Problem of the isoscalar axial-vector current in neutral-current phenomenology

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The matrix element of the isoscalar axial-vector current between nucleon states is not simply related to the corresponding coupling constant appropriate for analyzing inclusive reactions. The consequences for the theoretical analysis of elastic neutrino-proton scattering and of parity violation in atomic hydrogen and deuterium are discussed.

The standard phenomenological analysis[1,2] of neutral currents for hadrons involves four coupling constants:

$$N_\lambda = g_V V_\lambda^3 + g_A A_\lambda^3 + g_V' V_\lambda^s + g_A' A_\lambda^s + \cdots , \qquad (1)$$

where the superscripts s and 3 refer to isoscalar and isovector currents. In particular, the current of interest in this note, the isoscalar axial-vector current, is defined as

$$A_\lambda^s = \tfrac{1}{2}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d) . \qquad (2)$$

A completely equivalent set of four couplings u_L, d_L, u_R, and d_R describes the left- and right-handed couplings of u and d quarks. The centered dots in Eq. (1) represent additional isoscalar currents depending on the couplings s_L, s_R to strange quarks (as well as couplings to heavier quarks), but the contributions of these to usual observables are generally considered negligible.[3] On the basis of the parton model, it is possible to relate deep-inelastic neutrino cross sections to linear combinations of u_L^2, u_R^2, d_L^2, and d_R^2. For elastic νp and $\bar{\nu}p$ cross sections as well as for parity violation in atomic hydrogen and deuterium, it is necessary to calculate the matrix element of N_λ between nucleon states at or near $q^2=0$. The matrix elements of V_λ^3 and V_λ^s can be related to the isovector and isoscalar electromagnetic matrix elements, while the matrix elements of A_λ^3 can be determined from charged-current matrix elements via isospin. The purpose of this note is to point out that the matrix element of the isoscalar axial-vector component of N_λ cannot be easily related to g_A', although such a relation has been assumed in many previous analyses.[4-11]

The proton matrix element of an axial-vector current A_λ^i at $q^2=0$ is written as

$$\langle p|A_\lambda^i|p\rangle_{q^2=0} = G_A^i\, \bar{u}_p\gamma_\lambda\gamma_5 u_p . \qquad (3)$$

The equivalent equation for the usual charged current is

$$\langle p|A_\lambda^{1+i2}|n\rangle = G_A \bar{u}_p\gamma_\lambda\gamma_5 u_n .$$

One of the standard ways of treating the current A_λ^s is to assume an SU(6) type wave function for the proton made of u and d quarks, as in the bag model, which yields

$$G_A^s = \tfrac{3}{5}\times\tfrac{1}{2}G_A = 0.375 . \qquad (4)$$

An alternative is to use the quark model only for the purpose of formulating a kind of "Zweig rule,"

$$\langle p|\bar{s}\gamma_\lambda\gamma_5 s|p\rangle = 0 , \qquad (5)$$

so that

$$\langle p|A_\lambda^s|p\rangle = \tfrac{1}{2}\langle p|\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d - 2\bar{s}\gamma_\lambda\gamma_5 s|p\rangle$$
$$= \frac{\sqrt{6}}{2}\langle p|A_\lambda^8|p\rangle \qquad (6)$$

and then use SU(3) to give

$$G_A^s = \frac{\sqrt{6}}{2}G_A^8 = \tfrac{1}{2}(3F-D) = 0.23 \pm 0.02 , \qquad (7)$$

where F and D are determined from the Cabibbo analysis[12]:

$$F+D = 1.25 ,$$
$$D/(F+D) = 0.658 \pm 0.007 .$$

The difference between the results of Eqs. (4) and (7) has been noted as a problem[13]; however, we believe the problem is more basic than indicated by this difference.

The basic difficulty arises because the current we are considering, A_λ^s, has an anomaly and the current $\bar{s}\gamma_\lambda\gamma_5 s$, for which we want to assume Eq. (5), also has an anomaly. We do not believe that simple SU(6) quark-model arguments work for matrix elements of anomalous currents. In practice it is not necessary or appropriate to discuss anomalous currents since no renormalizable gauge model contains such a current. Within the framework of four quarks one need consider only the anomaly-free currents A_λ^8, defined by Eq. (6), and

$$A_\lambda^{15} \equiv \frac{1}{\sqrt{12}}(\bar{u}\gamma_\lambda\gamma_5 u + \bar{d}\gamma_\lambda\gamma_5 d + \bar{s}\gamma_\lambda\gamma_5 s - 3\bar{c}\gamma_\lambda\gamma_5 c) . \qquad (8)$$

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In addition, there may be currents involving only heavy quarks such as

$$A_\lambda^{tb} = \tfrac{1}{2}(\bar{t}\gamma_\lambda\gamma_5 t - \bar{b}\gamma_\lambda\gamma_5 b)\,. \qquad (9)$$

From this point of view there are two problems for neutral-current phenomenology: (1) Whereas in the inclusive reactions only the current A_λ^s enters (with small corrections as noted for the strange-quark neutral current), there is no way to divide the contribution A_λ^s from $\bar{s}\gamma_\lambda\gamma_5 s$ in the evaluation of the low-q^2 matrix element of A_λ^8. (2) While we can calculate relatively reliably the matrix elements, of A_λ^8 using SU(3), or η-pole dominance as discussed below, we have no good way of calculating A_λ^{15}, which involves a combination of heavy and light quarks.

The problem may be illustrated by studying the divergence of the axial-vector current. The standard analysis for the usual charged current may be written

$$\begin{aligned}\langle p|\partial_\lambda A_\lambda^{1+i2}|n\rangle_{q^2=0} &= \langle p|(m_u+m_d)\bar{u}\gamma_5 d|n\rangle_{q^2=0}\\ &= 2MG_A(\bar{u}_p\gamma_5 u_n)\\ &= \frac{\sqrt{2}\,g_{\pi NN} f_\pi m_\pi^2}{q^2+m_\pi^2}\bigg|_{q^2=0}(\bar{u}_p\gamma_5 u_n)\,,\end{aligned} \qquad (10)$$

where the last equality is pion partially conserved axial-vector current (PCAC) and yields the Goldberger-Treiman relation. In the chiral SU(2) $\times$SU(2) limit, which is quite good, $m_u = m_d = 0$, but G_A does not vanish because of the pion pole. If we apply the same analysis to A_λ^8, we have

$$\begin{aligned}\langle p|\partial_\lambda A_\lambda^8|p\rangle_{q^2=0} &= \frac{1}{\sqrt{6}}\langle p|(m_u+m_d)(\bar{u}\gamma_5 u+\bar{d}\gamma_5 d)\\ &\quad - 4m_s\bar{s}\gamma_5 s|p\rangle_{q^2=0}\\ &= 2MG_A^8\bar{u}_p\gamma_5 u_p\,,\end{aligned} \qquad (11)$$

where we have ignored a small isospin-violating piece[14] proportional to $(m_u - m_d)$. In the chiral SU(2)$\times$SU(2) limit the first term proportional to (m_u+m_d) appears to vanish since there is no isoscalar particle analogous to the pion. Historically the absence of this particle was called "the η problem"[15]; the resolution of the problem arises from the anomalous behavior of A_λ^s. If one were in addition to apply the "Zweig rule," Eq. (5), to the remaining term, one would erroneously deduce $G_A^8 = 0$ in contrast to the SU(3) result Eq. (7).

The validity of the Cabibbo analysis relating $\Delta S = 1$ decays to $\Delta S = 0$ decays[12] suggests that SU(3) is good for calculating matrix elements of axial-vector currents and that Eq. (10) can be extended to A_λ^{4+i5}. This yields the kaon Goldberger-Treiman relation, which is satisfied within the uncertainty of the strong-coupling constants. If we go a small step further and apply η PCAC to the evaluation of A_λ^8 we find

$$2MG_A^8 = \frac{g_{\eta NN} f_\eta m_\eta^2}{q^2+m_\eta^2}\bigg|_{q^2=0} = g_{\eta NN} f_\eta\,. \qquad (12)$$

If SU(3) is valid for $g_{\eta NN}$ and f_η then Eq. (12) gives the SU(3) result Eq. (7).[16] Looking back at Eq. (11), it is clear that the residue of the η pole proportional to m_η^2 arises almost entirely from the term proportional to m_s; an analogous relation between m_η^2 and m_s arises in the current-algebra analysis of pseudoscalar meson masses. The Zweig rule fails because the $\bar{s}s$ term couples to the nucleon via the η pole. Note in contrast for vector mesons $\bar{s}s$ couples only to ϕ which has little or no coupling to the nucleon.

We believe that the SU(3) result Eq. (7) provides a reasonable estimate for the matrix element of A_λ^8, although, as a consequence of η'-η mixing the SU(3) result is probably not so good for A_λ^8 as it appears to be for A_λ^{4+i5} and A_λ^{1+i2}. For currents involving heavy quarks such as A_λ^{tb} we can use the quantum-chromodynamics perturbation-theory result of Collins, Wilczek, and Zee,[17] which gives only a very small contribution for any reasonable value of m_t. The major source of uncertainty is the evaluation of the matrix element of A_λ^{15}. Two possible ways of calculating A_λ^{15} are instructive to consider: (1) Neglect the $\bar{c}c$ term and assume PCAC holds for η' with $g_{\eta' NN}$ given by SU(6). In this way one can essentially regain the Zweig rule. However, we do not believe that PCAC is meaningful for the η'.[18] (2) Neglect the η' pole completely and use an equation similar to that of Ref. 17. This requires extending QCD perturbation theory to the region of low-mass quarks where it is not expected to be valid.

The analysis of elastic νp and $\bar{\nu}p$ scattering depends on three form factors:

$$\langle p|N_\lambda|p\rangle = g_1(q^2)\,\bar{u}\gamma_\lambda\gamma_5 u + f_1(q^2)\,\bar{u}\gamma_\lambda u + if_2(q^2)\,\bar{u}\sigma_{\lambda\nu}u\,q_\nu\,.$$

The form factors f_1 and f_2 are directly related to electromagnetic form factors; in particular, the magnetic coupling constant is given by

$$f_1(0)+f_2(0) = 2.35\,g_V + 1.32\,g_V'\,. \qquad (13)$$

It is usually assumed that the q^2 dependence of g_1 is the same as for the axial-vector charged-current form factor and that

$$g_1(0) = \tfrac{1}{2}G_A g_A + G_A^s g_A' \qquad (14)$$

with G_A^s given by Eqs. (4) or (7). It is this last assumption which we believe is unfounded; instead, we write

$$g_1(0)=\tfrac{1}{2}G_A g_A + X, \qquad (15)$$

where X is the matrix element of the isoscalar axial-vector current and has no unique relation to g_A'. Our discussion above of A_λ^8 and A_λ^{15} suggests that X is considerably smaller than $\frac{1}{2}G_A$ unless A_λ^8 and A_λ^{15} have abnormally large couplings, but we do not believe anything more quantitative has been justified.

Proposed experiments on parity violation in atomic hydrogen and deuterium can determine four neutral-current couplings of electrons with nucleons:[19] C_{1p}, C_{1n}, C_{2p}, and C_{2n}. The couplings C_{1p} and C_{1n} involve the currents V_λ^3 and V_λ^s for which the usual arguments can be used. It is the couplings C_{2p} and C_{2n} that involve matrix elements of the isoscalar axial-vector current that must be reexamined. In particular, experiments on deuterium depend on the combination $C_{2p}+C_{2n}$ which in the standard analysis is directly proportional to g_A'. Thus in models in which $g_A'=0$ (as the Weinberg-Salam model) the standard analysis has $C_{2d}=0$ except for radiative corrections.[20] In contrast, we find C_{2d} is directly proportional to X, which cannot be related to the coupling g_A' relevant for the analysis[21] of parity violation in deep-inelastic electron scattering.

A large number of papers exist in which the coupling constants of Eq. (1) are evaluated on the basis of different neutrino experiments. These should now be reevaluated in the light of the problem we have discussed. Our tentative conclusions are the following:

(1) Several recent analyses[8-10] have attempted to combine four experimental results: inclusive ν and $\bar{\nu}$ total cross sections and elastic νp and $\bar{\nu} p$ cross sections to obtain four relations among the four coupling constants. According to the analysis of Claudson, Paschos, Strait, and Sulak,[9] these allow only two solutions (within fairly large experimental errors), one of which can be ruled out by qualitative considerations of exclusive single-pion production. Our discussion indicates that in a general phenomenological analysis one must fit five quantities: g_A, g_V, g_A', g_V', and X, where X is the matrix element of the isoscalar axial-vector current between nucleons. The data on elastic νp and $\bar{\nu} p$ scattering thus provide one relation rather than two constraining the four coupling constants relevant for inclusive reactions. The elastic scattering in the q^2 range that has been studied determines with greatest accuracy the axial coupling $g_1(0)$; however, this cannot be related to inclusive reactions. The data also determine with somewhat less accuracy the couplings f_1 and f_2, and thus provide one constraining relation between g_V and g_V'. To obtain quantitative results for all four couplings additional quantitative data must be used.

(2) Sehgal[22] has made an analysis of semi-inclusive pion production based on the parton fragmentation picture. Combining this with the inclusive cross sections, he determines values for u_L^2, d_L^2, u_R^2, and d_R^2. This yields four different solutions corresponding to different choices of relative signs. Two of these solutions can be ruled out by qualitative consideration of exclusive single-pion production.[23] The remaining two solutions, labeled A and B, have been compared to elastic νp and $\bar{\nu} p$ data which agree much better with A than with B.[7,8] It turns out that the major distinction between the models lies in their values for f_1 and f_2, and thus the use of elastic data to single out solution A is not vitiated by our discussion.[24]

(3) If we consider the more limited task of checking the Weinberg-Salam model, for which $g_A'=0$, the axial-vector neutral current can be written

$$A_\lambda^{\rm WS}=A_\lambda^3+\tfrac{1}{2}(\bar{c}\gamma_\lambda\gamma_5 c-\bar{s}\gamma_\lambda\gamma_5 s)+A_\lambda^{tb}$$
$$=A_\lambda^3+\tfrac{1}{6}(\sqrt{6}A_\lambda^8-2\sqrt{3}A_\lambda^{15})+A_\lambda^{tb}.$$

The major problem now lies in possible contributions of $\bar{s}\gamma_\lambda\gamma_5 s$ and higher-mass quark terms. Collins, Wilczek, and Zee[17] apply QCD perturbation theory directly to the second term; they note, however, that their result is very uncertain because of the low mass of the strange quark. From our point of view there is a contribution from A_λ^8 which increases g_1 by about 10%, but then there is a completely uncertain contribution from A_λ^{15}. Assuming this may be of the order of 15%, our best guess is

$$g_1^{\rm WS}=\tfrac{1}{2}G_A\times(1.1\pm 0.15). \qquad (16)$$

When the elastic νp and $\bar{\nu} p$ analysis is compared to the Weinberg-Salam model, it is found that the experimental value of g_1 agrees with the simple analysis based on Eq. (14) $[g_1(0)=\frac{1}{2}G_A]$. From our point of view, this agreement can at best serve as a qualitative success of the model. A quantitative test is possible, however, by comparing the constraints on g_V and g_V' with the model. It is clear from the analyses already made that the values of g_V and g_V' do agree with the model for a value of $\sin^2\theta_W$ in the neighborhood of 0.25 in agreement with the inclusive data. The fit, however, should be made using only the weak constraint of Eq. (16) on g_1, which leads to a somewhat larger error on $\sin^2\theta_W$ than would otherwise be calculated.

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[2]L. M. Sehgal [in *Neutrinos '78*, proceedings of the International Conference for Neutrino Physics and Astrophysics, Purdue, edited by E. C. Fowler (Purdue Univ. Press, Lafayette, Indiana, 1978), p. 253] provides a recent review and additional references.

[3]For the case of inclusive neutrino cross sections the contributions s_L^2 and s_R^2 are given in Eq. (20) of Sehgal (Ref. 2) and the uncertainties corresponding to reasonable limits on them are compounded with experimental errors. For the case of elastic cross sections, the problem of strange-quark and heavier-quark couplings is discussed in Ref. 17 and in this paper.

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[18]For the breaking of chiral SU(2)×SU(2) the pion is a "Goldstone boson" but the η is not; this is the meaning of the "η problem." (See the forceful remarks in Ref. 15.) For breaking chiral SU(3)×SU(3) π, κ, and η form an octet of "Goldstone bosons," and the η problem becomes the η' problem; that is, the η' is definitely not a ninth Goldstone boson.

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[24]This can be seen from Fig. 17 of L. R. Sulak *et al.*, in *Neutrinos '78* (Ref. 2), p. 355, as discussed by Paschos (Ref. 8). One can also see this by use of Eq. (13) which gives $f_1(0)+f_2(0)$ equal to 1.00 ± 0.3 for solution A and 2.25 ± 0.3 for solution B. The data analysis of Ref. 9 gives $f_1 + f_2 = 0.95^{+0.72}_{-0.27}$.