamma_\mu\frac{\tau_i}{2} u(p_2)\bar{u}(p_3)\Gamma_\rho^{(3)}(p_3, p_4, -q)\frac{\tau_i}{2} u(p_4)\Delta^{\mu\rho}(-q) \\ + \bar{u}(p_1)\Gamma_\mu^{(3)}(p_1, p_2, q)\frac{\tau_i}{2} u(p_2)\bar{u}(p_3)\gamma_\rho\frac{\tau_i}{2} u(p_4)\Delta^{\mu\rho}(q)\Big] + \text{box terms}\,,\end{aligned} \quad (3.2)$$

with $\Delta_{\mu\nu}^{(2)}$ and $\Gamma_\mu^{(3)}$ standing for the one-loop approximations of the two-point function $[\langle 0|T(A_\mu A_\nu)|0\rangle]$ and three-point function $[\langle 0|T(\psi\bar{\psi}A_\mu)|0\rangle]$, respectively. We have

$$\Delta_{\mu\nu}^{(2)} = -i\,\Delta_{\mu\sigma}\Pi^{\sigma\lambda}\Delta_{\lambda\nu}\,, \quad (3.3a)$$

with

$$\Pi^{\sigma\lambda} = A_1 g^{\sigma\lambda} + A_2 q^\sigma q^\lambda \quad (3.3b)$$

and

$$\Gamma_\mu^{(3)}(p_1, p_2, q) = F_1\gamma_\mu + F_2 p_\mu + F_3 q_\mu\,, \quad (3.3c)$$

$p_\mu = (p_1 + p_2)_\mu$ and $q_\mu = (p_2 - p_1)_\mu$.

A_1, A_2 and F_1, F_2, F_3 are the invariant amplitudes of the ρ self-energy and the ρNN vertex function, respectively. A_2 does not contribute to the S matrix because $\not{q}$ is sandwiched between the external spinors, and it turns out that $F_3 = 0$. For dimensional reasons, F_2 must be finite, and since there is no counterpart in the covariant, it must also be gauge independent by itself. This is confirmed by our analytic calculation (see Appendix A). We remark that, in general, $\Gamma_\mu^{(3)}$ is given by 12 invariant amplitudes. On-shell and between the positive-energy spinors, however, only 3 remain.

Gauge-dependent graphs contribute only to the amplitude multiplied with $\gamma_\mu^{(1)} \times \gamma^{\mu(2)}$. It reads

$$\frac{g_\rho^2}{q^2 - m_\rho^2}\left[2(Z_N - 1) - \frac{A_1(q^2) - A_1(m_\rho^2)}{q^2 - m_\rho^2} + 2F_1\right] + B(\xi)\,, \quad (3.4)$$

where $A_1(m_\rho^2)$ is the mass-renormalization counterterm and $B(\xi)$ comes from the box graphs. If one wants to study gauge invariance, it is advisable

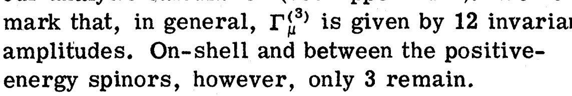

FIG. 3. Contributions to the self-energy of the nucleon. Here and in Figs. 4–7, stands for the ρ, --- stands for the π.

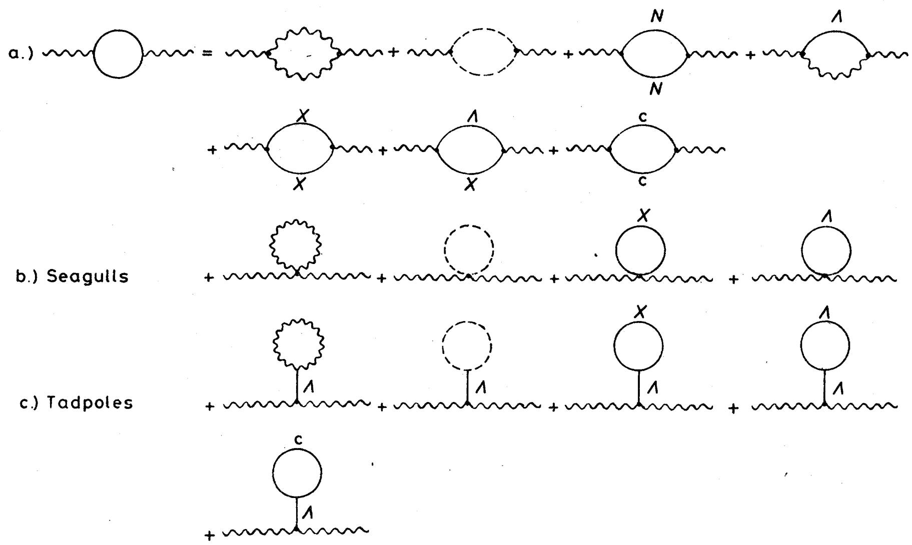

FIG. 4. Contributions to the self-energy of the ρ meson. Λ is the Higgs particle, X the "would-be" Goldstone boson, and c the Faddeev-Popov ghost.

to start with the decomposition of the triangle graphs $T(\xi)$ and box-graphs $B(\xi)$ as

$$T(\xi)=T(\xi=1)+t(\xi) \tag{3.5a}$$

and

$$B(\xi)=B(\xi=1)+b(\xi). \tag{3.5b}$$

Then because the denominators of the nucleon propagator cancel (only on-shell) against factors in the numerator, $t(\xi)$ and $b(\xi)$ simplify considerably, as shown in Fig. 8. The remaining ξ-dependent parts are easily seen to drop out (see Appendix A). It is not necessary to perform the one-dimensional integrations for that purpose. Finally, we mention that for the mass counterterm $A_1(m_\rho^2)$ to be gauge independent, the seagull and tadpole graphs of Figs. 4(b) and 4(c) have to be included.

One reason to perform the calculations with arbitrary ξ is to have a possibility to test the results. At first the ξ independence was checked analytically. Then the ξ-dependent part $b(\xi)$ of the box graphs was calculated numerically by evaluating (A5) and was compared for various ξ's with the second iteration of the Bethe-Salpeter equation—with the same result. As a further test we verified analytically the validity of the Slavnov-Taylor identity for the two-point function of the ρ meson, which relates $A_1+q^2A_2$ to a large number of unphysical amplitudes not appearing in the NN matrix element.

IV. PHYSICAL PARAMETERS

The definition of the renormalized coupling constants follows from

$$\langle \psi\bar{\psi}\phi\rangle_R = S_F^R S_F^R \Delta_\phi^R \Gamma^R g_R \tag{4.1a}$$

and

$$\langle \psi\bar{\psi}\phi\rangle_U = S_F^U S_F^U \Delta_\phi^U \Gamma^U g_U , \tag{4.1b}$$

R and U standing for the renormalized and unrenormalized quantities, respectively. Here (with the convention of Ref. 9)

$$\Gamma_{\mu i}^R(\psi\bar{\psi}\rho) = (F_1^R+2mF_2^R)\frac{\tau_i}{2}\gamma_\mu + F_2^R\frac{\tau_i}{2}i\sigma_{\mu\nu}q^\nu \tag{4.2a}$$

and

$$\Gamma_i^R(\psi\bar{\psi}\pi) = -iF_\pi^R\gamma_5\tau_i . \tag{4.2b}$$

We put

$$F_1^R(m_\rho^2)+2mF_2^R(m_\rho^2)=1 \tag{4.3a}$$

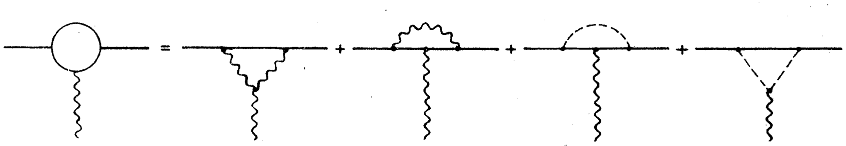

FIG. 5. Contributions to the ρNN vertex.

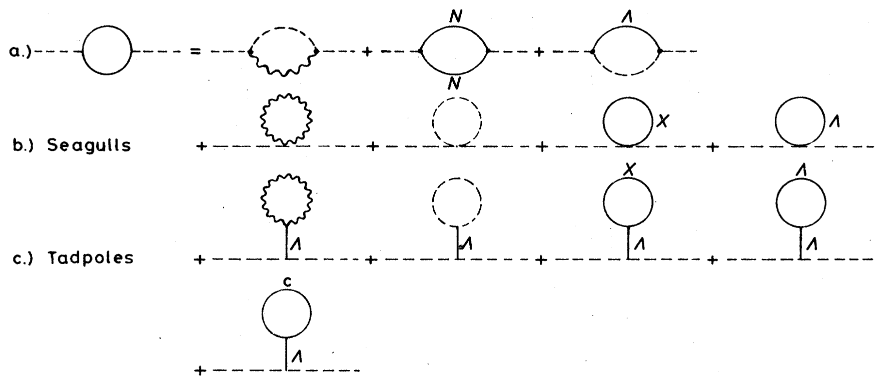

FIG. 6. Contributions to the self-energy of the pion.

and

$$F^R_\pi(m_\pi^2)=1\,, \tag{4.3b}$$

i.e., our renormalized coupling constants are the on-shell values of the three-point functions $NN\rho$ and $NN\pi$, respectively. As usual,

$$g^U=\frac{Z_1}{Z_N Z_\phi^{1/2}}\,g^R\,, \tag{4.4}$$

with

$$\psi_U=Z_N^{1/2}\psi_R\,,\quad \phi_U=Z_\phi^{1/2}\phi_R\,,\quad \Gamma^U=Z_1^{-1}\Gamma^R\,. \tag{4.5}$$

This is a gauge-invariant prescription since in the subtraction point all momenta are on-shell. Like real S-matrix elements also on-shell, two- and three-point functions of renormalized fields are gauge invariant. So $Z_1 Z_N^{-1} Z_\rho^{-1/2}$ has to be independent of ξ, which has been checked analytically. In the case of the ρ meson coupling to two pions as intermediate states, we have taken the real parts of the corresponding invariant amplitudes [see e.g., (4.3a)] as subtraction values.

Finally, we have the following expansions around $q^2=m_\rho^2$ and m_π^2, respectively:

$$A_1^U(q^2)=A_1^U(m_\rho^2)+(1-Z_\rho)(q^2-m_\rho^2)+\hat{A}_1(q^2) \tag{4.6}$$

and

$$F_1^U(q^2)+2mF_2^U(q^2)=Z_1^{-1}+\hat{F}_1(q^2)+2m\hat{F}_2(q^2)\,, \tag{4.7}$$

with $\hat{F}_i(m_\rho^2)=0$ and $\hat{A}_1(q^2)\sim O((q^2-m_\rho^2)^2)$.

If we insert (4.6) and (4.7) into the complete amplitude (i.e., second + fourth order) and utilize (4.4), all terms that become infinite for $n\to 4$ cancel, and the amplitude becomes unique. This procedure amounts to the usual subtractions with no radiative corrections on the external legs.

V. NUMERICAL RESULTS

Since it is well known that ω exchange gives a strong repulsive contribution to the NN force, we cannot expect to get proper phase shifts by considering only the ρ meson in addition to the pion. Moreover, the scalar [1/1] Padé approximant has deficiencies which have not much to do with the underlying model, like the negative sign of the $^1S_0$ and a spurious pole in the $^1D_2$—deficiencies which can be eliminated by working with matrix Padé approximants.[4] Therefore, we simply study at present the effect of the ρ meson in a few lower partial waves, namely $^3S_1$, $^3P_0$, and $^3P_1$. These are shown in Figs. 9 and 10.

Figure 10(a) shows the $^3P_0$ which has been obtained by adjusting g_ρ^R such that at 60 MeV (the position of the maximum), our phase shift agrees with the experimental value. The result is $g_\rho^{R^2}/4\pi=9.2$, where for the other parameters we have chosen $m_\rho^2=0.6735$ (in units of the nucleon mass) and $g_\pi^{R^2}/4\pi=14.2$, $m_\pi^2=0.022\,13$. The slight inversion of the $^3P_0$ at roughly the maximum of the phase shift as shown in Fig. 10(a) is considered our main result, and it is hoped that the inclusion of the

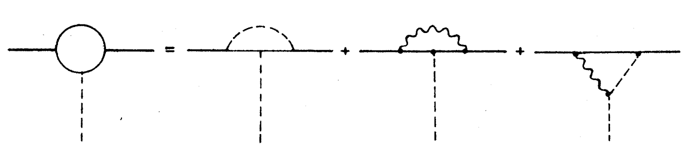

FIG. 7. Contributions to the πNN vertex.

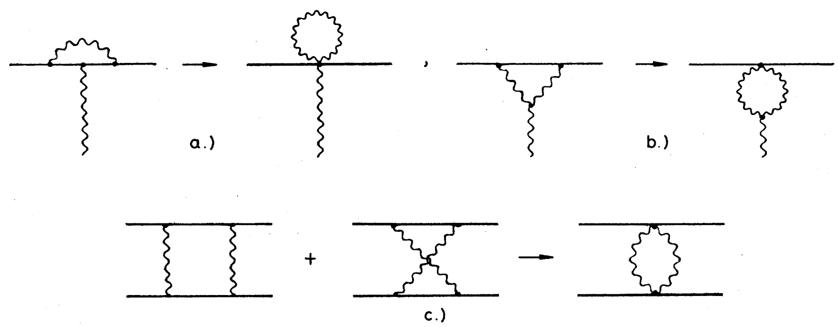

FIG. 8. Simplification of the on-shell ξ-dependent parts [see (3.5a), (3.5b)].

ω will bring further improvement. In the case of the $^3S_1$, two physical channels ($^3S_1$ and $^3D_1$) couple, and for that reason the calculation of a 2×2 matrix Padé approximant is possible. As shown in Fig. 9, the inclusion of the ρ meson yields further attraction at the higher energies in this partial wave.

The dependence of the phase shifts on the other parameters, namely λ_3 and m_Λ [see (2.2) and (2.5)] which remain after the subtractions are performed as described in Sec. III, is extremely small. Changing λ_3 from 1 to 10 and m_Λ from 5 m to 10 m (m is the nucleon mass), respectively, yields changes in the phase shifts which would not be visible in the drawings.

To conclude, we want to stress that clearly we do not regard it as surprising that the inclusion of the ρ meson by itself improves the $^3P_0$ phase shift, but rather consider it as important that this has been achieved in a renormalizable (cutoff-independent) model taking into account the fourth order.

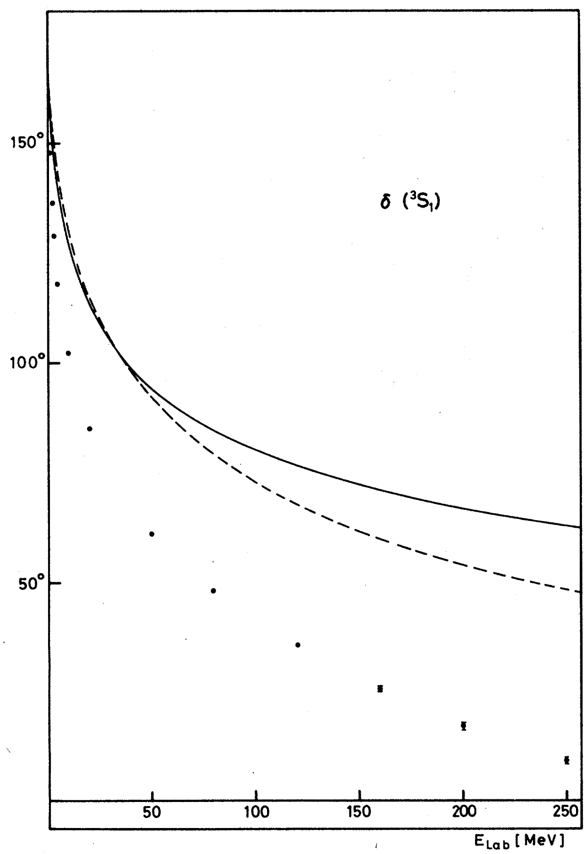

FIG. 9. $^3S_1$ phase shift; dashed line, only pion exchange; solid line, π and ρ exchange. The experimental data are taken from Ref. 10.

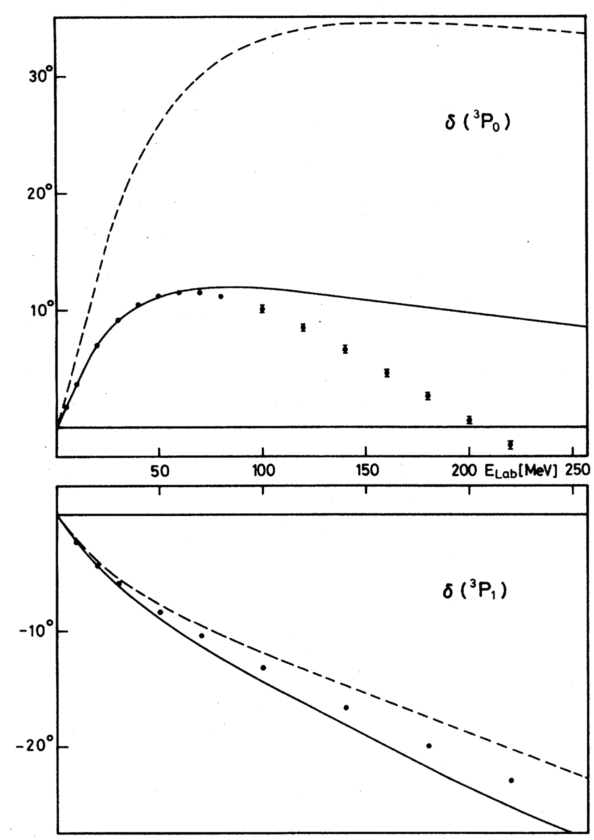

FIG. 10. Same as Fig. 9 for the $^3P_0$ and $^3P_1$.

APPENDIX A

Here we give in detail the regularized fourth-order contributions to (3.4) and (3.5b) for the ρ meson exchange. The normalization is $g_\rho^2/4\pi = 1$. We have

$$g_\rho^2(Z_N - 1) = -\frac{3}{4}\Big[-\Big(3+12\frac{m^2}{m_\rho^2-4m^2}\Big)\Big(1+\ln\frac{m^2}{m_\rho^2}\Big) - \Big(6+12\frac{m^2}{m_\rho^2-4m^2}+3\frac{m_\rho^2}{m^2}\Big) \times\Big(1+R_1(m^2, m_\rho^2, m^2) - \ln\frac{m_\rho^2}{\mu^2}\Big) + \xi - \xi\ln\xi + \xi\Big(\text{Reg} - \ln\frac{m_\rho^2}{\mu^2}\Big)\Big], \tag{A1}$$

for the ρ contribution to the renormalization constant of the nucleon. For the vertex functions, we obtain

$$\begin{aligned} g_\rho^2 F_1 = {} & -\frac{3}{4} - \frac{37}{12}\Big(R_1(m_\rho^2, m^2, m^2) - \ln\frac{m_\rho^2}{\mu^2}\Big) + \frac{3}{2}\Big(\text{Reg} - \ln\frac{m_\rho^2}{\mu^2}\Big) \\ & - [R_1(m^2, m^2, q^2) - R_1(m_\rho^2, m^2, m^2)]\Big(\frac{3}{4} - \frac{1}{2}\frac{m_\rho^2}{4m^2-q^2}\Big) \\ & + [R_1(m_\rho^2, m_\rho^2, q^2) - R_1(m_\rho^2, m^2, m^2)]\Big(\frac{2}{3} + 2\frac{2m^2-m_\rho^2}{4m^2-q^2}\Big) \\ & + J_2(m^2, m^2, m_\rho^2, q^2)\Big[-m_\rho^2 + m^2 + \frac{1}{2}\Big(\frac{m_\rho^4}{4m^2-q^2} - q^2\Big)\Big] \\ & + J_2(m_\rho^2, m_\rho^2, m^2, q^2)\Big(-4m_\rho^2 - 2m^2 + 2\frac{(2m^2-m_\rho^2)^2}{4m^2-q^2}\Big) + \frac{5}{4}\xi - \frac{19}{12}\xi\ln\xi + \frac{5}{4}\Big(\text{Reg} - \ln\frac{m_\rho^2}{\mu^2}\Big)\xi \\ & + \Big(R_1(m_\rho^2, \xi m_\rho^2, q^2) - \ln\frac{m_\rho^2}{\mu^2}\Big)\frac{\xi}{3} + [R_1(\xi m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, q^2)]\Big[\frac{\xi}{3}\frac{q^2}{m_\rho^2} - \frac{1}{12}\Big(\frac{q^2}{m_\rho^2}\Big)^2\Big] \\ & + [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, m_\rho^2, q^2)]\Big[-\frac{5}{3} + \frac{3}{2}\frac{q^2}{m_\rho^2} + \frac{1}{12}\Big(\frac{q^2}{m_\rho^2}\Big)^2\Big] \\ & - [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, 0)](1-\xi)^2\Big(\frac{m_\rho^2}{q^2} - 1\Big)\frac{1}{6} \end{aligned} \tag{A2}$$

and

$$\begin{aligned} g_\rho^2 F_2 = {} & -1 - \frac{3}{2}\Big[\frac{m_\rho^2}{m^2}\Big(1 + R_1(m_\rho^2, m^2, m^2) - \ln\frac{m_\rho^2}{\mu^2}\Big) - \Big(R_1(m_\rho^2, m^2, m^2) - \ln\frac{m^2}{\mu^2}\Big)\Big] \\ & + [R_1(m^2, m^2, q^2) - R_1(m_\rho^2, m^2, m^2)]\Big(\frac{1}{2} - 3\frac{m_\rho^2}{4m^2-q^2}\Big) \\ & + [R_1(m_\rho^2, m_\rho^2, q^2) - R_1(m_\rho^2, m^2, m^2]\Big(2 - 12\frac{2m^2-m_\rho^2}{4m^2-q^2}\Big) \\ & + J_2(m^2, m^2, m_\rho^2, q^2)\Big(2m_\rho^2 - 3\frac{m_\rho^4}{4m^2-q^2}\Big) \\ & + J_2(m_\rho^2, m_\rho^2, m^2, q^2)\Big(-4m_\rho^2 + 12m^2 - 12\frac{(2m^2-m_\rho^2)^2}{4m^2-q^2}\Big). \end{aligned} \tag{A3}$$

For the self-energy of the ρ meson, we get

$$
\begin{aligned}
g_\rho^2 A_1 = -2\Bigg\{ & \frac{2}{3} m^2 \left(R_1(m^2, m^2, q^2) - \ln \frac{m^2}{\mu^2}\right) \\
& + m_\rho^2 \left[\frac{17}{36} + \frac{3}{4}\frac{m_\rho^2}{m_\Lambda^2} + \left(\frac{17}{8} + \frac{9}{4}\frac{m_\rho^2}{m_\Lambda^2}\right)\left(\text{Reg} - \ln \frac{m_\rho^2}{\mu^2}\right) - \frac{1}{6}\left(\text{Reg} - \ln \frac{m_\pi^2}{\mu^2}\right)\right. \\
& \qquad + \frac{1}{3}\left(R_1(m^2, m^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right) + \frac{1}{2}\left(R_1(m_\rho^2, m_\Lambda^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right) \\
& \qquad \left. - \frac{33}{8}\left(R_1(m_\rho^2, m_\rho^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right) + \frac{1}{6}\left(R_1(m_\pi^2, m_\pi^2, q^2) - \ln \frac{m_\pi^2}{\mu^2}\right)\right] \\
& + m_\Lambda^2 \left[\frac{7}{24} + \frac{1}{24}\frac{m_\Lambda^2}{m_\rho^2} + \frac{3}{8}\left(\text{Reg} - \ln \frac{m_\Lambda^2}{\mu^2}\right) + \frac{1}{24}\ln \frac{m_\rho^2}{m_\Lambda^2}\right. \\
& \qquad \left. - \left(R_1(m_\rho^2, m_\Lambda^2, q^2) - \ln \frac{m_\Lambda^2}{\mu^2}\right)\left(\frac{1}{6} - \frac{1}{24}\frac{m_\Lambda^2}{m_\rho^2}\right)\right] \\
& + m_\pi^2 \left[6 \frac{m_\rho^2}{m_\Lambda^2}\frac{\lambda_3}{g_\rho^2}\left(1 + \text{Reg} - \ln \frac{m_\pi^2}{\mu^2}\right) - \frac{2}{3}\left(R_1(m_\pi^2, m_\pi^2, q^2) - \ln \frac{m_\pi^2}{\mu^2}\right)\right] \\
& + (q^2 - m_\rho^2)\left[\frac{5}{9} + \frac{7}{4}\left(\text{Reg} - \ln \frac{m_\rho^2}{\mu^2}\right) - \frac{1}{6}\left(\text{Reg} - \ln \frac{m_\pi^2}{\mu^2}\right)\right. \\
& \qquad + \frac{1}{3}\left(R_1(m^2, m^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right) - \frac{5}{4}\left(R_1(m_\rho^2, m_\rho^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right) \\
& \qquad + \frac{1}{24}[R_1(m_\rho^2, m_\Lambda^2, q^2) - R_1(m_\rho^2, m_\rho^2, q^2] + \frac{1}{6}\left(R_1(m_\pi^2, m_\pi^2, q^2) - \ln \frac{m_\pi^2}{\mu^2}\right) \\
& \qquad \left. - \frac{1}{24}[R_1(m_\rho^2, m_\Lambda^2, q^2) - R_1(m_\rho^2, m_\Lambda^2, 0)]\left(1 - \frac{m_\Lambda^2}{m_\rho^2}\right)^2 \frac{m_\rho^2}{q^2}\right] \\
& + (q^2 - m_\rho^2)\left[-\frac{1}{2}\xi + \frac{5}{6}\xi \ln\xi - \frac{1}{2}\left(\text{Reg} - \ln \frac{m_\rho^2}{\mu^2}\right)\xi - \left(R_1(m_\rho^2, \xi m_\rho^2, q^2) - \ln \frac{m_\rho^2}{\mu^2}\right)\frac{\xi}{3}\right. \\
& \qquad - [R_1(\xi m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, q^2)]\left(\frac{\xi}{6} - \frac{1}{24}\frac{q^2}{m_\rho^2}\right)\left(1 + \frac{q^2}{m_\rho^2}\right) \\
& \qquad + [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, m_\rho^2, q^2)]\left(\frac{5}{6} - \frac{17}{24}\frac{q^2}{m_\rho^2} - \frac{1}{24}\left(\frac{q^2}{m_\rho^2}\right)^2\right) \\
& \qquad \left. + [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, 0)](1-\xi)^2\left(\frac{m_\rho^2}{q^2} - 1\right)\frac{1}{12}\right]\Bigg\}, \qquad \text{(A4)}
\end{aligned}
$$

and for the ξ-dependent part of the box-graph contributions,

$$
\begin{aligned}
b(\xi) = \Bigg[& [R_1(m_\rho^2, m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, q^2)]\left(\frac{5}{3} + \frac{1}{12}\frac{q^2}{m_\rho^2}\right) \\
& + [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(\xi m_\rho^2, \xi m_\rho^2, q^2)]\left(\frac{\xi}{3} - \frac{1}{12}\frac{q^2}{m_\rho^2}\right) \\
& - [R_1(m_\rho^2, \xi m_\rho^2, q^2) - R_1(m_\rho^2, \xi m_\rho^2, 0)](1-\xi)^2 \frac{m_\rho^2}{6q^2}\Bigg]\frac{1}{m_\rho^2}. \qquad \text{(A5)}
\end{aligned}
$$

Here we have used the one-dimensional (R_1) and two-dimensional (J_2) integrals as defined in Appendix B. $\text{Reg} = 2/(4-n) + \text{const}$, where n is the space-time dimension. Furthermore, the mass parameter μ defines the mass scale and makes all arguments of logarithms (see also R_1) dimensionless. Since F_2 is finite, it does not contain the Reg term.

The above formulas are written down according to the following lines:

(1) One easily sees that the only μ dependence comes from terms like $\text{Reg} - \ln(m_x^2/\mu^2)$, m_x being some mass ($m_x = m_\rho, m_\pi, m_\Lambda$), and drops out after the subtractions are performed as described in Sec. IV.

(2) The ξ-dependent parts are separated, i.e., in all expressions (A1), (A2), (A4), (A5) the last lines containing the gauge parameter cancel completely when the invariant amplitudes are inserted into (3.4).

(3) It appears to be convenient to factor out $(q^2 - m_\rho^2)$ in A_1 as far as possible; in particular, one sees

that the whole ξ-dependent part contains this factor, which drops out after insertion of A_1 into (3.4), and it immediately shows that $A_1(m_\rho^2)$ is gauge independent.

APPENDIX B

From the integration over Feynman parameters, we get functions of the form

$$R_1(J, M_1^2, M_2^2, s) = \int_0^1 dz\, z^J \ln[-sz(1-z) + M_1^2 z + M_2^2(1-z)] \tag{B1}$$

and

$$J_2(I, J, M_1^2, M_2^2, M_3^2, s_1, s_2, s_3) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2\, y_1^I y_2^J [-y_1(1-y_1)s_1 - y_2(1-y_2)s_2 + y_1 y_2(s_1 + s_2 - s_3) + y_1(M_1^2 - M_3^2) + y_2(M_2^2 - M_3^2) + M_3^2]^{-1}. \tag{B2}$$

In Appendix A we used the abbreviations

$$R_1(M_1^2, M_2^2, s) = R_1(0, M_1^2, M_2^2, s) \tag{B3}$$

and

$$J_2(M_1^2, M_2^2, M_3^2, s) = J_2(0, 0, M_1^2, M_2^2, M_2^3, M^2, M^2, s). \tag{B4}$$

In the course of the calculations of our perturbative results, functions $R_1(I, M_1^2, M_2^2, s)$ and $J_2(I, J, M_1^2, M_1^2, M_3^2, s_1, s_2, s_3)$ with $I, J \neq 0$ arise. By recursive relations, which are too long to be written down here, they have been expressed by $R_1(0, M_1^2, M_2^2, s)$ and $J_2(0, 0, M_1^2, M_1^2, M_3^2, s_1, s_2, s_3)$.

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