A quantitative measure of the SU(4) × SU(4) breaking, SU(2) mass splitting of the charmed hadrons, and quark masses

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We investigate in a chiral SU(4) × SU(4) model the symmetry-breaking parameters, decay constants, D^+-D^0 mass difference, quark masses, and electromagnetic mass splitting of the charmed baryons by assuming (i) that matrix elements of the type $\langle 0|v_P|P\rangle$ are SU(4) invariant and alternatively (ii) that these are dominated by the quark mass term. The two approaches are compared in this work and conclusions presented. We also mention possible extensions to the chiral SU(5) × SU(5) scheme and discuss some of the associated problems.

I. INTRODUCTION

Long before the discovery of the charmed particles, the possibility of chiral SU(4) × SU(4) as an approximate symmetry of hadrons was explored by several authors.[1] The theoretical motivations behind such a study were the possibility of incorporating a quark-lepton symmetry by regarding the four basic quarks as a fundamental representation of SU(4) and shedding light on the existing ideas on SU(3) and other lower symmetries of strong interactions. Also, cosmic-ray experiments had revealed the existence of a new particle[2] with a rather large mass of about 1.78 GeV which could not be accomodated in a chiral SU(3) × SU(3) scheme.

The first authors to propose SU(4) × SU(4) as a symmetry group of strong interactions[1] used the postulated fourth quark in addition to the usual, u, d, and s and restricted the vacuum to be only SU(3) invariant in the chiral limit to derive two sum rules which were in good agreement with experimental meson masses and obtained a set of solutions for the symmetry-breaking parameters. In a similar scheme, the consequences of a broken-SU(4) × SU(4) were also investigated by Prasad.[3] He showed that the positivity conditions on the spectral functions of scalar and pseudoscalar densities contradicted some of the results of the previous authors and pointed out that SU(2) × SU(2) cannot be a good symmetry of the Hamiltonian if the vacuum is approximately SU(3) invariant.

With the detection of the charmed particles D and F, it is now generally believed that chiral SU(4) × SU(4) underlies the structure of the hadrons.[4] Some of the recent work on it includes a generalization[5] of the Gell-Mann, Oakes and Renner (GMOR) model[6] and a study of the symmetry-breaking parameters by two of the present authors.[4]

In this paper, we have considered chiral SU(4) × SU(4) in detail and have found that a precise knowledge of the matrix elements of the type $\langle O|v_P|P\rangle$, where P represents a pseudoscalar meson, is essential in order to understand how chiral symmetry is broken. A naive assumption that all $\langle O|v_P|P\rangle$ are equal does not take us far except for yielding the Gell-Mann–Okubo mass formula, and consistent results can be obtained only when one assumes that the quark mass term dominates this matrix element. We briefly review the basic theoretical framework in Sec. II and estimate the chiral SU(4) × SU(4) symmetry-breaking parameters in Sec. III from the standard approach of the GMOR model as well as the quark-mass-term-domination scheme alluded to above. As a consequence of this exercise, we calculate the D^+-D^0 mass difference in Sec. IV and follow it up in the next section with an estimation of the quark masses and electromagnetic mass splitting of the charmed baryons. Our conclusions are presented in Sec. VI wherein we also briefly mention possible extensions to the SU(5) × SU(5) scheme.

II. THEORY

The symmetry-breaking Hamiltonian density, by the simplest generalization of the GMOR model, can be taken as

$$H = u_0 + p u_8 + t u_{15}, \tag{1}$$

where p and t are parameters which break the symmetry and u_i together with v_i are a pair of scalar and pseudoscalar densities belonging to the $(4, \bar{4}) + (\bar{4}, 4)$ representation of chiral SU(4) × SU(4) which satisfy the following commutation relations:

$$\begin{aligned} [Q_i, u_j] &= i f_{ijk} u_k, \\ [Q_i, v_j] &= i f_{ijk} v_k, \\ [Q_i^5, u_j] &= -i d_{ijk} v_k, \end{aligned} \tag{2}$$

and

$$[Q_i^5, v_j] = i d_{ijk} u_k,$$

here Q_i and Q_i^5 are the vector and the axial-vector charges, respectively, and $i = 1, \ldots 15$ and $j = 0, 1, \ldots 15$.

The vector and the axial-vector current divergences are then given by

$$\begin{aligned} \partial_\mu V_i^\mu(x) &= (p f_{i8j} + t f_{i15j}) u^j(x)\,, \\ \partial_\mu A_i^\mu(x) &= -(d_{i0j} + p d_{i8j} + t d_{i15j}) v^j(x)\,, \end{aligned} \qquad (3)$$

where the structure constants f_{ijk} and d_{ijk} are listed in Table I and the parameters p and t are re-

TABLE I. Values of the SU(4) structure constants.

i	j	k	f_{ijk}	i	j	k	d_{ijk}
1	2	3	1	1	1	8	$1/\sqrt{3}$
1	4	7	1/2	1	1	15	$1/\sqrt{6}$
1	5	6	−1/2	1	4	6	1/2
1	9	12	1/2	1	5	7	1/2
1	10	11	−1/2	1	9	11	1/2
2	4	6	1/2	1	10	12	1/2
2	5	7	1/2	2	2	8	$1/\sqrt{3}$
2	9	11	1/2	2	2	15	$1/\sqrt{6}$
2	10	12	1/2	2	4	7	−1/2
3	4	5	1/2	2	5	6	1/2
3	6	7	−1/2	2	9	12	−1/2
3	9	10	1/2	2	10	11	1/2
3	11	12	−1/2	3	3	8	$1/\sqrt{3}$
4	5	8	$\sqrt{3}/2$	3	3	15	$1/\sqrt{6}$
4	9	14	1/2	3	4	4	1/2
4	10	13	−1/2	3	5	5	1/2
5	9	13	1/2	3	6	6	−1/2
5	10	14	1/2	3	7	7	−1/2
6	7	8	$\sqrt{3}/2$	3	9	9	1/2
6	11	14	1/2	3	10	10	1/2
6	12	13	−1/2	3	11	11	−1/2
7	11	13	1/2	3	12	12	−1/2
7	12	14	1/2	4	4	8	$-1/2\sqrt{3}$
8	9	10	$1/2\sqrt{3}$	4	4	15	$1/\sqrt{6}$
8	11	12	$1/2\sqrt{3}$	4	9	13	1/2
8	13	14	$-1/\sqrt{3}$	4	10	14	1/2
9	10	15	$\sqrt{2/3}$	5	5	8	$-1/2\sqrt{3}$
11	12	15	$\sqrt{2/3}$	5	5	15	$1/\sqrt{6}$
13	14	15	$\sqrt{2/3}$	5	9	14	−1/2
i	j	0	0	5	10	13	1/2
				6	6	8	$-1/2\sqrt{3}$
				6	6	15	$1/\sqrt{6}$
				6	11	13	1/2
				6	12	14	1/2
				7	7	8	$-1/2\sqrt{3}$
				7	7	15	$1/\sqrt{6}$
				7	11	14	−1/2
				7	12	13	1/2
				8	8	8	$-1/\sqrt{3}$
				8	8	15	$1/\sqrt{6}$
				8	9	9	$1/2\sqrt{3}$
				8	10	10	$1/2\sqrt{3}$
				8	11	11	$1/2\sqrt{3}$
				8	12	12	$1/2\sqrt{3}$
				8	13	13	$-1/\sqrt{3}$
				8	14	14	$-1/\sqrt{3}$
				9	9	15	$-1/\sqrt{6}$
				10	10	15	$-1/\sqrt{6}$
				11	11	15	$-1/\sqrt{6}$
				12	12	15	$-1/\sqrt{6}$
				13	13	15	$-1/\sqrt{6}$
				14	14	15	$-1/\sqrt{6}$
				15	15	15	$-\sqrt{2/3}$
				i	j	0	$1/\sqrt{2}\,\epsilon_{ij}$

lated to the quark masses as

$$p=\left(\frac{2}{3}\right)^{1/2}\frac{m_u+m_d-2m_s}{m_u+m_d+m_s+m_c}$$

and (4)

$$t=\frac{1}{\sqrt{3}}\,\frac{m_u+m_d+m_s-3m_c}{m_u+m_d+m_s+m_c}\ .$$

Using the PCAC (partial conservation of axial-vector current) relation

$$\partial_\mu A_i^\mu=f_iM_i^2\phi_i\ , \tag{5}$$

where f_i and M_i are the decay constant and mass of a pseudoscalar meson P_i, the vacuum-to-one-meson matrix elements of the axial-vector current divergences may be expressed in the low-energy limit as

$$\langle 0|\partial_\mu A_i^\mu|P_i\rangle=f_iM_i^2\ . \tag{6}$$

One can then combine Eqs. (3) with (6) to obtain the following relations between the symmetry-breaking parameters, the masses, and the decay constants of the pseudoscalar mesons π, K, D, and F:

$$\begin{aligned}
M_\pi^2&=-\frac{1}{f_\pi}\left(\frac{1}{\sqrt{2}}+\frac{p}{\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\langle 0|v_\pi|\pi\rangle\ ,\\
M_K^2&=-\frac{1}{f_K}\left(\frac{1}{\sqrt{2}}-\frac{p}{2\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\langle 0|v_K|K\rangle\ ,\\
M_D^2&=-\frac{1}{f_D}\left(\frac{1}{\sqrt{2}}+\frac{p}{2\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\langle 0|v_D|D\rangle\ ,
\end{aligned} \tag{7}$$

and

$$M_F^2=-\frac{1}{f_F}\left(\frac{1}{\sqrt{2}}-\frac{p}{\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\langle 0|v_F|F\rangle\ .$$

This set of equations forms the basis for determining the symmetry-breaking parameters.

III. SYMMETRY-BREAKING PARAMETERS

As evident from Eqs. (7), one needs to know the transformation properties of the matrix elements $\langle 0|v_P|P\rangle$ in order to estimate p and t.

Let us first reduce the pseudoscalar P occurring in $\langle 0|v_P|P\rangle$ for $P=\pi$, K, D, and F. We obtain

$$\begin{aligned}
M_\pi^2&=-\alpha_\pi\frac{\langle 0|u_0|0\rangle}{f_\pi^2}\left(\frac{1}{\sqrt{2}}+\frac{p}{\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{2}}+\frac{p'}{\sqrt{3}}+\frac{t'}{\sqrt{6}}\right)\ ,\\
M_K^2&=-\alpha_K\frac{\langle 0|u_0|0\rangle}{f_K^2}\left(\frac{1}{\sqrt{2}}-\frac{p}{2\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{2}}-\frac{p'}{2\sqrt{3}}+\frac{t'}{\sqrt{6}}\right)\ ,\\
M_D^2&=-\alpha_D\frac{\langle 0|u_0|0\rangle}{f_D^2}\left(\frac{1}{\sqrt{2}}+\frac{p}{2\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{2}}+\frac{p'}{2\sqrt{3}}-\frac{t'}{\sqrt{6}}\right)\ ,
\end{aligned} \tag{8}$$

and

$$M_F^2=-\alpha_F\frac{\langle 0|u_0|0\rangle}{f_F^2}\left(\frac{1}{\sqrt{2}}-\frac{p}{\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{2}}-\frac{p'}{\sqrt{3}}-\frac{t'}{\sqrt{6}}\right)\ ,$$

where p' and t' are $p'=\langle 0|u_8|0\rangle/\langle 0|u_0|0\rangle$ and $t'=\langle 0|u_{15}|0\rangle/\langle 0|u_0|0\rangle$ and α_P ($P=\pi$, K, D, and F) are soft-meson correction factors.

Following GMOR, if one now assumes that all $\langle 0|v_P|P\rangle$ are equal—which implies $\alpha_\pi/f_\pi=\alpha_K/f_K=\alpha_D/f_D=\alpha_F/f_F$ and also SU(3) as well as SU(4) invariance of the vacuum (i.e., $c'=0$, $t'=0$), one gets the following relations[5] ($m\equiv m_u=m_d$):

$$\frac{m_s}{m}=2\frac{M_K^2}{M_\pi^2}-1\ , \tag{9}$$

$$\frac{m_c}{m}=2\frac{M_D^2}{M_\pi^2}-1\ , \tag{10}$$

and the sum rule

$$M_K^2-M_\pi^2=M_F^2-M_D^2\ .$$

Equation (9) then enables one to predict the quark-mass ratios

$$m:m_s:m_c=1:25:360\ , \tag{11}$$

and for the symmetry-breaking parameters

$$p=-0.101\text{ and }t=-1.571 \tag{12}$$

from Eqs. (4).

The above considerations lead to the rather large value of 25 for the ratio m_s/m thus reducing considerably the effectiveness of the ansatz that matrix elements of the type $\langle 0|v_P|P\rangle$ are equal. This is because such a large number was found to be inconsistent with several experimental findings at the level of chiral SU(3) × SU(3).[7] Moreover, the solutions obtained in Eq. (12) are ruled out by the positivity conditions on the spectral-function sum rules.[3]

One of the reasons for the inconsistent results is that the above discussed scheme does not distinguish current quarks from the constituent ones. This becomes clear if one considers the following three-point function:

$$F_{ijk}(t)=\langle P_i(p)|u_j|P_k(p')\rangle\,, \tag{13}$$

where $t=p'-p$ and takes[6]

$$F_{ijk}(t)=\alpha(t)\delta_{j0}\delta_{ik}+\beta(t)d_{ijk}\,. \tag{14}$$

This procedure immediately leads to the equality of the matrix elements $\langle 0|v_P|P\rangle$ for $P=\pi,K,D,F$, the consequences of which have not been discussed earlier. But the assumption that

$$\langle P_i|u_j|P_k\rangle=\alpha\delta_{j0}\delta_{ik}+\beta d_{ijk} \tag{15}$$

is valid only if the u_i's transform as 15-plet under SU(4) strong. In view of the distinction between the current quarks and the constituent quarks, clearly one would not get correct estimates for p and t which are, respectively, SU(3) and SU(4) "current" breaking parameters.

A way around this difficulty is to transform the u_i's and the v_i's to the constituent-quark basis by applying the Melosh transformation,[8]

$$U=\exp\left[\frac{1}{2}\int d^4x\,\delta(x^+)\phi^\dagger(x)\tan^{-1}\left(\frac{\vec{\gamma}\cdot\vec{\nabla}}{k}\right)\phi(x)\right]. \tag{16}$$

By making use of this transformation, Fuchs showed that one could write

$$\langle 0|v_p|P\rangle=K(m_a+m_b)\,, \tag{17}$$

where (a,b) is the quark content of the meson P, and K is a constant, if one ignored the transverse motion of the quarks.[9] Equation (17), which is based on what is commonly known as the quark-mass-term-domination (QMD) hypothesis, can also be derived if we reduce the pseudoscalar meson P occurring in $\langle 0|v_P|P\rangle$ as shown in Eqs. (8), then saturate the vacuum matrix elements such as $\langle\bar{u}u\rangle_0$ with single-quark intermediate states[10] and assume that $f_P\propto\alpha_P$ (for $p=\pi$, K, D, and F). By considering light cone algebra and using somewhat different techniques, the above relation (17) was also obtained by Sazdjian and Stern.[11]

Assuming therefore the validity of Eq. (17) we get from Eq. (7)

$$\begin{vmatrix} 1 & 1 & 1 & f_\pi M_\pi^2/(2m) \\ 1 & -\frac{1}{2} & 1 & f_K M_K^2/(m+m_s) \\ 1 & \frac{1}{2} & -1 & f_D M_D^2/(m+m_c) \\ 1 & -1 & -1 & f_F M_F^2/(m_s+m_c) \end{vmatrix}=0\,, \tag{18}$$

which leads to the following sum rule:

$$\frac{f_F M_F^2}{m_s+m_c}-\frac{f_D M_D^2}{m+m_c}=\frac{f_K M_K^2}{m+m_s}-\frac{f_\pi M_\pi^2}{2m}\,. \tag{19}$$

In addition, we also have the relations

$$\frac{m_s}{m}=2\left(\frac{f_K}{f_\pi}\right)^{1/2}\frac{M_K}{M_\pi}-1$$

and (20)

$$\frac{m-m_s}{m_s+m_c}=\left(\frac{f_D}{f_F}\right)^{1/2}\frac{M_D}{M_F}-1\,.$$

Using experimental numbers for f_π and f_K which indicate that $f_\pi\approx f_K$ (=f say) and assuming that f_D will not be much different from f_F (Ref. 12), i.e., $f_D\approx f_F$ (=f' say) we obtain for the quark masses,

$$m:m_s:m_c=1:6:57.4\,. \tag{21}$$

This gives from Eqs. (4),

$$p=-0.125 \text{ and } t=-1.450\,. \tag{22}$$

It is worthwhile to note here the following points: (a) We have not assumed SU(3) and SU(4) invariance of vacuum anywhere (in fact, by following the techniques developed in Ref. 10, it can be shown that $p=p'$ and $t=t'$). (b) The assumption that quark mass term dominates $\langle 0|v_P|P\rangle$ reduces considerably the value of m_s/m from the GMOR prediction of $m_s/m\approx 25$; and (c) $m_s/m\approx 6$ is consistent with other determinations of m_s/m—using baryon mass formulas, fixed poles, σ terms, Goldberger-Treiman discrepancies, etc.[7]

The sum rule (19) enables us to predict the ratio f'/f which turns out to be

$$f'/f=4.8\,. \tag{23}$$

This estimate, although quite large in comparison with Ueda's prediction[13] of $f_D/f_\pi\sim 1-1.9$ and $f_F/f_\pi\sim 1.36-2.26$, is closer to Singer's estimate[14]: $f_D/f_\pi=f_F/f_\pi=2.8$. We note in this connection that while many authors have predicted $f'\lesssim f$ (Ref. 15) other estimates are available in the literature where the opposite has been claimed.[16] In particular, a chain of independent inequalities $f_F\gtrsim f_D>f_K>f_\pi$ was derived recently by Dominguez[17] by incorporating corrections to Goldberger-Treiman relation in chiral SU(2) × SU(2), SU(3) × SU(3), and SU(4) × SU(4).

In addition to f and f', it is also possible to estimate the ratio of the decay constants f_η, $f_{\eta'}$, and f_{η_c} of the isoscalars η, η', and η_c. The isoscalars in terms of the basis states $|8\rangle$, $|15\rangle$, and $|0\rangle$ can be written as[18]

$$\begin{pmatrix}\eta'\\ \eta\\ \eta_c\end{pmatrix}=U\begin{pmatrix}8\\ 15\\ 0\end{pmatrix}, \tag{24}$$

where U is given by

$$U=\begin{pmatrix} \cos\epsilon\,\sin\theta+\sin\epsilon\,\cos\delta\,\cos\theta & -\sin\epsilon\,\sin\theta+\cos\epsilon\,\cos\delta\,\cos\theta & \sin\delta\,\cos\theta \\ -\cos\epsilon\,\cos\theta+\sin\epsilon\,\cos\delta\,\sin\theta & \sin\epsilon\,\cos\theta+\cos\epsilon\,\cos\delta\,\sin\theta & \sin\delta\,\sin\theta \\ -\sin\epsilon\,\sin\delta & -\cos\epsilon\,\sin\delta & \cos\delta \end{pmatrix} \tag{25}$$

and $\epsilon=0°$, $\delta=60°$, and $\theta=\tan^{-1}(1/\sqrt{2})$ correspond to an ideal mixing.

If one now assumes that η_c is a pure $c\bar{c}$ state and that there is no mixing between η' and η as a first approximation, Eqs. (17), (21), (24), and (25) give

$$f_{\eta_c}/f_\eta=\frac{\sqrt{3}}{2}\frac{m_\eta{}^2}{m_{\eta_c}{}^2}\frac{m_c{}^2}{m_s{}^2-m^2}\approx 3.0\,, \tag{26}$$

$$f_{\eta_c}/f_{\eta'}=\left(\frac{3}{2}\right)^{1/2}\frac{m_{\eta'}{}^2}{m_{\eta_c}{}^2}\frac{m_c{}^2}{m_s{}^2+2m^2}\approx 11.8\,. \tag{27}$$

IV. D^+-D^0 MASS DIFFERENCE

Electromagnetic mass splitting of the charmed pseudoscalar mesons D^+ and D^0 has attracted considerable attention and several estimates exist, all ranging between 4 and 15 MeV.[19-24] While numbers close to the experimental value have been obtained by using Dashen's theorem[19] or by considering the MIT bag model,[20] a crude nonrelativistic assumption that the interaction distance between the quark is same for π, K, and D mesons has led to a rather unusually large number.[21] Other methods to estimate the mass difference include a generalization of Dashen's relation for K^+-K^0 mass difference together with strong PCAC (Ref. 22) and a chiral $SU(4)\times SU(4)$ scheme with broken isospin symmetry by assuming the so-called "smoothness condition".[23] These estimates, however, have turned out to be significantly smaller than the experimental findings.

By including an SU(2)-breaking term, one can write the symmetry-breaking Hamiltonian density as

$$H=u_0+ru_3+pu_8+tu_{15}\,, \tag{28}$$

where in addition to p and t, r is also a parameter but breaks isospin symmetry only.

It is now straightforward to get the following relations:

$$\begin{aligned} f_\pi M_\pi{}^2 &= -\left(\frac{1}{\sqrt{2}}+\frac{p}{\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\langle 0|v_\pi|\pi\rangle\,,\\ f_{K^+}M_{K^+} &= -\left(\frac{1}{\sqrt{2}}+\frac{r}{2}-\frac{p}{2\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\langle 0|v_{K^+}|K^+\rangle\,,\\ f_{K^0}M_{K^0}{}^2 &= -\left(\frac{1}{\sqrt{2}}-\frac{r}{2}-\frac{p}{2\sqrt{3}}+\frac{t}{\sqrt{6}}\right)\langle 0|v_{K^0}|K^0\rangle\,,\\ f_{D^0}M_{D^0}{}^2 &= -\left(\frac{1}{\sqrt{2}}+\frac{r}{2}+\frac{p}{2\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\langle 0|v_{D^0}|D^0\rangle\,,\\ f_{D^+}M_{D^+}{}^2 &= -\left(\frac{1}{\sqrt{2}}-\frac{r}{2}+\frac{p}{2\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\langle 0|v_{D^+}|D^+\rangle\,, \end{aligned} \tag{29}$$

and

$$f_F M_F{}^2=-\left(\frac{1}{\sqrt{2}}-\frac{p}{\sqrt{3}}-\frac{t}{\sqrt{6}}\right)\langle 0|v_F|F\rangle\,.$$

It then follows from Eqs. (17) and (29) that

$$\begin{aligned} \frac{m_u+m_d}{m_d+m_s} &= \frac{M_\pi}{M_K}\left(\frac{f_\pi}{f_K}\right)^{1/2},\\ \frac{m_s+m_c}{m_u+m_d} &= \frac{M_F}{M_\pi}\left(\frac{f_F}{f_\pi}\right)^{1/2}, \end{aligned} \tag{30}$$

and

$$M_{D^+}-M_{D^0}=\left[\left(\frac{f_K}{f_D}\right)^{1/2}M_{K^0}-\left(\frac{f_\pi}{f_D}\right)^{1/2}M_\pi\right]\frac{m_u-m_d}{m_u-m_s}\,, \tag{31}$$

where we have taken

$$f_{K^0}\approx f_{K^+}\equiv f_K$$

and

$$f_{D^0}\approx f_{D^+}\equiv f_D\,.$$

Equating all the decay constants as a first approximation, we obtain for the quark masses and the u_3 mass splitting for D^+, D^0,

$$m_u:m_d:m_s:m_c=1:1.1:64:23.6\,, \tag{32}$$

$$(M_{D^+}-M_{D^0})_{u_3}=3.9\ \text{MeV} \tag{33}$$

from Eqs. (30) and (31).

Additionally, however, there are finite second-order current-current contributions from the one-photon loop arising from the Coulomb interactions; in fact, Fritzsch has estimated this contribution to be as large as 2.6 MeV.[22] Including this Coulomb correction in our calculations, the electromagnetic mass difference for the D mesons turns out to be

$$M_{D^+}-M_{D^0}=6.5\ \text{MeV}, \tag{34}$$

which agrees with other theoretical estimates and is close to the recent experimental value of[25]

$$M_{D^+}-M_{D^0}=5.0\pm 0.8\ \text{MeV}. \tag{35}$$

Improved agreement with experiment is obtained, however, if we assume our estimate for f'/f given in Eq. (23), in this case we find

$$M_{D^+}-M_{D^0}=4.4\ \text{MeV}, \tag{36}$$

well within experimental errors.

V. QUARK MASSES

Equation (32) gives the ratio of the quark masses. In order to estimate the masses themselves, we need another equation or a sum rule in the various m_i. Such a relation can be obtained by considering baryon masses in the quark model. Assuming that only two-body forces are present, the mass of a baryon is given by

$$M_{B(i,j,k)} = m_i + m_j + m_k + V_{ij} + V_{jk} + V_{ki}\,, \quad i,j,k = u,d,s,c\,, \tag{37}$$

where m_α is the mass of the quark α, and $V_{\alpha\beta}$ is the binding energy resulting from the interaction between quarks α and β $(\alpha,\beta = i,j,k)$. This leads to the following relations[26]:

$$\begin{aligned} M(p) - M(n) &= m_u - m_d + V_{uu} - V_{dd}\,,\\ M(\Xi^-) - M(\Sigma^+) &= m_d + m_s - 2m_u + V_{ss} - V_{uu}\,,\\ M(\Xi^0) - M(\Sigma^-) &= m_u + m_s - 2m_d + V_{ss} - V_{dd}\,, \end{aligned} \tag{38}$$

which imply

$$\begin{aligned} m_d - m_u &= \tfrac{1}{2}[M(p) - M(n) + M(\Xi^-) \\ &\qquad - M(\Xi^0) + M(\Sigma^-) - M(\Sigma^+)] \\ &\approx 6.5\ \text{MeV} \end{aligned} \tag{39}$$

from the experimental data.[27]

This immediately leads, from Eqs. (32), to the estimates of the quark masses,[28]

$$m_u = 65\ \text{MeV}, \quad m_d = 72\ \text{MeV}, \quad m_s = 415\ \text{MeV}, \quad \text{and } m_c = 1535\ \text{MeV}. \tag{40}$$

One of the consequences of the above equation is that it enables us to predict the electromagnetic mass splittings of the charmed baryons. Using the Coleman-Glashow formula[29] and the observation that $V_{uu} + V_{dd} \approx 2V_{ud}$ gives the equal spacing sum rule,[30] we have

$$V_{uu} - V_{dd} = 5.1\ \text{MeV}, \quad V_{us} = V_{ds}\,. \tag{41}$$

By making an ansatz that $V_{uc} = V_{dc}$, we obtain mass splittings

$$\begin{aligned} M(C_1^0) - M(C_1^+) &= M(C_1^+) - M(C_1^{++}) = 3.9\ \text{MeV}\,,\\ M(X_d^+) - M(X_u^{++}) &= M(S^0) - M(S^+)\\ &= M(A^0) - M(A^+) = 6.5\ \text{MeV}\,, \end{aligned} \tag{42}$$

which can be verified when the experimental data are available.

VI. CONCLUSION AND DISCUSSIONS

In conclusion we note that the consequences of chiral-symmetry breaking can be explored if one has *a priori* knowledge of the transformation properties of the matrix elements of the type $\langle 0|v_P|P\rangle$. Two approaches which shed some light on this aspect have been considered in this paper. These are (i) a straightforward generalization of the GMOR model which implies that all $\langle 0|v_P|P\rangle$ are equal, and (ii) the domination of the matrix element by the quark mass term. It is found that the QMD hypothesis, which has given consistent results at the level of chiral SU(3) × SU(3), also gives satisfactory results for the enlarged chiral SU(4) × SU(4) group. In contrast, the former approach leads to several difficulties; the main reason is that it identifies current quarks with constituent quarks. It is therefore not very surprising that the estimates of p and t from the QMD scheme are significantly different from the predictions of an extended GMOR model. As to whether these QMD estimates for p and t are consistent with experimental findings can be tested when adequate data are available, e.g., scattering of charmed mesons off nuclei.

We have also examined the D^+-D^0 mass splitting which essentially consists of two parts: one due to the u_3 term in the Hamiltonian density and the other because of the Coulomb correction. Our estimate for this mass difference due to the u_3 term is, however, based on a conjecture that if all the decay constants f_π, f_K, f_D, f_F are not equal one would expect $f_\pi \approx f_K$ and $f_D \approx f_F$. By assuming with Fritzsch that the Coulomb correction can be as large as 2.6 MeV, we have found that our estimate for the D^+-D^0 mass splitting agrees remarkably well with the experimental data.

Thus we find that the quark-mass-term dominance of the $\langle 0|v_P|P\rangle$ matrix elements is a viable hypothesis for the study of the chiral SU(4) × SU(4) symmetry breaking. The recent discovery[31] of the Υ resonance has brought forth the possibility of an enlargement of the quark family to include at least five flavors, where the latest addition is the b quark whose $b\bar{b}$ bound state is believed to comprise the Υ. This necessitates an extension of the SU(4) to a large symmetry group, viz., SU(5). An extension of the present analysis is immediate and straightforward. Because of the rather larger mass splittings between the b-quark mesons and the strange mesons belonging to the chiral SU(3) × SU(3) subgroup one may assume SU(3) invariance to hold for the older mesons and the symmetry-breaking Hamiltonian density may then be written as

$$H = u_0 + g u_{15} + h u_{24}\,, \tag{43}$$

where g and h are symmetry-breaking parameters and u_i's belong to the $(5,\bar{5})+(\bar{5},5)$ representation of the chiral SU(5) × SU(5). One can then use the PCAC equation and consider as a first approximation that all the decay constants are equal in order to derive sum rules or estimate symmetry-breaking parameters.

However, it may be remarked in this connection that use of the PCAC equation given by Eq. (6) even as a first approximation would be injudicious unless one introduces suitable correction factors to take care of the huge extrapolation between the expected b-quark meson with mass about 5 Gev or higher to zero. Various forms can be chosen for these off-shell factors (e.g., first-order mass dependence can be introduced), and it would be an interesting problem to study the sensitivity of the soft-meson correction factors (SMF) in relation to the new particles. Moreover, such correction factors can be incorporated to investigate the various decays of the b-quark mesons in a manner discussed in Ref. 32. In particular, within this framework, i.e., chiral SU(5) × SU(5) breaking coupled with SMF, it would be worthwhile to study the $\Upsilon \to \eta_b \gamma$ decay where the η_b is the $b\bar{b}$ pseudoscalar partner of the Υ and to find out whether a similar problem like the rather low rate for the $\psi \to \eta_c \gamma$ exists (ψ and η_c are both believed to be $c\bar{c}$ bound states).

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