Determination of the chiral-SU(4) × SU(4)-breaking parameters

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We consider broken chiral SU(4) × SU(4) symmetry. From the observed mass spectrum of pseudoscalar charmed mesons, we are able to solve for the symmetry-breaking parameters of the theory. We find that both vacuum and Hamiltonian breaking play an important role as far as charmed states are concerned. Purely from the masses of D and F mesons we deduce the current-algebra mass ratio $m_c/m_s < 5$. This differs greatly from values obtained using linear or quadratic mass formulas. Considering η, η', and η_c mixing we further obtain a good solution with $m_c/m_s \simeq 3.2$ and $\langle \bar{c}c\rangle/\langle \bar{u}u\rangle \simeq 5.67$.

I. INTRODUCTION

Recent observation[1,2,3] of charmed pseudoscalar mesons $D^{\pm}$, D^0, F^+ as well as η_c prompts us to reexamine the question of how the chiral SU(4) ×SU(4) symmetry is broken.[4] Current ideas of strong interactions based on quantum chromodynamics (QCD) and unified theories of weak and electromagnetic interactions suggest that chiral SU(4) ×SU(4) symmetry is a global symmetry of the Lagrangian associated with the flavor group. Further, this symmetry is broken both by the vacuum and explicitly in the interaction Lagrangian by the quark mass terms which transform according to $(4,4^*)\oplus(4^*,4)$ representation. Our knowledge of quantum chromodynamics is not yet at a stage which will allow us to calculate directly the true vacuum of the chiral group. Nevertheless, we can use current-algebra techniques and the observed mass spectrum of the pseudoscalar mesons and their decay constants F_π, F_K, etc. to determine the properties of the vacuum as well as the mass ratios of the quarks.

The charm-quark-to-strange-quark mass ratio m_c/m_s in principle is simply calculated from the knowledge of the ratio of SU(4) breaking along the 15 direction to that along the 8 direction. Thus if the Hamiltonian for symmetry breaking is

$$H_{\text{symmetry breaking}} = -(\epsilon_0 u_0 + \epsilon_8 u_8 + \epsilon_{15} u_{15}), \qquad (1.1)$$

then since $m_u, m_d \ll m_s, m_c$ (Ref. 5)

$$\frac{m_c}{m_s} \simeq \frac{4}{3\sqrt{2}}\,\frac{\epsilon_{15}}{\epsilon_8} + \frac{1}{3}. \qquad (1.2)$$

However, the estimates for ϵ_{15}/ϵ_8 vary widely depending on whether we use first-order breaking formula to fit the masses or the masses squared. Thus if linear mass formula is employed we have $\epsilon_{15}/\epsilon_8 \simeq 9.7$, and if quadratic mass formula[6] is employed we find $\epsilon_{15}/\epsilon_8 \simeq 21.6$. These yield for the ratio m_c/m_s the values 9.5 and 20.7, respectively.

For vector mesons the linear formula fits better than the quadratic one, as

$$\begin{aligned} &M_{F^*} - M_{D^*} = M_{K^*} - M_\rho, \\ &\quad (0.13\text{ GeV}) \simeq (0.122\text{ GeV}), \\ &M_{F^*}^2 - M_{D^*}^2 = M_{K^*}^2 - M_\rho^2, \\ &\quad (0.54\text{ GeV}^2) \neq (0.203\text{ GeV}^2). \end{aligned} \qquad (1.3)$$

However, neither fit is satisfactory for pseudoscalar mesons, because they yield

$$\begin{aligned} &M_F - M_D = M_K - M_\pi, \\ &\quad (0.178\text{ GeV}) \neq (0.414\text{ GeV}), \\ &\text{or} \\ &M_F^2 - M_D^2 = M_K^2 - M_\pi^2, \\ &\quad (0.695\text{ GeV}^2) \neq (0.283\text{ GeV}^2). \end{aligned} \qquad (1.4)$$

Thus we need a more accurate treatment of chiral breaking to yield a more reliable estimate of m_c/m_s.

Work based on chiral SU(3) ×SU(3) breaking[7] had revealed that (a) the Lagrangian is approximately SU(2) ×SU(2) invariant leading to a small number for the ratios m_u/m_s, m_d/m_s, and (b) the vacuum is to a good approximation a SU(3) singlet implying approximate equality of decay constants $F_\pi \simeq F_K$, as well as the (mass)^2 octet broken formula for the pseudoscalar bosons. An extremely good solution[8] for all the parameters was obtained by solving the current-algebra equations which included a general η-η' mixing with a single hypothesis on equality of renormalization constants $\sqrt{Z}_i \equiv \langle 0|\bar{q}\lambda_i\gamma_5 q|P_i\rangle$, where P_i denotes the ith pseudoscalar meson. In this paper we wish to find a consistent set of solutions to the current-algebra equations using a similar technique in the case of SU(4) ×SU(4) symmetry. We have found, however, that the requirement that $\sqrt{Z}$ be SU(4) symmetric is completely at variance with the mass spectrum of the pseudoscalar mesons. We shall show that such an assumption leads to an extremely small D-F mass splitting if the decay constants are assumed equal, and if the latter requirement is given up, a realistic value for $F_K \simeq 1.28$ leads to $M_F < M_D$, which is quite unacceptable. In the present work, we use the value of M_D and M_F as inputs and find that Z_i's have large SU(4) breaking. Purely from SU(3) symmetry of Z_i's and the value of D and F meson masses, we establish that the ratio $m_c/m_s < 5$.

With further assumptions we are able to solve the coupled set of equations that characterize the model. We allow for a general mixing for η, η' and η_c. A surprise is that the vacuum is not an SU(4) singlet, although it is still to a good approximation SU(3) symmetric.

We have determined the ratios of quark masses and the decay constants. These are

$$\frac{m_s}{m_u} \simeq 33.5, \; \frac{m_c}{m_s} \simeq 3.2 , \tag{1.5}$$

and the decay constants are, in units of $F_\pi \simeq 92$ MeV,

$$F_K = 1.28, \quad F_D = 0.974, \quad F_F = 1.056 . \tag{1.6}$$

We could have started with the group U(4) ×U(4) instead of SU(4) ×SU(4). This would lead to the U(1) problem discussed by Weinberg.[9] As noted by 't Hooft,[10] this problem can be circumvented in QCD where the presence of instantons leads to an extra $U_A(1)$-breaking effective interaction $U = \det \bar{q}_i q_{jL} + \text{H.c.}$ We have added an extra term to the divergence of the A_μ^0 current to take into account this effect. The net result is that it is possible to consider the SU(4) ×SU(4) algebra itself and solve for the unknown parameters. No constraint is imposed on this form from the $\text{U}_A(1)$ sector. On the other hand, from the knowledge of the solution we can say something about the matrix elements of the $\text{U}_A(1)$-breaking term.

In Sec. II, we set up the basic equations of the model. The section also serves to define our notations. In Sec. III, we show how simple assumptions on equality of Z_i give unacceptable values and in Secs. IV and V we present our technique for solving the set of equations. A discussion of our results is contained in Sec. VI.

II. THE FUNDAMENTAL EQUATIONS

The strong-interaction Hamiltonian density is assumed to be of the form

$$H = H_0 - \epsilon_0 u_0 - \epsilon_8 u_8 - \epsilon_{15} u_{15} , \tag{2.1}$$

where H_0 is invariant under chiral SU(4) ×SU(4) symmetry, while the symmetry breaking terms u_i transform according to $(4^*, 4) \oplus (4, 4^*)$ representation. In terms of the quark model, these symmetry-breaking terms are merely the mass terms of the quarks. We shall neglect isospin-breaking effects due to lack of degeneracy of the mass of u and d quarks as well as conventional electromagnetic corrections, in this paper. The explicit relation between quark masses and ϵ_0, ϵ_8, and ϵ_{15} are easily found to be

$$m_u = m_d = -\left(\frac{\epsilon_0}{\sqrt{2}} + \frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}\right), \tag{2.2a}$$

$$m_s = -\left(\frac{\epsilon_0}{\sqrt{2}} - 2\frac{\epsilon_8}{\sqrt{3}} + \frac{\epsilon_{15}}{\sqrt{6}}\right), \tag{2.2b}$$

$$m_c = -\left(\frac{\epsilon_0}{\sqrt{2}} - 3\frac{\epsilon_{15}}{\sqrt{6}}\right). \tag{2.2c}$$

Note that if $m_u \neq m_d$, then (2.2a) should have average mass of light quarks, $m_l = \frac{1}{2}(m_u + m_d)$ on the left-hand side.

The generators of SU(4) × SU(4) can be expressed in terms of F^i and F_5^i, the vector and axial generators, which are defined as usual by

$$F^i(t) = \int_{x_0=t} d^3x V_0^i(x) , \qquad F_5^i(t) = \int_{x_0=t} d^3x A_0^i(x) . \qquad i = (0, \ldots, 15) \tag{2.3}$$

The scalar densities u_i $(\equiv \bar{q}\lambda_i q)$ and pseudoscalar densities v_i $(\equiv i\bar{q}\lambda_i \gamma^5 q)$ satisfy the equal-time commutation rules.

$$\begin{aligned}
&[F^i(t), u^j(x)]_{x_0=t} = i f_{ijk} u^k(x) ,\\
&[F^i(t), v^j(x)]_{x_0=t} = i f_{ijk} v^k(x) ,\\
&[F_5^i(t), u^j(x)]_{x_0=t} = -i d_{ijk} v^k(x) ,\\
&[F_5^i(t), v^j(x)]_{x_0=t} = i d_{ijk} u^k(x) .
\end{aligned} \tag{2.4}$$

The current divergences are given by

$$\partial^\mu V_\mu^i = -i[F^i(t), H(x)]_{x_0=t} , \tag{2.5a}$$

$$\partial^\mu A_\mu^i = -i[F_5^i(t), H(x)]_{x_0=t} . \tag{2.5b}$$

From Eqs. (2.1) and (2.5) these are found to be

$$\partial^\mu V_\mu^i = -\epsilon_8 f_{i8k} u^k - \epsilon_{15} f_{i15k} u^k , \tag{2.6a}$$

$$\partial^\mu A_\mu^i = \epsilon_0 d_{i0k} v^k + \epsilon_8 d_{i8k} v^k + \epsilon_{15} d_{i15k} v^k + \delta_{i0} V , \tag{2.6b}$$

where $V = -i[F_5^0, U]$ is a flavor singlet.

We take matrix elements of Eq. (2.7) between vacuum and single pseudoscalar or scalar meson states. The following conventional definitions are employed:

$$\langle 0|A_\mu^{1,2,3}|\pi\rangle = ip_\mu F_\pi ,$$
$$\langle 0|A_\mu^{4,5,6,7}|K\rangle = ip_\mu F_K ,$$
$$\langle 0|A_\mu^{9,10,11,12}|D\rangle = ip_\mu F_D ,$$
$$\langle 0|A_\mu^{13,14}|F\rangle = ip_\mu F_F ,$$
$$\langle 0|A_\mu^{i}|\eta,\eta',\eta_c\rangle = ip_\mu F^i_{\eta,\eta',\eta_c} \quad (i=0,\ 8,\ \text{or}\ 15), \tag{2.7}$$
$$\langle 0|V_\mu^{4}|\kappa\rangle = ip_\mu F_\kappa ,$$
$$\langle 0|V_\mu^{9,11}|S_{10,12}\rangle = -\langle 0|V_\mu^{10,12}|S_{9,11}\rangle = ip_\mu F_{S_D},$$
$$\langle 0|V_\mu^{13}|S_{14}\rangle = -\langle 0|V_\mu^{14}|S_{13}\rangle = ip_\mu F_{S_F} .$$

Our currents are so renormalized that $F_\pi \simeq 92$ MeV. States S_D and S_F are members of the scalar 15-plet.

$$\langle 0|v^{1,2,3}|\pi\rangle = \sqrt{Z_\pi} ,$$
$$\langle 0|v^{4,5,6,7}|K\rangle = \sqrt{Z_K} ,$$
$$\langle 0|v^{9,10,11,12}|D\rangle = \sqrt{Z_D} ,$$
$$\langle 0|v^{13,14}|F\rangle = \sqrt{Z_F} ,$$
$$\langle 0|v^{i}|\eta,\eta',\eta_c\rangle = \sqrt{Z^i_{\eta,\eta',\eta_c}} \quad (i=0,\ 8,\ \text{or}\ 15), \tag{2.8}$$
$$\langle 0|u^{4,5,6,7}|\kappa\rangle = \sqrt{Z_\kappa} ,$$
$$\langle 0|u^{9,10,11,12}|S_D\rangle = \sqrt{Z_{S_D}} ,$$
$$\langle 0|u^{13,14}|S_F\rangle = \sqrt{Z_{S_F}} ,$$
$$\langle 0|V|\eta,\eta',\eta_c\rangle = g_{\eta,\eta',\eta_c} .$$

We then find for pseudoscalar mesons

$$\frac{M_\pi^2 F_\pi}{\sqrt{Z_\pi}} = \frac{\epsilon_0}{\sqrt2} + \frac{\epsilon_0}{\sqrt3} + \frac{\epsilon_{15}}{\sqrt6} ,$$
$$\frac{M_K^2 F_K}{\sqrt{Z_K}} = \frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{2\sqrt3} + \frac{\epsilon_{15}}{\sqrt6} ,$$
$$\frac{M_D^2 F_D}{\sqrt{Z_D}} = \frac{\epsilon_0}{\sqrt2} + \frac{\epsilon_8}{2\sqrt3} - \frac{\epsilon_{15}}{\sqrt6} ,$$
$$\frac{M_F^2 F_F}{\sqrt{Z_F}} = \frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{\sqrt3} - \frac{\epsilon_{15}}{\sqrt6} , \tag{2.9}$$
$$M_{\eta_i}^2 F^8_{\eta_i} = \left(\frac{\epsilon_0}{\sqrt2} - \frac{\epsilon_8}{\sqrt3} + \frac{\epsilon_{15}}{\sqrt6}\right)\sqrt{Z^8_{\eta_i}} + \frac{\epsilon_8}{\sqrt6}\sqrt{Z^{15}_{\eta_i}} + \frac{\epsilon_8}{\sqrt2}\sqrt{Z^0_{\eta_i}} ,$$
$$M_{\eta_i}^2 F^{15}_{\eta_i} = \left(\frac{\epsilon_0}{\sqrt2} - 2\frac{\epsilon_{15}}{\sqrt6}\right)\sqrt{Z^{15}_{\eta_i}} + \frac{\epsilon_{15}}{\sqrt2}\sqrt{Z^0_{\eta_i}} + \frac{\epsilon_8}{\sqrt6}\sqrt{Z^8_{\eta_i}} ,$$
$$M_{\eta_i}^2 F^0_{\eta_i} = \frac{\epsilon_0}{\sqrt2}\sqrt{Z^0_{\eta_i}} + \frac{\epsilon_8}{\sqrt2}\sqrt{Z^8_{\eta_i}} + \frac{\epsilon_{15}}{\sqrt2}\sqrt{Z^{15}_{\eta_i}} + g_{\eta_i}$$
$$(\eta_i = \eta, \eta',\ \text{or}\ \eta_c) .$$

Similarly we find for scalar mesons

$$\frac{M_\kappa^2 F_\kappa}{\sqrt{Z_\kappa}} = \frac{\sqrt3}{2}\epsilon_8 ,$$
$$\frac{M_{S_D}^2 F_{S_D}}{\sqrt{Z_{S_D}}} = \frac{\epsilon_8}{2\sqrt3} + 2\frac{\epsilon_{15}}{\sqrt6} , \tag{2.10}$$
$$\frac{M_{S_F}^2 F_{S_F}}{\sqrt{Z_{S_F}}} = -\frac{\epsilon_8}{\sqrt3} + 2\frac{\epsilon_{15}}{\sqrt6} .$$

The above equations are exact consequences of the model. Further equations are obtained by single-particle saturation which becomes exact in the Nambu-Goldstone limit, i.e., $\epsilon_i \to 0$. Then the only breaking of symmetry is in the vacuum, resulting in massless bosons that saturate the commutation rules. The corrections to these results are expected to be of order ϵ and except for charmed states, we might expect these to be quite small. Here we assume the validity of all the relations and appeal to future experiments as a way of establishing them. Defining $\langle 0|u_i|0\rangle \equiv \delta_i$, we have for pseudoscalar mesons

$$F_\pi\sqrt{Z_\pi} = \frac{\delta_0}{\sqrt2} + \frac{\delta_8}{\sqrt3} + \frac{\delta_{15}}{\sqrt6} ,$$
$$F_K\sqrt{Z_K} = \frac{\delta_0}{\sqrt2} - \frac{\delta_8}{2\sqrt3} + \frac{\delta_{15}}{\sqrt6} ,$$
$$F_D\sqrt{Z_D} = \frac{\delta_0}{\sqrt2} + \frac{\delta_8}{2\sqrt3} - \frac{\delta_{15}}{\sqrt6} ,$$
$$F_F\sqrt{Z_F} = \frac{\delta_0}{\sqrt2} - \frac{\delta_8}{\sqrt3} - \frac{\delta_{15}}{\sqrt6} ,$$
$$F^8_\eta\sqrt{Z^0_\eta} + F^8_{\eta'}\sqrt{Z^8_{\eta'}} + F^8_{\eta_c}\sqrt{Z^0_{\eta_c}} = \frac{\delta_0}{\sqrt2} - \frac{\delta_8}{\sqrt3} + \frac{\delta_{15}}{\sqrt6} ,$$
$$F^8_\eta\sqrt{Z^0_\eta} + F^8_{\eta'}\sqrt{Z^0_{\eta'}} + F^8_{\eta_c}\sqrt{Z^0_{\eta_c}} = \frac{\delta_8}{\sqrt2} \tag{2.11}$$
$$F^8_\eta\sqrt{Z^{15}_\eta} + F^8_{\eta'}\sqrt{Z^{15}_{\eta'}} + F^8_{\eta_c}\sqrt{Z^{15}_{\eta_c}} = \frac{\delta_8}{\sqrt6} ,$$
$$F^{15}_\eta\sqrt{Z^0_\eta} + F^{15}_{\eta'}\sqrt{Z^0_{\eta'}} + F^{15}_{\eta_c}\sqrt{Z^0_{\eta_c}} = \frac{\delta_{15}}{\sqrt2} ,$$
$$F^{15}_{\eta'}\sqrt{Z^8_{\eta'}} + F^{15}_\eta\sqrt{Z^8_\eta} + F^{15}_{\eta_c}\sqrt{Z^8_{\eta_c}} = \frac{\delta_8}{\sqrt6} ,$$
$$F^{15}_\eta\sqrt{Z^{15}_\eta} + F^{15}_{\eta'}\sqrt{Z^{15}_{\eta'}} + F^{15}_{\eta_c}\sqrt{Z^{15}_{\eta_c}} = \frac{\delta_0}{\sqrt2} - 2\frac{\delta_{15}}{\sqrt6} .$$

Similar equations for scalar bosons can also be written. However, pole saturation cannot be justified for these because in the chiral limit the masses of scalar bosons diverge. For the sake of completeness, we only list them but shall not make use of them in this paper:

$$F_\kappa\sqrt{Z_\kappa}=\frac{\sqrt{3}}{2}\delta_8\,,$$
$$F_{S_D}\sqrt{Z_{S_D}}=\frac{\delta_8}{2\sqrt{3}}+2\frac{\delta_{15}}{\sqrt{6}}\,, \tag{2.12}$$
$$F_{S_F}\sqrt{Z_{S_F}}=-\frac{\delta_8}{\sqrt{3}}+2\frac{\delta_{15}}{\sqrt{6}}\,.$$

It is also possible to derive relations by considering commutators of generators with the divergences of currents, and then taking their vacuum expectation values. These relations are easily obtained from Eqs. (2.9) and (2.11) by eliminating $\sqrt{Z_i}$'s.

The basic problem we address to ourselves is to solve these equations with reasonable assumptions. The result will be to determine symmetry breaking parameters, i.e., ϵ_8/ϵ_0, ϵ_{15}/ϵ_0, and δ_8/δ_0, δ_{15}/δ_0 as well as decay constants F_i's. In the next section we examine some simple but experimentally inadmissible solutions and then in Secs. IV and V we make only the most plausible assumptions to solve these equations.

III. SIMPLE SOLUTIONS AND PROBLEMS

The set of equations obtained in Sec. II clearly involve too many unknown parameters to obtain a complete solution. In this section we shall make some simplifying assumptions to illustrate the difficulty in obtaining physically meaningful solutions. The simplest assumption is the generalization of the Gell-Mann, Oakes, and Renner (GOR)[7] solution to this enlarged group. The assumption is that the vacuum is a SU(4) singlet, i.e.,

$$\langle u_0\rangle\equiv\delta_0\neq 0\,,$$
$$\langle u_8\rangle\equiv\delta_8=0\,,\quad\langle u_{15}\rangle\equiv\delta_{15}=0\,, \tag{3.1}$$

and further that SU(4) symmetry is good for $\sqrt{Z}$'s, i.e., $Z_\pi=Z_K=Z_D=Z_F=Z_8=Z_{15}$. This is one of the solutions considered in Kandaswamy, Schechter, and Singer's paper in Ref. 4. Our point in discussing it here is to show that it is unsatisfactory. We also allow the possibility of mixing of the SU(4) $\underline{8}$ and $\underline{15}$ states because u_8 in the Hamiltonian mixes the 8 and the 15 components of the $\underline{15}$ representation. We shall, however, following GOR, neglect any singlet mixing, for simplicity. The general case will be considered next. Thus,

$$\sqrt{Z^8_\eta}=\sqrt{Z^{15}_{\eta_c}}=\sqrt{Z_\pi}\cos\theta\,,$$
$$\sqrt{Z^8_{\eta_c}}=-\sqrt{Z^{15}_\eta}=\sqrt{Z_\pi}\sin\theta\,, \tag{3.2}$$

A consistent set of solutions is then obtained to all the equations in Sec. II. The mass sum rules are

$$M_F{}^2-M_D{}^2=M_K{}^2-M_\pi{}^2\,,$$
$$4M_K{}^2-M_\pi{}^2=3(M_\eta{}^2\cos^2\theta+M_{\eta_c}{}^2\sin^2\theta)\,, \tag{3.3}$$
$$-(M_K{}^2-M_\pi{}^2)\tfrac{2}{3}\sqrt{2}=(M^2_{\eta_c}-M_\eta{}^2)\sin 2\theta\,,$$
$$9M_D{}^2+M_K{}^2-4M_\pi{}^2=6(M_\eta{}^2\sin^2\theta+M_{\eta_c}{}^2\cos^2\theta)\,.$$

These are four equations involving seven variables, six masses, and one mixing angle. A general feature of these equations is that because of the large η_c mass (~2.83 GeV) we have

$$M_D\simeq(\tfrac{2}{3})^{1/2}M_{\eta_c}\simeq 2.31\text{ GeV}\,,$$
$$M_F-M_D\simeq\frac{M_K{}^2}{M_F+M_D}\simeq 60\text{ MeV}\,. \tag{3.4}$$

The experimental value of M_D is, however, much lower, 1862 MeV,[2] while M_F-M_D is closer to 180 MeV.[3,2] The source of the problem can be traced to (mass)^2 sum rules that emerge with our simplifying assumptions, while the heavier masses are fitted better with a linear mass formula. Thus,

$$M_D\simeq\tfrac{2}{3}M_{\eta_c}\,. \tag{3.5}$$

Inclusion of singlet in our mixing scheme does not change the basic situation. The matrix $\sqrt{Z^i_{\eta,\eta',\eta_c}}$ ($i=0$, 8, or 15) is then a 3×3 orthogonal matrix, and Eq. (2.11) yields the solution (remembering $\delta_8=\delta_{15}=0$)

$$\frac{F^i_{\eta,\eta',\eta_c}}{F_\pi}=\frac{\sqrt{Z^i_{\eta,\eta',\eta_c}}}{\sqrt{Z_\pi}}\quad(i=0,\ 8,\text{ or }15)\,. \tag{3.6}$$

Now solving the Eq. (2.9) is equivalent to the diagonalization of the 3×3 η, η', η_c mass matrix. We identify the physical states as

$$|\eta\rangle=Z_\pi{}^{-1/2}[\sqrt{Z^0_\eta}\,|P_0\rangle+\sqrt{Z^8_\eta}\,|P_8\rangle+\sqrt{Z^{15}_\eta}\,|P_{15}\rangle]\,,$$
$$|\eta'\rangle=Z_\pi{}^{-1/2}[\sqrt{Z^0_{\eta'}}\,|P_0\rangle+\sqrt{Z^8_{\eta'}}\,|P_8\rangle+\sqrt{Z^{15}_{\eta'}}\,|P_{15}\rangle]\,,$$
$$|\eta_c\rangle=Z_\pi{}^{-1/2}[\sqrt{Z^0_{\eta_c}}\,|P_0\rangle+\sqrt{Z^8_{\eta_c}}\,|P_8\rangle+\sqrt{Z^{15}_{\eta_c}}\,|P_{15}\rangle]\,, \tag{3.7}$$

where $|P_i\rangle$ are SU(4)-symmetric unmixed states. Since operator V is a SU(4) singlet, it is reasonable to assume that matrix elements of V in lowest-order perturbation theory are

$$\langle 0|V|\eta,\eta',\eta_c\rangle\equiv g_{\eta,\eta',\eta_c}=\mu^2\sqrt{Z^0_{\eta,\eta',\eta_c}}\,, \tag{3.8}$$

where μ^2 is an arbitrary constant.

Equation (2.9) can be written as a matrix equation

$$\underline{M}^2\sqrt{Z}=\sqrt{Z}\,M^2\,,$$

where

$$\underline{M}^2=\begin{pmatrix} M_\eta{}^2 & 0 & 0 \\ 0 & M_{\eta'}{}^2 & 0 \\ 0 & 0 & M_{\eta_c}{}^2 \end{pmatrix} \tag{3.9a}$$

is a diagonal matrix,

$$M^2=\langle P_i|H|P_j\rangle$$

$$=\begin{bmatrix} \mu^2+\frac{\epsilon_0}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_{15}}{\sqrt{2}} \\ \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_0}{\sqrt{2}}-\frac{\epsilon_8}{\sqrt{3}}+\frac{\epsilon_{15}}{\sqrt{6}} & \frac{\epsilon_8}{\sqrt{6}} \\ \frac{\epsilon_{15}}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{6}} & \frac{\epsilon_0}{\sqrt{2}}-2\frac{\epsilon_{15}}{\sqrt{6}} \end{bmatrix} \tag{3.9b}$$

and

$$\sqrt{Z}=\begin{pmatrix} \sqrt{Z_\eta^0} & \sqrt{Z_\eta^8} & \sqrt{Z_\eta^{15}} \\ \sqrt{Z_{\eta'}^0} & \sqrt{Z_{\eta'}^8} & \sqrt{Z_{\eta'}^{15}} \\ \sqrt{Z_{\eta_c}^0} & \sqrt{Z_{\eta_c}^8} & \sqrt{Z_{\eta_c}^{15}} \end{pmatrix}. \tag{3.9c}$$

Equation (3.9b) is then written as

$$M^2=\begin{bmatrix} \mu^2+\frac{1}{2}(M_D{}^2+M_K{}^2) & -\frac{\sqrt{2}}{\sqrt{3}}(M_K{}^2-M_\pi{}^2) & \frac{1}{2\sqrt{3}}(M_K{}^2+2M_\pi{}^2-3M_D{}^2) \\ -\frac{\sqrt{2}}{\sqrt{3}}(M_K{}^2-M_\pi{}^2) & \frac{1}{3}(4M_K{}^2-M_\pi{}^2) & -\frac{\sqrt{2}}{3}(M_K{}^2-M_\pi{}^2) \\ \frac{1}{2\sqrt{3}}(M_K{}^2+2M_\pi{}^2-3M_D{}^2) & -\frac{\sqrt{2}}{3}(M_K{}^2-M_\pi{}^2) & \frac{1}{6}(9M_D{}^2+M_K{}^2-4M_\pi{}^2) \end{bmatrix}. \tag{3.9d}$$

This matrix is easily diagonalized as a function of μ^2.

Using $M_\pi=135$ MeV, $M_K=496$ MeV, and $M_D=1862$ MeV we, however, find that *no value* of μ^2 gives masses of η, η', and η_c that are close to the experimental values. All results being expressed in MeV, our results are

μ	M_η	M_{η_c}	$M_{\eta'}$
1909.2	549	2879	1569
1653.4	543	2802	1413
954.6	493	2677	953

Further, we still have $M_F-M_D=60$ MeV, which is far from the experimental value 180 MeV. Thus, we are forced to give up our assumption of vacuum being a SU(4) singlet. We next attempt a solution that admits nonvanishing δ_8 and δ_{15} though still preserve the SU(4) symmetry of Z's. The $\sqrt{Z}$ mixing matrix in Eq. (3.9c) is taken as

$$\sqrt{Z}=\sqrt{Z_\pi}\begin{pmatrix} -\sin\theta\cos\phi & \cos\theta & \sin\theta\sin\phi \\ -\sin\psi\sin\phi+\cos\theta\cos\phi\cos\psi & \sin\theta\cos\psi & -\sin\psi\cos\phi-\cos\theta\sin\phi\cos\psi \\ \cos\psi\sin\phi+\cos\theta\cos\phi\sin\psi & \sin\theta\sin\psi & \cos\psi\cos\phi-\cos\theta\sin\phi\sin\psi \end{pmatrix}. \tag{3.10}$$

The set of Eqs. (2.9), (2.11) can now be solved numerically on a computer. This approach, however, leads to a problem of mass reversal, i.e., M_D comes out greater than M_F. The source of the problem can be seen easily. From Eqs. (2.9) and (2.11) we have, since Z's are equal,

$$F_F - F_D = F_K - F_\pi ,$$
$$M_F{}^2F_F - M_D{}^2F_D = M_K{}^2F_K - M_\pi{}^2F_\pi . \tag{3.11}$$

Simplifying we obtain

$$M_F{}^2 - M_D{}^2 \simeq \frac{F_K}{F_D}\left[M_K{}^2 - M_F{}^2\left(\frac{F_K-1}{F_K}\right)\right]. \tag{3.12}$$

We then see that for $F_K \simeq 1.28$ the right-hand side is negative provided F_D has the same sign as F_K. This is expected from SU(4) symmetry and also found to be true in the numerical solution. This fact actually leads to the erroneous prediction $m_F < m_D$ by Ueda[11] and Vaughn,[11] who analyzed the linear σ model which corresponds to the assumption of $\sqrt{Z_i}$'s being equal.

We are forced, thus, to abandon the assumption of equality of $\sqrt{Z}$'s.[12] Nevertheless, from SU(3) $\times$SU(3) solution we know that the equality for $\sqrt{Z}$'s among SU(3) members is a good assumption. Thus we can retain the restrictive assumption $\sqrt{Z_\pi}=\sqrt{Z_K} = \sqrt{Z_8}$ and $\sqrt{Z_D}=\sqrt{Z_F}$. The simplest assumption to make now is that the vacuum is a SU(4) singlet, i.e., $\delta_8=\delta_{15}=0$. We can assume that $\sqrt{Z}$'s are SU(4)-broken along the the 15 direction, i.e.,

$$\langle 0|v_\alpha|P_\beta\rangle = c_1\delta_{\alpha\beta} + c_2\delta_{\alpha 0}\delta_{\beta 0} + c_3 d_{15\alpha\beta} , \tag{3.13}$$

where $|P_\beta\rangle$ are pure SU(4) states. The states, η, η', η_c are taken as linear combinations of $|P_0\rangle$, $|P_8\rangle$, and $|P_{15}\rangle$. The Eqs. (2.9) and (2.11) can now be solved on a computer, and we again find no acceptable solution. We may also note that Kandaswamy, Schechter, and Singer[4] have assumed instead

$$\frac{F_\pi}{\sqrt{Z_\pi}} = \frac{F_K}{\sqrt{Z_K}} = \frac{F_D}{\sqrt{Z_D}} = \frac{F_F}{\sqrt{Z_F}} . \tag{3.14}$$

Since this implies

$$M_F{}^2 - M_D{}^2 = M_K{}^2 - M_\pi{}^2 , \tag{3.15}$$

which is badly violated, this solution also seems unacceptable. We thus are forced to consider the very general case of symmetry breaking in Z's as well as in the vacuum. In the next section we shall see the restriction that emerges from purely D and F masses on the nature of symmetry breaking.

IV. CONSTRAINTS FROM D AND F MESON MASSES

In this section we obtain powerful constraints on the solution to Eqs. (2.9) and (2.11) that arise purely from our knowledge of D and F meson masses and the weak assumptions that the wave-function renormalization constants $\sqrt{Z}$'s are SU(3) symmetric. The latter is verified to a good extent from previous work on SU(3)$\times$SU(3) breaking. Consider the subset of Eqs. (2.9) and (2.11) which arise from π, K, D, and F meson pole saturation. We set $\sqrt{Z_\pi}=\sqrt{Z_K}$ and $\sqrt{Z_D}=\sqrt{Z_F}$.

We prefer to write these equations in terms of the masses of quarks and quark expectation values. Relations between ϵ_i and masses are given in Eqs. (2.2) and the vacuum expectation values δ_i are related to $\langle\bar{q}q\rangle$ by

$$\delta_i \equiv \langle \bar{q}\lambda_i q\rangle ,$$

$$\begin{aligned}
-\frac{2M_\pi{}^2F_\pi}{\sqrt{Z_\pi}} &= m_u + m_d = 2m_u ,\\
-\frac{2M_K{}^2F_K}{\sqrt{Z_\pi}} &= m_u + m_s ,\\
-\frac{2M_D{}^2F_D}{\sqrt{Z_D}} &= m_u + m_c ,\\
-\frac{2M_F{}^2F_F}{\sqrt{Z_D}} &= m_s + m_c ,\\
F_\pi\sqrt{Z_\pi} &= \langle\bar{u}u\rangle + \langle\bar{d}d\rangle = 2\langle\bar{u}u\rangle ,\\
F_K\sqrt{Z_\pi} &= \langle\bar{u}u\rangle + \langle\bar{s}s\rangle ,\\
F_D\sqrt{Z_D} &= \langle\bar{c}c\rangle + \langle\bar{u}u\rangle ,\\
F_F\sqrt{Z_D} &= \langle\bar{c}c\rangle + \langle\bar{s}s\rangle .
\end{aligned} \tag{4.1}$$

Note that all quark masses on the right-hand side are the bare masses or so-called current-algebra quark masses. This follows from the fact that the divergence of the axial-vector current is proportional to the bare masses in a Lagrangian theory.

We assume the masses of mesons $M_\pi = 135$ MeV, $M_K = 496$ MeV, $M_D = 1862$ MeV, $M_F = 2039.5$ MeV, and the ratio $F_K/F_\pi = 1.28$. The following relations can now be easily derived:

$$\begin{aligned}
\frac{m_F{}^2}{m_D{}^2} &= \frac{\left(\frac{m_c}{m_s}+1\right)\left(\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}+1\right)}{\left(\frac{m_c}{m_s}+\frac{m_u}{m_s}\right)\left(\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}+\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}\right)} ,\\
\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle} &= 2\frac{F_K}{F_\pi} - 1 = 1.56 ,\\
\frac{m_s}{m_u} &= 2\frac{M_K{}^2F_K}{F_\pi{}^2F_\pi} - 1 \simeq 33.4 .
\end{aligned} \tag{4.2}$$

Thus to a good approximation

$$\frac{M_F{}^2}{M_D{}^2} = \frac{\left(\frac{m_c}{m_s}+1\right)\left(\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}+1\right)}{\frac{m_c}{m_s}\left(\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}+1.56\right)} \simeq 1.2 . \tag{4.3}$$

Note that this relation is insensitive to assumed equality of m_u and m_d.

We plot the ratio m_c/m_s as a function $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ in Fig. 1. Since $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle$ is positive, we expect $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ to be positive and large as the symmetry breaking arises from large m_c. As $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle \to \infty$ we observe $m_c/m_s \to 5$. Thus for all physically meaningful values of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ we deduce the condition

$$m_c/m_s < 5\,. \tag{4.4}$$

This conclusion is very different from the result that follows from the quadratic mass formula[6] that yields $m_c/m_s = 20.7$, or the linear mass formula which gives $m_c/m_s = 9.5$. Some support for a small value for m_c/m_s comes from consideration of the renormalization group in QCD where Georgi and Politzer[13] have deduced the value $m_c/m_s \simeq 4$. We can make further progress only after an estimate of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ or equivalently the ratio δ_{15}/δ_0.

A model for vacuum breaking which assumes a linear relation

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 + m_q A \tag{4.5}$$

would yield

$$\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle} = 1 + \frac{m_c}{m_s}\left(\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle} - 1\right) = 1 + (0.56)\frac{m_c}{m_s}\,. \tag{4.6}$$

The solution to Eqs. (4.3) and (4.6) lead to $m_c/m_s \simeq 2.4$ and $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle = 2.3$. However, this value for m_c/m_s seems rather low, and linear breaking cannot be justified. In the next section we shall

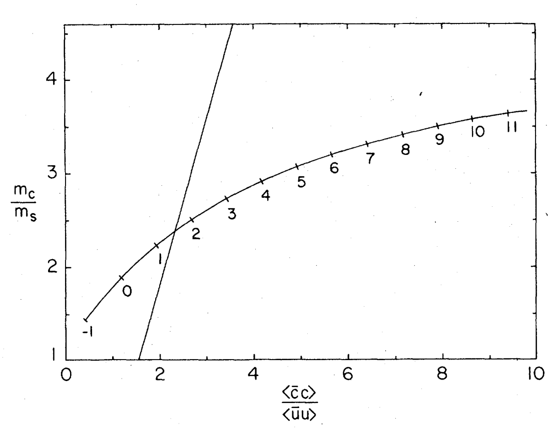

FIG. 1. The curve represents the variation of m_c/m_s as a function of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ which is obtained by using the masses of D and F mesons as input. The numbers on the curve are the values of the parameter K defined to be $\delta_{15}/\sqrt{2}\,\delta_8$. The straight line represents linear breaking for the vacuum expectation values.

consider the remaining equations involving η, η', η_c mixing and solve for $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$. Our conclusion is that $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle \gtrsim 5.6$. This leads to

$$m_c/m_s \gtrsim 3.2\,. \tag{4.7}$$

V. GENERAL SOLUTION

In this section we obtain a phenomenological solution of the Eqs. (2.9) and (2.11) by considering the equations involving η, η', and η_c mixing in addition to constraints obtained in the last section.

Reviewing, we find that the equations resulting from considerations of π, K, D, and F mesons have yielded a considerable amount of information. They involve six equations with nine unknowns, and we obtain values of the six symmetry-breaking parameters, ϵ_0, ϵ_8, ϵ_{15}, δ_0, δ_8, and δ_{15} and the decay constants F_D and F_F if we know one unknown which can be chosen to be $K \equiv \delta_{15}/\sqrt{2}\,\delta_8$. The value of K in terms of $\langle\bar{c}c\rangle/\langle\bar{u}u\rangle$ can be written as

$$K \equiv \frac{\delta_{15}}{\sqrt{2}\,\delta_8} = \frac{3}{4}\,\frac{[\langle\bar{c}c\rangle/2\langle\bar{u}u\rangle - \frac{1}{2} - \frac{1}{3}(F_K - 1)]}{[\langle\bar{s}s\rangle/2\langle\bar{u}u\rangle - \frac{1}{2}]} = 2.6786[\langle\bar{c}c\rangle/2\langle\bar{u}u\rangle - 0.593]\,, \tag{5.1}$$

where we expect K to be a large and positive number.

We now turn to the η mixing Eqs. (2.9) and (2.11) and obtain solutions as a function of K. We shall see that not all values of K are allowed.

Consider Eq. (2.9). We can eliminate F's which are involved linearly in favor of $\sqrt{Z}$'s. It is useful here to define new variables $x_1, x_2, x_3, \ldots, x_6$:

$$\begin{aligned}
x_1 &= \frac{Z^0_\eta}{M_\eta^{\,2}} + \frac{Z^0_{\eta'}}{M_{\eta'}^{\,2}} + \frac{Z^0_{\eta_c}}{M_{\eta_c}^{\,2}}\,,\\
x_2 &= \frac{Z^8_\eta}{M_\eta^{\,2}} + \frac{Z^8_{\eta'}}{M_{\eta'}^{\,2}} + \frac{Z^8_{\eta_c}}{M_{\eta_c}^{\,2}}\,,\\
x_3 &= \frac{Z^{15}_\eta}{M_\eta^{\,2}} + \frac{Z^{15}_{\eta'}}{M_{\eta'}^{\,2}} + \frac{Z^{15}_{\eta_c}}{M_{\eta_c}^{\,2}}\,,\\
x_4 &= \frac{(Z^0_\eta Z^8_\eta)^{1/2}}{M_\eta^{\,2}} + \frac{(Z^0_{\eta'} Z^8_{\eta'})^{1/2}}{M_{\eta'}^{\,2}} + \frac{(Z^0_{\eta_c} Z^8_{\eta_c})^{1/2}}{M_{\eta_c}^{\,2}}\,,\\
x_5 &= \frac{(Z^0_\eta Z^{15}_\eta)^{1/2}}{M_\eta^{\,2}} + \frac{(Z^0_{\eta'} Z^{15}_{\eta'})^{1/2}}{M_{\eta'}^{\,2}} + \frac{(Z^0_{\eta_c} Z^{15}_{\eta_c})^{1/2}}{M_{\eta_c}^{\,2}}\,,\\
x_6 &= \frac{(Z^8_\eta Z^{15}_\eta)^{1/2}}{M_\eta^{\,2}} + \frac{(Z^8_{\eta'} Z^{15}_{\eta'})^{1/2}}{M_{\eta'}^{\,2}} + \frac{(Z^8_{\eta_c} Z^{15}_{\eta_c})^{1/2}}{M_{\eta_c}^{\,2}}\,.
\end{aligned} \tag{5.2}$$

The new equations which take the place of η-mixing equations in Eq. (2.11) are then written in a matrix form as

$$\begin{bmatrix} 0 & \frac{\epsilon_0}{\sqrt{2}}-\frac{\epsilon_8}{\sqrt{3}}+\frac{\epsilon_{15}}{\sqrt{6}} & 0 & \frac{\epsilon_8}{\sqrt{2}} & 0 & \frac{\epsilon_8}{\sqrt{6}} \\ \frac{\epsilon_8}{\sqrt{2}} & 0 & 0 & \frac{\epsilon_0}{\sqrt{2}}-\frac{\epsilon_8}{\sqrt{3}}+\frac{\epsilon_{15}}{\sqrt{6}} & \frac{\epsilon_8}{\sqrt{6}} & 0 \\ 0 & 0 & \frac{\epsilon_8}{\sqrt{6}} & 0 & \frac{\epsilon_8}{\sqrt{2}} & \frac{\epsilon_0}{\sqrt{2}}-\frac{\epsilon_8}{\sqrt{3}}+\frac{\epsilon_{15}}{\sqrt{6}} \\ \frac{\epsilon_{15}}{\sqrt{2}} & 0 & 0 & \frac{\epsilon_8}{\sqrt{6}} & \frac{\epsilon_0}{\sqrt{2}}-2\frac{\epsilon_{15}}{\sqrt{6}} & 0 \\ 0 & \frac{\epsilon_8}{\sqrt{6}} & 0 & \frac{\epsilon_{15}}{\sqrt{2}} & 0 & \frac{\epsilon_0}{\sqrt{2}}-2\frac{\epsilon_{15}}{\sqrt{6}} \\ 0 & 0 & \frac{\epsilon_0}{\sqrt{2}}-2\frac{\epsilon_{15}}{\sqrt{6}} & 0 & \frac{\epsilon_{15}}{\sqrt{2}} & \frac{\epsilon_8}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \frac{\delta_0}{\sqrt{2}}-\frac{\delta_8}{\sqrt{3}}+\frac{\delta_{15}}{\sqrt{6}} \\ \frac{\delta_8}{\sqrt{2}} \\ \frac{\delta_8}{\sqrt{6}} \\ \frac{\delta_{15}}{\sqrt{2}} \\ \frac{\delta_8}{\sqrt{6}} \\ \frac{\delta_0}{\sqrt{2}}-2\frac{\delta_{15}}{\sqrt{6}} \end{bmatrix} . \tag{5.3}$$

An examination of the ϵ matrix reveals that the determinant of the matrix is zero, and actually only five of the six equations are linearly independent. So, it is possible to find the values of the five of the six x's as a function of one of them (chosen as x_2) and δ's, which are known for any given value of K.

Now, since

$$0 \le Z^8_{\eta_i} \le Z_\pi \text{ and } \sum_i Z^8_{\eta_i} = Z^8 = Z_\pi$$
$$(\eta_i = \eta,\ \eta',\ \text{or } \eta_c), \tag{5.4}$$

the bounds on x_2/Z_π are known to be

$$\frac{1}{M_{\eta_c}{}^2} \le \frac{x_2}{Z_\pi} \le \frac{1}{M_\eta{}^2}. \tag{5.5}$$

Further, x_2/Z_π must be close to $1/M_\eta{}^2$ because η is known to be nearly an octet.

Once the x's are known we have six equations [Eq. (5.2)] for the nine Z's. There is one constraint that $\sum_{\eta_i} Z^8_{\eta_i} = Z_\pi$. So we need to postulate two more reasonable constraints to solve for the individual Z's. Although there exist a lot of choices to select two such constraints, we, here, investigate the one that seems the most reasonable. We demand that η and η' do not contain any charmed quarks. Since $\bar{c}c \propto \bar{q}(\lambda_0 - \sqrt{3}\lambda_{15})q$, this requirement leads to

$$\langle 0 | (v_0 - \sqrt{3} v_{15}) | \eta\rangle = 0 \quad \text{or} \quad \sqrt{Z^0_\eta} = \sqrt{3}\sqrt{Z^{15}_\eta}$$

and

$$\langle 0 | (v_0 - \sqrt{3} v_{15}) | \eta'\rangle = 0 \quad \text{or} \quad \sqrt{Z^0_{\eta'}} = \sqrt{3}\sqrt{Z^{15}_{\eta'}}. \tag{5.6}$$

Then the Z's and from them the F's are obtained as functions of $K = \delta_{15}/\sqrt{2}\delta_8$ and x_2. The solutions are found numerically by choosing particular values of K and letting x_2/Z_π vary near $1/M_\eta{}^2$. It was found that the set of equations yield consistent physical solutions only for a very narrow range of x_2. Besides, the solutions do not vary much in this range. This practically makes the whole set of solutions depend only on the value of K. The least value of K for which solutions were found is around 6. We present a table (Table I) to show the variation of the solutions as a function of K. $K=6$ implies a large SU(4) vacuum breaking ($\delta_{15} < 0$; $\delta_{15} \simeq 8.5\,\delta_8$) compared to the almost symmetric vacuum found in broken-chiral-SU(3)×SU(3) models.

Although we cannot determine the value of K from these equations, we feel that $K = 6-7$ represents a reasonable solution. Larger values of K would mean extremely large SU(4) breaking in the vacuum which would not be reasonable in view of the validity of approximate SU(4) classification of states. We list below the various symmetry-breaking parameters, as well as the decay constants that emerge from our solution (complete solution can be found in Appendix A):

$$K=6,\quad \frac{\delta_{15}}{\delta_8} \simeq 8.5,\quad \frac{m_c}{m_s} \simeq 3.2,$$

$$\frac{\langle \bar{c}c\rangle}{\langle \bar{u}u\rangle} \simeq 5.67,\quad \frac{\langle \bar{s}s\rangle}{\langle \bar{u}u\rangle} \simeq 1.56,\quad \frac{m_s}{m_u} \simeq 33.5,$$

$$\frac{F_D}{F_\pi} \simeq 0.974,\quad \frac{F_F}{F_\pi} \simeq 1.056,\quad \frac{F_K}{F_\pi} \simeq 1.28.$$

We notice that although there is large SU(4) breaking both in the Hamiltonian and in the vacuum, F's retain their approximate SU(4) symmetry. This prediction can be tested experimentally by direct measurements of F_D and F_F.

From our solutions we can also obtain the expectation value of the operator V between vacuum

TABLE I. Symmetry-breaking parameters as a function of $K \equiv \delta_{15}/\sqrt{2}\,\delta_8$.

$K=\frac{\delta_{15}}{\sqrt{2}\delta_8}$	F_D/F_π	F_F/F_π	$\frac{\sqrt{Z_D}}{\sqrt{Z_\pi}}$	$\frac{m_s}{m_u}$	$\frac{m_c}{m_s}$	$\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}$	$\frac{\langle\bar{c}c\rangle}{\langle\bar{u}u\rangle}$
0	0.430	0.540	2.545		1.884		1.186
1	0.541	0.644	2.711		2.233		1.933
2	0.642	0.793	2.867		2.508		2.679
3	0.734	0.827	3.015		2.730		3.426
4	0.819	0.908	3.157		2.914		4.173
5	0.899	0.984	3.292		3.068		4.919
6	0.974	1.056	3.421	33.5	3.199	1.56	5.666
7	1.045	1.124	3.547		3.311		6.413
8	1.113	1.189	3.667		3.410		7.159
9	1.177	1.251	3.784		3.496		7.906
10	1.238	1.310	3.898		3.572		8.653
11	1.297	1.367	4.008		3.640		9.399
12	1.354	1.422	4.115		3.701		10.146

and η, η', or η_c states. From Eq. (2.9) we find, in GeV^3

$$g_\eta = 0.0231\,,$$
$$g_{\eta'} = -0.6191\,,$$
$$g_{\eta_c} = 0.0014\,,$$

Since these are expectation values of SU(4) singlet operator which arises from QCD effects, it may be possible to verify them from direct calculation in the future. Here we observe that the contribution from η and η_c are small because these states are not predominately singlets, while η' is large, as expected.

VI. RESULTS

We have found a good phenomenological solution for the broken-chiral-SU(4) ×SU(4) model that incorporates the masses of the charmed pseudoscalar mesons D and F and η_c exactly. The values for the symmetry-breaking parameters reveal that the vacuum is not a SU(4) singlet and a large value for the ratio of the vacuum expectations of the scalar densities, u_{15} to u_8 was observed. Further, the renormalization constants $\sqrt{Z_i}$'s for the pseudoscalar meson wave functions are found not to be SU(4) symmetric, although the SU(3) symmetry is preserved.

From the observed D and F meson masses we reached a strong constraint on the mass ratio m_c/m_s of the 'current' quarks $m_c/m_s < 5$. With two more plausible assumptions, namely that η and η' do not contain any charm quarks, we obtain $m_c/m_s \gtrsim 3.2$, $F_D \gtrsim 0.974\,F_\pi$, and $F_F \gtrsim 1.056\,F_\pi$. This value of m_c/m_s comes very close to the value Georgi and Politzer[13] found from renormalization-group considerations in QCD. This value differs sharply from the linear or quadratic mass fitting for SU(4) multiplets, both of which give much larger values for this ratio.

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APPENDIX A

In Secs. IV and V we have discussed how the solution to Eqs. (2.9) and (2.11) is obtained. Our inputs were the masses $M_\pi = 135$ MeV, $M_K = 496$ MeV, $M_D = 1.862$ GeV, $M_F = 2.0395$ GeV, $M_\eta = 549$ MeV, $M_{\eta'} = 958$ MeV, $M_{\eta_c} = 2.83$ GeV; the decay constants of π and K mesons are in units of F_π, $F_\pi = 1$, $F_K = 1.28$. Here we list the complete set of parameters for our solution with $K \equiv \delta_{15}/\sqrt{2}\,\delta_8 = 6$ in units of $\sqrt{Z_\pi}$, F_π, and M_π:

$$\epsilon_0 = 50.52,\quad \epsilon_8 = -18.80,\quad \epsilon_{15} = -58.47,$$
$$\delta_0 = 3.262,\quad \delta_8 = -0.3233,\quad \delta_{15} = -2.4743,$$
$$\langle\bar{s}s\rangle/\langle\bar{u}u\rangle = 1.56,\quad \langle\bar{c}c\rangle/\langle\bar{u}u\rangle = 5.67,$$
$$m_s/m_u = 33.5,\quad m_c/m_s = 3.2;$$

$F_\pi = 1$

$F_K = 1.28, \quad F_D = 0.974, \quad F_F = 1.056,$

$F^8_\eta = 1.456, \quad F^8_{\eta'} = 0.0611, \quad F^8_{\eta_c} = 0,$

$F^{15}_\eta = -0.4998, \quad F^{15}_{\eta'} = -0.0233, \quad F^{15}_{\eta_c} = -1.019$

$F^0_\eta = -0.0346, \quad F^0_{\eta'} = -7.369, \quad F^0_{\eta_c} = 0.5884;$

$\sqrt{Z_\pi} = 1$

$\sqrt{Z_K} = 1, \quad \sqrt{Z_D} = 3.42, \quad \sqrt{Z_F} = 3.42,$

$\sqrt{Z^0_\eta} = -0.1507, \quad \sqrt{Z^0_{\eta'}} = -0.1542, \quad \sqrt{Z^0_{\eta_c}} = 1.981,$

$\sqrt{Z^8_\eta} = 0.943, \quad \sqrt{Z^8_{\eta'}} = 0.016, \quad \sqrt{Z^8_{\eta_c}} = -0.333,$

$\sqrt{Z^{15}_\eta} = -0.087, \quad \sqrt{Z^{15}_{\eta'}} = -0.089, \quad \sqrt{Z^{15}_{\eta_c}} = -4.416,$

$g_\eta = 13.748, \quad g_{\eta'} = -369.26, \quad g_{\eta_c} = 0.8106.$

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