Comments on the bremsstrahlung model of Brodsky and Gunion

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Brodsky and Gunion (BG) have proposed a bremsstrahlung model of particle production in which the total hadron multiplicity $\langle n \rangle$ increases as $O(\ln^2 s)$. Standard facts of hadron-hadron scattering indicate that bremsstrahlung in the form proposed by BG cannot be the dominant production mechanism at current energies. Two refinements are suggested which enable bremsstrahlung to become a dominant production mechanism at current energies. Both lead to $\langle n \rangle = O(\ln s)$. We also comment on some criticisms by BG of standard hadron models. However, we have no criticism of the BG model as applied to large-Q^2 $lp \to lx$ and $e^+e^- \to X$ processes.

Brodsky and Gunion (BG)[1] have proposed a universal particle production mechanism for hadron-hadron scattering, e^+e^- annihilation, and ep inelastic scattering. The idea is to utilize gluon bremsstrahlung as the underlying mechanism for particle production, following the initial separation of a primary radiating $q\bar{q}$ pair. A color-confinement cutoff Λ is included as a lower bound for gluon frequencies ω, which makes all quantities finite. QED formulas with $e^2/4\pi$ replaced by $4\alpha/3$ are utilized to obtain qualitative results. The most prominent result is the unusual prediction that the total hadron multiplicity $\langle n\rangle$, assumed to be proportional to the bremsstrahlung gluon multiplicity $\langle n_g\rangle$, has the asymptotic form

$$\langle n\rangle \approx \frac{4\alpha}{3\pi} \ln^2 s\,. \tag{1}$$

Adding the assumption that α is constant (BG choose $\alpha = 0.47$), one obtains $\langle n\rangle = O(\ln^2 s)$. This is in fact consistent with hadron multiplicity data, as is the more traditional $\langle n\rangle = O(\ln s)$ leading behavior.[2]

We point out in this note that the bremsstrahlung model as proposed by BG cannot be the dominant production mechanism at current energies.[3] This arises from two not-unrelated problems. The first is that the average bremsstrahlung transverse momentum $\bar{k}_T = \langle k_T^2\rangle^{1/2}$ in the BG model is unbounded in s. The second is that total cross sections in this model have a manifestly non-Regge-pole behavior. In fact, unconventional j-plane cuts are present. Now all conventional models of Regge cuts are associated with the existence of long-range correlations in rapidity in inelastic states. Since correlations in hadron-hadron scattering are in fact dominately short range, this is usually taken to imply that j-plane cuts do not play a dominant role. Whether the BG model, with its unusual j-plane cuts, actually does or does not possess long-range correlations is currently under investigation. This and other matters including rapidity distributions will appear in a future publication.[4]

We shall suggest two ways by which bremsstrahlung can play a major role in particle production at current energies without the above-mentioned problems. The first is to provide a natural scale of ~1 GeV by constructing a multiperipheral hadron cluster model, with each cluster being produced by bremsstrahlung. This model is described in detail in Ref. 5. The second is simply to impose a k_T cutoff in order to bound $\bar{k}_T$, and to repair the BG total cross section behavior by treating α as the effective running coupling $\alpha = \alpha_{\rm eff}(s) = O(\ln s)^{-1}$. Both these suggestions lead to bounded $\bar{k}_T$ behavior with s, and multiplicities of the standard form $\langle n\rangle = O(\ln s)$ rather than $O(\ln^2 s)$ emerge.

Before proceeding, we make several remarks:

(1) Hadron-hadron scattering to a good first approximation possesses the attributes of limited transverse momentum, short-range order in rapidity, and the dominance of j-plane poles over j-plane cuts.[6] This is all embodied in the statement that production amplitudes are predominantly limited-k_T and Fredholm-multiperipheral in limited-mass clusters. This produces $\langle n\rangle = O(\ln s)$, not $O(\ln^2 s)$ as in the BG model. Thus one must investigate whether this $\ln^2 s$ behavior of $\langle n\rangle$ leads to associated problems in hadron-hadron scattering.

(2) The confinement of quarks in $e^+e^- \to X$ seems to require either non-k_T-cutoff or nonmultiperipheral final states, with long-range correlations in rapidity.[7] Moreover, the axial-gauge perturbative quantum-chromodynamics structure equivalent to the $Q^2 \to \infty$ one-loop renormalization-group results is a *non*-k_T-cutoff multipheripheral model. (Here the scale of k_T is set by Q).[7a] Since this is very different from the canonical situation in hadron-hadron scattering, it is hard at least for us to see how any universal production mechanism could be true. BG claim[1] that their model produces long-range correlations in color but not flavor. We disagree with this claim for reasons which will

be presented below.

(3) An important aspect of the BG model as stated by them is that the rising contribution to the hadron multiplicity is flavor independent.[1] This is in direct contradiction to present energy multiplicity data in hadron-hadron scattering which exhibit striking flavor-dependent delayed quasithreshold excitations [i.e., for strangeness ($K\bar{K}$ pairs), charm, and also baryon number].[8] Hence quark mass effects must be included in the BG model. It is then a challenge to this model (and any other proposed universal production mechanism) to simultaneously describe the very different behaviors of the excitation of a new flavor in $e^+e^- \to X$ and in hadron-hadron scatterings.

(4) BG argue[1] that their model is preferred because multiperipheral-Regge dynamics will have problems in describing multiplicities in $lp \to lX$ ($l = e, \mu$) and in $e^+e^- \to X$. While the BG model may indeed be applicable to large Q^2 $lp \to lX$ and $e^+e^- \to X$ scatterings, we disagree with their evaluation of standard hadronic models as applied to these processes, specifically, with respect to the following:

(a) BG argue that multiperipheral models will produce strong Q^2 variations in $\langle n\rangle(lp \to lX)$ which are not observed. They argue that $\langle n\rangle$ should change by a factor of 2 in multiperipheral models between $x=0$ and $x=1$ because the cylinder (planar) amplitudes should dominate at $x=0$ ($x=1$). However, the factor 2 which is commonly quoted for the ratio of multiplicities of cylinder/planar amplitudes is a leading-trajectory weak-coupling result. In principle there is no reason to trust these assumptions away from $x \approx 0$ because $t_{\min}$ effects generally spoil this Regge picture and particularly the weak-coupling approximation. Moreover, the natural variable in multiperipheral models is not $x = \nu/Q^2$ but $\tilde{x} = \nu/(m_c^2 + Q^2)$ where $m_c \approx (1\text{ GeV})$ is a typical hadron cluster mass.[9] This scale tends to wipe out the Q^2 dependence in $\langle n\rangle = O(\ln\tilde{x})$ predicted by the multiperipheral approach at small Q^2, where such models are to be trusted.

(b) BG argue that a "gluon-bound-state Pomeron" model of diffraction implies a ratio of $\frac{9}{4}$ between multiplicities in hadron-hadron scattering and $e^+e^- \to X$, which if evaluated at the same $s = Q^2$ is not observed. First, it is not clear that one should choose to compare at $s = Q^2$.[10] Second, recent arguments indicate that there is little connection between the existence of a quantum-chromodynamics gluon bound state in a theory without quarks, and the observed nature of diffractive scattering and hadronic final states.[11] Finally, hadron-hadron final states are predominantly k_T-cutoff and multiperipheral, and this, as mentioned above, is not the case in $e^+e^- \to X$.

These remarks are the only ones we shall make about $lp \to lX$ and $e^+e^- \to X$ processes. From now on we shall restrict our attention to hadron-hadron scattering.

We now proceed to the details of the BG model as applied to hadron-hadron scattering. The maximum gluon energy $\omega_{\max}$ (the "energy resolution") is treated by BG as $O(\sqrt{s})$, the total c.m. energy. This is the crucial point, and we shall return to it at the end. Specifically one takes $\omega_{\max} = \xi\sqrt{s}/2$ where $\xi(s) \le 1$. Two possibilities are $\xi(s) = 2/\langle n_g\rangle$ and $\xi(s) = \text{constant}$. The mean transverse momentum $\bar{k}_T = \langle k_T^2\rangle^{1/2}$ of the emitted gluons, and by assumption of the produced hadrons, is then

$$\bar{k}_T \approx \xi\sqrt{s}\left[2\ln\left(\frac{s}{m_q^2}\right)\ln\left(\frac{\xi^2 s}{4\Lambda^2}\right)\right]^{-1/2}, \qquad (2)$$

where m_q is the quark mass. Hence $\bar{k}_T = O(\sqrt{s})$ is unbounded in s. Technically, the angular peaking for gluon emission along the quark direction is only $\langle\theta^2\rangle = O(\ln s)^{-1}$ rather than $O(m_q^2/s)$, and that is not enough to bound $\bar{k}_T$. One may ask whether some logarithmic suppression due to $\xi(s) = 2/\langle n_g\rangle = O(\ln s)^{-2}$ can bring $\bar{k}_T$ from Eq. (2) into agreement with the nearly s-independent experimental $(\bar{k}_T)_{\text{expt}}$. The answer appears to be no. Iterating Eq. (2) once yields

$$\bar{k}_T \approx \frac{3\pi}{4\alpha}\frac{(2s)^{1/2}}{\left\{\ln\left(\frac{s}{m_q^2}\right)\ln\left[\frac{9\pi^2 s}{16\alpha^2\Lambda^2\ln^4(s/s_0)}\right]\right\}^{3/2}}. \qquad (3)$$

Taking $\Lambda = m_\pi$ for the color-confinement cutoff, $m_q = 300$ MeV for a constituent quark, $s_0 = 1\text{ GeV}^2$ for the internal scale, and $\alpha = 0.47$ yields a result for $\bar{k}_T$ which increases by about a factor of 2 between $E_{\text{lab}} = 10$ and 200 GeV. However, the data for $(\bar{k}_T)_{\text{expt}}$ only increase by 15% over the same range.[2]

These remarks show that if bremsstrahlung of the above type exists, it cannot be the major component of particle production at current energies. However, since $(\bar{k}_T)_{\text{expt}}$ does rise slowly with s, some mixture of the above unbounded $\bar{k}_T$ due to bremsstrahlung and a constant $\bar{k}_T$ (due, e.g., to a conventional multiperipheral source) could very well be consistent with the data.[12]

Of course,[13] one could always introduce an *ad hoc* k_T cutoff $(k_T^2)_{\max}$. In this case

$$\langle n_g\rangle \approx \frac{8\alpha}{3\pi}\int_\Lambda^{\omega_{\max}}\frac{d\omega}{\omega}\int_0^{(k_T^2)_{\max}}\frac{dk_T^2}{(k_T^2 + 4m_q^2\omega^2/s)} \qquad (4)$$

$$\approx \frac{1}{2}\,\frac{4\alpha}{3\pi}\ln^2 s\ . \qquad (5)$$

The dependence on $(k_T^2)_{\max}$ is in the nonleading term. Asymptotically, half the gluons come from $k_T^2 > (k_T^2)_{\max}$ and half from $k_T^2 < (k_T^2)_{\max}$ in Eq. (1).

It is interesting in this regard that the kinematical constraint of momentum conservation seems to introduce an effective k_T cutoff.[14]

We next discuss the total cross section $\sigma_{\text{tot}}(s)$. Consistent with the BG treatment of gluon emission as being Poisson, σ_{tot} is given by the following QED formula which keeps only infrared-singular terms and incorporates energy conservation[15]:

$$\sigma_{\text{tot}}(s) = \sigma_0 \exp\left[\frac{4\alpha}{3\pi}\ln^2\left(\frac{s}{\Lambda_{\text{UV}}^2}\right)\right] \times \int dx f(x) \exp\left[\alpha g(x)\ln\left(\frac{s}{m_q^2}\right)\right] \quad . \tag{6}$$

Here σ_0 is the primary cross section, unconnected with the other two factors due to bremsstrahlung, and therefore presumably irrelevant in the discussion. In particular, we would not expect σ_0 to cancel either of the other two factors in σ_{tot}.

The parameter Λ_{UV} in Eq. (6) is an ultraviolet cutoff, presumably independent of s and not obviously related to the "energy resolution" $\omega_{\max} = O(\sqrt{s}\,)$, except that $\omega_{\max} < \Lambda_{\text{UV}}$ for those s where the bremsstrahlung formalism is being trusted. BG wants this to be through Fermilab and CERN ISR energies. Hence, Eq. (6) predicts the behavior of $\sigma_{\text{tot}}(s)$ through Fermilab and ISR energies. This is problematic for two reasons:

(1) The factor $\exp[(4\alpha/3\pi)\ln^2 s] \approx O(s^{\ln s})$ strongly violates the Froissart bound and cannot be present in a viable representation of the data. It is easy to see that this undesirable feature is connected to $\langle n\rangle = O(\ln^2 s)$. This is because, for a Poisson distribution, $\langle n_g\rangle = \alpha\, \partial \ln\sigma_{\text{tot}}/\partial\alpha$, which produces Eq. (1) for $\langle n\rangle$ once this factor for σ_{tot} is inserted. In order to eliminate this factor, one must essentially choose $\Lambda_{\text{uv}} = O(\omega_{\max})$. As emphasized by Weinberg,[15] this would seem hard to justify *a priori*. This choice requires knowledge of the infrared nonleading terms which have nothing to do with the leading $O(\ln^2 s)$ behavior of $\langle n\rangle$ proposed by BG. However, it is true that retaining the infrared nonleading k^2 terms in virtual gluon propagators does result in $\Lambda_{\text{uv}} = O(\omega_{\max})$. If one trusts keeping these infrared nonleading terms and neglecting others due to nonbremsstrahlung effects, then this difficulty in σ_{tot} is resolved.

(2) The second factor in σ_{tot} is a superposition of powers of s, corresponding to a j-plane cut.[16,17] This cut is of an unconventional type not constructed through iteration of a Regge pole through unitarity as, for example, in the Reggeon field theory or the eikonal model. Thus the predicted form Eq. (6) for σ_{tot} contradicts the standard and successful lore of hadron σ_{tot} as due to dominant j-plane poles and weak j-plane cuts. We remind the reader that j-plane cuts and long-range order in rapidity for production processes are related in all conventional models of j-plane cuts. As we have said, long-range correlations may be relevant for e^+e^- annihilation[7] but not for hadron scattering where a Fredholm multiperipheral short-range-order picture seems to dominate. Technically, the longitudinal- and transverse-momentum components of each produced cluster must roughly decouple from each other, and this does not happen for singular models.[16]

We now suggest two ways to retain gluon bremsstrahlung as a primary underlying particle production mechanism in hadron-hadron scattering at current energies but without the problems encountered by the BG model. Both suggestions lead to $\langle n\rangle = O(\ln s)$.

The first way, described in Ref. 5, is to cut off the maximum gluon frequency $\omega_{\max}$ by some hadronic scale $m_c \approx 1$ GeV. This is at least esthetically pleasing, since bremsstrahlung is *a priori* only a low-energy approximation. Thus the $\omega_{\max} = O(\sqrt{s}\,)$ assumption of BG is avoided, and the resulting $\bar{k}_T$ is bounded. The scale m_c can be naturally associated with an average cluster mass in a multiperipheral cluster model. This automatically produces the desirable good first approximation of such a model, and in addition leads to predictions for the clusters. Moreover, the observed flavor dependence of $\langle n\rangle$ is easily accommodated. As with any k_T-cutoff multiperipheral model, this bremsstrahlung-cluster multiperipheral model can only be valid *a priori* in hadron-hadron scattering, small-Q^2 $lp \to lX$ scattering, and not at all in $e^+e^- \to X$.

The second suggestion, staying more within the BG framework, is to choose α as the effective running coupling rather than the renormalized coupling. Thus

$$\alpha = \alpha_{\text{eff}}(s) = c[\ln(s/\lambda^2)]^{-1}$$

leads to

$$\langle n\rangle = O(\alpha_{\text{eff}}\ln^2 s) = O(\ln s)\,.$$

This assumption means that loop corrections are imagined to be summed using the renormalization group, resulting in the replacement $\alpha \to \alpha_{\text{eff}}$ but leaving intact the Poisson distribution. The total cross section becomes

$$\sigma_{\text{tot}}(s) = O(s^{4c/3\pi})\sigma_0\,, \tag{7}$$

which again, ignoring σ_0, is a bare-Pomeron-pole-like behavior $O(s^{\alpha - 1})$. If we take the flavor-SU(4) value $c = 12\pi/25$ then $\alpha = 1.64$. This is too high to fit the data, but σ_0 could perhaps intervene to lower the power of s, and at least Eq. (7) can be unitarized by j-plane cuts in standard fashions. Note that this Regge behavior arises from the $O(s^{\alpha \ln s})$ term of Eq. (6).

This result is rather like the situation found by Cardy[16] in $\phi^3(D=6)$ theory. Here, Regge-pole behavior is reinstated after loop corrections are added to an otherwise Regge-cut-behaved σ_{tot}, provided that asymptotic freedom is present.

The Regge-pole behavior in Eq. (7) is similar to that found[18] by Pancheri-Srivastava in that it arises from keeping only the infrared leading $(k\cdot p)^{-1}$ terms in virtual propagators. However, there is a technical difference. In Ref. 18 $\omega_{max} = O(m)$ is taken as being bounded by some mass and α is a constant, so $\alpha \ln\omega_{max} = O(1)$. In the above modified BG model, $\omega_{max} = O(\sqrt{s})$ and $\alpha = \alpha_{eff}(s)$ so again $\alpha \ln\omega_{max} = O(1)$.

The situation is like a non-k_T-cutoff multiperipheral model in which each new rung yields rapidity and k_T phase-space factors $(\ln s)(\ln s)$, along with a canceling $(\ln s)^{-1}$ from $\alpha_{eff}(s)$. This produces the canonical $\ln s$ for each rung and the Regge behavior of Eq. (7).

Note added in proof. We urge experimentalists to measure the rapidity correlation function for produced hadrons in $e^+e^- \to X$ and $lp \to lX$. It would be most interesting to see whether, in contrast to hadron-hadron scattering, dominant long-range order in rapidity is observed in these processes.

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