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Particle dissociation on a composite target

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We discuss the Drell-Hiida-Deck (DHD) mechanism for particle dissociation on a composite target. Working in an approximation scheme of the eikonal type (linearized propagators), terms describing interaction with more than one target nucleon are easily included. Several interesting features emerge. Due to cancellations, the only important diagrams are those describing dissociation before the target is encountered. Disregarding cancellations, internal production vanishes like the longitudinal momentum transfer. There is thus no intranuclear cascading at high rapidities. A careful analysis of the rescattering terms for the coherent process $Nd \to (N\pi)d$ reveals that the target is much more transparent than one would naively think ($\sigma_2 = \sigma_{NN} + \sigma_{\pi N}$), though not as transparent as indicated by experiments. This suggests that the DHD mechanism is not the dominant mechanism for exclusive particle production on nuclei at high energies.

I. INTRODUCTION

When a proton beam scatters off some target, there is a large cross section for producing pions. This is common to all strongly interacting beams, and in many ways similar to bremsstrahlung. A simple mechanism for strong dissociation was proposed by Drell and Hiida[1] for the case of nucleon dissociation, and by Deck[2] for the case of pion dissociation.

In general, the Drell-Hiida-Deck (DHD) model is quite successful in describing the gross features of low-mass production on proton targets.[3] However, the interpretation of these low-mass enhancements is not clear, for a couple of reasons. First, the quark model predicts meson resonances close to threshold. The low-mass peaks in $\pi \to 3\pi$ and $K \to K\pi\pi$ are mainly $J^P = 1^+$, but the failure to find resonance behavior in the $J^{PC} = 1^{++}$ waves has been a persistent problem.[4] For the pion excitation (A_1), a possible solution has recently been suggested by several authors.[5] They argue that the direct production of A_1 is negligible, but that it is formed in final-state interactions. The deduced mass and width of the A_1 agree well with data on $\tau \to (3\pi) + \nu_\tau$.[6] This solution to the problem should be contrasted with the recent phase-shift analysis by Pernegr *et al.*[7] of data on nuclear 3π production. They conclude that the 1^+ wave is resonant. This would seem to indicate that when produced on nuclei, the DHD mechanism is strongly suppressed, leaving resonance production as the dominant mode. However, the puzzling point is that the nuclear data are consistent with the elementary production strength being only slightly smaller than for production on hydrogen.[8]

The second reason the interpretation of the low-mass enhancement is unclear is that when produced on nuclei, these low-mass states appear to have very small cross sections for interacting with target nucleons.[9,10] It has been argued[11] that if the DHD mechanism were dominant, the cross section (σ_2) should be of the order of the sum of the elementary cross sections. In particular, in the case $\pi \to \rho\pi$, one would expect[11] $\sigma_{A_1 N} \gtrsim 1.7\sigma_{\pi N}$, which is definitely ruled out by the experiments.[9,11]

We want here to discuss in some detail the simplest example of DHD dissociation on a composite target, $Nd \to (N\pi)d$, paying particular attention to the rescattering, i.e., contributions where both target nucleons participate. If the deuteron is to remain bound, the two nucleons tend to share the overall momentum transfer. This, however, can take place in a variety of ways. To second order in the scattering amplitudes, the beam nucleon can scatter off both nucleons, the pion can scatter off both nucleons, or the beam nucleon can scatter off one nucleon and the pion off the other one. Because of cancellations, the first type of terms is not important, whereas the last two are of comparable magnitude. We also discuss terms where either of the target nucleons interacts with both the outgoing particles. A careful analysis of the rescattering terms can reveal to what extent the DHD mechanism is important.

The pion can be produced before, between, or after the two scatterings on the target nucleons. An evaluation of these amplitudes therefore provides a quantitative insight into the spacetime structure of this kind of particle production. We shall in particular show that internal production is strongly suppressed for low masses and/or high energies, irrespective of the energy loss of the leading particle. This phenomenon is related to the suppression of bremsstrahlung from a high-energy electron going through an extended medium, first discussed (incorrectly) by Williams.[12] Feinberg and Pomeranchuk[13] pointed out that this suppression of particle production at high energies depends upon the mean free path of the particle going through the medium. Therefore, it sets in at much lower energies for $N \to N\pi$ in nuclear matter than for $e \to e\gamma$ in atomic matter.

II. DISSOCIATION ON A SIMPLE TARGET

We consider first briefly the dissociation $N \to N\pi$ on a simple target. Like for bremsstrahlung, the only role of the target is to take a little momentum, thereby restoring the energy-momentum balance. This can be achieved in three ways, as illustrated by the diagrams in Fig. 1.

Let us then see how the three diagrams add. We take a pseudoscalar πNN coupling. One usually[3] represents vertex corrections by some form factor. However, this form factor is slowly varying, and since we are interested in the limit of small momentum transfers, we shall ignore it. (This allows us to use a very simple notation for the terms involving two target nucleons.) In scattering off the target nucleons, the quasireal nucleons and pions will be treated as real particles. For a fixed final mass M close to threshold, the momentum transfer required to get it on shell is at high energies very small. We take the elastic scattering to be spin independent, and may then write

$$\begin{aligned} M_a &= \bar{u}_f M_{NN}(E_f, \vec{q})\, \frac{(\not{p}+m)}{p^2-m^2}\, i\sqrt{2}g\gamma^5 u_i\,, \\ M_b &= \bar{u}_f\, i\sqrt{2}g\gamma^5\, \frac{(\not{p}'+m)}{p'^2-m^2}\, M_{NN}(E_i, \vec{q})\, u_i\,, \\ M_c &= \bar{u}_f\, i\sqrt{2}g\gamma^5 u_i\, \frac{1}{k'^2-\mu^2}\, M_{\pi N}(\omega, \vec{q})\,. \end{aligned} \tag{2.1}$$

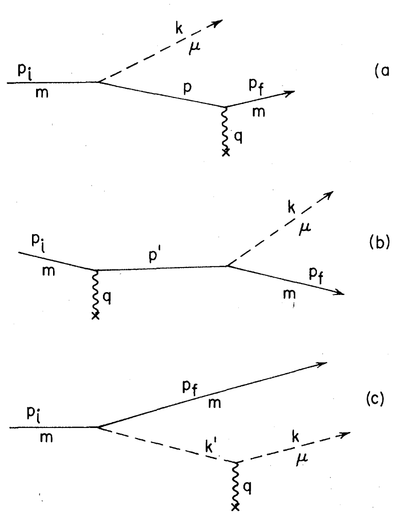

FIG. 1. Diagrams and kinematical variables for the DHD mechanism on a simple target.

Neglecting terms $\sim\vec{q}$, and identifying M_{NN} and $M_{\pi N}$ in terms of scattering amplitudes, we find

$$\begin{aligned} M_a &= -\mathcal{A}\, \frac{f_{NN}(E_f, \vec{q})}{2E_f\, q_L}\,, \\ M_b &= +\mathcal{A}\, \frac{f_{NN}(E_i, \vec{q})}{2E_i\, q_L}\,, \\ M_c &= -\mathcal{A}\, \frac{f_{\pi N}(\omega, \vec{q})}{2\omega q_L}\,, \\ \mathcal{A} &= 8\pi m i\sqrt{2}g\bar{u}_f\gamma^5 u_i\,. \end{aligned} \tag{2.2}$$

The above simple structure is essentially the one arrived at by Stodolsky.[14] However, he did not consider the term we here refer to as M_b, and it was Ross and Yam[15] who first realized that the terms M_a and M_b are comparable and of opposite signs. We should like to stress that this cancellation comes about (as in the case of bremsstrahlung) because the elastic scattering amplitude (at fixed momentum transfer) is proportional to the energy.

A detailed, quantitative study of the baryon-exchange terms has been performed by Cutler and Berger.[16] They allow for the off-shell-to-on-shell NN scattering amplitudes to be different from the on-shell-to-on-shell amplitudes, and find the cancellation to depend somewhat upon the form adopted for these amplitudes. This sensitivity to the form of the off-shell amplitude probably comes from the fact that the virtual nucleon in the two cases is off the mass shell by amounts which are comparable, but of opposite signs.

III. DISSOCIATION ON A TWO-NUCLEON TARGET

In dealing with the scattering of extremely relativistic particles off weakly bound target nucleons, it is convenient to do an adiabatic approximation. The energy transferred to the target nucleons is negligible, and we may regard them as fixed during the scattering process.[17] In fact, we shall first write down the amplitudes for a fixed separation of the target nucleons. This allows us to get some insight into the space-time structure of the dissociation. Relevant deuteron results are presented later. Since we are mainly interested in qualitative features, we shall work in the limit $\sigma_{NN}(E_i) = \sigma_{NN}(E_f)$.

We distinguish three types of terms, (i) single-scattering terms (one of the target nucleons interacts with either of the fast particles), (ii) double-scattering terms (both target nucleons interact with either or both of the fast particles, but none of them interacts with more than one fast particle), and (iii) correction terms (either or both target nucleons interact with both the fast particles). The latter are known as "absorption corrections" in the case of a single proton target.[18]

A. Single-scattering terms

The single-scattering diagrams are for either target nucleon given in Fig. 1. There are thus six terms, out of which four will cancel pairwise in the limit $\sigma_{NN}(E_i)=\sigma_{NN}(E_f)$. We are left with

$$\mathfrak{M}^{(1)}(\vec{r})=\bar{u}_f\, i\sqrt{2}\, g\, \gamma^5 u_i\, \frac{4\pi f_{\pi N}(\omega,\vec{q})}{k'^2-\mu^2} \times (e^{-i\vec{q}\cdot\vec{r}/2}+e^{i\vec{q}\cdot\vec{r}/2}) = -2\pi X\omega^{-1} f_{\pi N}(\omega,\vec{q})(e^{-i\vec{q}\cdot\vec{r}/2}+e^{i\vec{q}\cdot\vec{r}/2}), \qquad (3.1)$$

with

$$X\equiv i\sqrt{2}\, g\bar{u}_f\gamma^5 u_i\, q_L^{-1}. \qquad (3.2)$$

For coherent dissociation off a deuteron target, the two exponentials give a form factor, and the dependence upon $\vec{q}$ will be given by $2f_{\pi N}(\omega,\vec{q})S(\frac{1}{2}\vec{q})$, like for πd elastic scattering. However, in the region of double scattering, the dissociation amplitude becomes very different from the elastic πd amplitude.

B. Double-scattering terms

The double-scattering terms are given by the six diagrams in Fig. 2, plus those where the target nucleons are interchanged. We do not distinguish between whether the target nucleon is a proton or a neutron, so interchanging them amounts to reversing $\vec{r}$,

$$\mathfrak{M}^{(2)}(\vec{r})=\mathfrak{M}_a^{(2)}(\vec{r})+\mathfrak{M}_b^{(2)}(\vec{r})+\cdots+\mathfrak{M}_f^{(2)}(\vec{r}) + \{\vec{r}\to-\vec{r}\}\ .$$

For the first diagram, we find

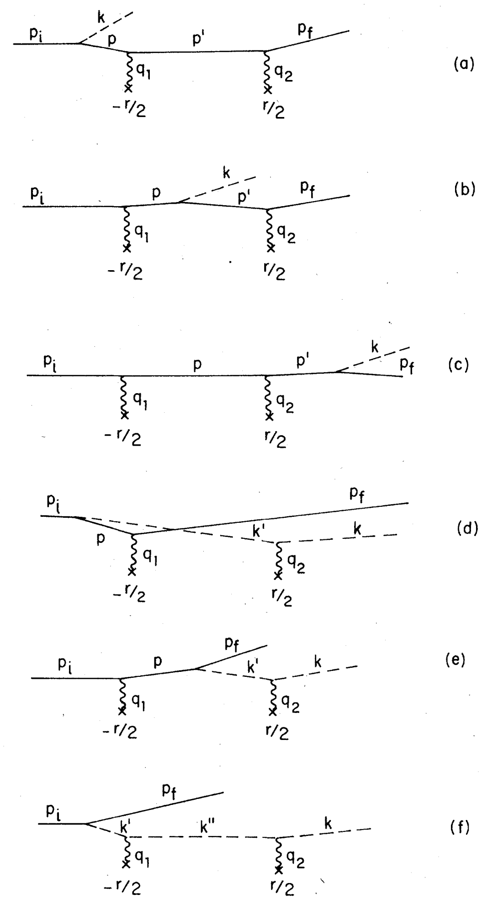

FIG. 2. The simplest diagrams describing DHD production on a two-nucleon target, with both target nucleons participating.

$$\mathfrak{M}_a^{(2)}(\vec{r})=\bar{u}_f\int\frac{d^3q_1}{(2\pi)^3}\, 4\pi f_{NN}(E_f,\vec{q}_2)e^{i\vec{q}_2\cdot\vec{r}/2}\,\frac{i}{p'^2-m^2+i\epsilon} \times 4\pi f_{NN}(E_f,\vec{q}_1)e^{-i\vec{q}_1\cdot\vec{r}/2}\,\frac{i}{p^2-m^2+i\epsilon}\, i\sqrt{2}\, g\gamma^5 u_i\,,$$

where $p=p_i-k$, $\vec{q}_1=\vec{p}-\vec{p}'$, $\vec{q}_2=\vec{p}'-\vec{p}_f$, $\vec{q}=\vec{q}_1+\vec{q}_2$, $E_p=E_{p'}=E_f$. Neglecting terms $\sim\vec{q}_1$ and $\vec{q}_2$, we have replaced the numerators $(\not{p}+m)$ and $(\not{p}'+m)$ by $2m$.

At small momentum transfers we may parametrize the amplitudes as

$$f_{NN}(E,\vec{q})=(iE\sigma_{NN}/4\pi)e^{-a_N\vec{q}^2/2}\,,$$

where we have neglected the real part. If we now linearize the propagators, $p'^2-m^2+i\epsilon\simeq-2E_f q_{2L}+i\epsilon$, $p^2-m^2+i\epsilon\simeq-2E_f q_L+i\epsilon$, the integration over transverse momentum transfers factors out, and may be expressed in terms of

$$G_{NN}(\vec{s};\vec{q}_T)=\int\frac{d^2p}{(2\pi)^2}\, e^{i\vec{p}\cdot\vec{s}}\exp\{-\tfrac{1}{2}a_N[(\tfrac{1}{2}\vec{q}_T+\vec{p})^2+(\tfrac{1}{2}\vec{q}_T-\vec{p})^2]\}\ , \qquad (3.3)$$

where $\vec{r}=\{\vec{s},z\}$, $\vec{q}=\{\vec{q}_T,q_L\}$.

The z values of interest are large compared with the range of the NN force, so we may further approximate

$$\int \frac{dq_{1L}}{2\pi} \frac{\exp\{-\frac{1}{2}a_N[(q_L - q_{1L})^2 + q_{1L}{}^2] - iq_{1L}z\}}{-q_L + q_{1L} + i\epsilon} \simeq -i\theta(z)e^{-a_N q_L{}^2/2}e^{-iq_L z}, \tag{3.4}$$

and get

$$\mathfrak{M}_a^{(2)}(\vec{r}) \simeq \tfrac{1}{4} iX\sigma_{NN}{}^2 G_{NN}(\vec{s};\vec{q}_T)e^{-a_N q_L{}^2/2}\theta(z)e^{-iq_L z/2}. \tag{3.5}$$

The longitudinal separation z enters only in an overall phase factor $e^{-iq_L z/2}$ and the θ function. All diagrams describing external production [2(a), 2(c), 2(d), 2(f)] give amplitudes of this structure. However, for internal production [diagrams 2(b) and 2(e)] this is not so.

Consider next diagram 2(b). Linearizing the propagators, and integrating over transverse momentum transfers, we now get

$$\mathfrak{M}_b^{(2)}(\vec{r}) = \tfrac{1}{4} Xq_L\sigma_{NN}{}^2 G_{NN}(\vec{s};\vec{q}_T)e^{iq_L z/2}\int \frac{dq_{1L}}{2\pi} \frac{\exp\{-\frac{1}{2}a_N[(q-q_1)_L{}^2 + q_{1L}{}^2] - iq_{1L}z\}}{(q_{1L} + i\epsilon)(q_{1L} - q_L + i\epsilon)}. \tag{3.6}$$

This integrand has two poles on the same side of the real axis, separated by q_L, and the approximation (3.4) leads to

$$\mathfrak{M}_b^{(2)}(\vec{r}) \simeq -\tfrac{1}{4} iX\sigma_{NN}{}^2 G_{NN}(\vec{s};\vec{q}_T)e^{-a_N q_L{}^2/2}\theta(z)(e^{-iq_L z/2} - e^{iq_L z/2}). \tag{3.7}$$

For small values of $q_L z$, $|\mathfrak{M}_b^{(2)}(\vec{r})|$ will vanish relative to $|\mathfrak{M}_a^{(2)}(\vec{r})|$ like $q_L z$. This result can be obtained qualitatively from the uncertainty principle. However, it is worth stressing that the momentum conjugate to $\vec{r}$ is the overall momentum transfer. Neither the momentum $\vec{p}_i - \vec{p}_f$ lost by the incident particle nor the momentum $\vec{k}$ of the produced particle enters explicitly.

The remaining terms are evaluated in the same manner. We find

$$\mathfrak{M}_c^{(2)}(\vec{r}) \simeq -\tfrac{1}{4} iX\sigma_{NN}{}^2 G_{NN}(\vec{s};\vec{q}_T)e^{-a_N q_L{}^2/2}\theta(z)e^{iq_L z/2}, \tag{3.8}$$

$$\mathfrak{M}_d^{(2)}(\vec{r}) \simeq \tfrac{1}{4} iX\sigma_{\pi N}\sigma_{NN}G_{N\pi}(\vec{s};\vec{q}_T)[e^{-a_N q_L{}^2/2}\theta(z)e^{-iq_L z/2} + e^{-a_\pi q_L{}^2/2}\theta(-z)e^{iq_L z/2}], \tag{3.9}$$

$$\mathfrak{M}_e^{(2)}(\vec{r}) \simeq -\tfrac{1}{4} iX\sigma_{\pi N}\sigma_{NN}G_{N\pi}(\vec{s};\vec{q}_T)\theta(z)(e^{-a_N q_L{}^2/2}e^{-iq_L z/2} - e^{-a_\pi q_L{}^2/2}e^{iq_L z/2}), \tag{3.10}$$

$$\mathfrak{M}_f^{(2)}(\vec{r}) \simeq \tfrac{1}{4} iX\sigma_{\pi N}{}^2 G_{\pi\pi}(\vec{s};\vec{q}_T)e^{-a_\pi q_L{}^2/2}\theta(z)e^{-iq_L z/2}. \tag{3.11}$$

In analogy with (3.3), we have defined

$$\begin{aligned} G_{N\pi}(\vec{s};\vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2} e^{i\vec{p}\cdot\vec{s}} \exp[-\tfrac{1}{2}a_N(\tfrac{1}{2}\vec{q}_T + \vec{p})^2]\ \exp[-\tfrac{1}{2}a_\pi(\tfrac{1}{2}\vec{q}_T - \vec{p})^2], \\ G_{\pi\pi}(\vec{s};\vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2} e^{i\vec{p}\cdot\vec{s}} \exp\{-\tfrac{1}{2}a_\pi[(\tfrac{1}{2}\vec{q}_T + \vec{p})^2 + (\tfrac{1}{2}\vec{q}_T - \vec{p})^2]\}. \end{aligned} \tag{3.12}$$

Both terms $\mathfrak{M}_b^{(2)}(\vec{r})$ and $\mathfrak{M}_e^{(2)}(\vec{r})$ that describe internal production vanish for $q_L z \to 0$, in agreement with qualitative expectations based upon the uncertainty principle.

In contrast to the analogous first-order terms, $\mathfrak{M}_a^{(2)}(\vec{r})$ and $\mathfrak{M}_c^{(2)}(\vec{r})$ do *not* cancel unless $q_L z \to 0$. But their sum is precisely cancelled by $\mathfrak{M}_b^{(2)}(\vec{r})$,

$$\mathfrak{M}_a^{(2)}(\vec{r}) + \mathfrak{M}_b^{(2)}(\vec{r}) + \mathfrak{M}_c^{(2)}(\vec{r}) = 0, \tag{3.13}$$

so no terms $\propto\sigma_{NN}{}^2$ will survive.

Summing all terms, we get

$$\mathfrak{M}^{(2)}(\vec{r}) \simeq \tfrac{1}{4} iX\sigma_{\pi N}e^{-a_\pi q_L{}^2/2}[\sigma_{NN}G_{N\pi}(\vec{s};\vec{q}_T)e^{iq_L z/2} + \sigma_{\pi N}G_{\pi\pi}(\vec{s};\vec{q}_T)\theta(z)e^{-iq_L z/2} + \{\vec{r} \to -\vec{r}\}]. \tag{3.14}$$

C. Correction terms

There are corrections to the above results due to the fact that either or both of the target nucleons can interact with both the fast particles. Terms involving only one target nucleon are shown in Fig. 3. When we describe each interaction by the physical scattering amplitude, no pair of particles is allowed to interact more than once. There is thus no term where the nucleon scatters before and after emission. One might likewise argue that diagram 3(b) should be omitted. Actually, for small q_L its contribution vanishes relative to the dominant term like $a_\pi q_L{}^2$, and the reason we include it is to cancel a small contribution from diagram 3(a). The sum of the two diagrams thus gives a very simple result. (For the corrections to the double-scattering terms, the inclusion of diagrams of this type leads to tremendous simplifications.)

The two diagrams give

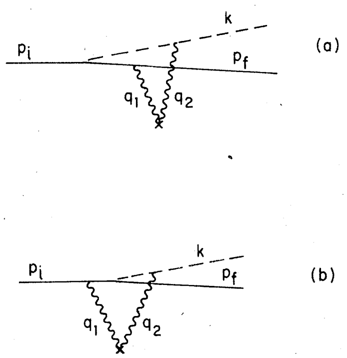

FIG. 3. Absorptive corrections to the single-scattering terms.

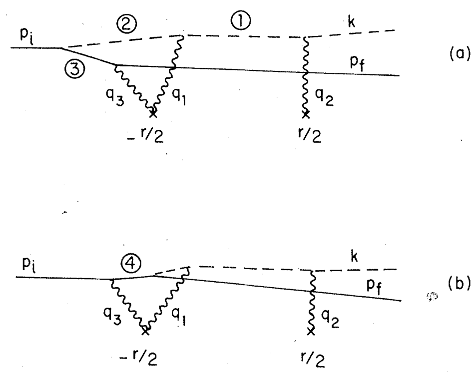

FIG. 4. Absorptive corrections to the double-scattering terms. Contributions proportional to $\sigma_{\pi N}{}^2\sigma_{NN}$.

$$\mathfrak{M}^{(1)}_{\text{corr}}(\vec{r})=\bar{u}_f\int\frac{d^3q_1}{(2\pi)^3}\,4\pi f_{\pi N}(\omega,\vec{q}_2)e^{-i\vec{q}_2\cdot\vec{r}/2}\,\frac{i}{(k+q_2)^2-\mu^2+i\epsilon}$$
$$\times\left[4\pi f_{NN}(E_f,\vec{q}_1)e^{-i\vec{q}_1\cdot\vec{r}/2}\,\frac{i}{(p_f+q_1)^2-m^2+i\epsilon}\right.$$
$$\left.+4\pi f_{NN}(E_i,\vec{q}_1)e^{-i\vec{q}_1\cdot\vec{r}/2}\,\frac{i}{(p_i-q_1)^2-m^2+i\epsilon}\right]i\sqrt{2}\,g\gamma^5u_i+\{\vec{r}\to-\vec{r}\}\,,$$

with $\vec{q}_2=\vec{q}-\vec{q}_1$. For the transverse part we use the definition (3.12), and get

$$\mathfrak{M}^{(1)}_{\text{corr}}(\vec{r})\simeq\tfrac{1}{4}iX\sigma_{\pi N}\sigma_{NN}G_{N\pi}(0;\vec{q}_T)\,e^{-a_\pi q_L{}^2/2}(e^{-i\vec{q}\cdot\vec{r}/2}+e^{i\vec{q}\cdot\vec{r}/2})\,, \tag{3.15}$$

where the transverse integral is

$$G_{N\pi}(0;\vec{q}_T)=\frac{1}{2\pi}\,\frac{1}{a_\pi+a_N}\,\exp\left(-\frac{1}{2}\,\frac{a_\pi a_N}{a_\pi+a_N}\,\vec{q}_T{}^2\right). \tag{3.16}$$

This is a significant correction. Let $\xi^{(1)}$ be the ratio $\mathfrak{M}^{(1)}_{\text{corr}}(\vec{r})/\mathfrak{M}^{(1)}(\vec{r})$. Then

$$\xi^{(1)}(\vec{q})\simeq-\frac{1}{4\pi}\,\frac{\sigma_{NN}}{a_\pi+a_N}\,\exp\left(\frac{1}{2}\,\frac{a_\pi{}^2}{a_\pi+a_N}\,\vec{q}_T{}^2\right). \tag{3.17}$$

This type of correction has recently been considered by Berger and Pirilä[18] for $Np\to(N\pi)p$, who, doing a more detailed analysis arrive at essentially the same relative strength. With $a_\pi=8$ (GeV/c)$^{-2}$, $a_N=10$ (GeV/c)$^{-2}$, and $\sigma_{NN}=40$ mb, we find

$$\xi^{(1)}(\vec{q})\simeq-0.45\exp[3.6\ (\text{GeV}/c)^{-2}\times\vec{q}_T{}^2]\,. \tag{3.18}$$

For proton targets, this correction has two desired features. (i) The cross section for nucleon dissociation is reduced by a factor of 2 to 3,[18,19] and (ii) it produces for low masses a dip around $\vec{q}^{\,2}\sim0.3$ (GeV/c)^2.[18]

The basic diagrams giving amplitudes proportional to $\sigma_{\pi N}{}^2\sigma_{NN}$ are given in Fig. 4. In addition, there are two analogous diagrams, where the fast nucleon interacts with the second target nucleon (obtained by the replacement $e^{-i\vec{q}_3\cdot\vec{r}/2}\to e^{i\vec{q}_3\cdot\vec{r}/2}$). Finally, there are four diagrams obtained by interchanging the two target nucleons.

The contribution from diagram 4(a) is

$$\mathfrak{M}^{(2)}_{\text{corr},\pi\pi N,a}(\vec{r})=iXq_L\int\frac{d^3q_1}{(2\pi)^3}\,\frac{d^3q_2}{(2\pi)^3}\,4\pi f_{\pi N}(\omega,\vec{q}_2)e^{i\vec{q}_2\cdot\vec{r}/2}\,\frac{i}{(k+q_2)^2-\mu^2+i\epsilon}\,4\pi f_{\pi N}(\vec{\omega},\vec{q}_1)$$
$$\times\frac{i}{(k+q_1+q_2)^2-\mu^2+i\epsilon}\,4\pi f_{NN}(E_f,\vec{q}_3)e^{-i\vec{q}_3\cdot\vec{r}/2}\,\frac{i}{(p_f+q_3)^2-m^2+i\epsilon}$$

where $\vec{q}_3=\vec{q}-\vec{q}_1-\vec{q}_2$. When we linearize the propagators, all energy factors in the amplitudes cancel against those of the propagators. Diagrams 4(a) and 4(b) thus contain the same integral, apart from the "reduced propagators." The encircled numbers in Fig. 4 label the different reduced propagators that appear. They are

$$d_1^{-1}=-2q_{2L}+i\epsilon,\quad d_2^{-1}=-2(q_1+q_2)_L+i\epsilon,\quad d_3^{-1}=-2q_{3L}+i\epsilon,\quad d_4^{-1}=2q_{3L}+i\epsilon\,. \tag{3.19}$$

Summing both combinations of reduced propagators, we get

$$\frac{1}{8}\,\frac{2\pi i}{q_L}\,\delta(q_{3L})\,\frac{1}{q_{1L}-q_L+i\epsilon}\,.$$

The fact that this has to vanish unless $q_{3L}=0$, can also be seen from the cancellation encountered in Sec. II. The sum of contributions from all eight diagrams is

$$\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi\pi N}(\vec{r})\simeq -\tfrac{1}{4}\,iX\sigma_{\pi N}{}^2\sigma_{NN}G_{\pi\pi N}(\vec{s};\vec{q}_T)e^{-a_\pi q_L{}^2/2}[\theta(z)e^{-iq_Lz/2}+\theta(-z)e^{iq_Lz/2}]\,, \tag{3.20}$$

where we have defined for the transverse integral

$$G_{\pi\pi N}(\vec{s};\vec{q}_T)=\int\frac{d^2p}{(2\pi)^2}\,\frac{d^2p'}{(2\pi)^2}\,e^{i\vec{p}\cdot\vec{s}}\exp\{-\tfrac{1}{2}a_\pi[(\tfrac{1}{2}\vec{q}_T+\vec{p})^2+(\tfrac{1}{2}\vec{q}_T-\vec{p}\,')^2]\}\exp[-\tfrac{1}{2}a_N(\vec{p}\,'-\vec{p})^2]$$

$$=\int\frac{d^2p}{(2\pi)^2}\,e^{i\vec{p}\cdot\vec{s}}\exp[-\tfrac{1}{2}a_\pi(\tfrac{1}{2}\vec{q}_T+\vec{p})^2]G_{N\pi}(0;\tfrac{1}{2}\vec{q}_T-\vec{p})\,. \tag{3.21}$$

The basic set of diagrams leading to amplitudes $\propto\sigma_{\pi N}\sigma_{NN}{}^2$ is given in Fig. 5. In addition, there are the analogous diagrams with the pion interacting with the second nucleon (obtained by $e^{-i\vec{q}_3\cdot\vec{r}/2}\to e^{i\vec{q}_3\cdot\vec{r}/2}$), and the ones with the target nucleons interchanged ($\vec{r}\to-\vec{r}$). Terms $\propto\sigma_{\pi N}\sigma_{NN}{}^2$ are given by

$$\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi NN}(\vec{r})\simeq -\tfrac{1}{4}\,iX\sigma_{\pi N}\sigma_{NN}{}^2G_{NN\pi}(\vec{s};\vec{q}_T)e^{-a_\pi q_L{}^2/2}\cos(\tfrac{1}{2}q_Lz)\,, \tag{3.22}$$

where

$$G_{NN\pi}(\vec{s};\vec{q}_T)=\int\frac{d^2p}{(2\pi)^2}\,\frac{d^2p'}{(2\pi)^2}\,e^{i\vec{p}\cdot\vec{s}}\exp\{-\tfrac{1}{2}a_N[(\tfrac{1}{2}\vec{q}_T+\vec{p})^2+(\tfrac{1}{2}\vec{q}_T-\vec{p}\,')^2]\}\exp[-\tfrac{1}{2}a_\pi(\vec{p}\,'-\vec{p})^2]$$

$$=\int\frac{d^2p}{(2\pi)^2}\,e^{i\vec{p}\cdot\vec{s}}\exp[-\tfrac{1}{2}a_N(\tfrac{1}{2}\vec{q}_T+\vec{p})^2]G_{N\pi}(0;\tfrac{1}{2}\vec{q}_T-\vec{p})\,. \tag{3.23}$$

The basic diagrams giving amplitudes $\propto\sigma_{\pi N}{}^2\sigma_{NN}{}^2$ are given in Fig. 6. To these we have to add the terms with the q_3 and q_4 lines interchanged, and those with the target nucleons interchanged. The total amplitude of this order is

$$\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi\pi NN}(\vec{r})\simeq\tfrac{1}{16}\,iX\sigma_{\pi N}{}^2\sigma_{NN}{}^2G_{\pi\pi NN}(\vec{s};\vec{q}_T)e^{-a_\pi q_L^2/2}\,[\theta(z)e^{-iq_Lz/2}+\theta(-z)e^{iq_Lz/2}]\,, \tag{3.24}$$

where the transverse part can be written as

$$G_{\pi\pi NN}(\vec{s};\vec{q}_T)=\int\frac{d^2t}{(2\pi)^2}\,G_{NN}(\vec{s};\tfrac{1}{2}\vec{q}_T+\vec{t})G_{\pi\pi}(\vec{s};\tfrac{1}{2}\vec{q}_T-\vec{t})$$

$$=\int\frac{d^2p}{(2\pi)^2}\,e^{i\vec{p}\cdot\vec{s}}G_{N\pi}(0;\tfrac{1}{2}\vec{q}_T+\vec{p})G_{N\pi}(0;\tfrac{1}{2}\vec{q}_T-\vec{p})\,. \tag{3.25}$$

Consider now the total correction to the double-scattering terms

$$\mathfrak{M}^{(2)}_{\mathrm{corr}}(\vec{r})=\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi\pi N}(\vec{r})+\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi NN}(\vec{r})+\mathfrak{M}^{(2)}_{\mathrm{corr},\,\pi\pi NN}(\vec{r})\,,$$

and the ratio $\xi^{(2)}=\mathfrak{M}^{(2)}_{\mathrm{corr}}(\vec{r})/\mathfrak{M}^{(2)}(\vec{r})$. As $q_Lz\to0$, the following limit is approached,

$$\xi^{(2)}(\vec{s};\vec{q}_T)=-\,\frac{\sigma_{\pi N}\sigma_{NN}G_{\pi\pi N}(\vec{s};\vec{q}_T)+\sigma_{NN}{}^2G_{NN\pi}(\vec{s};\vec{q}_T)-\tfrac{1}{4}\sigma_{\pi N}\sigma_{NN}{}^2G_{\pi\pi NN}(\vec{s};\vec{q}_T)}{2\sigma_{NN}G_{N\pi}(\vec{s};\vec{q}_T)+\sigma_{\pi N}G_{\pi\pi}(\vec{s};\vec{q}_T)}\,. \tag{3.26}$$

To gain some insight into the $\vec{s}$ and $\vec{q}$ dependence of this ratio, consider the case $a_\pi=a_N=a$. Then

$$G_{N\pi}(\vec{s};\vec{q}_T)=\frac{1}{4\pi a}\exp\left(-\frac{1}{4}\,\frac{\vec{s}^{\,2}}{a}-\frac{1}{4}\,a\vec{q}_T{}^2\right)\,,$$

$$G_{\pi\pi N}(\vec{s};\vec{q}_T)=\frac{1}{4\pi a}\,\frac{1}{3\pi a}\exp\left(-\frac{1}{3}\,\frac{\vec{s}^{\,2}}{a}-\frac{1}{6}\,a\vec{q}_T{}^2-\frac{i}{6}\,\vec{s}\cdot\vec{q}_T\right)\,, \tag{3.27}$$

$$G_{\pi\pi NN}(\vec{s};\vec{q}_T)=\left(\frac{1}{4\pi a}\right)^2\frac{1}{2\pi a}\exp\left(-\frac{1}{2}\,\frac{\vec{s}^{\,2}}{a}-\frac{1}{8}\,a\vec{q}_T{}^2\right)\,,$$

and

$$\xi^{(2)}(\vec{s};\vec{q}_T)=-\,\frac{\sigma_{NN}}{8\pi a}\,\frac{1}{1+\frac{1}{2}\alpha}\left\{\frac{4}{3}(1+\alpha)\exp\left[\frac{1}{12}a\left(\vec{q}_T-\frac{i\vec{s}}{a}\right)^2\right]-\alpha\,\frac{\sigma_{NN}}{8\pi a}\exp\left(\frac{1}{8}\,a\vec{q}_T{}^2-\frac{1}{4}\,\frac{\vec{s}^{\,2}}{a}\right)\right\}\,, \tag{3.28}$$

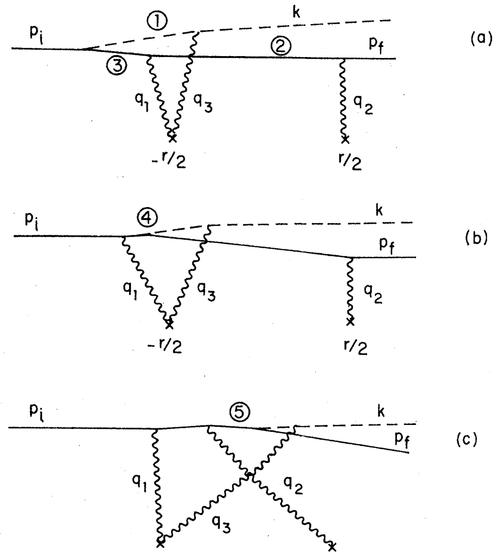

FIG. 5. Absorptive corrections to the double-scattering terms. Contributions proportional to $\sigma_{\pi N}\sigma_{NN}^2$.

where $\alpha = \sigma_{\pi N}/\sigma_{NN}$.

For fixed transverse separation $\vec{s}$, the ratio increases with $\vec{q}_T$, as does $\xi^{(1)}(\vec{q}_T)$, whereas for fixed momentum transfer the ratio decreases with $\vec{s}$ (the correction terms are of shorter transverse range than the ordinary double-scattering terms).

For small transverse separation, $\vec{s}^2 \lesssim 4a \simeq 1.6$ fm^2, and for small momentum transfers, $\vec{q}_T^{\,2} \lesssim 8/a$ $\simeq 0.8$ (GeV/c)^2, and with $\alpha \simeq \frac{2}{3}$, this becomes

$$\xi^{(2)}(0,0) \simeq 1.45\,\xi^{(1)}(0). \tag{3.29}$$

The corrections to the double-scattering terms are in this limit much more important than the corrections to the single-scattering terms. Since the corrections are negative, this implies that there will be relatively little double scattering (in the sense that both target nucleons participate) at small $\vec{s}$.

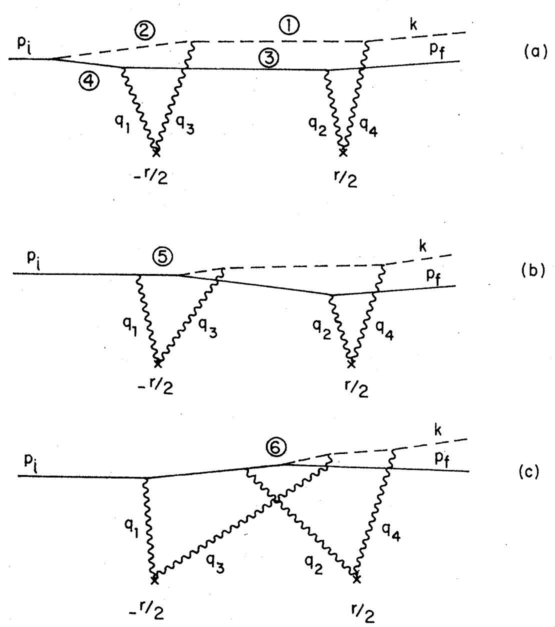

FIG. 6. Absorptive corrections to the double-scattering terms. Contributions proportional to $\sigma_{\pi N}^2\sigma_{NN}^2$.

IV. THE NOTION OF A FORMATION ZONE

It was pointed out by Landau and Pomeranchuk[20] that particle production cannot be localized to an arbitrary accuracy. They considered soft bremsstrahlung and found that for a particle of mass m and energy E to emit a photon of energy ω ($\omega \ll E$) at small angles, the essential time interval would have to be of the order

$$t \sim E^2/m^2\omega. \tag{4.1}$$

The length corresponding to such a characteristic production time has become known as the formation zone.

The importance of the formation zone has more recently been stressed by Stodolsky,[21] who defines it for nonzero radiation angles as the inverse propagator,

$$L_{\text{LPS}} = \frac{1}{\omega - \vec{\beta}\cdot\vec{k}}. \tag{4.2}$$

Here $\vec{\beta}$ is the velocity of the emitting particle, and LPS denotes Landau-Pomeranchuk-Stodolsky.

We have seen in Sec. III that production can only take place over an interval z provided $zq_L \gtrsim 1$ [compare the matrix elements $\mathfrak{M}_b^{(2)}(\vec{r})$ and $\mathfrak{M}_e^{(2)}(\vec{r})$, Eqs. (3.7) and (3.10)]. We therefore find it more natural to define the formation zone as

$$L \equiv \frac{1}{q_L} \simeq \frac{2E_i}{M^2 - m^2}. \tag{4.3}$$

The quantities (4.2) and (4.3) are in general different. For small angles and extremely relativistic particles, we find

$$L_{\text{LPS}}^{-1} = \tfrac{1}{2}\omega\left(\theta^2 + \frac{m^2}{E_i^2} + \frac{\mu^2}{\omega^2}\right), \tag{4.4}$$

$$L^{-1} = \tfrac{1}{2}\omega\frac{E_i}{E_f}\left(\theta^2 + \frac{m^2}{E_i^2} + \frac{\mu^2}{\omega^2}\,\frac{E_f}{E_i}\right). \tag{4.5}$$

For small energy losses ($E_i/E_f \to 1$) the two quantities become identical. However, they will for large energy losses and hard radiation,

$$\frac{\mu^2}{\omega^2}\,\frac{E_f}{E_i} \lesssim \frac{m^2}{E_i^2} \ll 1,$$

differ by essentially a factor E_i/E_f, our formation zone being the shorter one.

The dependence upon the produced mass and transverse momentum is more apparent in the form[22]

$$L_{\text{LPS}} \simeq \frac{2\omega}{(mx)^2 + k_T^2 + \mu^2}\,, \tag{4.6}$$

$$L \simeq (1-x)\,\frac{2\omega}{(mx)^2 + k_T^2 + (1-x)\mu^2}\,, \tag{4.7}$$

where $x = k_L/E_i \simeq \omega/E_i$.

For fixed $\vec{k}_T$ and E_i, the quantity (4.7) is maximal for

$$x_m = (\eta + \eta^2)^{1/2} - \eta, \quad \eta = \frac{\mu^2 + \vec{k}_T^{\,2}}{m^2 - \mu^2}\,.$$

For $\vec{k}_T = 0$, this amounts to the condition that the outgoing particles travel with parallel and equal velocities. Obviously, this minimizes their invariant mass, and thus maximizes the formation zone. On the other hand, maximizing (4.6) corresponds to having the emitted particle travel with the same speed as the incident particle. This is only correct if its energy is small compared to the incident energy ($x \ll 1$).

V. COHERENT DISSOCIATION $Nd \to (N\pi)d$

The coherent dissociation on the deuteron is given by

$$\mathfrak{M}_{\text{coh}}^{M'M} = \langle d,M' | \mathfrak{M}^{(1)}(\vec{r}) + \mathfrak{M}^{(1)}_{\text{corr}}(\vec{r}) + \mathfrak{M}^{(2)}(\vec{r}) + \mathfrak{M}^{(2)}_{\text{corr}}(\vec{r}) | d,M\rangle\,, \tag{5.1}$$

where $|d,M\rangle$ is the deuteron ground state for spin projection M.

We define the deuteron form factor by

$$S^{M'M}(\vec{q}) = \langle d,M' | e^{i\vec{q}\cdot\vec{r}} | d,M\rangle\,. \tag{5.2}$$

A related quantity will in general also be needed

$$\begin{aligned} S_\theta^{M'M}(q_L, \vec{q}_T) &= 2\langle d,M' | e^{iq_L z} e^{i\vec{q}_T\cdot\vec{s}}\,\theta(-z) | d,M\rangle \\ &= S^{M'M}(q_L, \vec{q}_T) \\ &\quad - i\langle d,M' | \sin(q_L|z|) e^{i\vec{q}_T\cdot\vec{s}} | d,M\rangle\,. \end{aligned} \tag{5.3}$$

The target nucleon that is first encountered will in our approximation take all the longitudinal momentum transfer. This leads to terms involving the latter form factor.

Suppressing spin projection indices, the expectation value (5.1) can be written as

$$\begin{aligned} \mathfrak{M}_{\text{coh}} = -iX\sigma_{\pi N} e^{-a_\pi q_L^2/2} \{ & [e^{-a_\pi \vec{q}_T^{\,2}/2} - \tfrac{1}{2}\sigma_{NN} G_{N\pi}(0;\vec{q}_T)] S(\tfrac{1}{2}\vec{q}) \\ & - \tfrac{1}{4}[2\sigma_{NN} K_{N\pi}(\tfrac{1}{2}q_L, \vec{q}_T) + \sigma_{\pi N} K^\theta_{\pi\pi}(\tfrac{1}{2}q_L, \vec{q}_T) - \sigma_{NN}^2 K_{NN\pi}(\tfrac{1}{2}q_L, \vec{q}_T) \\ & - \sigma_{\pi N}\sigma_{NN} K^\theta_{\pi\pi N}(\tfrac{1}{2}q_L, \vec{q}_T) + \tfrac{1}{4}\sigma_{\pi N}\sigma_{NN}^2 K^\theta_{\pi\pi NN}(\tfrac{1}{2}q_L, \vec{q}_T)]\}\,, \end{aligned} \tag{5.4}$$

where we have defined

$$\begin{aligned} K_{N\pi}(q_L, \vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2}\, S(q_L, \vec{p}) \exp[-\tfrac{1}{2}a_N(\tfrac{1}{2}\vec{q}_T + \vec{p})^2] \exp[-\tfrac{1}{2}a_\pi(\tfrac{1}{2}\vec{q}_T - \vec{p})^2]\,, \\ K^\theta_{\pi\pi}(q_L, \vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2}\, S_\theta(q_L, \vec{p}) \exp\{-\tfrac{1}{2}a_\pi[(\tfrac{1}{2}\vec{q}_T + \vec{p})^2 + (\tfrac{1}{2}\vec{q}_T - \vec{p})^2]\}\,, \\ K_{NN\pi}(q_L, \vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2}\, S(q_L, \vec{p}) \exp[-\tfrac{1}{2}a_N(\tfrac{1}{2}\vec{q}_T + \vec{p})^2] G_{N\pi}(0; \tfrac{1}{2}\vec{q}_T - \vec{p})\,, \\ K^\theta_{\pi\pi N}(q_L, \vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2}\, S_\theta(q_L, \vec{p}) \exp[-\tfrac{1}{2}a_\pi(\tfrac{1}{2}\vec{q}_T + \vec{p})^2] G_{N\pi}(0; \tfrac{1}{2}\vec{q}_T - \vec{p})\,, \\ K^\theta_{\pi\pi NN}(q_L, \vec{q}_T) &= \int \frac{d^2p}{(2\pi)^2}\, S_\theta(q_L, \vec{p}) G_{N\pi}(0; \tfrac{1}{2}\vec{q}_T + \vec{p}) G_{N\pi}(0; \tfrac{1}{2}\vec{q}_T - \vec{p})\,. \end{aligned} \tag{5.5}$$

Data on processes of this type are usually analyzed assuming a somewhat different physical picture. The interaction between the incident particle and one of the target nucleons is supposed to lead to the produced system (in its initial stage). The incident particle and the produced system (which we denote N^*) may scatter elastically before or after production, respectively.[23] If the elementary production is described by

$$\tilde{M}(\vec{q}) = \tilde{M}(0)\, e^{-a_{\text{pr}}\vec{q}^{\,2}/2}\,,$$

then for coherent production on a deuteron,

$$\tilde{\mathfrak{M}}_{\text{coh}} = \tilde{M}(0)\{2e^{-a_{\text{pr}}\vec{q}^{\,2}/2} S(\tfrac{1}{2}\vec{q}) - \tfrac{1}{2}[\sigma_{NN} K^\theta_{N\text{pr}}(-\tfrac{1}{2}q_L, \vec{q}_T) + \sigma_{N^*N} K_{\text{pr}N^*}(\tfrac{1}{2}q_L, \vec{q}_T)]\}\,, \tag{5.6}$$

where

$$K^{\theta}_{N\mathrm{pr}}(q_L,\vec{q}_T)=\int \frac{d^2p}{(2\pi)^2}\, S_\theta(q_L,\vec{p})\exp[-\tfrac{1}{2}a_N(\tfrac{1}{2}\vec{q}_T+\vec{p})^2]\exp[-\tfrac{1}{2}a_{\mathrm{pr}}(\tfrac{1}{2}\vec{q}_T-\vec{p})^2]\,,$$
$$K^{\theta}_{\mathrm{pr}N^*}(q_L,\vec{q}_T)=\int \frac{d^2p}{(2\pi)^2}\, S_\theta(q_L,\vec{p})\exp[-\tfrac{1}{2}a_{\mathrm{pr}}(\tfrac{1}{2}\vec{q}_T+\vec{p})^2]\exp[-\tfrac{1}{2}a_{N^*}(\tfrac{1}{2}\vec{q}_T-\vec{p})^2]\,. \tag{5.7}$$

We shall refer to Eq. (5.6) as the KM (Kölbig-Margolis) amplitude.

The quantity of greatest interest is σ_{N^*N}. In related experiments $[nA\to(p\pi^-)A$[24] and $pd\to Xd$[25]$]$ this is found to be small compared to σ_{NN}, and definitely much smaller than $\sigma_{NN}+\sigma_{\pi N}$. This has traditionally[11] been taken as an argument against the DHD mechanism.

For a comparison of (5.4) with (5.6), let us specialize to the case $q_L=0$ (i.e., $q_LR\ll 1$, where R is the deuteron radius). The form factors (5.2) and (5.3) are then the same.

Let us for a moment consider the case of a "large" deuteron, i.e., let the ranges of the forces be negligible compared to the size of the deuteron (a_π, a_N, a_{pr}, $a_{N^*}\ll R^2$). The $\vec{p}$ and $\vec{q}_T$ dependences in Eqs. (5.5) and (5.7) then decouple. Using[26]

$$\int \frac{d^2p}{(2\pi)^2}\, S(\vec{p})=\frac{1}{2\pi}\left\langle\frac{1}{r^2}\right\rangle\,, \tag{5.8}$$

we find

$$K_{N\pi}(\vec{q}_T)\simeq\frac{1}{2\pi}\left\langle\frac{1}{r^2}\right\rangle\exp[-\tfrac{1}{8}(a_N+a_\pi)\vec{q}_T^{\,2}]\,, \tag{5.9}$$

etc. With

$$\tilde{a}^{-1}=a_\pi^{-1}+a_N^{-1},\quad \bar{a}=\tfrac{1}{2}(a_\pi+a_N),\quad \zeta=\frac{\sigma_{NN}}{2\pi(a_\pi+a_N)},\quad t'=-\vec{q}_T^{\,2}\,, \tag{5.10}$$

Eq. (5.4) takes in this limit the simple form

$$\mathfrak{M}_{\mathrm{coh}}\simeq -iX\sigma_{\pi N}\Big\{(e^{a_\pi t'/2}-\tfrac{1}{2}\zeta e^{\tilde{a}t'/2})S(\tfrac{1}{4}t') - \frac{1}{8\pi}\left\langle\frac{1}{r^2}\right\rangle[2\sigma_{NN}e^{\bar{a}t'/4}+\sigma_{\pi N}e^{a_\pi t'/4}-\zeta(\sigma_{NN}e^{(a_N+\tilde{a})t'/8}+\sigma_{\pi N}e^{(a_\pi+\tilde{a})t'/8})+\tfrac{1}{4}\zeta^2\sigma_{\pi N}e^{\tilde{a}t'/4}]\Big\}\,, \tag{5.11}$$

whereas Eq. (5.6) becomes

$$\mathfrak{M}_{\mathrm{coh}}\simeq 2\tilde{M}(0)\Big\{\exp(\tfrac{1}{2}a_{\mathrm{pr}}t')S(\tfrac{1}{4}t')-\frac{1}{8\pi}\left\langle\frac{1}{r^2}\right\rangle[\sigma_{NN}\exp(\tfrac{1}{8}(a_N+a_{\mathrm{pr}})t')+\sigma_{N^*N}\exp(\tfrac{1}{8}(a_{N^*}+a_{\mathrm{pr}})t')]\Big\}\,. \tag{5.12}$$

In general, no value of σ_{N^*N} will make the two amplitudes the same for all values of t', due to their complex t' dependence. However, with a suitable choice of σ_{N^*N}, they may in a limited range of t' look very similar. Consider the near-forward direction, where the dominant part of the amplitude goes like $S(\frac{1}{4}t')$. In comparison, the rescattering terms are essentially t' independent. The relative amount of rescattering will be the same, provided $\sigma_{N^*N}=54$ mb. Without the correction terms ($\zeta\to 0$), the corresponding value would have been $\sigma_{N^*N}=\sigma_{NN}+\sigma_{\pi N}=65$ mb. As discussed in Sec. III, this *increased* transparency is due to the correction terms being relatively more important for the double-scattering than for the single-scattering terms.

Effects due to the size of the deuteron being comparable to the range of the forces, do only quantitatively modify the above result. For an investigation of these effects, we take a realistic multi-Gaussian parametrization[27] of the deuteron wave function with 6.7% d state. The slopes of the elastic amplitudes are fixed to $a_\pi=8$ (GeV/c)$^{-2}$, $a_N=a_{N^*}=10$ (GeV/c)$^{-2}$.

To compare the double-scattering contribution to the cross sections as determined from the matrix elements (5.4) and (5.6), we consider the ratio

$$R(t')=\frac{d\sigma_{Nd\to(N\pi)d}(t',\ldots)}{d\sigma_{Np\to(N\pi)p}(t',\ldots)}\,\frac{d\sigma_{Np\to Np}(t')}{d\sigma_{Nd\to Nd}(t')}\,. \tag{5.13}$$

The dots denote any set of variables that gives a complete description of the two-body $N\pi$ final state, e.g., mass and decay distribution variables, $M_{N\pi}$, $\cos\theta_{\mathrm{GJ}}$, ϕ_{GJ}. In the limit considered, $q_L\to 0$, the above ratio does not depend upon kinematical variables other than t'. The evaluation of the deuteron cross sections is outlined in the Appendix.

In Fig. 7 we compare $R_{\mathrm{DHD}}(t')$ with $R_{\mathrm{KM}}(t')$ for three values of $a_{\mathrm{pr}}=15$, 10, and 5 (GeV/c)$^{-2}$, and for three values of $\sigma_{N^*N}=20$, 40, and 60 mb. Be-

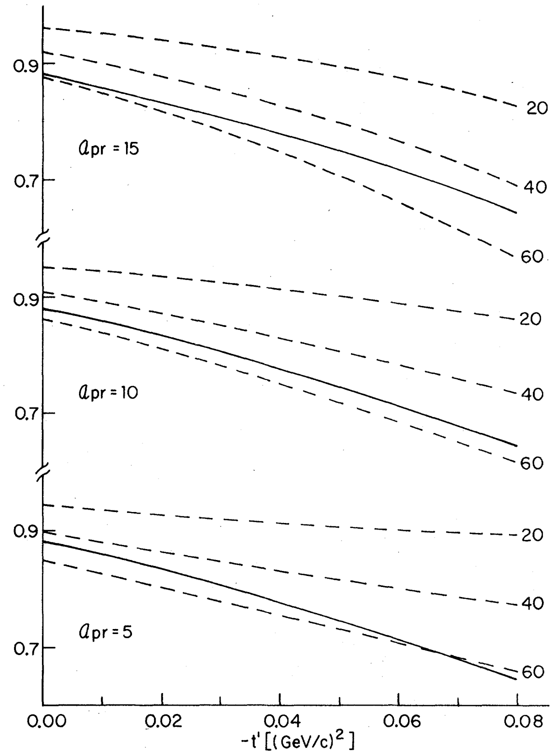

FIG. 7. The rescattering ratio $R(t')$ as defined by Eq. (5.13). Solid curves: $R_{\rm DHD}$. Dashed curves: $R_{\rm KM}$. Three production slopes [15, 10, and 5 $({\rm GeV}/c)^{-2}$] and three values of σ_{N^*N} (20, 40, and 60 mb) are considered for $R_{\rm KM}$.

cause of the neglect of transverse parts of the propagators, only small values of $|t'|$ are considered.

The value of σ_{N^*N} that corresponds to the rescattering given by the DHD model depends significantly on $a_{\rm pr}$ and t'. At $t'=0$, the equivalent σ_{N^*N} decreases with decreasing $a_{\rm pr}$ but at larger $|t'|$ the situation is reversed. Typical values are around 50 mb. Although being significantly below the value $\sigma_{N^*N}=\sigma_{NN}+\sigma_{\pi N}$, this seems still too high compared with the value $\sigma_{N^*N}\simeq \frac{1}{2}\sigma_{NN}$ favored by the CERN ISR data.[10] This suggests that the DHD mechanism is not the dominant production mechanism at these energies.

VI. CONCLUDING REMARKS

We have presented a qualitative way of analyzing Drell-Hiida-Deck-type dissociation on a composite target, with some detailed results for the process $Nd\to(N\pi)d$.

For low masses produced at high energies, the only relevant diagrams are those describing dissociation before the target is encountered. There is thus no intranuclear cascading of the fast particles, they are produced before the target is encountered, like for production on hydrogen.

If no target nucleon were to interact with more than one fast particle, this dissociation mechanism would lead to a strong absorption of the produced system, $\sigma_2=\sigma_{NN}+\sigma_{\pi N}$. This is in clear contradiction with the data. However, both the fast particles may hit the same target nucleon. This leads to an increased transparency of the target, since these correction terms are more important for the double-scattering than for the single-scattering terms. ("Single scattering" and "double scattering" refer to how many target nucleons interact with the fast particles.)

Given the rules for how certain terms cancel, this increased importance of the correction terms for the double scattering is essentially due to combinatorics.

The increased transparency may alternatively be thought of as being due to the structure of the target. A different approach,[28] also allowing more than one fast particle to hit the target, but neglecting its structure, also leads to the too high value $\sigma_2=\sigma_{NN}+\sigma_{\pi N}$.

Because of the different structure of the double-scattering terms in the DHD and the KM pictures, the equivalent σ_2 becomes somewhat $|t'|$-dependent. For $|t'|=0$, typical values are around 50 mb. This value is still too high for a favorable comparison with the preliminary analysis of the ISR data.[10] If these small values of σ_2 are confirmed in the final analysis of the data at small $|t'|$, one would have to conclude that the DHD mechanism is not the dominant production mechanism at these energies.

The DHD mechanism is only qualitatively capable of describing the hydrogen data. One might question the omission of interactions of the fast particles among themselves. Judging from the spectrum of resonances, one would expect the relevant time scale to be of the order of 1/(100–400 MeV) in the rest frame of the dissociating system. At high energies, it should therefore be permissible to do an adiabatic approximation, i.e., to treat the dissociation products as frozen (noninteracting) in their c.m. frame while the interaction with the target nucleons takes place. Accordingly, the interaction of the produced particles among themselves should not be more important for dissociation on a composite target than for dissociation on hydrogen. It thus seems unlikely that this mutual interaction of the fast particles is responsible for the small values of σ_2, as argued recently.[29]

Having left out the transverse parts of the propagators, the decay distribution is not calculable. But at small t', where single scattering dominates, the decay distribution should be essentially as for production on hydrogen (at the same t'). In the double-scattering region, on the other hand, the decay distribution might be quite different from the corresponding distribution for production on hydrogen (at $t'/4$). Such a deviation, if found, would be a clear indication against the KM picture.

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APPENDIX

We write the deuteron wave function as

$$\Psi^M(\vec{r}) = \frac{1}{\sqrt{4\pi}}\,\frac{1}{r}\left(u(r) + \frac{1}{\sqrt{8}}\,S_{12}(\hat{r})w(r)\right)\chi_1^M\,,$$

where

$$S_{12}(\hat{r}) = 3(\vec{\sigma}_n\cdot\hat{r})(\vec{\sigma}_p\cdot\hat{r}) - 1 = 6(\vec{s}\cdot\hat{r})^2 - 4$$

($\vec{s}$ is here the deuteron spin) and χ_1^M is the spin function. The form factor then becomes

$$S_{M'M}(\vec{q}) = \chi_1^{M'\dagger}(S_S(q) - S_{12}(\hat{q})S_Q(q))\chi_1^M\,.$$

$S_S(q)$ and $S_Q(q)$ are the spherical and quadrupole form factors, respectively.

The K functions [Eqs. (5.5) and (5.7)] are for $q_L = 0$ of the form

$$K^{M'M} = \chi_1^{M'\dagger}\int\frac{d^2p}{(2\pi)^2}\,[S_S(p) - S_{12}(\hat{p})S_Q(p)] \times \exp(-\alpha\vec{q}_T^{\,2} - \beta\vec{p}^{\,2} - \gamma\vec{p}\cdot\vec{q}_T)\chi_1^M\,,$$

where $\gamma \neq 0$ if the two slopes involved are different.

Integration over the angle leads to modified Bessel functions,

$$\frac{1}{2\pi}\int d\phi\,\exp(-\gamma\vec{p}\cdot\vec{q}_T) = I_0(\gamma p q_T)\,,$$

$$\frac{1}{2\pi}\int d\phi\,\exp(-\gamma\vec{p}\cdot\vec{q}_T)S_{12}(\hat{p}) = I_2(\gamma p q_T)S_{12}(\hat{q}_T) - \tfrac{1}{2}[I_0(\gamma p q_T) - I_2(\gamma p q_T)]S_{12}(\hat{z})\,.$$

With a multi-Gaussian fit to $u(r)$ and $w(r)$, also the integral over p can be done analytically. The matrix elements (5.4) and (5.6) take the form

$$\mathfrak{M}_{\rm coh}^{M'M} = \mathfrak{M}_0\,\chi_1^{M'\dagger}[a + bS_{12}(\hat{q}_T) + cS_{12}(\hat{z})]\chi_1^M\,,$$

where $\mathfrak{M}_0$ is the matrix element for production on hydrogen.

The unpolarized cross section can be expressed in terms of

$$\tfrac{1}{3}\,\mathrm{Tr}\,\mathfrak{M}_{\rm coh}^\dagger\mathfrak{M}_{\rm coh} = \mathfrak{M}_0^\dagger\mathfrak{M}_0\{|a|^2 + 8[|b|^2 + |c|^2 - \mathrm{Re}(bc^*)]\}\,.$$

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