Quark-parton model from dual topological unitarization

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Topology, which occurs in the topological expansion of quantum chromodynamics (QCD) and in the dual topological unitarization (DTU) schemes, allows us to establish a quantitative correspondence between QCD and the dual S-matrix approaches. This topological correspondence, proposed by Veneziano and made more explicit in a recent paper for current-induced reactions, provides a clarifying and unifying quark-parton interpretation of soft inclusive processes. Precise predictions for inclusive cross sections in hadron-hadron collisions, structure functions of hadrons, and quark fragmentation functions including absolute normalizations are shown to agree with data. On a more theoretical ground the proposed scheme suggests a new approach to the confinement problem.

I. INTRODUCTION

The theoretical understanding of hadrons has made considerable progress during the last few years. The common belief is now that the strong-interaction theory is a non-Abelian gauge field theory, the so-called quantum chromodynamics[1] (QCD), The popularity of this scheme is such that it is the starting point of almost all theoretical and phenomenological investigations about hadrons. The ambition of the QCD program is to derive the physics of hadrons from the interactions of their constituents—the quarks and gluons. This program has already been extremely successful in application to processes involving large momentum transfers: scaling, asymptotic freedom, scaling violations, heavy "quarkonium," etc. However, in describing soft (low-$p_\perp$) hadronic collisions, it is faced with severe difficulties related to nonperturbative and infrared problems.

The successes of the QCD program must not obscure the developments of another approach to the theory of hadrons, the dual S-matrix theory. This approach corresponds to another ambitious program, the so-called bootstrap program, in which all properties of hadrons are derived from self-consistency constraints, and which excludes any fundamental particles or any hierarchy among hadrons. This program has also been recently successful in its most developed form, i.e., the dual topological unitarization scheme[2] (DTU): understanding duality as an order property of the S matrix, violations of the Okubo-Zweig-Iizuka (OZI) rule, etc.

We stress that, in fact, DTU is the only available and calculable approach to soft hadronic collisions. However, it also encounters severe difficulties in understanding asymptotic freedom, pointlike constituents, currents, etc.

It is interesting to observe that the domains of applicability of the two programs are complementary. For instance, whereas perturbative QCD works for large-momentum-transfer processes, DTU provides a quantitative description of low-$p_\perp$ interactions. There have been several attempts to give a QCD-parton interpretation to soft hadronic processes. Although some successes have been reached, the situation is rather confused because of the *ad hoc* or empirical assumptions needed.[3] On the other hand, it seems difficult to apply Regge phenomenology to current-induced reactions.

Actually, there have been attempts to relate dynamical descriptions in different kinematical regions or to find relations between parton and Regge descriptions of the same process. The investigations in this direction are mainly twofold. The first one, the so-called universality picture,[4] inspired by a naive quark-parton model, is based on an approximate equality of mean multiplicities and inclusive spectra in current-induced and in soft hadronic processes.

The second approach was proposed by Veneziano[5] in order to unify QCD and DTU schemes. It turns out that the crucial tool for this unification is topology, the dynamical role of which has been emphasized in the phenomenology of strong interactions.[6] This second approach has been used in a recent paper of Hayot and two of us[7] (hereafter referred to as CTHP), where the extension of DTU to current-induced reactions is proposed.

The present work is a continuation of this unification program. Section II is devoted to a general discussion of unification ideas. First we clarify what we mean by topological correspondence and argue that the available multiplicity data, which are usually presented as an argument against topological correspondence, cannot discriminate between this approach and the universality picture.

In Sec. III, after establishing a kind of dictionary between parton-QCD and DTU descriptions, a model for parton interpretation of soft one-particle inclusive processes is developed. Comparison with data and relations to other models are presented.

In the conclusion we propose the study of the equivalence between QCD and DTU as a program to approach the confinement problem. We show how the CTHP paper and the present one are the first contributions to this program and we present as a series of questions what we believe are the next steps.

II. TOPOLOGICAL CORRESPONDENCE: GENERAL DISCUSSION

A. Topology in QCD and DTU

The complementarity of different approaches (QCD, DTU, Reggeon field theory) and the fact that their unification can be very fruitful has been pointed out by Veneziano.[5] In fact, the scheme proposed by Veneziano provides the general framework for the present paper. It is thus useful to review it briefly.

On the one hand, DTU consists in decomposing S-matrix elements into an infinite series of topological components characterized by the number of handles of the dual diagram representing each component. The more complex the topology is (higher number of handles), the more complex are the singularities associated with the topological component. Because the simpler singularities are stronger (e.g., poles are simplest and strongest), this topological expansion is expected to converge rapidly. It turns out[2] that all nonlinear unitarity constraints are only concentrated at the planar level (so-called planar bootstrap) and they allow one to determine the planar component of the scattering amplitude. Higher terms, with higher topology, in the topological expansion are calculable from lower terms through linear equations.

On the other hand, 't Hooft[8] proposed to rearrange the QCD perturbative expansion into the so-called $1/N$ expansion of QCD, where N is the number of colors N_c at fixed number of flavors N_f. It leads to a perturbative expansion in the number of quark loops and in a topological parameter (the genus or number of handles of a surface on which QCD Feynman diagrams can be drawn) that is very similar to the perturbative expansion of dual field theories.

Veneziano[5] has shown that, using the $1/N$ expansion for N_c/N_f fixed (instead of N_f fixed), one can establish a one-to-one correspondence between certain classes of QCD diagrams and DTU topological components of the S matrix. This expansion leads now to a perturbative expansion only in the number of handles. The basic conjecture about nonperturbative QCD is that the confinement already occurs at the planar level. This conjecture amounts to assuming that the nonperturbative summations of all QCD diagrams with fixed number of handles (with infinite number of closed color and flavor loops) give rise to the particular component of the S matrix in DTU with the same number of handles. In particular, the summation of all planar QCD diagrams gives rise to the planar contribution in DTU, that is, to the one for which the planar bootstrap works.

Actually, it has been shown[9] that the Veneziano conjecture is satisfied for QCD in $1+1$ dimensions (QCD_2). This is the reason why QCD_2 provides an excellent theoretical laboratory, that is, a quark-parton model, compatible not only with the expectations of gauge theories but also with those of dual S-matrix theory.

In order to stress the dynamical role attributed to the topology we call this one-to-one correspondence topological, which is explicitly stated in the two following points:

(i) The parton-inspired QCD and DTU descriptions are equivalent; i.e., one can exploit all successes of both approaches to solve their respective outstanding problems.

(ii) The topology gives the connection between certain classes of QCD and DTU diagrams; i.e., processes represented by diagrams with the same topology give rise to the same distributions.

The "correspondence" means that one can try to establish a "dictionary" which translates the description in one language (QCD or DTU) to the other. The topological correspondence means that if one tries to extend one of the QCD or DTU approaches to new kinematical regions, or to describe processes which up to now were solely described in the other approach, one must be able to do it continuously starting from the conventional description, provided the topology is the same.

B. Topological universality of jets

The first quantitative consequences of topological correspondence can be looked for by studying the final-state distributions in lepton-hadron and soft hadron-hadron collisions. It is well known that jets are observed in both types of collisions. Phenomenologically, jets are defined as collections of particles with small transverse momenta with respect to a given direction, called the jet axis. Jets are clearly observed in e^+e^- hadrons, $lN\to$ hadrons, and in large-$p_\perp$ hadron-hadron collisions. But according to the

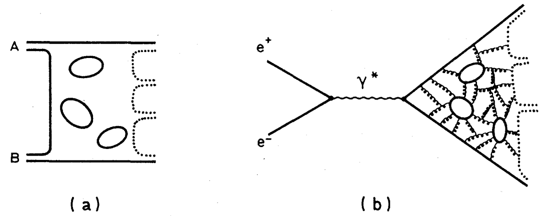


FIG. 1. (a) Dual diagram for the planar part of $A + B \to$ hadrons cross section. (b) QCD diagram for planar jets in $e^+e^- \to$ hadrons. In this figure, as in all others, we represent by dotted lines the quark lines forming the hadrons over which the inclusive summation is done.

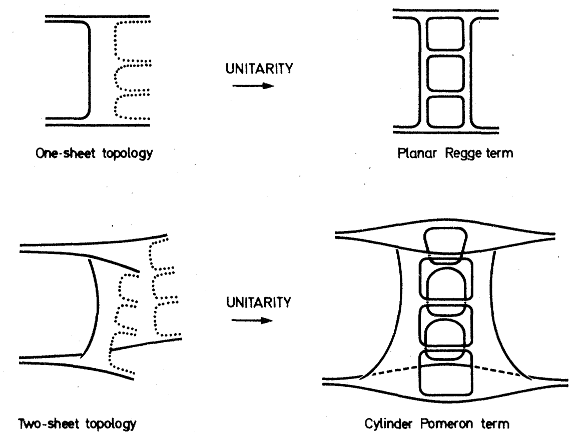


FIG. 2. The two components arising in DTU.

phenomenological definition of jets, soft hadronic collisions also produce jets with jet axis along the direction of incoming particles. Empirically and unexpectedly it has been observed that the structure of jets is similar in hard and soft processes. This property, called jet universality, is interpreted in different ways.

From the topological correspondence it follows that one has to compare processes with the same topology. In the leading order of the $1/N$ expansion of QCD, jets produced in, say, the annihilation of $e^+e^- \to$ hadrons are associated with planar topology. Therefore, they have to be compared with the corresponding planar part of the hadron-hadron final state, i.e., produced by the planar Regge term of the multiparticle production mechanism in the DTU scheme. This is illustrated in Fig. 1.

Let us at this point clarify the terminology we will use. According to the conventional terminology, we equal the number of jets to the number of "initial" quarks which then convert into hadrons; e.g., both diagrams in Fig. 1 have two jets in the final state. From the topological point of view, these two planar jets are associated with one dual sheet in the dual diagram.

Because the topology is the same, the final-state distribution of hadrons produced in e^+e^- annihilation has to be equal to the one expected for the Regge component of hadron-hadron reactions. More generally, particles in two planar jets (or associated with one dual sheet) are distributed universally (topological universality of jets), irrespective of how the jets are produced (or the dual sheet is generated). This assumption is in contrast with the universality picture[4] which associates the final-state distributions in $e^+e^- \to$ hadrons with the total distribution in hadron-hadron collisions and not with their planar components only.

Now one knows that in hadron-hadron collisions one can separate (as shown in Fig. 2) two components of multiparticle production (with different distributions) leading, through unitarity, to Reggeon and Pomeron exchanges. Since the Pomeron contribution is dominant at high energy, one expects the two-sheet topology to dominate the multiparticle production in hadron collisions; one would thus expect that the predictions of topological universality and of the universality picture are drastically different and that comparison with experiment should allow one to discriminate between the two schemes.

The most discussed aspect of this correspondence concerns the mean multiplicity. Dual models predict the mean multiplicity to be proportional to the number of sheets, and thus to the number of jets. The experimental observation that the mean hadronic multiplicity in e^+e^- annihilation (one-sheet structure or two jets) is roughly equal to the mean multiplicity in, say, proton-proton collisions[10] is often presented as an argument against topological universality, since the pp reaction is dominated by the two-sheet (four jets) topology.

In the Appendix we present a rather detailed discussion of this question. In fact we show that the analysis only of mean multiplicity data is not conclusive.

III. TOPOLOGICAL CORRESPONDENCE: PARTON INTERPRETATION OF SOFT INCLUSIVE PROCESSES

A. Formulation of the problem

We want now to proceed to a systematic discussion of topological correspondence. Let us recall that the main goal of this approach is to extend QCD and DTU schemes outside of their usual regions of applicability. The CTHP paper is a contribution in this direction. The purpose of CTHP is to extend the DTU approach to current-induced reactions. The method proposed in that paper consists of specifying the hadronic system with which one can replace a large-Q^2 current (electromagnetic or weak) in inclusive lepton-hadron processes. Through this method, DTU, at the lowest order, is shown to lead to all the expectations of naive quark-parton models: Bjorken scaling in deep-inelastic lepton-hadron collisions, asymptotic constancy of the ratio $R=\sigma(e^+e^-\rightarrow \text{hadrons})/\sigma(e^+e^-\rightarrow \mu^+\mu^-)$, scaling in one-particle inclusive production in e^+e^- annihilation. Moreover, arguments based on DTU allow one to get information about the scaling functions (structure functions of hadrons and/or fragmentation functions): power behavior near $x=0$ and $x=1$ which is compatible with the data at moderate values of Q^2 and with popular expectations like Regge exchange near $x=0$ and spectator-quark-counting rules near $x=1$;[11] relative normalization near $x=0$ and $x=1$; relative normalization of sea to valence contributions.

To summarize: One can say that the main result of CTHP is that the quark lines of duality diagrams can be interpreted as the quarks of a workable and predictive quark-parton model.

Now, we turn to the problem of a parton interpretation of soft hadronic collisions, which is of great practical interest: Is it possible to use the information about structure functions obtained in lepton-hadron collisions in order to interpret the bulk of hadronic data, i.e., low-$p_\perp$ data? On the other hand, is it possible to use soft hadronic collisions to get information about the structure of hadrons?

B. Situation of parton interpretations of soft processes

Several authors[3] have attacked this problem with some success. But a convincing picture has not yet emerged because of the lack of systematics and the need of *ad hoc* or empirical assumptions.

It is now well known that if one wants to have any chance to interpret hadronic collisions in parton language, one has to focus on inclusive processes. Models to interpret inclusive soft hadronic reactions can be classified into two families: fragmentation models and recombination models. In fragmentation models a quark is "liberated" out of an excited multiparton state and then fragments into the observed hadrons. In recombination models a fast quark (or an antiquark) recombines with a soft antiquark (or a quark) of the sea of partons. Until now these two mechanisms have been considered as orthogonal or competing. In the present section we show how the one-sheet topology (Reggeon component) can be interpreted as a parton fragmentation mechanism and the two-sheet topology (Pomeron component) can be interpreted as recombination.

C. A dictionary

It will be useful to first establish a "dictionary" between QCD-parton and DTU descriptions. For this purpose it is enough to consider the planar part of the total cross section.

1. Wee-valence-quark exchange and Reggeon exchange

In the parton picture[2] the energy dependence of the total cross section is understood as a wee-parton exchange. This process, which takes a time typical of strong interactions (1 F in length units), is followed by a "hadronization" process during which all the remaining partons form final hadrons. Now, the exchange of a valence quark leads (in the sense of $1/N$ expansion of QCD) to the planar part of the total cross section. Because the hadronization takes a time proportional to the momentum in the center-of-mass system[4] (which could be much larger than 1 F) and appears with probability equal to 1, one can factorize out this process from the planar part of the total cross section and write

$$\sigma^{\text{planar}}_{AB\rightarrow X}(s)\simeq\left(\int xG^{A}_{\text{val}}(x)\frac{dx}{x}\right)\left(\int xG^{B}_{\text{val}}(x)\frac{dx}{x}\right)\simeq s^{\alpha_\omega-1}. \tag{1}$$

The functions $G^{(i)}_{\text{val}}(x)\simeq x^{-\alpha_\omega}$ describe the distribution of valence quarks in the ith colliding hadron, and integrals are taken over the wee region.

In DTU language, the planar part of the total cross section is given by a sum of all planar dual diagrams. The planar unitarity (illustrated in Fig. 3) allows one to express $\sigma^{\text{planar}}_{AB\rightarrow X}(s)$ as a discontinuity of a Regge exchange

$$\sigma^{\text{planar}}_{AB\rightarrow X}(s)=g^2 s^{\alpha_{ii}(0)-1}. \tag{2}$$

Comparing formulas (1) and (2), we get the first item of our dictionary:

The exchange of a wee-valence quark in the parton picture corresponds to a Reggeon exchange; i.e., the intercept of Regge trajectory $\alpha_{ii}(0)$ is related to the distribution of the valence quarks in the wee region $\alpha_{ii}(0)=\alpha_\omega$. In other words, the quark line exchanged in the corre-

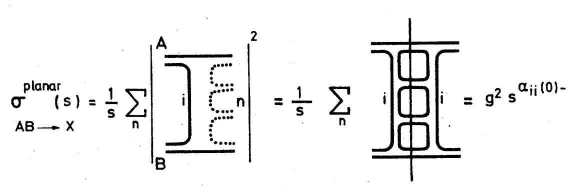


FIG. 3. Planar unitarity.

sponding duality diagram can be identified with the wee-valence quark exchanged in the parton language.

2. Hadronization and planar bootstrap

In the parton picture the hadronization of partons has all the properties of a confinement mechanism: It takes a long time, it factorizes out from the initial quark exchange, and it appears with probability equal to 1 when one sums over all possible final hadron states. In the case of wee-valence-quark exchange, hadronization corresponds to the confinement assumed to appear at the planar level of QCD.

From the DTU point of view, a similar mechanism seems to hold with planar unitarity. Planar bootstrap constraints say that the sum over all possible planar loops in duality diagram, i.e., the sum over all possible final hadron states in multiproduction amplitude, gives 1 and disappears from the expression for $\sigma^{\text{planar}}_{AB\to X}(s)$.

Having identified Regge exchange with the wee-quark exchange, we are led to the following second point of our dictionary:

The properties of the hadronization process of the planar configuration of partons, left over after the wee-valence-quark exchange, can be expressed in terms of planar unitarity in the DTU language.

3. Quark jets and planar hadron jets

In Sec. II we identified particles produced in quark jets seen in the $e^+e^-\to$ hadrons process with particles produced in planar hadron-hadron collisions, i.e., associated with one dual sheet (c.f. Fig. 1).

A satisfactory description of the particle production mechanism depicted in Fig. 1(b) is provided by a multi-Regge exchange model in the DTU approach. The above identification means that one can apply the same description to jets in $e^+e^-\to$ hadrons provided care is taken, since now the jet axis is not along the direction of the colliding particles (as it is in h-h collisions).

However, for our purpose it is more interesting to use the common parton description of e^+e^- annihilation and apply it to hadron-hadron collisions. In the parton picture $e^+e^-\to$ hadrons is described as a two-step process: First a pair of $q\bar{q}$ is created, and then they fragment into hadrons. This leads to the third point of our dictionary:

One can reinterpret the planar part of the $hh\to$ hadrons process as a two-to-two-body process $hh\to q\bar{q}$ at $t\simeq 0$ (wee-valence-quark exchange takes almost no momentum transfer), followed by the same fragmentation process $q\bar{q}\to$ hadrons as in

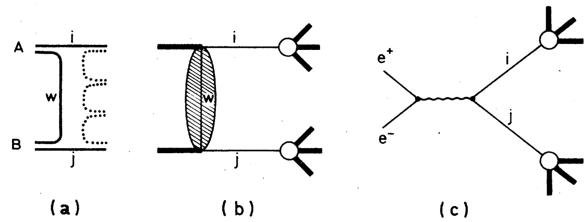


FIG. 4. Topological correspondence between $\sigma^{\text{PLANAR}}_{AB\to X}$ and $\sigma_{e^+e^-\to q\bar{q}\to X}$. (a) Dual diagram; (b) parton interpretation of (a); (c) current-induced process giving rise to the same distribution as in (b).

$e^+e^-\to q\bar{q}\to$ hadrons (see Fig. 4). In particular, one can use the same quark fragmentation function $D_{q\to h}$ as measured in e^+e^- processes to describe inclusive distribution of hadrons produced in planar hadron-hadron collisions.

D. Inclusive processes; one-sheet topology

Consider the inclusive process $A+B\to H+X$, where H is in the fragmentation region of the incoming particle B, i.e., $0.2\lesssim x\lesssim 0.9$, x being the Feynman variable of H; A, B, and H are mesons. We shall use the notation

$$f_{\underset{A}{B\to H}}(s,x,p_\perp)=E_H\frac{d^3\sigma}{d^3p_H}(B\underset{A}{\to}H) \tag{3}$$

for the invariant inclusive cross section.

In terms of dual diagrams, there are in general six diagrams contributing (at the lowest order of DTU) to the one-particle inclusive distribution $f_{\underset{A}{B\to H}}$, which are depicted in Fig. 5. Diagrams 5(a) and 5(b) have one-sheet structure; diagrams 5(c)–5(f) have two-sheet structure. All these

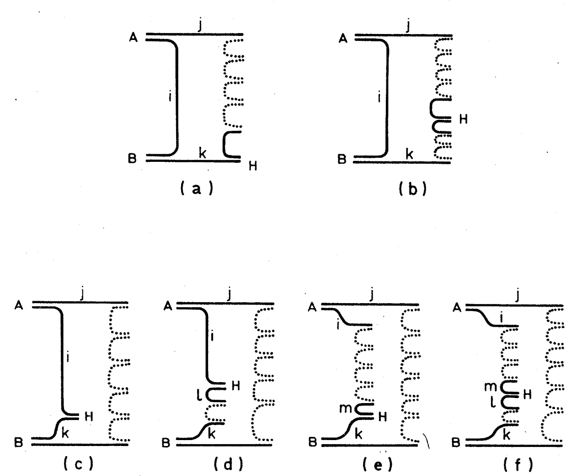


FIG. 5. Dual diagrams contributing to one-particle inclusive processes. (a), (b): one-sheet topology; (c)–(f): two-sheet topology.

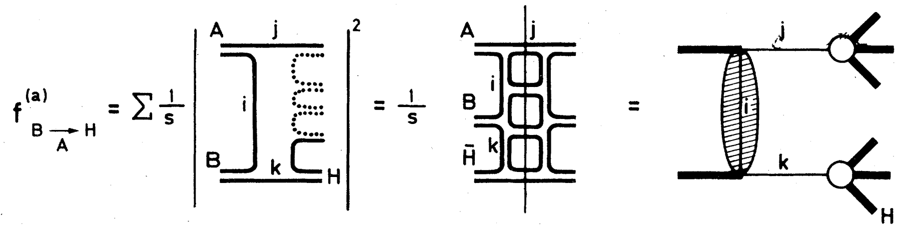


FIG. 6. Parton interpretation of process of Fig. 5(a).

diagrams are described in principle using the Mueller-Regge formalism[13] and planar unitarity constraints.

Let us study diagrams with one-sheet topology, i.e., diagrams 5(a) and 5(b). Planar unitarity with ordinary Mueller-Regge analysis leads to the Regge behavior for the diagram 5(a)

$$f^{(a)}_{\underset{A}{B\to H}} = g^2 s^{\alpha_{ii}(0)-1}\beta(x,p_\perp)\,, \tag{4}$$

where $s=(p_A+p_B)^2$, $M^2=(p_A+p_B-p_H)^2$, $x=1-M^2/S\simeq P_H^{\parallel}/P_B$, and $\beta(x,p_\perp)$ is a Feynman scaling invariant distribution. The dictionary established in Sec. III C allows us to give the following parton interpretation of (4): $g^2 s^{\alpha_{ii}(0)-1}$ is, as before, the cross section of the process corresponding to the wee-valence-quark exchange ($i=\omega$), whereas $\beta(x,p_\perp)$ is the fragmentation function of quark k into hadron H (which contains the quark k) plus anything (see Fig. 6). Since wee-quark exchange implies $t\simeq 0$, the quark k just keeps all the momentum of particle B. It thus follows that

$$z=\frac{P_H^{\parallel}}{P_k^{\parallel}}=\frac{P_H^{\parallel}}{P_B}=x\,. \tag{5}$$

Therefore, integrating over $p_\perp$, we are led to identify $\int\beta(x,p_\perp)d^2p_\perp$ with the usual direct fragmentation function of quark k, $xD^{\rm dir}_{k\to H}(x)$. By a direct (nondirect) quark fragmentation function we mean the process when a produced hadron H contains (does not contain) the original quark k:

$$\int\beta(x,p_\perp)d^2p_\perp = xD^{\rm dir}_{k\to H}(z=x)\,. \tag{6}$$

Combining (5) and (6), we get

$$xD^{\rm dir}_{k\to H}(x)=\frac{\int f^{(a)}_{\underset{A}{B\to H}}(s,x,p_\perp)d^2p_\perp}{\sigma^{\rm planar}_{AB\to X}(s)}\,. \tag{7}$$

Similarly, diagram 5(b) is identified with the nondirect quark fragmentation function $D^{\rm nd}_{k\to H}(x)$ (see Fig. 7):

$$\int f^{(b)}_{\underset{A}{B\to H}}(s,x,p_\perp)d^2p_\perp = g^2 s^{\alpha_{ii}(0)-1}xD^{\rm nd}_{k\to H}(x=z)\,. \tag{8}$$

The results (7) and (8) are extremely similar to those of QCD_2.[9]

Let us now check our predictions (7) and (8) against existing experimental data. As already noticed, diagrams 5(a) and 5(b) and 5(c)–5(f) contribute, respectively, to the Reggeon and the Pomeron exchange. Since this last contribution is unchanged by the replacement of particles by antiparticles, it is possible to extract planar contributions by considering differences of line-reversed inclusive cross sections in such a way that the Pomeron diagrams can be eliminated. The following combinations were already used in previous works[14,15]:

$$zD^{\rm nd}_{u\to\pi^-}(z)=zD^{\rm nd}_{d\to\pi^+}(z)=\frac{\int f_{\underset{p}{\pi^-\to\pi^+}}d^2p_\perp-\int f_{\underset{p}{\pi^+\to\pi^-}}d^2p_\perp}{\sigma_{\pi^-p}(s)-\sigma_{\pi^+p}(s)}\,, \tag{9}$$

$$zD^{\rm dir}_{s\to\bar K^0}(z)=zD^{\rm dir}_{\bar s\to K^0}(z)=\frac{\int f_{\underset{p}{K^-\to\bar K^0}}d^2p_\perp-\int f_{\underset{p}{K^+\to K^0}}d^2p_\perp}{\sigma_{K^-p}(s)-\sigma_{K^+p}(s)}\,. \tag{10}$$

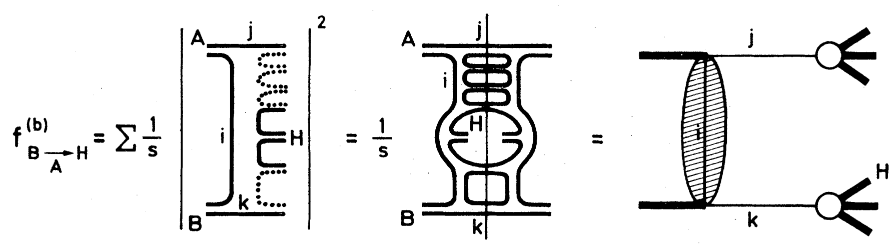


FIG. 7. Parton interpretation of process of Fig. 5(b).

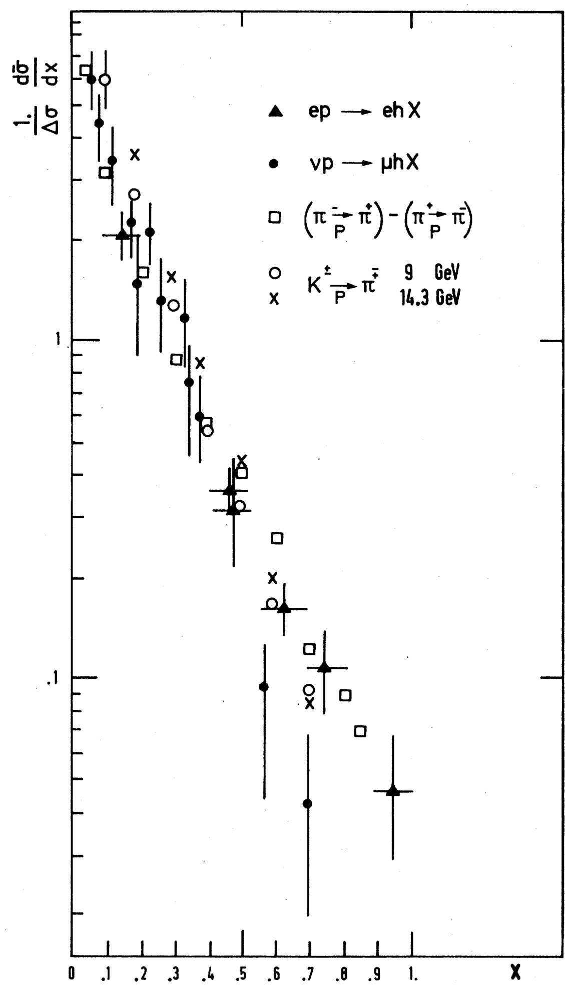


FIG. 8. $D^{\rm nd}_{s\to\pi^+}$ calculated from $K^\pm \underset{p}{\to} \pi^\mp$ data compared with $D^{\rm nd}_{u\to\pi^-}$ extracted from $\pi^\pm \underset{p}{\to} \pi^\mp$ data (Ref. 14), and with lepton data.

For the discussion, we will use also data[16] on $K^\pm p \to \pi^\mp X$, where the pion is in the fragmentation region of the kaon:

$$zD^{\rm nd}_{s\to\pi}(z) = zD^{\rm nd}_{\bar{s}\to\pi}(z)$$

$$= \frac{\int f_{K^- \underset{p}{\to} \pi^+} d^2p_\perp - \int f_{K^+ \underset{p}{\to} \pi^-} d^2p_\perp}{\sigma_{K^-p}(s) - \sigma_{K^+p}(s)} . \qquad (11)$$

It is expected[17] that the two nondirect fragmentation functions $D^{\rm nd}_{u\to\pi^-}(z)$ and $D^{\rm nd}_{s\to\pi^-}(z)$ should be similar because in both cases the same quark pair has to be created. Therefore, in Fig. 8, we compare results of an analysis made for (9) in Ref. 14 and our calculations for (11) with the existing data taken from various lepton-induced reactions. The points taken from lepton-hadron and hadron-hadron collisions at different energies show (within experimental errors) universal

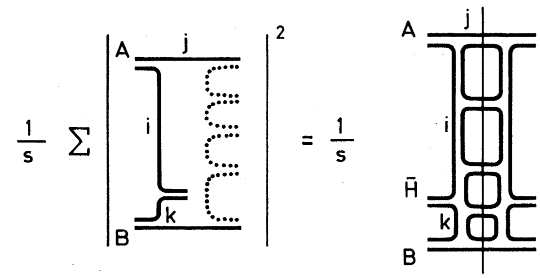


FIG. 9. Mueller-Regge analysis of process of Fig. 5(c).

x behavior, as it should be if it is to be interpreted as a quark fragmentation function.

The formula (10) is discussed in Ref. 15, where it is shown that using data on $K^\pm p \to K^0, \bar{K}^0 + X$ at 16 and 32 GeV one gets all the right properties of $xD^{\rm dir}_{\bar{s}\to K^0}$ expected from the parton picture.

E. Inclusive processes; two-sheet topology

Though one obtains good results with the quark fragmentation model for the Reggeon (one-sheet) component in hadron-hadron reactions, the dominant part of the inclusive cross section is given by diagrams with two-sheet topology [diagrams 5(c)–5(f)].

1. Hadron structure functions

We start the discussion with diagram 5(c). According to standard Mueller-Regge analysis, the inclusive cross section described by diagram 5(c) is given by the discontinuity of the forward $3\to 3$ amplitude $A+\bar{H}+B \to A+\bar{H}+B$ (see Fig. 9). In the fragmentation region of particle B the discontinuity has the form

$$f^{(c)}_{B\underset{A}{\to}H}(s,x,p_\perp) = g^2 |u|^{\alpha_{ii}(0)-1} \gamma(x,p_\perp) , \qquad (12)$$

where $u=(p_A - p_H)^2$ and now $\alpha_{ii}(0)$ is the intercept of trajectory exchanged between A and $\bar{H}$.

Thanks to the discussion in Sec. III C 1 we can interpret $g^2|u|^{\alpha_{ii}(0)-1}$ as the planar total cross section of the wee-quark-exchange subprocess $A+k \to j+H$. Since, as before, wee-quark exchange takes no momentum transfer, the momentum of H is equal to the momentum of the quark k; i.e., x is equal to the Bjorken variable x_{Bj} of the quark k. Therefore $|u| = xs = x_{Bj}s$ is also equal to the center-of-mass energy squared of subprocess $A+k \to j+H$, and the planar total cross section of this subprocess, through crossing, is equal to $\sigma^{\rm planar}_{A\bar{H}\to X}(xs)$. Integrating over $p_\perp$, we thus interpret $\int \gamma(x,p_\perp) d^2p_\perp$ as the distribution of valence quark k inside the hadron B with

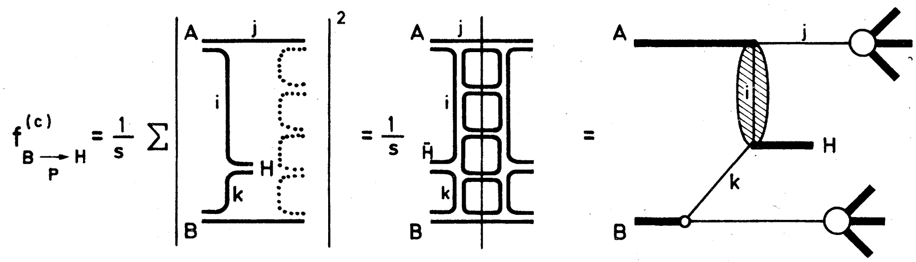


FIG. 10. Parton interpretation of process of Fig. 5(c).

momentum fraction $x_k = x_H$, and write

$$\int \gamma(x,p_\perp)d^2p_\perp = xG^{\text{val}}_{B\to k}(x) = \frac{\int f^{(c)}_{\underset{A}{B\to H}}(s,x,p_\perp)d^2p_\perp}{\sigma^{\text{planar}}_{A\bar{H}\to X}(xs)}. \tag{13}$$

In other words, the process shown in diagram 5(c) is interpreted as a recombination of a fast valence quark k of hadron B with a wee quark i into hadron H. Moreover, the variable $|u|$ is equivalent to Q^2 in deep-inelastic scattering [square of momentum transfer to quark k (see Fig. 10], and thus we recover the result of CTHP.

Similarly, one can interpret diagram 5(d) as a recombination of a sea quark l of hadron B with a valence quark i of hadron A (quark i is wee because H is considered to be in the fragmentation region of B):

$$\int f^{(d)}_{\underset{A}{B\to H}}(s,x,p_\perp)d^2p_\perp = \sigma^{\text{planar}}_{A\bar{H}\to X}(xs)xG^{\text{sea}}_{B\to l}(x). \tag{14}$$

Note that, though diagrams 5(c) and 5(d) have two-sheet topology and give rise, through unitarity, to the Pomeron term, the formulas (12) and (14) are not scale invariant [they contain the factor $|u|^{\alpha_{ii}(0)-1} \simeq (xs)^{\alpha_{ii}(0)-1}$]. This is due to the fact that the two-sheet topology is not yet "fully developed"; i.e., particles forming the second sheet [in the case 5(c) it is just one particle which is observed] occupy a finite interval in rapidity when $s\to\infty$.

2. *f promotion*

The remaining two diagrams 5(e) and 5(f) cannot be simply interpreted. The main difference between them and diagrams 5(c) and 5(d) is that now particle H does not contain a wee-valence quark of A but a quark, denoted as m, which could carry some momentum. So there is no reason to expect that $p_H = p_k$ in 5(e) [or $p_H = p_l$ in 5(f)] and, consequently, that the spectrum of particle H could be simply related to the structure function of B.

However, diagrams 5(c)–5(f) have the same topology corresponding to the two-sheet structure. It is natural to expect that the DTU scheme can relate these contributions quantitatively.

Let us consider diagram 5(e). We are now faced with the problem of the development of two-sheet topology. The DTU scheme tells us that adding the two-sheet contributions to the one-sheet contribution just amounts to changing the energy dependence from Reggeon to Pomeron behavior and to changing the overall normalization by a factor of order $1/N_f$. This effect is known as the promotion of the planar f Reggeon to the cylinder Pomeron. In order to generalize f-promotion arguments to inclusive processes, it is convenient to treat the one-particle inclusive cross section as a total cross section for a quasi-two-body collision $A+(B\bar{H})$ at an energy equal to M^2.

We thus rewrite Eq. (12) as

$$f^{(c)}_{\underset{A}{B\to H}} = g^2(M^2)^{\alpha_{ii}(0)-1}F(x,p_\perp), \tag{15}$$

where

$$g^2(M^2)^{\alpha_{ii}(0)-1} = \sigma^{\text{planar}}_{A(B\bar{H})\to X}(M^2), \tag{16}$$

and

$$F(x,p_\perp) = \gamma(x,p_\perp)\left(\frac{|u|}{M^2}\right)^{\alpha_{ii}(0)-1} = \gamma(x,p_\perp)\left(\frac{1-x}{x}\right)^{\alpha_{ii}(0)-1} \tag{17}$$

is to be interpreted as the probability of finding a fragment $(B\bar{H})$ into hadron B with momentum fraction $1-x$. Therefore, adding contributions of Fig. 5(e) to those of Fig. 5(c) does not affect this fragmentation distribution but changes only α_{ii} to α_P in Eq. (15). We thus get (see Fig. 11)

$$f^{(c)+(e)}_{\underset{A}{B\to H}} = \frac{g^2}{N_f}(M^2)^{\alpha_P(0)-1}F(x,p_\perp). \tag{18}$$

Integrating over $p_\perp$, recollecting Eqs. (12), (15), (17), and (18), and assuming $\alpha_P(0)=1$, one gets

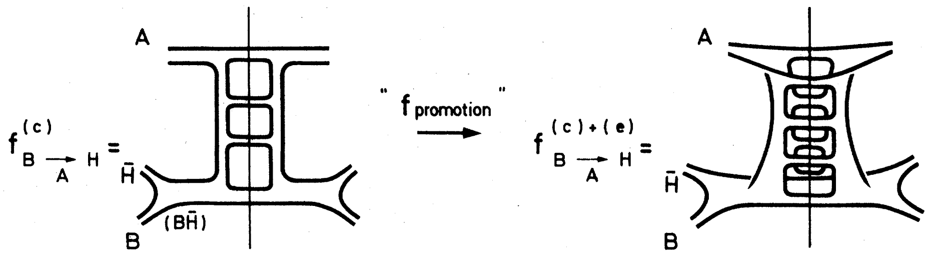


FIG. 11. f-promotion mechanism in one-particle-inclusive cross section.

$$f^P_{\text{val}}(x) \equiv \int f^{(c)+(e)}_{\substack{B\to H\\A}} d^2p_\perp = \frac{g^2}{N_f}\left(\frac{1-x}{x}\right)^{1-\alpha_R} x G^{\text{val}}_{B\to k}(x)\,. \tag{19}$$

Applying the same reasoning to contributions of Fig. 5(d) and 5(f), we get

$$f^P_{\text{sea}}(x) \equiv \int f^{(d)+(f)}_{\substack{B\to H\\A}} d^2p_\perp = \frac{g^2}{N_f}\left(\frac{1-x}{x}\right)^{1-\alpha_R} x G^{\text{sea}}_{B\to l}(x)\,. \tag{20}$$

For particle H with quark content $(a\bar{b})$ we can now express the dominant part of the full one-particle inclusive distribution in terms of distributions of quarks a and $\bar{b}$ inside B by adding all contributions of Fig. 5(c) to 5(f):

$$f^P_{\substack{B\to H\\A}}(x) = \frac{g^2}{N_f}\left(\frac{1-x}{x}\right)^{1-\alpha_R} x[G_{B\to a}(x) + G_{B\to \bar{b}}(x)]\,. \tag{21}$$

The only dependence of Eq. (21) on particle A is in g^2, which, we recall, is the normalization of $\sigma^{\text{planar}}_{A\bar{H}\to X}(xs)$.

It is interesting to note that through Eqs. (19)–(21) we are able to express the dominant part of the inclusive cross section in terms of structure functions. In the language of recombination models we can interpret the new factor $[(1-x)/x]^{1-\alpha_R}$, which is *predicted from DTU arguments*, as accounting for the spread of momentum of the quark m which appears in diagrams 5(e) and 5(f) and for the recombination probability. We finally stress that the absolute normalization is predicted.

3. Analysis of $pp\to\pi^\pm, K^\pm$ inclusive reactions

There exist good data on proton fragmentation in π and K mesons. The formalism described above can naturally be extended to mesons produced in proton-proton collisions, leading to ($\alpha_R = \frac{1}{2}$)

$$\begin{aligned}
x\frac{d\sigma}{dx}(pp\to\pi^+ + X) &= \frac{g_{\pi p}^2}{N_f}\left(\frac{1-x}{x}\right)^{1/2}[xu(x) + x\bar{d}(x)]\,,\\
x\frac{d\sigma}{dx}(pp\to\pi^- + X) &= \frac{g_{\pi p}^2}{N_f}\left(\frac{1-x}{x}\right)^{1/2}[x\bar{u}(x) + xd(x)]\,,\\
x\frac{d\sigma}{dx}(pp\to K^+ + X) &= \frac{g_{Kp}^2}{N_f}\left(\frac{1-x}{x}\right)^{1/2}[xu(x) + x\bar{s}(x)]\,,\\
x\frac{d\sigma}{dx}(pp\to K^- + X) &= \frac{g_{Kp}^2}{N_f}\left(\frac{1-x}{x}\right)^{1/2}[x\bar{u}(x) + xs(x)]\,,
\end{aligned} \tag{22}$$

where we use common notations for quark distributions in the proton. In Fig. 12, we present $x(d\sigma/dx)$ for $pp\to\pi^\pm x$ calculated from (22) and

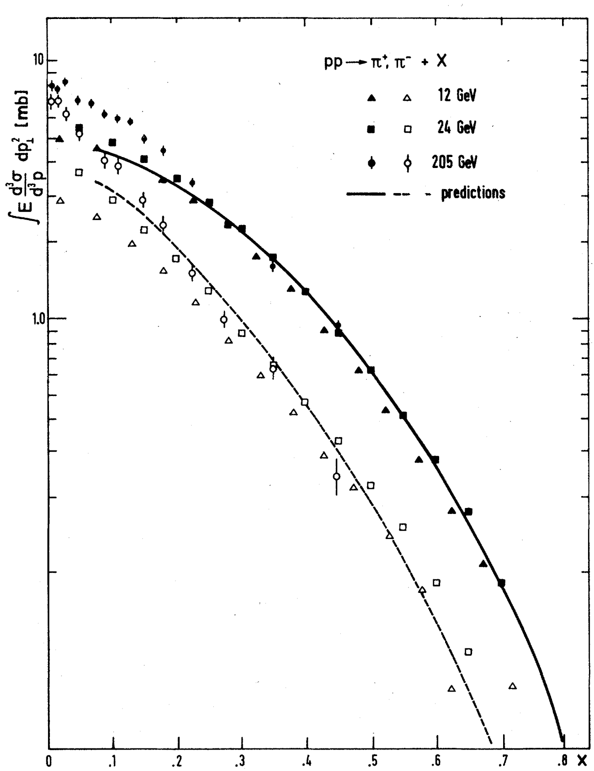


FIG. 12. $x(d\sigma/dx)$ calculated from formula (22), compared with data taken from Ref. 18.

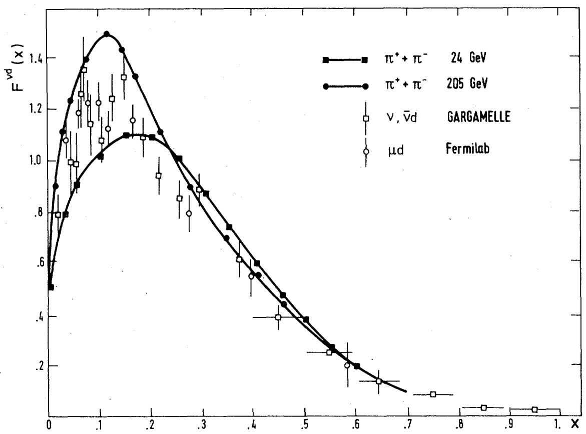


FIG. 13. $x(u+\bar{u}+d+\bar{d})$ distribution calculated from $pp \to \pi^{\pm}$ data compared with lepton data from Ref. 21.

compare with data at 12, 24, and 205 GeV/c.[18] We have used the Field-Feynman parametrization for structure functions,[17] and $g_{\pi p}{}^2$ was taken to be 20.8 mb [the planar f contribution to $\sigma_{\text{tot}}^{\pi p}$ is parametrized in the form $\sigma_{\pi p}^{\text{planar}}(s) = 20.8 s^{-1/2}$ mb GeV (Ref. 19)]. The agreement with data, including the absolute normalization, is very satisfactory for $N_f = 2.87$. This number is in remarkable agreement with the DTU expectation for N_f, the "effective" number of flavors usually taken between 2.5 and 3.[20] Note that the factor $[(1-x)/x]^{1/2}$ is very important in describing almost all the fragmentation region. The discrepancy in the small-x region between high-energy data and our curves calculated with the Field-Feynman parametrization is due to violation of Feynman scaling at high energy.

However, we can also proceed in the opposite way, that is, to determine quark distributions from hadronic data. Using the same data for $pp \to \pi^{\pm} x$ and the same numbers for N_f and $g_{\pi p}{}^2$, we have calculated $x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]$. In Fig. 13 we compare the resulting distribution with leptonic data on deuteron[21] which precisely probe this structure function. The interesting fact to note is the growth with energy of this distribution in the small-x region. It is very tempting to compare this behavior of experimental data with the Bjorken scaling-violation effect measured in deep-inelastic scattering. Using the relation $Q^2 = |u| = xs$, we plot in Fig. 14 the distribution $x[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)]$ as a function of Q^2 at fixed x. The experimental points shown in Fig. 14 are taken from a compilation of data.[22] The agreement is quite good, indicating a possibility of the same origin for both Feynman and Bjorken scaling violations.

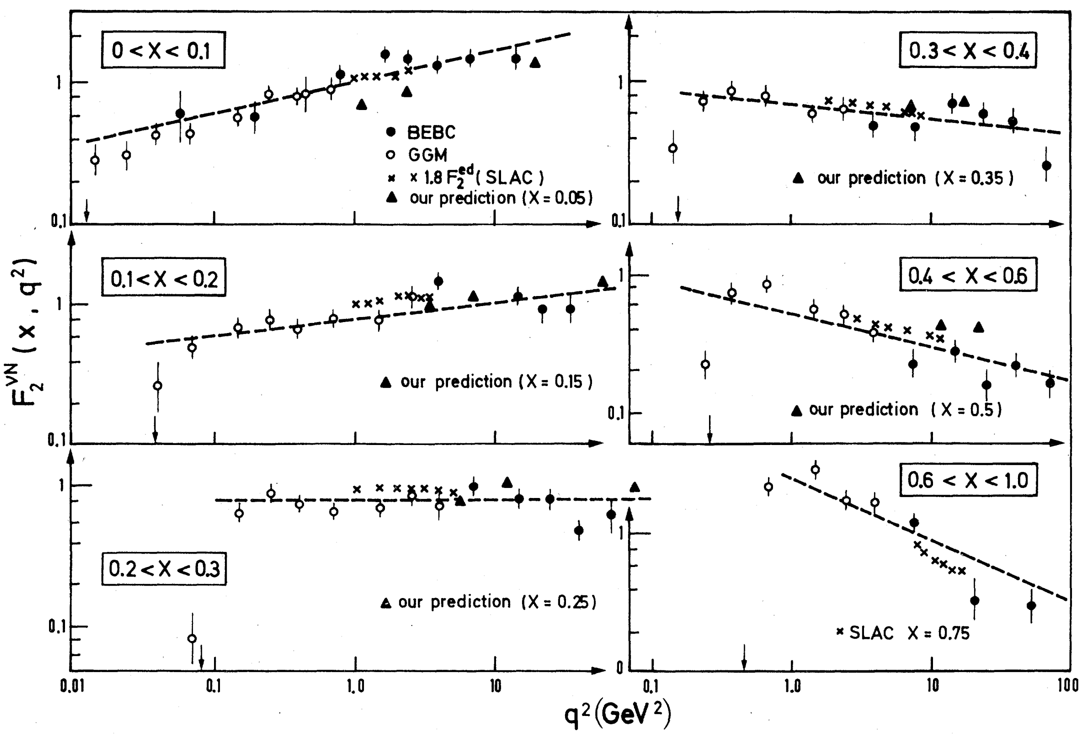


FIG. 14. Comparison of Bjorken- and Feynman-scaling-violation patterns. Black triangles are our predictions from hadronic data at fixed values of x, $Q^2 = xs$.

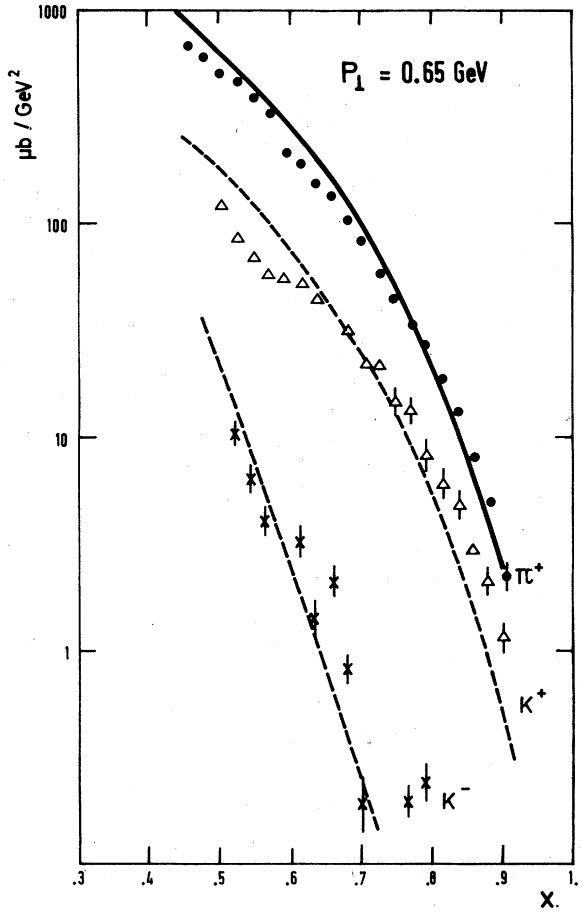


FIG. 15. Comparison of $x(d\sigma/dx)$ for $K^{\pm}$ (broken line) and π^+ (solid line) production with data taken from Ref. 23.

As for kaon production, in Fig. 15, the inclusive cross section $x(d\sigma/dx)$ [calculated also from (22) using the Field-Feynman parametrization for structure functions] is compared with the recent data[23] for $pp\to K^{\pm}+X$ at $\sqrt{s}=45$ GeV and fixed $p_\perp=0.65$ GeV. The absolute normalization is arbitrary because data are taken at fixed $p_\perp$. The agreement with K^+ data is better than with K^- data.

The factor ≈ 4 between π^+ and K^+ data is usually interpreted as due to the smaller probability of finding in the sea a strange quark than a nonstrange one. This fact allows one also to understand qualitatively why the experimental distribution of K^+ is a bit broader than for π^+'s (both are mainly related to the distribution of up quarks in the proton times $[(1-x)/x]^{1/2}$). The relative smallness of strange quarks in the sea means that they are more concentrated in the low-x region [strange quarks are almost wee ($x_s<\bar{x}_u$)]. We thus expect a smaller effect due to the spread of momentum of sea quarks for K^+ than for π^+ production. This has to be confirmed by more quantitative analysis.

The broader experimental K^- distribution compared to our predictions can be understood as a reasonance effect, K^- being the decay product of a directly produced resonance which contains a valence quark of the proton (K^- cannot be directly produced from valence quarks). Though being depreciated by a two-step process, K^- production via resonances dominates in the large-x region over direct production from a sea quark. The resonance effect can be neglected in other cases where the allowed direct production from the valence quark is overwhelming in the large-x region.

IV. CONCLUSIONS AND OUTLOOK

A. Summary of results

In Table I we summarize the dictionary of topological correspondence; namely, we show for some inclusive processes their parton interpretations and the lepton-hadron processes giving rise to the same final-state distributions.

We want to emphasize here the main features of the proposed scheme.

(i) We derive a quark-parton model for soft hadronic processes, which is compatible with the results of CTHP.

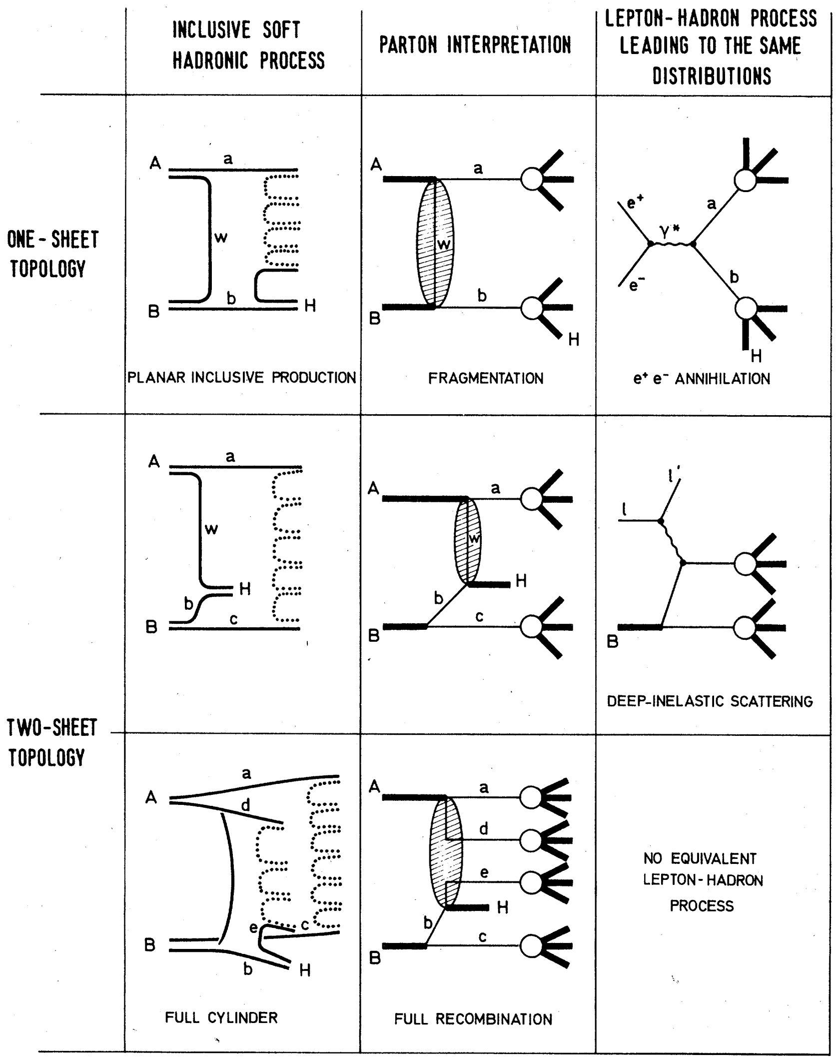
(ii) Topology allows one to classify and to clarify the parton mechanism associated with soft hadronic inclusive processes: One-sheet topology is associated with the *fragmentation* mechanism, whereas two-sheet topology is associated with the *recombination* mechanism.

This interpretation is to be compared with models proposed till now, where the full one-particle inclusive cross section has been interpreted as a recombination[3(a)-3(c), 3(f)] *or* fragmentation[3(e)] mechanism.

The one-sheet contribution has been extracted from data by taking the difference between "line-reversed" one-particle inclusive cross sections. One can thus compare quark fragmentation functions extracted from current-induced reactions and from some specific soft hadronic processes. The comparison is quite good *in the full range of the z variable*.

We have generalized the f-promotion mechanism (which is one of the basic results of DTU for total cross section) to one-particle inclusive cross sections. This allows us to express the dominant part of the one-particle inclusive cross section (two-sheet topology) in terms of hadron structure functions. Comparison with data is quite good. DTU arguments allow one to solve the problem of

TABLE I. Duality diagrams and their parton interpretation for inclusive soft hadronic processes.



absolute and relative normalization.

(iii) In this comparison we have made the empirical observation that scaling violations, which are known to occur in soft hadronic processes, are similar to the ones observed (and predicted from perturbative QCD) in structure functions. This similarity is quantitative once one uses the topological correspondence to define Q^2 in hadronic processes.

B. Further investigations

In order to proceed in the investigation of further consequences of the topological correspondence we have to stress the theoretical significance of the phenomenological successes which we have acheived. Our essential result is that DTU, at the lowest order, can be interpreted as a genuine quark-parton model, satisfying in particular exact scale invariance. Although it is a great achievement (*a priori* duality seems orthogonal to pointlike-parton physics), this result cannot be more than an approximation, since one knows that scale invariance is actually broken, and that perturbative QCD provides a coherent description of this breaking. The above-mentioned empirical observation about scaling violations suggests the direction in which we can proceed: One knows, in effect, that Feynman scaling violations can be described in terms of long-range correlations, Pomeron corrections,[24] all effects which appear in higher topologies in DTU. Therefore, if it is true that violations of Bjorken scaling and Feynman scaling are equivalent, one can hope to transform our quark-parton model into a QCD quark-parton model by taking into account higher orders in DTU. This program raises a series of challenging questions. In particular, it has been noticed[25] that scaling violations in QCD occur already at the planar level (and that, in some gauges, leading violations come even totally from planar diagrams). This observation seems in contradiction with the expectation described just above; it is clear that any further development of the QCD-DTU equivalence program requires solving this contradiction, and answering a series of connected questions:

(i) What is color in DTU?

(ii) Where are gluons in DTU?

(iii) Is there a connection between Pomeron exchange and gluon exchange?[26]

(iv) Are there gluon jets in soft hadronic processes?

(v) How can one extend DTU to large-$p_\perp$ hadronic processes?

(vi) How can one understand the empirical observation that scaling violations are similar in soft processes and in current-induced processes?

(vii) How can one include baryons in the topological correspondence schemes?[27]

C. Theoretical outlook

Suppose now that one would be able to establish or to prove complete equivalence between QCD and DTU. What will it prove? What will it teach us about fundamentals of hadron theory?

It is well admitted that QCD is a candidate to be a real theory of hadrons, that is, a method of deriving all properties of hadron structure and interactions from a well defined renormalizable Lagrangian. Less agreed upon is the fact that DTU is also a candidate to be a real theory of hadrons. Usually one considers DTU as a semiphenomenological approach (Veneziano ansatz) or as needing to rely on an underlying field theory (for example, string model of hadrons). But the recent developments in ordered S-matrix theory suggest that DTU can be formulated as a real theory with its own axioms (S-matrix theoretical axioms plus the axiom about the role of order), with its own logic (self-consistency giving rise to constraints capable of uniquely determining scattering amplitudes) with predictive power (coherent and global description of exclusive soft hadronic reactions).

Now, if one admits that both QCD and DTU have the characteristics of real theories, then if one proves their equivalence one learns something important. In our opinion, this important thing is intimately related to confinement.

Up to now, all attempts to prove confinement in QCD in four dimensions have failed. More and more theorists are convinced that the confinement problem is not a technicality, that it is related to deep conceptual questions (unobservability of the fundamental blocks of matter and proof of this unobservability).

Actually, complete equivalence between QCD and DTU would mean that (i) all properties of hadrons (or Reggeons) in DTU can be derived from QCD, and (ii) all properties of QCD partons (quarks and gluons) can be derived from DTU. We think that such an equivalence can provide an extremely general proof of confinement. This formulation of the confinement problem has the important advantage of exhibiting its "epistemological" implication. In effect everybody agrees that there should exist only one theory of hadrons. If one finds two equivalent theories it would necessarily mean that neither one nor the other is the theory. It would mean that a *new theory*, implying *new concepts*, is needed to account for this equivalence of the former theories and to predict *new* effects.

Finally, we want to stress that our first results are very promising not only on a phenomenological but also on a theoretical ground. We have pointed out the outstanding role of the planar bootstrap in determining completely the planar amplitudes which are the starting point of DTU. Now planar amplitudes are the ones for which confinement is expected to occur. We thus establish a relation between the confinement problem and the bootstrap or self-consistency dynamics. This, we think, provides a *new* promising insight into the confinement problem.

ACKNOWLEDGMENTS

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APPENDIX A: MEAN MULTIPLICITY

In dual models, the proportionality of the mean multiplicity to the number of sheets occurs only asymptotically. As was extensively discussed by Dias de Deus and Jadach[14] and Veneziano,[28] taking into account the unequal partition of energy into jets and the question of what is the relevant energy at which the comparison is made (total energy, energy available for particle production, etc.),

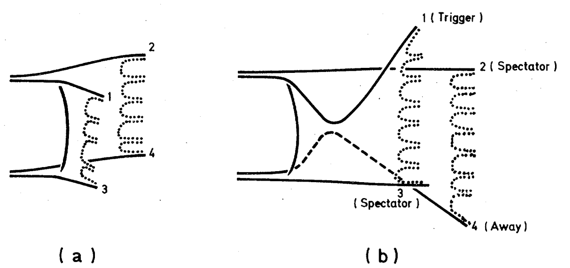

FIG. 16. Two-sheet (four-jet) topology in meson-meson scattering. (a) low $p_\perp$; (b) high $p_\perp$.

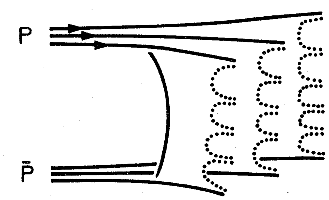

FIG. 17. Three-sheet topology in $p\bar{p}$ annihilation.

one can explain the accidental similarity of all multiplicities independently of the number of sheets involved. The same argument[14] accounts for the smallness of the Q^2 dependence in the deep-inelastic multiplicity data. Therefore, one can say that, at present energies, data on multiplicities do not allow one to discriminate between the universality picture and topological universality. However, higher-energy data should provide a clear experimental test.

APPENDIX B: MULTIPLICITY ASSOCIATED WITH HIGH-$p_\perp$ HADRONIC COLLISIONS

It will be interesting to study the multiplicity associated with high-$p_\perp$ hadron collisions. The hadron final state in pp or πp collisions has, at high energy, a two-sheet (or four-jet) structure. In low-$p_\perp$ events, all jets are elongated along the scattering axis. In high-$p_\perp$ reactions one can expect that two of those jets are rotated to give rise to the "trigger" and "away" jets, whereas the two others give rise to the "spectator" jets (see Fig. 16). Therefore, topological universality predicts the same total multiplicity in low-$p_\perp$ and

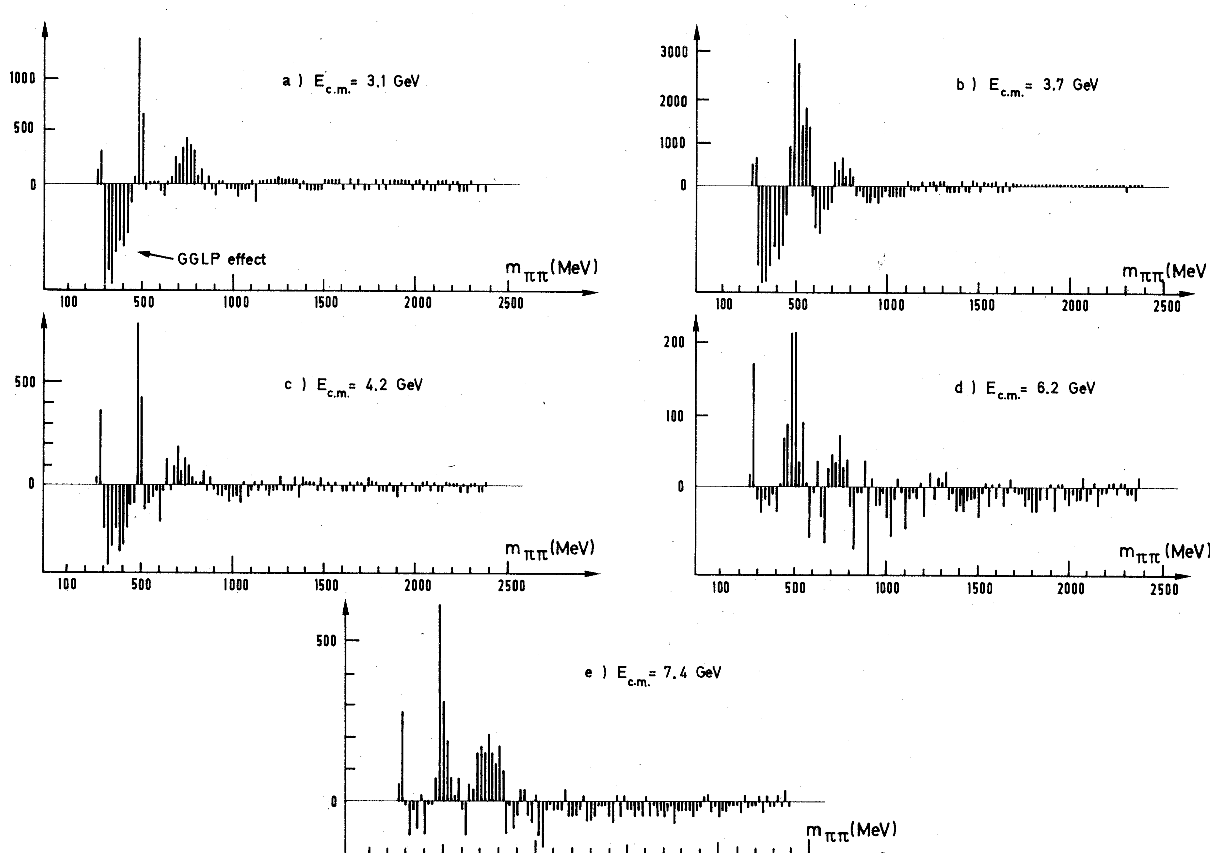

FIG. 18. Data showing the energy dependence of the GGLP effect. In the abscissas is the dipion mass and in the ordinates the number of unlike minus the number of like pion pairs normalized to the same total population.

in high-$p_\perp$ hadronic collisions. This prediction is different from the one of the universality picture (Brodsky and Gunion, Ref. 4), since in this scheme only two jets are produced in soft collisions, whereas four jets are explicitly seen in large-$p_\perp$ collisions. Experimental information[29] (which is difficult to get because usually the experimental acceptance does not cover 4π of solid angle) seems to favor topological universality, that is, no change in the total mean multiplicity from soft to hard four-jet reactions. We do not present this an an argument in favor of topological universality, but simply to argue that the analysis of existing data on mean multiplicity is not enough to rule out any of the two schemes.

APPENDIX C: GOLDHABER-GOLDHABER-LEE-PAÏS EFFECT AND RESONANCE PRODUCTION

Giovanini and Veneziano[30] have pointed out that a more sensitive confirmation of the topological structure of hadronic final states could be found in two-pion correlation data. If topology, in fact, has something to do with production mechanism, the Goldhaber-Goldhaber-Lee-Pais[31] (GGLP) effect for like pions (which is due to Bose-Einstein statistics) is expected to be maximal in $\bar{p}p$ annihilation (three sheets; see Fig. 17), intermediate in pp and πp scattering (two sheets), and minimal in $e^+e^- \to$ hadrons (one sheet). On the contrary, the correlation function

$$R = \frac{1}{\sigma_{in}} E_1 E_2 \frac{d^6\sigma}{d^3p_1 d^3p_2} \Big/ \left(\frac{1}{\sigma_{in}} E_1 \frac{d^3\sigma}{d^3p_1}\right)\left(\frac{1}{\sigma_{in}} E_2 \frac{d^3\sigma}{d^3p_2}\right) - 1 \tag{A1}$$

for unlike pions ($\pi^+\pi^-$, $\pi^+\pi^0$, $\pi^-\pi^0$) is expected to be maximal for e^+e^- and minimal for $p\bar{p}$ annihilation. These predictions have to be compared with data at sufficiently high energy where the jet structure is fully developed. From e^+e^- analysis we know that jet structure appears for $E_{\rm c.m.} \gtrsim 6$ GeV, so it would mean at least $p_{\rm lab} \simeq 200$ GeV for $p\bar{p}$. Unfortunately, at such a high energy one cannot extract $p\bar{p}$ annihilation events.

However, an analysis of the GGLP effect in e^+e^- quoted by Veneziano in Ref. 28 (see Fig. 18) shows clearly the disappearance of the GGLP effect for the largest measured energies (corresponding to the development of planar $q\bar{q}$ structure). It is interesting to note a well marked GGLP effect at the ψ and at the ψ', which would imply an important nonplanarity in the decay of heavy quarkonium. It would be interesting to look for the reappearance of the GGLP effect at the Υ, since it has been shown that the decay of this particle does not go through a $q\bar{q}$ system.[32]

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