PHYSICAL REVIEW D VOLUME 19, NUMBER 11 1 JUNE 1979

Duality, exchange-degeneracy breaking, and exotic states

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(Received 23 August 1978)

We study the connection between exchange-degeneracy breaking and multiquark states within the framework of a highly constrained dual approach. We show that M_4 (baryonium) states emerge at the daughter trajectory level as a consequence of small exchange-degeneracy breaking in the meson-meson system ($\sim\delta$) and larger exchange-degeneracy breaking of the baryon trajectories in the meson-baryon system ($\sim\epsilon$). The M_4 states are coupled weakly to external mesons in proportion to the breaking parameter δ. Assuming M_4 couplings to $\bar{B}B$ channels are strong, as determined by duality with normal mesons in the $\bar{B}B$ system, consistency requires $\epsilon\sim\sqrt{\delta}$, thereby relating the larger breaking of baryon trajectories to the violation of the Okubo-Zweig-Iizuka-type rule for M_4. It is shown that exotic baryon states, B_5, also emerge from this scheme at the daughter level and that dibaryons will appear at the second daughter level.

I. INTRODUCTION

The old question of whether or not there exist multiquark exotic states has been revived in the wake of the many current successes of the simple quark model. The existence of such states has been predicted in a multitude of different theoretical contexts—as a consequence of duality,[1] in antinucleon-nucleon potential models,[2] in dual string models,[3,4] quark potential models,[5] dual unitarization schemes,[6,7] and bag models of quantum-chromodynamics (QCD) quark confinement.[8,9] That such a variety of approaches to hadron dynamics requires exotic states is compelling. But perhaps most compelling is a recent accumulation of experimental evidence for the production of narrow high-mass mesonic states in various baryon-antibaryon channels,[10,11] a circumstance envisaged by Rosner[1] ten years ago, when he suggested the existence of certain exotic mesons as a means of satisfying duality in baryon-antibaryon scattering. Such mesons would be primarily two-quark–two-antiquark systems and would couple strongly to baryon-antibaryon channels, with decays into normal mesons suppressed by a generalized Okubo-Zweig-Iizuka- (OZI-) type rule, first stated concisely by Freund, Waltz, and Rosner[3] (in their second rule to which we will refer as the FWR rule). With meson decays suppressed and possible high values of spin,[5,8] the $qq\bar{q}\bar{q}$ states would be narrow states when their masses are near $\bar{B}B$ thresholds, as now seen in experimental data. These states are now referred to variously as baryonium[6] or M_4 (Ref. 7) or simply exotic mesons. They will consist of states within normal flavor representations, called *cryptoexotic states*,[8] and states belonging to purely exotic flavor representations [e.g., 10-, $\overline{10}$-, and 27-plets for mesons in SU(3)].

Accepting that multiquark exotic states exist, a systematic means of determining their quantum numbers, masses, decay widths, and production cross sections must be developed. Several approaches have been pursued thus far.[5-9] In our view, however, the original connection between duality and exotic states[1] provides a fruitful point of departure. Let us review the situation regarding exotic mesons. Exotic mesons must couple to $\bar{B}B$ in order to consistently cancel out dibaryon states. Yet in the solutions to meson-meson (MM) and meson-baryon (MB) duality constraints, SU(3) or SU(4) flavor symmetry, crossing invariance, and exchange degeneracy can be maintained without exotic meson poles[12] coupling to MM. This circumstance is considered to be a manifestation of planarity and the FWR rule.[13] Such a solution necessitates a large spectrum of exchange-degenerate baryon trajectories.[14] However, the experimental situation indicates sizeable breaking of this exchange degeneracy, and the possible absence of some of the required baryon multiplets. The breaking of exchange degeneracy is required by unitarity, and ultimately may be calculable from a dual unitarization program.[6,7,15-18] If the baryon trajectories are not exactly exchange degenerate, the FWR rule cannot be maintained and exotic meson exchanges must contribute to the MB amplitudes.[7,13,19] Factorization of Regge residues then implies the presence of these same exotic meson exchanges and resonances in the MM system. Then, in turn, the normal nonet meson trajectories must not be exactly exchange degenerate, as expected from calculations of lowest-order topological corrections to the singlets (cylinder corrections).[16,17] This chain of inferences implies a strong connection between duality and exotic states and between exchange-degeneracy breaking and violation of the FWR rule. We will exploit these connections to determine some of the properties of exotic mesons, and in the process, exotic baryons of the $qqqq\bar{q}$ form (B_5) and dibaryons of the six-quark form will appear as well.

Our basic approach is as follows. We will first construct exchange-degenerate dual resonance spectra for MM, MB, and $\bar{B}B$ scattering, satisfying the constraints of flavor symmetry and crossing invariance, with minimal exoticity (i.e., only M_4 states in $\bar{B}B$), then exchange degeneracy will be broken while duality, crossing, and flavor symmetry are maintained. This necessitates the introduction of exotic states in each of the systems considered, whose couplings are related to the breaking parameter. Using an explicit dual resonance model for the MM system,[20,21,22] we will find that the exotic M_4 states must appear on the first daughter trajectory of the leading vector and tensor trajectories, as anticipated in the topological-expansion approach[7] and suggested by exotic-exchange reactions.[19] Carrying this result through to the $\bar{B}B$ system then implies that the M_4 states be dual to the normal states and vice versa, even with exact exchange degeneracy.

This latter property of the $\bar{B}B$ system agrees with a phenomenological study of finite-mass sum rules for off-shell $\bar{B}B$ scattering.[23] Complete consistency among MM, MB, and $\bar{B}B$ scattering requires a rich spectrum of M_4 states on the daughter trajectories, which contain almost exchange-degenerate pairs of 27-plets, in terms of SU(3)-flavor representations. One of the 27-plets violates the FWR rule strongly, the other weakly. This indicates a possible relationship between our approach and a QCD bag-model approach, in which two kinds of baryonium states arise,[9] although we do not consider color constraints.

In order to keep the duality constraints manageable, we have assumed exact SU(3) flavor symmetry throughout, and allow only pseudoscalar mesons and spin-$\frac{1}{2}$ octet baryons as external particles. Then we need consider only the leading natural-parity mesons (and their first daughters) as exchanges. We cannot study the full SU(6) structure of the spectra, however. Furthermore, to stay as general as possible, we have not constructed explicit dual resonance models for the MB or $\bar{B}B$ systems, preferring to avoid problems of parity doubling and sattelites, but have solved the general duality constraint relations. This leaves overall coupling strengths for each exotic system unspecified, until they are fixed phenomenologically.

In breaking exchange degeneracy, we assume some effective parallel displacement of multiplet trajectories without evoking any particular dynamical mechanism, realizing that such breaking may actually be dependent on the square of the mass.[16] We expect that the essential features of the crossed-channel spectrum will not be altered significantly by this simplification. A simple relation that results in our scheme between the amount of displacement of opposite-signature trajectories and the size of FWR-rule violation may also be modified in a real dynamical calculation. But the attractiveness of this result and the implication that baryon exchange degeneracy is broken more severely, and in proportion to the square root of the breaking parameter, may be more general properties that will emerge in a complete dynamical scheme. Furthermore, the application of existing dual unitarization programs to baryon states remains incomplete.[12,15]

The paper is organized in order of increasing spin complexity. In Sec. II we construct an explicit dual, SU(3)- and crossing-symmetric model for MM scattering with exchange-degeneracy breaking introduced in such a way that the exact exchange-degenerate spectrum[21] is obtained in the limit that the trajectory displacement parameter, δ, goes to zero. Requiring positive residues for the emerging exotic daughter trajectories leaves two related M_4 27-plets and forces the $10+\overline{10}$ states to decouple.

The MB system is studied in Sec. III, by first considering the simplest exchange-degenerate baryon solution to the complete crossing-symmetric, spin-invariant amplitudes as proposed by Equchi[14] and Fukugita.[12] Breaking exchange degeneracy for both meson exchanges, by δ, and baryon resonances, by ϵ, requires some additional baryon singlets in order to maintain the same M_4 spectrum. As a result, the splitting parameters must be related by $\epsilon \sim \sqrt{\delta}$, and B_5 states must appear, presumably at the daughter level. The couplings of the various exotics to normal states are then determined, relative to one another.

To close the system, $\bar{B}B$ is studied in Sec. IV. The notion of BB duality is used to relate exchange-degenerate M_4 couplings to $\bar{B}B$ to normal-meson couplings, for natural-parity combinations of helicity amplitudes. Consistency imposes restrictions on the coupling parameters. The relative strengths of the two different 27-plets and the $10+\overline{10}$ states are obtained. Breaking exchange degeneracy forces dibaryons to emerge, and, without an explicit calculation, their general properties are discussed.

Finally, in Sec. V, the resulting exotic spectra are discussed[24] and compared with other theoretical approaches. How this scheme relates to dual unitarization is discussed, along with the question of color symmetry. The obvious extension to a more complete treatment is considered, along with some more indirect approaches to the exotic spectra through duality.

II. THE MESON-MESON SYSTEM

The fully crossing-symmetric, SU(3)-invariant, exchange-degenerate solution to the nonexotic duality constraints for pseudoscalar-meson nonet scattering is well known.[21] It involves the exchange or resonance formation of the tensor-meson nonet, exchange degenerate with the ideally mixed vector octet. The solution can be expressed suggestively in terms of Veneziano functions[20] and SU(3) representations as

$$\begin{aligned} A(s,t,u)=\beta_M^{\,2}\,[&(C_s+D_s)V(\alpha(s),\alpha(t))\\ &+(C_s-D_s)V(\alpha(s),\alpha(u))\\ &+(C_u-D_u)V(\alpha(u),\alpha(t))]\,, \end{aligned} \tag{2.1}$$

where

$$V(\alpha(x),\alpha(y))=\frac{\Gamma(1-\alpha(x))\Gamma(1-\alpha(y))}{\Gamma(1-\alpha(x)-\alpha(y))},$$

for $x, y=s,\ t,$ or u, and $\alpha(x)=\alpha_0+\alpha' x$. The factor $\beta_M^{\,2}$ is an overall coupling strength and

$$\begin{aligned} C_x&=\tfrac{8}{3}\,[1]_x+\tfrac{20}{3}\,[8_{ss}]_x\,,\\ D_x&=12[8_{aa}]_x\,, \end{aligned} \tag{2.2}$$

with the notation $[R]_x$ corresponding to the Rth irreducible SU(3) representation in the x $=s$-, t-, or u-channel decomposition.[14] The particular combinations of representations are crossing eigenvectors[25]:

$$\begin{aligned} &C_s+D_s=C_t+D_t,\quad C_s-D_s=C_u+D_u,\\ &C_u-D_u=C_t-D_t\,, \end{aligned} \tag{2.3}$$

and thereby give rise to nonexotic poles in all channels. The C_x gives the residues of the even-signature tensor-trajectory poles in the x channel; the D_x gives the odd-signature vector-trajectory pole residues. These symmetric and antisymmetric couplings will arise for all the daughter poles as well as the leading poles on the trajectory.

The crossing eigenvectors $C_x \pm D_x$ are in one-to-one correspondence with the simple quark duality diagrams[26] for meson-meson scattering with only quark-antiquark intermediate states in the appropriate pair of channels. This relation is indicated in Fig. 1, and results from the correspondences

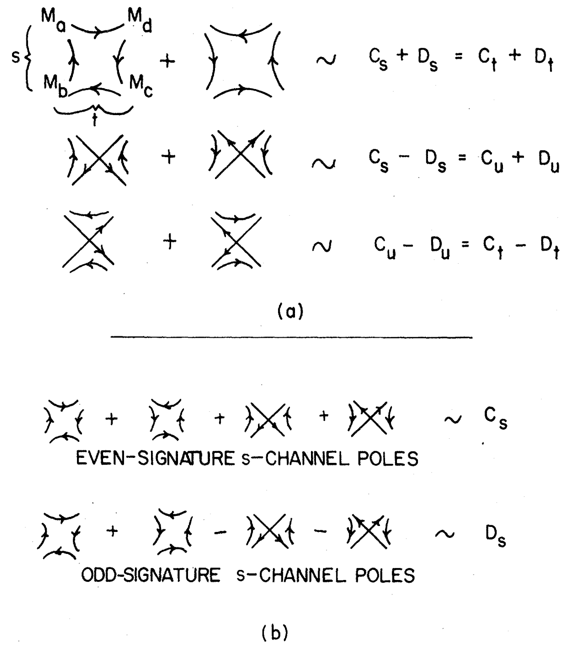

FIG. 1. Quark duality diagrams for pole residues. (a) Planar diagrams for s-t, s-u, u-t dual amplitudes. (b) Diagrams for s-channel pole residues on leading trajectory.

$$\begin{aligned} C_s+D_s&\sim\langle M_aM_bM_cM_d\rangle+\langle M_aM_dM_cM_b\rangle\,,\\ C_s-D_s&\sim\langle M_aM_bM_dM_c\rangle+\langle M_aM_cM_dM_b\rangle\,,\\ C_u-D_u&\sim\langle M_aM_cM_bM_d\rangle+\langle M_aM_dM_bM_c\rangle\,, \end{aligned} \tag{2.4}$$

where M_a is the pseudoscalar-nonet matrix for external meson a, and $\langle\ \rangle$ is the trace. It is therefore evident that C_s+D_s, C_s-D_s, C_u-D_u are *planar* in s-t, s-u, and u-t channels, respectively.

Now we consider breaking the exchange degeneracy of the vector and tensor trajectories. We do this, without breaking SU(3), by supposing that the tensor trajectory is displaced upward by a constant amount, $\alpha_+(s)=\alpha(s)+\delta/2$, and the vector trajectory downward, $\alpha_-(s)=\alpha(s)-\delta/2$. Whether this breaking is due to topological corrections to the planar diagrams[7,16,17] or some other unitarity corrections will not concern us here, nor will we consider the possibility that δ is a function of s.[16] Although these dynamical questions are of crucial importance, it is our view that the *relationship* between the gross features of the hadron spectrum and the breaking of exchange degeneracy may be comprehensible from more general considerations, as we will show.

Given the two trajectories in each channel, we next require that duality constraints still be satisfied; i.e., that exchanges in one channel are dual to resonances in the imaginary part of the crossed-channel amplitude in a local sense. Hence the resonance poles on $\alpha_+(s)$ are dual to both $\alpha_+(t)$ *and* $\alpha_-(t)$ Regge exchanges; and correspondingly for the resonance poles on $\alpha_-(s)$. Where a single Veneziano function represented the s-t dual amplitude [in Eq. (2.1)], now four will be required, each having an SU(3) factor.

Let $C_s^{(+)}$ ($C_s^{(-)}$) be the SU(3) coefficient of the $\alpha_+(s), \alpha_+(t)$ ($\alpha_+(s), \alpha_-(t)$) dual amplitude expressed in terms of s-channel representations and let $D_s^{(+)}$ ($D_s^{(-)}$) be the SU(3) coefficient of the $\alpha_-(s), \alpha_+(t)$ ($\alpha_-(s), \alpha_-(t)$) dual amplitude. Then, to guarantee that the leading trajectory poles have the desired SU(3) structure and that the exchange-degenerate solution [Eq. (2.1)] is obtained in the limit that $\delta \to 0$, we require

$$C_s^{(+)} + C_s^{(-)} = C_s, \quad D_s^{(+)} + D_s^{(-)} = D_s,$$

and

$$C_s^{(+)} + D_s^{(+)} = C_t, \quad C_s^{(-)} + D_s^{(-)} = D_t. \tag{2.5}$$

Along with these coefficients, there will be analogous SU(3) coefficients for the s-u and u-t dual amplitudes, but these latter will be obtained by imposing s-t, s-u, and u-t crossing symmetry on the full amplitude. The amplitude will thereby have the form

$$\begin{aligned} A(s,t,u) = \beta_M{}^2\{ & [C_s^{(+)} V(\alpha_+(s), \alpha_+(t)) + C_s^{(-)} V(\alpha_+(s), \alpha_-(t)) + D_s^{(+)} V(\alpha_-(s), \alpha_+(t)) + D_s^{(-)} V(\alpha_-(s), \alpha_-(t))] \\ & + [s\text{-}u \text{ terms}] + [u\text{-}t \text{ terms}]\}, \end{aligned} \tag{2.6}$$

with the additional constraints that $C_s^{(+)}$ be an even eigenvector under s-t crossing [with all other SU(3) coefficients related via Eqs. (2.3) and (2.5)]. This last requirement follows from the symmetry of the dual function under interchange of its arguments, and the requirement of overall crossing symmetry.

The most general crossing even eigenvector that satisfies the additional requirement that there be no abnormal charge-conjugation contributions (i.e., no $[8_{as}]$ or $[8_{sa}]$ terms) has the form

$$\begin{aligned} C_s^{(+)} &= \{a_1[1]_s + a_s[8_{ss}]_s + (15a_1 - 6a_s - 6a_{10})[27]_s\} + \{(-8a_1 + 5a_s + 4a_{10})[8_{aa}]_s + a_{10}([10]_s + [\overline{10}]_s)\} \\ &\equiv E_s + F_s, \end{aligned} \tag{2.7}$$

where the first term in brackets, E_s, must be even signature while the second, F_s, must be odd,[27] and the coefficients a_1, a_s, a_{10} are unspecified. The full amplitude thereby has the structure

$$\begin{aligned} A(s,t,u) = \beta_M{}^2\{ & [(E_s + F_s) V(\alpha_+(s), \alpha_+(t)) + (C_s - E_s - F_s) V(\alpha_+(s), \alpha_-(t)) \\ & \quad + (C_t - E_s - F_s) V(\alpha_-(s), \alpha_+(t)) + (D_s - C_t + E_s + F_s) V(\alpha_-(s), \alpha_-(t))] \\ & + [(E_s - F_s) V(\alpha_+(s), \alpha_+(u)) + (C_s - E_s + F_s) V(\alpha_+(s), \alpha_-(u)) \\ & \quad + (C_u - E_s + F_s) V(\alpha_-(s), \alpha_+(u)) + (D_u - C_s + E_s - F_s) V(\alpha_-(s), \alpha_-(u))] \\ & + [(E_u - F_u) V(\alpha_+(u), \alpha_+(t)) + (C_u - E_u + F_u) V(\alpha_+(u), \alpha_-(t)) \\ & \quad + (C_t - E_u + F_u) V(\alpha_-(u), \alpha_+(t)) + (-D_t - C_u + E_u - F_u) V(\alpha_-(u), \alpha_-(t))]\}. \end{aligned} \tag{2.8}$$

Note that $E_x \pm F_x$ have the same crossing properties as $C_x \pm D_x$ [Eq. (2.3)],

$$E_s + F_s = E_t + F_t, \quad E_s - F_s = E_u + F_u,$$

$$E_u - F_u = E_t - F_t. \tag{2.9}$$

The fully crossing-symmetric dual amplitude with broken exchange degeneracy has the desired structure on the *leading trajectories* in any channel; $\alpha_+(s)$ has only even-signature poles with residues proportional to $\beta_M{}^2 C_s$ and $\alpha_-(s)$ has only odd-signature poles with residues proportional to $\beta_M{}^2 D_s$. However, the structure of the daughters is significantly different from the leading trajectories. This can be seen by expanding the dual amplitudes in terms of their poles and the parameter δ. For $\alpha_+(s)$ near J, for example,

$$\begin{aligned} & V(\alpha_+(s), \alpha_\pm(t)) \\ & \simeq \frac{1}{(J-1)!\,[J - \alpha_+(s)]} \\ & \quad \times \{[\alpha(t)]^J + \tfrac{1}{2}J(J-1\pm\delta)[\alpha(t)]^{J-1} + \cdots\}, \end{aligned} \tag{2.10}$$

where the remaining terms are lower powers of $\alpha(t)$ and of *order* δ^2. Then there will be poles at the first daughter level, whose residues are *proportional to* δ, that would not be present in the exchange-degenerate case. Furthermore, these new poles *must* contain exotic SU(3) representations.

The explicit pole structure, down to the first daughter level, that arises from Eq. (2.8) follows: Up to the overall factor

$$\frac{2\beta_M{}^2}{(J-1)![J-\alpha_\pm(s)]}$$

and with $z_s=\cos\theta_s$, $b_J=2k_s{}^2\alpha'=\frac{1}{2}[J-\alpha(4\mu^2)]$, we have the residues of the poles on the leading trajectories and the first daughter trajectories:

$\alpha_+(s)=J$ even:

$$C_s(b_Jz_s)^J+F_sJ\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.11a}$$

$\alpha_+(s)=J$ odd:

$$C_sJ[\alpha_0-b_J+\tfrac{1}{2}(J-1-\delta)](b_Jz_s)^{J-1}+E_sJ\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.11b}$$

$\alpha_-(s)=J$ odd:

$$D_s(b_Jz_s)^J+(C_t-\tfrac{1}{2}D_s-E_s)J\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.11c}$$

$\alpha_-(s)=J$ even:

$$D_sJ[\alpha_0-b_J+\tfrac{1}{2}(J-1)](b_Jz_s)^{J-1}-F_sJ\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.11d}$$

where the remaining terms are of higher order in δ and at the second daughter level or below. Note that with $\alpha_0=\frac{1}{2}$, the term $\alpha_0-b_J+\frac{1}{2}(J-1)$ is equal to $\frac{1}{2}\alpha(4\mu^2)$, where μ is the average (unsplit) pseudoscalar-octet mass. The daughter poles consist of terms that were present in the absence of exchange-degeneracy breaking [the C_s term in (2.11b) and the D_s term in (2.11d)], which we will call normal daughters, and terms that arise from the splitting and are proportional to δ. These latter will contain exotic SU(3) representations and will be called exotic daughters or M_4 states. Note that each daughter term satisfies the correct charge-conjugation and signature relation, including the term in (2.11c), since

$$C_t-\tfrac{1}{2}D_s=\tfrac{7}{6}[1]_s+\tfrac{2}{3}[8_{ss}]_s+\tfrac{27}{2}[27]_s, \tag{2.12}$$

is an even-charge-conjugation set of representations.

To further specify the residues of the exotic daughters, which still depend on the parameters a_1, a_s, a_{10} in E_s and F_s [Eq. (2.7)], we require that those residues be positive. For the odd-signature poles of (2.11a) and (2.11d), *positivity* forces the $[10]+[\overline{10}]$ poles to decouple; a_{10} must vanish. The coefficient of the $[8_{aa}]_s$ term must be positive in (2.11a), but is unconstrained in (2.11d) where it can mix with the normal daughter term. Similarly the coefficients of the $[1]_s$ and $[8_{ss}]_s$ terms must be positive in (2.11c) but are unconstrained in (2.11b) due to mixing with the normal daughter terms. Lastly, the coefficients of $[27]_s$ must be positive on both exotic daughters. The resulting positivity constraints are compatible and can be expressed as

$$\begin{aligned}&a_{10}=0,\\&\tfrac{2}{3}\geq a_s\geq 0,\\&\tfrac{5}{8}a_s\geq a_1\geq\tfrac{2}{5}a_s.\end{aligned} \tag{2.13}$$

These constraints leave most of the E_s and F_s terms in (2.11) small compared to the leading terms.

The residues of the various poles in the expansion can be written explicitly in terms of the remaining SU(3) representations, in order to see the effects of the preceding constraints.

$\alpha_+(s)=J$ even:

$$\{\tfrac{8}{3}[1]_s+\tfrac{20}{3}[8_{ss}]_s\}(b_Jz_s)^J+(-8a_1+5a_s)[8_{aa}]_sJ\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.14a}$$

$\alpha_+(s)=J$ odd:

$$\{[\tfrac{8}{3}(\alpha_0-b_J+\tfrac{1}{2}(J-1))-\delta(\tfrac{4}{3}-a_1)][1]_s+[\tfrac{20}{3}(\alpha_0-b_J+\tfrac{1}{2}(J-1))-\delta(\tfrac{10}{3}-a_s)][8_{ss}]_s+3\delta(5a_1-2a_s)[27]_s\}\times J(b_Jz_s)^{J-1}+\cdots; \tag{2.14b}$$

$\alpha_-(s)=J$ odd:

$$12[8_{aa}]_s(b_Jz_s)^J+\{(\tfrac{7}{6}-a_1)[1]_s+(\tfrac{2}{3}-a_s)[8_{ss}]_s+[\tfrac{27}{2}-3(5a_1-2a_s)][27]_s\}J\delta(b_Jz_s)^{J-1}+\cdots; \tag{2.14c}$$

$\alpha_-(s)=J$ even:

$$[12(\alpha_0-b_J+\tfrac{1}{2}(J-1))+\delta(8a_1-5a_s)][8_{aa}]_sJ(b_Jz_s)^{J-1}+\cdots. \tag{2.14d}$$

Now it can be seen that with the constraints (2.13) satisfied, at least one 27-plet must be present at the daughter level. This can be understood as the mechanism by which duality is restored in the presence of exchange-degeneracy breaking in this model. In essence, exchange-degeneracy breaking has mixed the u-t planar diagrams in with the s-t and s-u diagrams, thereby introducing $(qq\bar{q}\bar{q})$ or M_4 intermediate states into the s-channel amplitudes as indicated in Fig. 2. For arbitrary allowed values of a_1 and a_s, a plethora of new states is generated at the daughter level: a $C=-$, odd-signature octet at $\alpha_\pm(s)-1$, a $C=+$, even-signature singlet, octet, and 27-plet at $\alpha_\pm(s)-1$. Furthermore, additional terms (proportional to δ) have appeared as modifications to the normal daughter residues, which may be interpreted as additional poles; a $C=+$, even-signature singlet and octet at $\alpha_+(s)-1$, a $C=-$, odd-signature octet at $\alpha_-(s)-1$. The states are shown in Fig. 3. Whether or not these latter poles are to be treated as separate entities, which can mix with the normal daughters, depends on the requirements on the spectra that arise in the meson-baryon system, to be considered in the next section.

The residues of the poles are listed in Table I. With the constraints (2.13) on a_1 and a_s, the residues are strongly bounded. The largest coupling is that of the 27-plet on $\alpha_- - 1$. Even with δ on the order of $\frac{1}{10}$, this pole can couple with as much

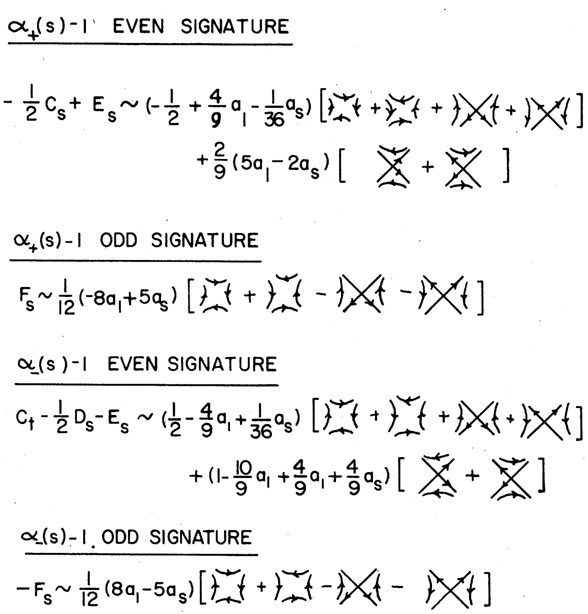

FIG. 2. Exotic pole residues on daughter trajectories, proportional to exchange-degeneracy-breaking parameter δ.

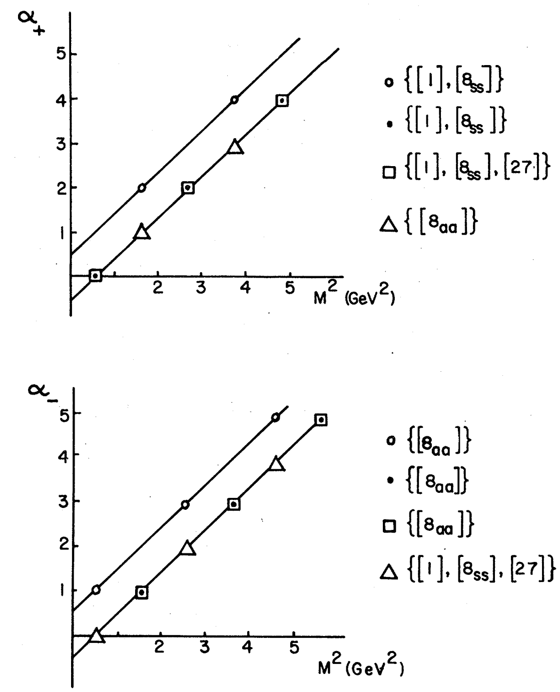

FIG. 3. The meson spectrum after exchange-degeneracy breaking. O and ● represent parent and normal daughter poles, respectively; □ and △ represent exotic daughter poles. Note that ● and □ have the same M^2 value.

as half the strength of the normal singlet tensor meson, while its partner—the 27-plet on $\alpha_+ - 1$—has less than $\frac{1}{3}$ its strength, at most. For the particular choice of $a_1 = a_s = 0$, the spectrum of M_4 states has its simplest form; even-signature singlet, octet, and 27-plet on $\alpha_- - 1$. However, the values of a_1 and a_s must be consistent with the requirements from meson-baryon scattering, which we will explore.

The most important features of the preceding results are that these new states, that we identify with M_4 or baryonium states, couple to ordinary mesons with *strengths proportional to* δ, the exchange-degeneracy breaking parameter. Thus the coupling of these M_4 states to mesons is obtained as a "planarity-breaking" effect, whereas the coupling to $\bar{B}B$ channels will be of normal strength. We anticipate that this simple connection between the breaking of the generalized OZI rule and the breaking of exchange degeneracy will be a feature of more realistic models as well. Furthermore, in our model, the M_4 states lie on trajectories one unit below the normal trajectories, as expected in several other schemes for multiquark states.[6,7,19] This has interesting implications for duality in the $\bar{B}B$ system, as we will see.

TABLE I. Poles in meson-meson system.

Trajectory	Signature	Representation[a]	Meson-meson residues	Bounds on new pole residues: Minimum (a_1, a_s)	Bounds on new pole residues: Maximum (a_1, a_s)
α_+	+	1	1 (normalized)		
α_+	+	8_{ss}	$\frac{5}{2}$		
α_-	−	8_{aa}	$\frac{9}{2}$		
$\alpha_+ - 1$	−	$\underline{8}_{aa}$	$\frac{\delta}{b_J}(2J-1)(-8a_1+5a_s)$	0 at (0, 0)	$\frac{6}{5}\frac{\delta}{b_J}(2J-1)$ at $(\frac{4}{15},\frac{2}{3})$
$\alpha_+ - 1$	+	1	$\frac{1}{b_J}(2J-1)\left[\left(\alpha_0-b_J+\frac{J-1}{2}\right)-\frac{1}{2}\delta\left(1-\frac{3}{4}a_1\right)\right]$		
$\alpha_+ - 1$	+	$\underline{8}_{ss}$	$\frac{1}{b_J}(2J-1)\left[\frac{5}{2}\left(\alpha_0-b_J+\frac{J-1}{2}\right)-\frac{5}{4}\delta\left(1-\frac{3}{10}a_s\right)\right]$		
$\alpha_+ - 1$	+	$\underline{27}$	$\frac{\delta}{b_J}(2J-1)\frac{9}{8}(5a_1-2a_s)$	0 at (0, 0)	$\frac{21}{16}\frac{\delta}{b_J}(2J-1)$ at $(\frac{5}{12},\frac{2}{3})$
$\alpha_- - 1$	+	$\underline{1}$	$\frac{\delta}{b_J}(2J-1)\frac{3}{8}\left(\frac{7}{6}-a_1\right)$	$\frac{9}{32}\frac{\delta}{b_J}(2J-1)$ at $(\frac{5}{12},\frac{2}{3})$	$\frac{7}{16}\frac{\delta}{b_J}(2J-1)$ at (0, 0)
$\alpha_- - 1$	+	$\underline{8}_{ss}$	$\frac{\delta}{b_J}(2J-1)\frac{3}{8}\left(\frac{2}{3}-a_s\right)$	0 at $(a_1,\frac{2}{3})$	$\frac{1}{4}\frac{\delta}{b_J}(2J-1)$ at (0, 0)
$\alpha_- - 1$	+	$\underline{27}$	$\frac{\delta}{b_J}(2J-1)\frac{3}{8}\left[\frac{27}{2}-3(5a_1-2a_s)\right]$	$\frac{135}{32}\frac{\delta}{b_J}(2J-1)$ at $(\frac{5}{12},\frac{2}{3})$	$\frac{81}{16}\frac{\delta}{b_J}(2J-1)$ at (0, 0)
$\alpha_- - 1$	−	8_{aa}	$\frac{1}{b_J}(2J-1)\left[\frac{9}{2}\left(\alpha_0-b_J+\frac{J-1}{2}\right)+\frac{3}{8}\delta(8a_1-5a_s)\right]$		

[a]Underlined terms have residues proportional to δ.

III. THE MESON-BARYON SYSTEM

An SU(3)-invariant, exchange-degenerate solution to duality constraints, that satisfies crossing constraints as well, is more difficult to obtain for the meson-baryon system.[12] Constructing explicit dual dynamical functions for the spin amplitudes, requires complicated parametrization of combinations of Veneziano amplitudes, leaves considerable ambiguity, obscures the underlying symmetry, and requires more baryon states than have been observed experimentally. Therefore, rather than considering explicit dual models as a starting point, we will consider the SU(3) structure of the invariant amplitudes for a simple solution to the dual, crossing-symmetry constraints. Such a solution was obtained several years ago by Eguchi.[14]

Eguchi's solution begins with seven baryon trajectories; four exchange degenerate trajectories with positive-signature times parity, or $\tau P=+$, called the "I" terms, and three $\tau P=-$ exchange-degenerate trajectories labelled "II," in the following pattern:

$$\begin{aligned} &\mathrm{I}:\ (8_{2/3}\oplus 10)^+_\alpha \longleftrightarrow (1\oplus 8_\infty)^-_\gamma \quad (\tau P=+)\,, \\ &\mathrm{II}:\ (8_{-1/3})^+_\beta \longleftrightarrow (8_{-1/3}\oplus 10)^-_\delta \quad (\tau P=-)\,, \end{aligned} \tag{3.1}$$

where superscripts are the signature and subscripts on the octets are the F/D values. The relative couplings of these various representations to the octet meson-octet baryon external particles are fixed so that neither I nor II contribute to exotic representations in the t channel and so that I crosses into -II in the u channel. Hence, the "s-u dual" part of the invariant amplitudes, $A'(s,u)$ and $B(s,u)$ are combinations of $\dot{\mathrm{II}}-\mathrm{I}$ and $\mathrm{II}+\mathrm{I}$, respectively. All couplings to mesons or baryons are fixed thereby.

Eguchi's solution is given explicitly by

$$\begin{aligned} \mathrm{I} &= \tfrac{3}{2}[1]_s + \tfrac{25}{12}[8_{+2/3}]_s + \tfrac{1}{4}[8_\infty]_s + \tfrac{5}{2}[10]_s \\ &= 2[1]_t + \tfrac{5}{2}[8_{ss}]_t + \tfrac{15}{2}[8_{aa}]_t \\ &\quad - \tfrac{3}{2}\sqrt{5}[8_{as}]_t - \tfrac{5}{2}\sqrt{5}[8_{sa}]_t\,, \\ \mathrm{II} &= \tfrac{1}{12}[8_{-1/3}]_s + \tfrac{3}{12}[8_{-1/3}]_s + 10[10]_s \\ &= 2[1]_t - 5[8_{ss}]_t + 3[8_{aa}]_t + 3\sqrt{5}[8_{as}]_t - \sqrt{5}[8_{sa}]_t \end{aligned} \tag{3.2}$$

for the s-t dual parts of the invariant amplitudes, and

$$\begin{aligned} \overline{\mathrm{I}} &= -\tfrac{3}{2}[1]_s + \tfrac{25}{12}[8_{+2/3}]_s - \tfrac{1}{4}[8_\infty]_s + \tfrac{5}{2}[10]_s \\ &= \tfrac{1}{12}[8_{-1/3}]_u - \tfrac{3}{12}[8_{-1/3}]_u - 10[10]_u\,, \\ \overline{\mathrm{II}} &= -\tfrac{1}{12}[8_{-1/3}]_s + \tfrac{3}{12}[8_{-1/3}]_s + 10[10]_s \\ &= \tfrac{3}{2}[1]_u - \tfrac{25}{12}[8_{+2/3}]_u + \tfrac{1}{4}[8_\infty]_u - \tfrac{5}{2}[10]_u \end{aligned} \tag{3.3}$$

for the s-u dual parts of the invariant amplitudes, where the definition

$$\begin{aligned} [8_{F/D}]_s &= \tfrac{20}{3}D^2[8_{ss}]_s + 12F^2[8_{aa}]_s \\ &\quad + 4\sqrt{5}\,FD([8_{as}]_s + [8_{sa}]_s)\,, \\ F+D &= 1\,, \end{aligned} \tag{3.4}$$

has been used. The crossing matrices of Rebbi and Slansky[25] have been used, with the phases of u-channel representations altered to correspond to Eguchi's choices ($[8_{as}]_u$, $[8_{aa}]_u$, $[10]_u$, $[\overline{10}]_u$ all acquire a factor of -1). The couplings of the t-channel meson poles are obtained from $\mathrm{II}\pm\mathrm{I}$, corresponding to s-channel helicity nonflip and flip couplings, $\bar{f}_{+\pm}$ with the normalization given by

$$\begin{aligned} &s^+[1]_t + d^+\sqrt{5}[8_{ss}]_t - 3f^+[8_{sa}]_t \\ &\quad - d^-\sqrt{5}[8_{as}]_t + 3f^-[8_{aa}]_t\,, \end{aligned} \tag{3.5}$$

where superscripts + and − refer to even-signature (tensor) and odd-signature (vector) poles, the f/d ratios become $-\frac{7}{3}$ and $+\frac{1}{3}$ for the nonflip and flip couplings, respectively, for both T and V, in rough agreement with phenomenology.

The problem with this or any other solution to the meson-baryon duality and crossing constraints is the necessity of unobserved baryon multiplets in the spectrum. Furthermore, the exchange degeneracy of the observed baryon states is significantly broken. The two defects could be connected, in that the unobserved states may become weakly coupled or their masses may become larger or their widths may become very broad as exchange degeneracy is broken by some dynamical mechanism. As with the meson-meson system, we will *assume* that such a breaking occurs, by whatever means, and require that the resulting exchange-degeneracy-breaking pattern satisfies the duality and crossing constraints.

We first assume, for simplicity, that a single parameter, ϵ, determines the breaking of the exchange-degenerate baryon trajectories (3.1). The pattern of breaking will elevate 8_α and 1_γ on the $\tau P=+$ trajectory, $\alpha_{\mathrm{I}}(s)$, to $\alpha_{\mathrm{I}}^+(s)=\alpha_{\mathrm{I}}(s)+\epsilon/2$; lower 10_α (unobserved at $J=\frac{1}{2}$) and 8_α to $\alpha_{\mathrm{I}}^-(s)=\alpha_{\mathrm{I}}(s)-\epsilon/2$; raise 8_β and 10_δ on the $\tau P=-$trajectory to $\alpha_{\mathrm{II}}^+(s)=\alpha_{\mathrm{II}}(s)+\epsilon/2$; lower 8_δ (unobserved at $J=\frac{3}{2}$) to $\alpha_{\mathrm{II}}^-(s)=\alpha_{\mathrm{II}}(s)-\epsilon/2$.[18] To guarantee that the spectrum of meson states in the crossed t channel contain the same representations (in particular, no $10+\overline{10}$ states) as in the meson-meson system (as required by factorization), it is necessary that some additional baryons be introduced onto the broken trajectories. The simplest way to do this, without vitiating the desirable features of the scheme, is to introduce additional singlets on $\alpha_{\mathrm{I}}^-(s)$, $\alpha_{\mathrm{II}}^+(s)$ and $\alpha_{\mathrm{II}}^-(s)$. These singlets can mix with the same signature octets and thereby lead to observable and possibly desirable effects. The signatures of these additional singlets are not determined by the t-channel desiderata and will be left free, to begin with. The broken pattern thereby becomes

$$\begin{aligned} \mathrm{I}+&: \ (8_{2/3})^+_\alpha \longleftrightarrow (1)^-_\gamma \ , \\ \mathrm{I}-&: \ (10)^+_\alpha \longleftrightarrow (1\oplus 8_\infty)^-_\gamma \ , \\ \mathrm{II}+&: \ (1\oplus 8_{-1/3})^+_\beta \longleftrightarrow (10)^-_\delta \ , \\ \mathrm{II}-&: \ (1)^+_\beta \longleftrightarrow (8_{-1/3})^-_\delta \ , \end{aligned} \tag{3.6}$$

where the singlets' signatures may be mixed.

The SU(3) factors that multiply s-t dual amplitudes for the s-channel poles corresponding to the four baryon trajectories are then

$$\begin{aligned} \alpha^+_{\mathrm{I}}(s)&: \ V_s = 2[1]_s + \tfrac{25}{12}[8_{+2/3}]_s \ ; \\ \alpha^-_{\mathrm{I}}(s)&: \ W_s = -\tfrac{1}{2}[1]_s + \tfrac{1}{4}[8_\infty]_s + \tfrac{5}{2}[10]_s \ ; \\ \alpha^+_{\mathrm{II}}(s)&: \ Y_s = -\tfrac{1}{2}[1]_s + \tfrac{3}{12}[8_{-1/3}]_s + 10[10]_s \ ; \\ \alpha^-_{\mathrm{II}}(s)&: \ Z_s = \tfrac{1}{2}[1]_s + \tfrac{1}{12}[8_{-1/3}]_s \ ; \end{aligned} \tag{3.7}$$

where

$$\mathrm{I} = V_s + W_s \text{ and } \mathrm{II} = Y_s + Z_s \,, \tag{3.8}$$

in order to obtain the degenerate limit of Eq. (3.2). The negative coefficient for the singlet contributions to W_s and Y_s will be altered by mixing, presumably to restore positivity for the residues. With exchange degeneracy broken, as in the meson-meson case, each s-t dual term, I and II, is now split into four terms, since both s and t channel trajectories are split. Hence we write:

$$\begin{aligned} \mathrm{I} &= \mathrm{I}_+ + \mathrm{I}'_+ + \mathrm{I}_- + \mathrm{I}'_- \ , \\ \mathrm{II} &= \mathrm{II}_+ + \mathrm{II}'_+ + \mathrm{II}_- + \mathrm{II}'_- \ , \end{aligned} \tag{3.9}$$

where $\mathrm{I}_\pm$ are SU(3) coefficients of the $(\alpha^\pm_{\mathrm{I}}(s), \alpha_+(t))$ dual dynamical amplitudes, $\mathrm{I}'_\pm$ are coefficients of the $(\alpha^\pm_{\mathrm{I}}(s), \alpha_-(t))$ dual amplitudes, $\mathrm{II}_\pm$ are the coefficients of the $(\alpha^\pm_{\mathrm{II}}(s), \alpha_+(t))$ dual amplitudes, and II'_+ are coefficients of the $(\alpha^\pm_{\mathrm{II}}(s), \alpha_-(t))$ dual amplitudes. To obtain the chosen pattern of s-channel poles, these new coefficients must satisfy

$$\begin{aligned} &\mathrm{I}_+ + \mathrm{I}'_+ = V_s, \quad \mathrm{I}_- + \mathrm{I}'_- = W_s \ , \\ &\mathrm{II}_+ + \mathrm{II}'_+ = Y_s \ , \quad \mathrm{II}_- + \mathrm{II}'_- = Z_s \ . \end{aligned} \tag{3.10}$$

The form taken by these new coefficients is constrained by various requirements of duality and crossing, as we will see.

The leading t-channel poles have residues which must be consistent with the meson-meson system. Thus $(\mathrm{II}_+ + \mathrm{II}_-)^\pm(\mathrm{I}_+ + \mathrm{I}_-)$ must correspond to pure tensor-meson nonflip and flip couplings for the *leading* $\alpha_+(t)$ poles, and $(\mathrm{II}'_+ + \mathrm{II}'_-)^\pm(\mathrm{I}'_+ + \mathrm{I}'_-)$ must correspond to pure vector-meson nonflip and flip couplings for the *leading* $\alpha_-(t)$ poles. This separation is easily accomplished by identifying these combinations of II's and I's with the appropriate terms in the exchange-degenerate expressions in Eq. (3.2). We obtain

$$\begin{aligned} (\mathrm{II}_+ + \mathrm{II}_-) + (\mathrm{I}_+ + \mathrm{I}_-) &= 4[1]_t - \tfrac{5}{2}[8_{ss}]_t - \tfrac{7}{2}\sqrt{5}[8_{sa}]_t \\ &= N_+ \ ; \\ (\mathrm{II}_+ + \mathrm{II}_-) - (\mathrm{I}_+ + \mathrm{I}_-) &= \tfrac{15}{2}[8_{ss}]_t + \tfrac{3}{2}\sqrt{5}[8_{sa}]_t \\ &= F_+ \ ; \\ (\mathrm{II}'_+ + \mathrm{II}'_-) + (\mathrm{I}'_+ + \mathrm{I}'_-) &= \tfrac{21}{2}[8_{aa}]t + \tfrac{3}{2}\sqrt{5}[8_{as}]_t \\ &= N_- \ ; \\ (\mathrm{II}'_+ + \mathrm{II}'_-) - (\mathrm{I}'_+ + \mathrm{I}'_-) &= -\tfrac{9}{2}[8_{aa}]_t + \tfrac{9}{2}\sqrt{5}[8_{as}]_t \\ &= F_- \ ; \end{aligned} \tag{3.11}$$

where $N_\pm(F_\pm)$ represent the nonflip (flip) coupling of the even-signature or odd-signature mesons. With these relations, and (3.10), we have

$$\begin{aligned} &\mathrm{I}'_+ = V_s - \mathrm{I}_+, \quad \mathrm{I}_- = \tfrac{1}{2}(N_+ - F_+) - \mathrm{I}_+ , \\ &\mathrm{I}'_- = W_s - \tfrac{1}{2}(N_+ - F_+) + \mathrm{I}_+ \ ; \\ &\mathrm{II}'_+ = Y_s - \mathrm{II}_+, \quad \mathrm{II}_- = \tfrac{1}{2}(N_+ + F_+) - \mathrm{II}_+ , \\ &\mathrm{II}'_- = Z_s - \tfrac{1}{2}(N_+ + F_+) + \mathrm{II}_+ \ ; \end{aligned} \tag{3.12}$$

with

$$V_s + W_s = \tfrac{1}{2}(N_+ + N_- - F_+ - F_-)$$

and

$$Y_s + Z_s = \tfrac{1}{2}(N_+ + N_- + F_+ + F_-) \,.$$

The residues of the t-channel exotic daughter poles that are generated by exchange-degeneracy breaking will be determined by the differences of the coefficients, so that the $\alpha_+(t)$ daughter residue for nonflip is

$$(\mathrm{II}_+ - \mathrm{II}_-) + (\mathrm{I}_+ - \mathrm{I}_-) = 2(\mathrm{II}_+ + \mathrm{I}_+) - N_+ \,, \tag{3.13a}$$

and for flip coupling is

$$(\mathrm{II}_+ - \mathrm{II}_-) - (\mathrm{I}_+ - \mathrm{I}_-) = 2(\mathrm{II}_+ - \mathrm{I}_+) - F_+ \,, \tag{3.13b}$$

while for the $\alpha_-(t)$ daughter, the nonflip coupling is

$$\begin{aligned} &(\mathrm{II}'_+ - \mathrm{II}'_-) + (\mathrm{I}'_+ - \mathrm{I}'_-) \\ &\quad = -2(\mathrm{II}_+ + \mathrm{I}_+) + 2(Y_s + V_s) - N_- \,, \end{aligned} \tag{3.13c}$$

and for flip coupling is

$$\begin{aligned} &(\mathrm{II}'_+ - \mathrm{II}'_-) - (\mathrm{I}'_+ - \mathrm{I}'_-) \\ &\quad = -2(\mathrm{II}_+ - \mathrm{I}_+) + 2(Y_s - V_s) - F_- \,. \end{aligned} \tag{3.13d}$$

It is assumed that these residues will each be proportional to the breaking parameter ϵ, since, as in the meson-meson system, the first-order breaking corrections involve differences of dual functions and hence daughter poles. Now these exotic daughter terms should correspond to the exotic daughter spectrum in the meson-meson system [Eqs. (2.14)]. To compare these spectra, let

$$2(\mathrm{II}_+ + \mathrm{I}_+) = n_1[1]_t + n_{ss}[8_{ss}]_t + \sqrt{5}\, n_{sa}[8_{sa}]_t + \sqrt{5}\, n_{as}'[8_{as}]_t + n_{aa}[8_{aa}]_t + n_{27}[27]_t ,$$

and (3.14)

$$2(\mathrm{II}_+ - \mathrm{I}_+) = f_1[1]_t + f_{ss}[8_{ss}]_t + \sqrt{5} f_{sa}[8_{sa}]_t + \sqrt{5} f_{as}[8_{as}]_t + f_{aa}[8_{aa}]_t + f_{27}[27]_t ,$$

and expand $Y_s \pm V_s$ into t-channel representations

$$\begin{aligned} Y_s + V_s &= \tfrac{25}{8}[1]_t - \tfrac{11}{4}[8_{ss}]_t - \tfrac{13}{4}\sqrt{5}[8_{sa}]_t + \tfrac{3}{4}\sqrt{5}[8_{as}]_t + \tfrac{33}{4}[8_{aa}]_t + \tfrac{27}{8}[27]_t , \\ Y_s - V_s &= \tfrac{3}{8}[1]_t - \tfrac{31}{4}[8_{ss}]t + \tfrac{3}{4}\sqrt{5}[8_{sa}]_t + \tfrac{19}{4}\sqrt{5}[8_{as}]_t - \tfrac{19}{4}[8_{aa}]_t - \tfrac{63}{8}[27]_t . \end{aligned} \tag{3.15}$$

Then the exotic daughter residues become

$\alpha_+(t) - 1$, nonflip, even signature:

$$(n_1 - 4)[1]_t + (n_{ss} + \tfrac{5}{2})[8_{ss}]_t + (n_{sa} + \tfrac{7}{2})\sqrt{5}[8_{sa}]_t + n_{27}[27]_t ;$$

$\alpha_+(t) - 1$, nonflip, odd signature:

$$n_{as}'\sqrt{5}[8_{as}]_t + n_{aa}'[8_{aa}]_t ;$$

$\alpha_+(t) - 1$, flip, even signature:

$$f_1[1]_t + (f_{ss} + \tfrac{15}{2})[8_{ss}]_t + (f_{sa} - \tfrac{3}{2})\sqrt{5}[8_{sa}]_t + f_{27}[27]_t ;$$

$\alpha_+(t) - 1$, flip, odd signature:

$$f_{as}\sqrt{5}[8_{as}]_t + f_{aa}[8_{aa}]_t ; \tag{3.16}$$

$\alpha_-(t) - 1$, nonflip, even signature:

$$(-n_1 + \tfrac{25}{4})[1]_t + (-n_{ss} - \tfrac{11}{2})[8_{ss}]_t + (-n_{sa} - \tfrac{13}{2})\sqrt{5}[8_{sa}]_t + (-n_{27} + \tfrac{27}{4})[27]_t ;$$

$\alpha_-(t) - 1$, nonflip, odd signature:

$$-n_{as}'\sqrt{5}[8_{as}]_t + (-n_{aa}' + 6)[8_{aa}]_t ;$$

$\alpha_-(t) - 1$, flip, even signature:

$$(-f_1 + \tfrac{3}{4})[1]_t + (-f_{ss} - \tfrac{31}{2})[8_{ss}]_t + (-f_{sa} + \tfrac{3}{2})\sqrt{5}[8_{sa}]_t + (-f_{27} - \tfrac{63}{4})[27]_t ;$$

$\alpha_-(t) - 1$, flip, odd signature:

$$(-f_{as} + 5)\sqrt{5}[8_{as}]_t + (-f_{aa} - 5)[8_{aa}]_t .$$

These daughter residues are to be compared with those in the meson-meson system of Table I. As with the latter, we expect normal daughters to occur as well: $C=+$, even-signature singlet and octet on $\alpha_+ - 1$, and $C=-$, odd-signature octet on $\alpha_- - 1$. Then the corresponding terms in (3.16) may be interpreted as order ϵ corrections to these normal daughters, or as additional poles. Assuming the former is the simpler interpretation, since the number of new mesons in the spectrum is much smaller. Furthermore, the simplest exotic spectrum allowed in the meson-meson system then can be chosen for the meson-baryon system: $C=+$, even-signature singlet, octet, and 27-plet on $\alpha_- - 1$. This choice of simplicity corresponds to

$$\begin{aligned} 0 &= n_{27}' = n_{as} = n_{aa} , \\ 0 &= f_{27} = f_{as} = f_{aa} , \end{aligned} \tag{3.17}$$

with the other parameters undetermined.

The couplings of the M_4 states to the baryons can be extracted from the residues in (3.16) by factoring out the M_4 couplings to mesons in Table I. Since we have not used a specific dynamical model for the meson-baryon system, the strength of an M_4 coupling to baryons can not be compared directly to an ordinary meson coupling; only the relative strengths of the various M_4 states can be compared. To illustrate the procedure, we first consider the normal meson couplings.

With the overall strength of the meson-meson amplitude β_M^2, and the singlet tensor-meson residue [on $\alpha_+(t)$] normalized to $1\times\beta_M^2$, the effective couplings of the normal mesons to pseudoscalar mesons become (see Table I)

$$g^+_{MM1} = \beta_M, \quad g^+_{MM8} = (\tfrac{5}{2})^{1/2}\beta_M, \quad g^-_{MM8} = \frac{3}{\sqrt{2}}\beta_M , \tag{3.18}$$

where the superscripts refer to signature. For the meson-baryon system, the residues of the normal poles are proportional to $N_\pm$ and $F_\pm$ (3.11). Let the overall normalization be chosen so that the singlet nonflip residue is $1\times\beta_M\beta_B$. Then we have

$$\begin{aligned} &g^+_{MM1}g^+_n(1) = \beta_M\beta_B, \quad g^+_{MM8}g^+_n(8_s) = -\tfrac{5}{8}\beta_M\beta_B, \\ &g^+_{MM8}g^+_n(8_a) = -\tfrac{7}{8}\sqrt{5}\,\beta_M\beta_B , \\ &g^+_{MM1}g^+_f(1) = 0, \quad g^+_{MM8}g^+_f(8_s) = -\tfrac{15}{8}\beta_M\beta_B , \\ &g^+_{MM8}g^+_f(8_a) = \tfrac{3}{8}\sqrt{5}\,\beta_M\beta_B , \\ &g^-_{MM8}g^-_n(8_s) = \tfrac{3}{8}\sqrt{5}\,\beta_M\beta_B, \quad g^-_{MM8}g^-_n(8_a) = \tfrac{21}{8}\beta_M\beta_B , \\ &g^-_{MM8}g^-_f(8_s) = \tfrac{9}{8}\sqrt{5}\,\beta_M\beta_B, \quad g^-_{MM8}g^-_f(8_a) = -\tfrac{9}{8}\beta_M\beta_B , \end{aligned} \tag{3.19}$$

where $g^{(+\text{ or }-)}_{(n\text{ or }f)}(R)$ refers to the even or odd signature, nonflip or flip coupling of the meson belonging to the R representation, symmetrically (d-type coupling) or antisymmetrically (f-type coupling) to $\bar{B}B$. Factoring out the meson-meson couplings, we obtain

$$g_n^+(1)=\beta_B,\quad g_n^+(8_s)=-\frac{\sqrt{5}}{4\sqrt{2}}\beta_B,\quad g_n^+(8_a)=-\frac{7}{4\sqrt{2}}\beta_B\,,$$
$$g_f^+(1)=0,\quad g_f^+(8_s)=-\frac{3\sqrt{5}}{4\sqrt{2}}\beta_B,\quad g_f^+(8_a)=+\frac{3}{4\sqrt{2}}\beta_B\,,$$
$$g_n^-(8_s)=\frac{\sqrt{5}}{4\sqrt{2}}\beta_B,\quad g_n^-(8_a)=\frac{7}{4\sqrt{2}}\beta_B\,, \tag{3.20}$$
$$g_f^-(8_s)=\frac{3\sqrt{5}}{4\sqrt{2}}\beta_B,\quad g_f^-(8_a)=-\frac{3}{4\sqrt{2}}\beta_B\,,$$

or, using (2.5) to define the f and d couplings, with $f+d=1$,

$$g_n^+(8)=\frac{1}{3\sqrt{2}}\beta_B=g_M^-(8),\quad \left(\frac{f}{d}\right)_n^{\pm}=-\frac{7}{3}\,,$$
$$g_f^+(8)=-\frac{1}{\sqrt{2}}\beta_B=g_f^-(8),\quad \left(\frac{f}{d}\right)_f^{\pm}=+\frac{1}{3}\,, \tag{3.21}$$

as expected from Eguchi's model.

We now repeat this calculation for the exotic daughter or M_4 spectrum, restricting ourselves to those poles that do not appear as normal daughters (i.e., the poles in Table I whose residues are proportional to δ). Hence, we consider $(8)^-$ and $(27)^+$ on $\alpha_+(t)-1$, and $(1,8,27)^+$ on $\alpha_-(t)-1$. Normalizing the singlet residue to $\delta\gamma_M{}^2\frac{1}{16}(7-6a_1)$, the resulting M_4 couplings to mesons will be

$$h_{MM1}^+=\sqrt{\delta}\,\gamma_M\tfrac{1}{4}(7-6a_1)^{1/2}\,,$$
$$h_{MM8}^+=\sqrt{\delta}\,\gamma_M\frac{1}{2\sqrt{2}}(2-3a_s)^{1/2}\,,$$
$$h_{MM27}^+=\sqrt{\delta}\,\gamma_M\tfrac{3}{4}(9-10a_1+4a_s)^{1/2}\,, \tag{3.22}$$
$$k_{MM8}^-=\sqrt{\delta}\,\gamma_M(-8a_1+5a_s)^{1/2}\,,$$
$$k_{MM27}^+=\sqrt{\delta}\,\gamma_M\tfrac{3}{4}\sqrt{2}\,(5a_1-2a_s)^{1/2}\,,$$

where γ_M is related to β_M in the explicit Veneziano model we have used by $\gamma_M=[(2J-1)/b_J]^{1/2}$ the h couplings refer to the $\alpha_-(t)-1$ daughters and the k couplings refer to the $\alpha_+(t)-1$ daughters. The meson-baryon system residues, obtained from Eq. (3.16), are normalized so that the singlet nonflip residue is $\epsilon\gamma_M\gamma_B\frac{1}{4}(25-4M_1)$. Then the singlet nonflip coupling to $\bar{B}B$ will be

TABLE II. Exotic-meson couplings.

Trajectory	Signature	Representation	Coupling to mesons (units of $\sqrt{\delta}\,\gamma_M$)	Couplings to $\bar{B}B$ $\left(\text{units of } \frac{\epsilon}{\sqrt{\delta}}\gamma_B\right)$
$\alpha_- - 1$	+	1	h_{MM1}^+: $\frac{1}{4}(7-6a_1)^{1/2}$	$h_n^+(1)$: $\frac{(25-4n_1)}{(7-6a_1)^{1/2}}$
				$h_f^+(1)$: $\frac{(3-4f_1)}{(7-6a_1)^{1/2}}$
$\alpha_- - 1$	+	8	h_{MM8}^+: $\frac{1}{4}\sqrt{2}(2-3a_s)^{1/2}$	$h_n^+(8s)$: $-\sqrt{2}\,\frac{(11+2n_{ss})}{(2-3a_s)^{1/2}}$
				$h_n^+(8a)$: $-\sqrt{10}\,\frac{(13+2n_{sa})}{(2-3a_s)^{1/2}}$
				$h_f^+(8s)$: $-\sqrt{2}\,\frac{(31+2f_{ss})}{(2-3a_s)^{1/2}}$
				$h_f^+(8a)$: $\sqrt{10}\,\frac{(3-2f_{sa})}{(2-3a_s)^{1/2}}$
$\alpha_- - 1$	+	27	h_{MM27}^+: $\frac{3}{4}(9-10a_1+4a_s)^{1/2}$	$h_n^+(27)$: $\frac{1}{3}\,\frac{(27-4n_{27})}{(9-10a_1+4a_s)^{1/2}}$
				$h_f^+(27)$: $-\frac{1}{3}\,\frac{(63+4f_{27})}{(9-10a_1+4a_s)^{1/2}}$
$\alpha_+ - 1$	−	8	k_{MM8}^-: $(-8a_1+5a_s)^{1/2}$	$k_n^-(8s)$: $\sqrt{5}\,\frac{n_{as}}{(-8a_1+5a_s)^{1/2}}$
				$k_n^-(8a)$: $\frac{n_{aa}}{(-8a_1+5a_s)^{1/2}}$
				$k_f^-(8s)$: $\sqrt{5}\,\frac{f_{as}}{(-8a_1+5a_s)^{1/2}}$
				$k_f^-(8a)$: $\frac{f_{aa}}{(-8a_1+5a_s)^{1/2}}$
$\alpha_+ - 1$	+	27	k_{MM27}^+: $\frac{3}{4}\sqrt{2}(5a_1-2a_s)^{1/2}$	$k_n^+(27)$: $\frac{2}{3}\sqrt{2}\,\frac{n_{27}}{(5a_1-2a_s)^{1/2}}$
				$k_f^+(27)$: $\frac{2}{3}\sqrt{2}\,\frac{f_{27}}{(5a_1-2a_s)^{1/2}}$

$$h_n^+(1) = \frac{\epsilon}{\sqrt{\delta}} \gamma_B \frac{25 - 4M_1}{(7-6a_1)^{1/2}}, \tag{3.23}$$

while the singlet flip coupling becomes

$$h_f^+(1) = \frac{\epsilon}{\sqrt{\delta}} \gamma_B \frac{3 - 4f_1}{(7-6a_1)^{1/2}}. \tag{3.24}$$

The remaining couplings to $\bar{B}B$ are listed in Table II, along with the meson-meson couplings. Note that the resulting (f/d) ratios for the octets become [using Eq. (3.5)]:

$$\left(\frac{f}{d}\right)^+_{\text{nonflip}} = -\frac{5}{3}\left(\frac{13+2n_{sa}}{11+2n_{ss}}\right), \qquad \left(\frac{f}{d}\right)^+_{\text{flip}} = \frac{5}{3}\left(\frac{3-2f_{sa}}{31+2f_{ss}}\right), \tag{3.25}$$

for the even-signature octet on $\alpha_-(t)-1$,

$$\left(\frac{f}{d}\right)^-_{\text{nonflip}} = -\frac{1}{3}\frac{n_{aa}}{n_{as}}, \qquad \left(\frac{f}{d}\right)^-_{\text{flip}} = -\frac{1}{3}\frac{f_{aa}}{f_{as}}, \tag{3.26}$$

for the odd-signature octet on $\alpha_+(t)-1$. The overall octet coupling strengths become

$$h_n^+(8) = \frac{\epsilon}{\sqrt{\delta}}\gamma_B \frac{2\sqrt{2}}{3\sqrt{5}(2-3a_s)^{1/2}}(16 - 3n_{ss} + 5n_{sa}),$$
$$h_f^+(8) = -\frac{\epsilon}{\sqrt{\delta}}\gamma_B \frac{2\sqrt{2}}{3\sqrt{5}(2-3a_s)^{1/2}}(54 + 3f_{ss} - 5f_{sa}), \tag{3.27}$$

for the even-signature octet, and

$$k_n^-(8) = \frac{\epsilon}{\sqrt{\delta}}\gamma_B \frac{(-3n_{as}+n_{aa})}{3(-8a_1+5a_s)^{1/2}},$$
$$k_f^-(8) = \frac{\epsilon}{\sqrt{\delta}}\gamma_B \frac{(-3f_{as}+f_{aa})}{3(-8a_1+5a_s)^{1/2}}, \tag{3.28}$$

for the odd-signature octet.

Obviously, any further statement about these couplings requires knowledge of the remaining free parameters. The n's and f's will be involved in the residues for baryon-antibaryon scattering, and will be constrained thereby. However, the exotic couplings are generated by $\bar{B}B$ duality with exchange degeneracy *unbroken*. This was one of the original motivations for postulating exotic states.[1] In one current view,[23] exotic resonances are dual to normal-meson Regge exchanges, and normal resonances are dual to exotic Regge exchanges, in correspondence with the quark duality diagrams, Fig. 4. Coupled with our result that the exotic mesons lie on a trajectory one unit down from the normal mesons, we will investigate this construct in greater detail below, but, for the moment, we observe that, as a consequence of $\bar{B}B$ duality, the coupling of exotic mesons to baryons is of the same order of magnitude as the coupling of normal mesons to baryons. Hence, the factor $\epsilon/\sqrt{\delta}$, appearing in the couplings of M_4 states to $\bar{B}B$, must be of *order unity*. This is one of the most interesting consequences of our approach.

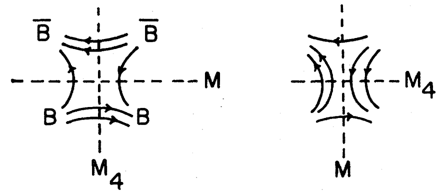

FIG. 4. Duality in $\bar{B}B$ scattering.

Recall that δ is a measure of the splitting between the vector and tensor trajectories. Taking the ρ and A_2 trajectories as representative of the broken SU(3) octets, the separation of their intercepts (as determined by a Regge fit to $\pi^-p \to \pi^0 n$ vs $\pi^-p \to \eta n$) is about $\frac{1}{10}$ or 10%.[29] This implies, in our scheme, a suppression by $(\frac{1}{10})^{1/2}$ in the couplings of M_4 states to mesons, or a suppression of $\frac{1}{10}$ in partial widths into mesons, as compared to normal meson widths. Furthermore, since $\epsilon \sim \sqrt{\delta}$, the splitting of the baryon trajectories is predicted to be of order 0.3 or 30%, which is in rough agreement with the larger exchange-degeneracy breaking among the observed baryon trajectories. Hence, the observed pattern of exchange-degeneracy breaking and the small coupling of exotic mesons to ordinary mesons (i.e., the breaking of the FWR rule for baryonium) are intimately connected. The accuracy and simplicity of this result suggests that this will be a feature that transcends the crudeness of our model.

Another point concerning the couplings to mesons should be noted. For the spectrum that we have chosen, the coupling to mesons of the "h-type" exotic 27-plet [Eq. (3.22)] is an order of magnitude bigger than the cryptoexotic singlet and octet coupling due to numerical constants alone, and the constraints on the parameters a_1 and a_s [Eq. (2.13)]. Hence the "h-type" exotic states will be broad in comparison to the cryptoexotic states—a result that may explain the difficulty of seeing exotic flavor states experimentally.

A final point concerns the breaking of exchange degeneracy in the meson-meson system. If we had assumed that *only* the baryon spectrum is broken, we would have obtained the same exotic meson spectrum as in (3.16), with $\alpha_+ = \alpha_-$. Hence a 27-plet would have to appear in the t channel of the meson-baryon system. Factorization would then require a 27-plet *daughter* coupling to meson-meson. But the only way for this pole to appear in an exchange-degenerate dual meson-meson system would be as a *leading pole*. This inconsistency is resolved by splitting the exchange degeneracy of the mesons, as we have done.

Next we consider the baryonic channels for the exchange-degeneracy-broken form postulated in (3.6). The s-u dual amplitudes must give rise to residues for the same spectrum as in (3.7), with appropriate sign changes for signature. The corresponding SU(3) factors are then

$$\begin{aligned}
\alpha_{\mathrm{I}+}(s):&\ \ \overline{V}_s=(\eta-2)[1]_s+\tfrac{25}{12}[8_{+2/3}]_s\,,\\
\alpha_{\mathrm{I}-}(s):&\ \ \overline{W}_s=(-\eta+\tfrac{1}{2})[1]_s-\tfrac{1}{4}[8_\infty]_s+\tfrac{5}{2}[10]_s\,,\\
\alpha_{\mathrm{II}+}(s):&\ \ \overline{Y}_s=(\xi-\tfrac{1}{2})[1]_s+\tfrac{3}{12}[8_{-1/3}]_s-10[10]_s\,,\\
\alpha_{\mathrm{II}-}(s):&\ \ \overline{Z}_s=(-\xi+\tfrac{1}{2})[1]_s-\tfrac{1}{12}[8_{-1/3}]_s\,,
\end{aligned}\tag{3.29}$$

where $\eta(\xi)$ is a measure of the undetermined strength of coupling of even- (odd-) signature singlets to $\alpha_{\mathrm{I}\pm}(s)$ ($\alpha_{\mathrm{II}\pm}(s)$), and the degenerate limit constraints are satisfied by

$$\bar{\mathrm{I}}=\overline{V}_s+\overline{W}_s,\quad \overline{\mathrm{II}}=\overline{Y}_s+\overline{Z}_s\,.\tag{3.30}$$

The s-u crossing-symmetry requirements in the degenerate limit are

$$\overline{V}_s+\overline{W}_s=-(\overline{Y}_u+\overline{Z}_u)$$

and

$$\overline{Y}_s+\overline{Z}_s=-(\overline{V}_u+\overline{W}_u)\,,\tag{3.31}$$

where $\overline{Y}_u$ is the same linear combination of representations as $\overline{Y}_s$ [in (3.29)], but in the u channel rather than the s channel.

Each s-u dual term, $\bar{\mathrm{I}}$ and $\overline{\mathrm{II}}$, is now split into four terms,

$$\begin{aligned}
\bar{\mathrm{I}}&=\bar{\mathrm{I}}_{s+}+\bar{\mathrm{I}}'_{s+}+\bar{\mathrm{I}}_{s-}+\bar{\mathrm{I}}'_{s-}\,,\\
\overline{\mathrm{II}}&=\overline{\mathrm{II}}_{s+}+\overline{\mathrm{II}}'_{s+}+\overline{\mathrm{II}}_{s-}+\overline{\mathrm{II}}'_{s-}\,,
\end{aligned}\tag{3.32}$$

where $\bar{\mathrm{I}}_{s\pm}$ are SU(3) coefficients of the $(\alpha_{\mathrm{I}\pm}(s),\ \alpha_{\mathrm{II}+}(u))$ dual dynamical amplitudes, expressed in terms of s-channel representations, $\bar{\mathrm{I}}'_{s\pm}$ are coefficients of $(\alpha_{\mathrm{I}\pm}(s),\ \alpha_{\mathrm{II}}-(u))$, $\overline{\mathrm{II}}_{s\pm}$ are coefficients of $(\alpha_{\mathrm{II}\pm}(s),\ \alpha_{\mathrm{I}+}(u))$, and $\overline{\mathrm{II}}'_{s\pm}$ are coefficients of $(\alpha_{\mathrm{II}\pm}(s), \alpha_{\mathrm{I}-}(u))$. These terms must satisfy:

$$\begin{aligned}
&\bar{\mathrm{I}}_{s+}+\bar{\mathrm{I}}'_{s+}=\overline{V}_s,\quad \bar{\mathrm{I}}_{s-}+\bar{\mathrm{I}}'_s=\overline{W}_s\,,\\
&\overline{\mathrm{II}}_{s+}+\overline{\mathrm{II}}'_{s+}=\overline{Y}_s\,,\quad \overline{\mathrm{II}}_{s-}+\overline{\mathrm{II}}'_s=\overline{Z}_s\,.
\end{aligned}\tag{3.33}$$

The s-u crossing requirements on these terms can be expressed as the condition that $\bar{\mathrm{I}}_{s+}, \bar{\mathrm{I}}'_{s+}, \bar{\mathrm{I}}_{s-}, \bar{\mathrm{I}}'_{s-}$ when expanded in terms of *u-channel representations*, have the same form as $-\overline{\mathrm{II}}_{s+}, -\overline{\mathrm{II}}_{s-}, -\overline{\mathrm{II}}'_{s+}, -\overline{\mathrm{II}}'_{s-}$, respectively, expressed in terms of s-channel representations. This is illustrated by the following. Suppose

$$\bar{\mathrm{I}}_{s+}=\sum_{R=1}^{27}C_+^R[R]_s$$

and

$$\overline{\mathrm{II}}_{s+}=\sum_{R=1}^{27}d_+^R[R]_s\,.$$

The crossing matrix, $X^{(su)}$, allows the expansion of $\bar{\mathrm{I}}_{s+}$ into u-channel representations $[P]_u$, so that

$$\bar{\mathrm{I}}_{s+}=\sum_{R=1}^{27}\sum_{P=1}^{27}C_+^R X_{RP}^{(SU)}\eta_P[P]_u\,,$$

where $\eta_P=-1$ for $[8_{as}]_u, [8_{aa}]_u, [10]_u, [\overline{10}]_u$. Then crossing symmetry is implemented by requiring

$$\sum_{R=1}^{27}C_+^R X_{RP}^{(SU)}\eta_P=-d_+^P$$

which is equivalent to $\bar{\mathrm{I}}_{s+}=-\overline{\mathrm{II}}_{u+}$. Then all the $\overline{\mathrm{II}}_s$ terms [in (3.32)] are determined through crossing of the $\bar{\mathrm{I}}_s$ terms. Furthermore, because of relations (3.33) and the crossing requirements,

$$\begin{aligned}
&\bar{\mathrm{I}}_{s+}+\bar{\mathrm{I}}_{s-}=-\overline{Y}_u,\quad \bar{\mathrm{I}}'_{s+}+\bar{\mathrm{I}}'_{s-}=-\overline{Z}_u\,,\\
&\overline{\mathrm{II}}_{s+}+\overline{\mathrm{II}}_{s-}=-\overline{V}_u,\quad \overline{\mathrm{II}}'_{s+}+\overline{\mathrm{II}}'_{s-}=-\overline{W}_u\,,
\end{aligned}\tag{3.34}$$

so that

$$\begin{aligned}
&\bar{\mathrm{I}}_{s-}=-\overline{Y}_u-\bar{\mathrm{I}}_{s+},\quad \bar{\mathrm{I}}'_{s+}=\overline{V}_s-\bar{\mathrm{I}}_{s+}\,,\\
&\bar{\mathrm{I}}'_{s-}=\overline{W}_s+\overline{Y}_u+\bar{\mathrm{I}}_{s+}\,,\\
&\overline{\mathrm{II}}_{s+}=-\bar{\mathrm{I}}_{u+}\,,\ \overline{\mathrm{II}}'_{s+}=\overline{Y}_s+\bar{\mathrm{I}}_{u+}\,,\\
&\overline{\mathrm{II}}_{s-}=-\overline{V}_u+\bar{\mathrm{I}}_{u+}\,,\quad \overline{\mathrm{II}}'_{s-}=-\overline{W}_u-\overline{Y}_s-\bar{\mathrm{I}}_{u+}\,,
\end{aligned}\tag{3.35}$$

which leaves $\bar{\mathrm{I}}_{s+}$ the only independent term.

The leading normal poles have the required SU(3) factors, by construction:

$$\begin{aligned}
&\alpha_{\mathrm{I}+}(s),\ \text{even signature:}\\
&\quad(\mathrm{I}_++\mathrm{I}'_+)+(\bar{\mathrm{I}}_{s+}+\bar{\mathrm{I}}'_{s+})=V_s+\overline{V}_s=\eta[1]_s+\tfrac{25}{6}[8_{+2/3}]_s\,;\\
&\alpha_{\mathrm{I}+}(s)\,,\ \text{odd signature:}\\
&\quad(\mathrm{I}_++\mathrm{I}'_+)-(\bar{\mathrm{I}}_{s+}+\bar{\mathrm{I}}'_{s+})=V_s-\overline{V}_s=(2-\eta)[1]_s\,;\\
&\alpha_{\mathrm{I}-}(s),\ \text{even signature:}\\
&\quad(\mathrm{I}_-+\mathrm{I}'_-)+(\bar{\mathrm{I}}_{s-}+\bar{\mathrm{I}}'_{s-})=W_s+\overline{W}_s=-\eta[1]_s+5[10]_s\,;\\
&\alpha_{\mathrm{I}-}(s),\ \text{odd signature:}\\
&\quad(\mathrm{I}_-+\mathrm{I}'_-)-(\bar{\mathrm{I}}_{s-}+\bar{\mathrm{I}}'_s)=W_s-\overline{W}_s=(\eta-1)[1]_s+\tfrac{1}{2}[8_\infty]_s\,;\\
&\alpha_{\mathrm{II}+}(s),\ \text{even signature:}\\
&\quad(\mathrm{II}_++\mathrm{II}'_+)+(\overline{\mathrm{II}}_{s+}+\overline{\mathrm{II}}'_{s+})=Y_s+\overline{Y}_s\\
&\qquad\qquad=(\xi-1)[1]_s+\tfrac{1}{2}[8_{-1/3}]_s\,;\\
&\alpha_{\mathrm{II}+}(s),\ \text{odd signature:}\\
&\quad(\mathrm{II}_++\mathrm{II}'_+)-(\overline{\mathrm{II}}_{s+}+\overline{\mathrm{II}}'_{s+})=Y_s-\overline{Y}_s=-\xi[1]_s+20[10]_s\,;\\
&\alpha_{\mathrm{II}-}(s),\ \text{even signature:}\\
&\quad(\mathrm{II}_-+\mathrm{II}'_-)+(\overline{\mathrm{II}}_{s-}+\overline{\mathrm{II}}'_{s-})=Z_s+\overline{Z}_s=-(\xi-1)[1]_s\,;\\
&\alpha_{\mathrm{II}-}(s),\ \text{odd signature:}\\
&\quad(\mathrm{II}_-+\mathrm{II}'_-)-(\overline{\mathrm{II}}_{s-}+\overline{\mathrm{II}}'_{s-})=Z_s-\overline{Z}_s=\xi[1]_s+\tfrac{1}{6}[8_{-1/3}]_s\,.
\end{aligned}\tag{3.36}$$

The additional daughter poles that appear because of exchange-degeneracy breaking will have residues involving differences of s-t dual functions with factors like $\mathrm{I}_+ - \mathrm{I}'_+$, and differences of s-u dual functions, like $\bar{\mathrm{I}}_{s+} - \bar{\mathrm{I}}'_{s+}$. In the absence of a particular dual model, we might expect, as with the meson-meson system, that the contribution of differences of s-t terms will be of order δ [since $\alpha_+(t)$ and $\alpha_-(t)$ are involved], and the s-u terms will be of order ϵ [since $\alpha_{\mathrm{I}\pm}(u)$ or $\alpha_{\mathrm{II}\pm}(u)$ are involved], which is in turn, of order $\sqrt{\delta}$. This leads to some confusion with regard to how these daughter residues are to be extracted (and whether the *couplings* to ordinary meson-baryon states are of order $\delta^{1/2}$ or $\delta^{1/4}$). The confusion may have its origin in the problem of removing parity doublets, which arises in baryon channels when amplitudes are functions of s and u rather than $\sqrt{s}$ and $\sqrt{u}$. Without a specific model, we can only say that the additional daughter residues will be linear combinations of the s-t factors and the s-u factors with coefficients that will depend on spins, the parameters of the particular model, and δ and ϵ. But this statement alone allows us to investigate the SU(3) structure of these additional terms.

The contributions to both signatures of daughter poles arising from the exchange-degeneracy breaking of the s-t dual amplitudes will involve

$$\begin{aligned}
&\alpha_{\mathrm{I}+}(s):\ \ \mathrm{I}_+ - \mathrm{I}'_+ = 2\,\mathrm{I}_+ - 2[1]_s - \tfrac{25}{12}[8_{+2/3}]_s\,;\\
&\alpha_{\mathrm{I}-}(s):\ \ \mathrm{I}_- - \mathrm{I}'_- = -2\,\mathrm{I}_+ + \tfrac{13}{8}[1]_s + \tfrac{5}{2}[8_{ss}]_s + \tfrac{7}{2}[8_{aa}]_s + \tfrac{5}{2}\sqrt{5}\,([8_{as}]_s + [8_{sa}]_s) + \tfrac{25}{4}[10]_s - \tfrac{15}{4}[\overline{10}]_s + \tfrac{135}{8}[27]_s\,;\\
&\alpha_{\mathrm{II}+}(s):\ \ \mathrm{II}_+ - \mathrm{II}'_+ = 2\mathrm{II}_+ + \tfrac{1}{2}[1]_s - \tfrac{1}{4}[8_{-1/3}]_s - 10[10]_s\,;\\
&\alpha_{\mathrm{II}-}(s):\ \ \mathrm{II}_- - \mathrm{II}'_- = -2\mathrm{II}_+ - \tfrac{5}{4}[1]_s + \tfrac{23}{4}[8_{ss}]_s - \tfrac{5}{4}[8_{aa}]_s + \tfrac{5}{4}\sqrt{5}\,([8_{as}]_s + [8_{sa}]_s) + \tfrac{25}{2}[10]_s + \tfrac{15}{2}[\overline{10}]_s + \tfrac{27}{4}[27]_s\,.
\end{aligned} \tag{3.37}$$

The corresponding contributions due to the s-u dual amplitudes involve

$$\begin{aligned}
&\alpha_{\mathrm{I}+}(s):\ \ \bar{\mathrm{I}}_{s+} - \bar{\mathrm{I}}'_{s+} = 2\bar{\mathrm{I}}_{s+} + (2-\eta)[1]_s - \tfrac{25}{12}[8_{+2/3}]_s\,;\\
&\alpha_{\mathrm{I}-}(s):\ \ \bar{\mathrm{I}}_{s-} - \bar{\mathrm{I}}'_{s-} = -2\bar{\mathrm{I}}_{s+} + (\eta - \tfrac{1}{4}\xi - \tfrac{29}{8})[1]_s - 2(\xi - 6)[8_{ss}]_s + 2(\xi + \tfrac{5}{2})[8_{aa}]_s + 4\sqrt{5}\,([8_{as}]_s + [8_{sa}]_s)\\
&\qquad\qquad + \tfrac{5}{2}(\xi + \tfrac{1}{2})[10]_s + \tfrac{5}{2}(\xi - \tfrac{3}{2})[\overline{10}]_s - \tfrac{27}{4}(\xi - \tfrac{1}{6})[27]_s\,;\\
&\alpha_{\mathrm{II}+}(s):\ \ \overline{\mathrm{II}}_{s+} - \overline{\mathrm{II}}'_{s+} = -2\bar{\mathrm{I}}_{u+} - (\xi - \tfrac{1}{2})[1]_s - \tfrac{1}{4}[8_{-1/3}]_s + 10[10]_s\,;\\
&\alpha_{\mathrm{II}-}(s):\ \ \overline{\mathrm{II}}_{s-} - \overline{\mathrm{II}}'_{s-} = 2\bar{\mathrm{I}}_{u+} + (\xi - \tfrac{1}{4}\eta - \tfrac{1}{4})[1]_s - 2(\eta - \tfrac{49}{8})[8_{ss}]_s + 2(\eta - \tfrac{11}{8})[8_{aa}]_s - \tfrac{1}{4}\sqrt{5}\,([8_{as}]_s + [8_{sa}]_s)\\
&\qquad\qquad + \tfrac{5}{2}(\eta - 8)[10]_s + \tfrac{5}{2}[\overline{10}]_s - \tfrac{27}{4}(\eta + \tfrac{1}{3})[27]_s\,.
\end{aligned} \tag{3.38}$$

The terms I_+, II_+, $\bar{\mathrm{I}}_{s+}$ are unspecified in the preceding. Whatever the form of these terms, it is clear that exotic representations $[\overline{10}]$ and $[\overline{27}]$ will remain at the first daughter level. Whether these exotic baryons (B_5) will appear with both signatures on the daughter trajectory of $\alpha_{\mathrm{I}+}$, $\alpha_{\mathrm{I}-}$, $\alpha_{\mathrm{II}+}$, or $\alpha_{\mathrm{II}-}$, will depend on the choice of parameters and the particular dynamical realization of duality.

For the simplest choice of new parameters, $\mathrm{I}_+ = 0$, $\mathrm{II}_+ = 0$, $\bar{\mathrm{I}}_{s+} = 0$, the $[\overline{10}]$ and $[27]$ baryons will appear on the lower trajectory daughters, $\alpha_{\mathrm{I}-}$ and $\alpha_{\mathrm{II}-}$, with both signatures, in general. Hence, from our point of view, exotic baryon states that couple to the ordinary, nonexotic meson and baryon states, must appear at the daughter level in order to maintain duality in the presence of exchange-degeneracy breaking of the ordinary mesons and baryons. Because the leading baryon trajectories are split more strongly than the mesons, the coupling of the exotic baryons to nonexotic meson-baryon channels may be close to full strength (i.e., $\delta \sim 10\%$, $\epsilon \sim 30\%$, $\epsilon^{1/2} \sim 50\%$). So, such baryons will be broad and difficult to disentangle in nonexotic meson-baryon channels.

IV. THE BARYON-ANTIBARYON SYSTEM

The difficulty of the implementation of duality constraints for the $\bar{B}B$ system is the original impetus for postulating exotic mesons.[1] To recapitulate the argument, in the $\bar{B}B$ system, the s- and t-channels are identical and the u channel is the dibaryon channel, for which there were no known prominent resonances (at least until recently[30]). The s-u duality constraints thereby require strong exchange degeneracy among the meson states in the s channel in order to cancel the imaginary part of the u-channel amplitude. If these mesons are chosen to be the nonexotic vector and tensor nonets, then crossing into the t channel produces additional exotic meson states which must also fall into strong exchange-degenerate patterns.

The s-t crossing symmetry then requires these exotic states in both channels, with couplings to $\bar{B}B$ of normal hadronic strength. That such states were not seen *below the $\bar{B}B$ thresholds* lead to the speculation that they did not couple to ordinary mesons.

We have seen that exotic mesons are "generated" by the breaking of exchange degeneracy in the MM and MB systems, and that they appear at the first daughter level, weakly coupled to MM. Consistency thus requires that these be the same states that appear in $\bar{B}B$. In order that they agree, the meson states in $\bar{B}B$ must satisfy a form of duality. That is, the leading nonexotic mesons on $\alpha(s)$ are dual to exotic mesons on $\alpha(t)-1$ in the crossed channel and vice-versa. This is suggested by the quark duality diagrams of Fig. 4 and by the analysis of finite mass sum rules in certain inclusive processes.[23] An immediate implication of this hypothesis is that an exotic-exchange reaction, such as $\bar{p}p\to\bar{\Sigma}^-\Sigma^-$ or $\bar{\Xi}^{0,-}\Xi^{0,-}$, has asymptotic behavior determined by $\alpha(t)-1$.

An example of such duality, in the absence of spin complications, is the amplitude

$$[N_s\alpha(t)+M_s\alpha(s)]B(1-\alpha(s),1-\alpha(t)), \tag{4.1}$$

where B is the beta function, N_s is a combination of nonexotic representations in the s channel, and $M_s=N_t$ includes exotic representations in the s channel generated by crossing the nonexotic states from the t channel. By considering the pole structure or the asymptotic behavior in either channel, it can be seen that N_s is proportional to the residue of the exchange-degenerate poles on $\alpha(s)$, while M_s is proportional to the residue of poles on $\alpha(s)-1$. This example is included in order to clarify our notion of duality, and to serve as a guide in discussing the $\bar{B}B$ system. We will not construct an explicit dual dynamical model, but will examine the SU(3) structure of the system, as we did in the MB system without breaking exchange degeneracy. Our primary purpose here is to use the $\bar{B}B$ system to provide additional constraints on the couplings of the exotic mesons to $\bar{B}B$.

The scattering amplitude for $\bar{B}B\to\bar{B}B$ reactions consists of 32 independent terms which are combinations of spin- and SU(3)-reduced amplitudes,[32] satisfying P and C or T constraints. These amplitudes may be chosen to be invariant amplitudes, definite spin amplitudes (as in Ref. 32), or helicity amplitudes. The Reggeon couplings obtained from the MB system were expressed in terms of s-channel helicities, so that it is efficacious to work directly with helicity amplitudes. Furthermore, we are interested in couplings of factorizable natural-parity exchanges and resonances, which reduces the number of amplitudes. We will implement the duality hypothesis by assuming the normal singlet and octet mesons are exchanged in the t channel and so generate s-channel exotic states. Because of SU(3) crossing, the s-channel states belong to the $[1]$, $[8_{ss}]$, $[8_{aa}]$, $([8_{as}]+[8_{sa}])$, $([10]+[\overline{10}])$, and $[27]$ representations, and these, in turn, can only contribute to five out of seven possible independent linear combinations of helicity amplitudes, which reduces the helicity structure to that of the antinucleon-nucleon reactions. Finally, then, the natural-parity-exchange contributions to the s-channel helicity amplitudes at asymptotic energies can be written in terms of the couplings obtained from Eguchi's model in Eq. (3.20). The amplitudes (to first order in t and leading order in s) are thereby of the form

$$\begin{aligned}
&f_{++,++}\simeq f_{+-,+-}\\
&\quad\sim\sum_R\{[g_n^+(R)]^2+[g_n^-(R)]^2\}[R]_t s^{\alpha(t)},\\
&f_{++,--}\simeq -f_{+-,-+}\\
&\quad\sim\frac{t}{4M^2}\sum_R\{[g_f^+(R)]^2+[g_f^-(R)]^2\}[R]_t s^{\alpha(t)},\\
&f_{++,+-}\simeq f_{-+,++}\simeq -f_{+-,++}\simeq -f_{++,-+}\\
&\quad\sim-\frac{\sqrt{-t}}{2M}\sum_R\{g_n^+(R)g_f^+(R)\\
&\qquad\qquad+g_n^-(R)g_f^-(R)\}[R]_t s^{\alpha(t)},
\end{aligned}\tag{4.2}$$

where the sum is over nonexotic SU(3) representations, the notation for the couplings is identical to that in Eq. (3.20), and the helicity amplitudes are in standard notation.[31] Because of the CP constraints,[32] $[8_{as}]_t$ and $[8_{sa}]_t$ must appear in the even combination $([8_{as}]_t+[8_{sa}]_t$ so that the nonflip $\times$ flip term must be symmetrized accordingly. The requirements of strong exchange degeneracy are

$$g_n^+(R)=-g_n^-(R),\quad g_f^+(R)=-g_f^-(R), \tag{4.3}$$

which are satisfied by the couplings, providing the additional singlet of odd signature (which does not couple to MM) is introduced for completeness.

Now duality requires that the asymptotic amplitudes of Eq. (4.2) be built up by M_4 exotic resonances on $\alpha(s)-1$ [e.g., the M_s term in Eq. (4.1)]. The net M_4 residues are thus proportional to the expressions in Eq. (4.2) with the representations crossed into the s channel. These residues will be expressed in terms of direct channel helicities, but to compare with the corresponding crossed-channel exchange couplings, the helicities must be crossed into the t channel. In other words, we want those combinations of s-channel helicity amplitudes that give rise to pure nonflip or flip vertices in the crossed channel at corresponding asymptotic energies in the crossed channel. Such combinations are easily expressed,[31] for natural-parity resonances, as

$$\begin{aligned}
&\text{nonflip-nonflip: } f_{++,++}\,,\\
&\text{nonflip-flip: } f_{++,++}+\frac{2M}{\sqrt{-t}}f_{++,+-}\,,\\
&\text{flip-flip: } f_{++,++}+\frac{4M}{\sqrt{-t}}f_{++,+-}+\frac{4M^2}{t}f_{++,--}\,.
\end{aligned}\tag{4.4}$$

Then the M_4 resonance couplings of Table II saturate these combinations of amplitudes, by the duality hypothesis, at the daughter level. Hence for the M_4 resonances,

$$\begin{aligned}
&\sum_R \{[h_n^+(R)]^2+[h_n^-(R)]^2+[k_n^+(R)]^2+[k_n^-(R)]^2\}[R]_s \sim f_{++,++}\,,\\
&\sum_R \{[h_f^+(R)]^2+[h_f^-(R)]^2+[k_f^+(R)]^2+[k_f^-(R)]^2\}[R]_s\\
&\qquad\sim f_{++,++}+\frac{4M}{\sqrt{-t}}f_{++,+-}+\frac{4M^2}{t}f_{++,--}\,,\\
&\sum_R \{h_n^+(R)h_f^+(R)+h_n^-(R)h_f^-(R)+k_n^+(R)k_f^+(R)\\
&\qquad+k_n^-(R)k_f^-(R)\}[R]_s \sim f_{++,++}+\frac{2M}{\sqrt{-t}}f_{++,+-}\,,
\end{aligned}\tag{4.5}$$

where the $h^\pm(R)$'s ($k^\pm(R)$'s) are the couplings of the even- or odd-signature M_4 states in the $[R]$ representation lying on $\alpha_- - 1$ ($\alpha_+ - 1$), as expressed in Table II, and it is assumed for this discussion that $\alpha_+ - \alpha_- \to 0$.

Before implementing the duality constraints we must decide what the spectrum of M_4 states will be in Eq. (4.5). To begin with, every representation in $8\otimes 8$ must be present, since the expansion of the exchange terms of Eq. (4.2), with the couplings fixed [Eq. (3.20)], populates all of the possible s-channel representations. Hence the $([10]+[\overline{10}])$ terms must be present, although such states do not couple to the MM system. Such exotic states would obey the FWR rule exactly in our scheme. Furthermore, additional odd-signature states, that would couple to pseudoscalar–vector-meson systems, must be introduced to cancel out dibaryons in the u channel. Hence there must be odd-signature 27-plets $[h^-(27), k^-(27)]$ to accompany the two even-signature 27-plets $[h^+(27), k^+(27)]$ that arose in the MM and MB systems. Although one odd-signature 27-plet might be sufficient, it is more natural to parallel the strong exchange-degeneracy pattern of the normal mesons by requiring two, with

$$\begin{aligned}
&h_n^+(27)=-h_n^-(27),\quad h_f^+(27)=-h_f^-(27)\,,\\
&k_n^+(27)=-k_n^-(27),\quad k_f^+(27)=-k_f^-(27)\,.
\end{aligned}\tag{4.6}$$

For the octets and singlets, the situation is more complicated due to possible mixings and confusion with the daughters of the normal mesons. We will therefore concentrate on the exotic representations in implementing duality constraints.

Expanding the exchange contributions to the helicity amplitudes [Eq. (4.2)], with fixed couplings [Eq. (3.20)], into s-channel representations and equating these with the M_4 resonance contributions [Eqs. (4.5)] gives the duality constraints on the 27-plet,

$$\begin{aligned}
&\{[h_n^+(27)]^2+[k_n^+(27)]^2\}G_1\simeq\tfrac{225}{128}G_2\,,\\
&\{[h_f^+(27)]^2+[k_f^+(27)]^2\}G_1\simeq\tfrac{9}{32}G_2\,,\\
&\{[h_n^+(27)h_f^+(27)]+[k_n^+(27)k_f^+(27)]\}G_1\simeq\tfrac{45}{64}G_2\,.
\end{aligned}\tag{4.7}$$

The overall strengths G_1 and G_2 on the left- and right-hand sides are unspecified without a dynamical model, but the relative strengths G_2/G_1 will be fixed by these constraints. The coefficients h and k are functions of n_{27}, f_{27}, and $(5a_1-2a_s)$, as determined from MB duality in Table II. With $0<5a_1-2a_s\leqslant\frac{3}{4}$, as required for meson residue positivity in Eq. (2.13), the nonlinear constraints (4.7) yield unique solutions for n_{27}, f_{27}, and G_2/G_1, for each choice of $5a_1-2a_s$. Over the full range of allowed $5a_1-2a_s$ values,

$$\begin{aligned}
&n_{27}\simeq-100,\quad f_{27}\simeq-40\,,\\
&(G_2/G_1)^{1/2}\simeq 74/(5a_1-2a_s)^{1/2}\,,
\end{aligned}\tag{4.8}$$

are good approximations within about 8%. The resulting couplings are approximately

$$\begin{aligned}
&h_n^+(27)\simeq 49,\quad h_f^+(27)\simeq 12\,,\\
&k_n^+(27)\simeq -94/(5a_1-2a_s)^{1/2}\,,\\
&k_f^+(27)\simeq -38/(5a_1-2a_s)^{1/2}\,.
\end{aligned}\tag{4.9}$$

Thus the $\bar{B}B$ couplings of the $(\alpha_+ - 1)$ 27-plet exotic, or the $k(27)$, is at least twice as large as the coupling of the $(\alpha_- - 1)$ 27-plet, exotic, or the $h(27)$. Stated another way, the $k(27)$ exotic resonances contribute more to the building up of the normal exchanges in the duality sense.

There are two important implications of these results. First, we see that there must be at least two 27-plets in order to satisfy the meson and baryon duality constraints. Since the coupling of $k(27)$ to mesons is proportional to $(5a_1-2a_s)^{1/2}$ (see Table II), this multiplet is more weakly coupled to mesons, but more strongly coupled to baryons than its counterpart, $h(27)$. The limit $5a_1=2a_s$ is not allowed, however, since the $k(27)$ does not couple to mesons in that limit, which requires $n=f=0$ to satisfy factorization and does not allow $\bar{B}B$ constraints to be satisfied. The $k(27)$ will thus be a candidate for narrow states in the $\bar{B}B$ system near threshold, whereas the $h(27)$ will be broad. Neither of these will be narrow far above the relevant thresholds.

Duality also requires $([10]+[\overline{10}])$ multiplets to couple to $\bar{B}B$, the relevant constraints being

$$\begin{aligned}
&\{[h_n^+(10)]^2+[k_n^+(10)]^2\}G_1 \simeq \tfrac{75}{64}G_2\,,\\
&\{[h_f^+(10)]^2+[k_f^+(10)]^2\}G_1 \simeq \tfrac{15}{16}G_2\,, \qquad (4.10)\\
&2\{[h_n^+(10)h_f^+(10)]+[k_n^+(10)k_f^+(10)]\}G_1 \simeq \tfrac{45}{32}G_2\,,
\end{aligned}$$

where the h's and k's are the couplings for two positive-signature $(10+\overline{10})$ multiplets. Because of factorization, two such multiplets must be present, at both signatures. Since these constraints do not determine the h's and k's completely, it is convenient to treat (h_n, k_n) and (h_f, h_f) as two 2-vectors, $\vec{\mathrm{N}}$ and $\vec{\mathrm{F}}$. Then the constraints imply [along with G_2/G_1 from (4.8)]

$$\begin{aligned}
|\vec{\mathrm{N}}| &= \{[h_n^+(10)]^2+[k_n^+(10)]^2\}^{1/2}\\
&\simeq 80/(5a_1-2a_s)^{1/2}\,,\\
|\vec{\mathrm{F}}| &= \{[h_f^+(10)]^2+[k_f^+(10)]^2\}^{1/2}\\
&\simeq 71/(5a_1-2a_s)^{1/2}\,,\\
\arccos&\left(\frac{\vec{\mathrm{N}}\cdot\vec{\mathrm{F}}}{|\vec{\mathrm{N}}||\vec{\mathrm{F}}|}\right)=48^\circ\,. \qquad (4.11)
\end{aligned}$$

Comparing with the sizes of the 27-plet couplings, Eq. (4.9), both h and k $(10+\overline{10})$ couplings to $\bar{B}B$ are comparable to the $k(27)$ in magnitude. The $(10+\overline{10})$'s, which do not couple to mesons in our scheme, are thus possible narrow exotic states near the $\bar{B}B$ threshold.

In all of the above discussion we have assumed exact exchange degeneracy for the $\bar{B}B$ states. Consistency with MM and MB requires that the degeneracy be broken. While such an investigation has begun, it is considerably more complicated, due to the spin structure. Certain qualitative results must emerge, however. By breaking exchange degeneracy while maintaining duality, the imaginary part of the u-channel amplitudes will become nonzero. Hence dibaryon states must emerge, with couplings proportional to the same splitting parameter, $\sqrt{\delta}$. These states will lie at the second daughter level, since the exact exchange-degeneracy limit must restore duality between leading poles and first daughter mesons. To maintain s-u duality when the first daughter degeneracy is broken would require these second-daughter states in the u channel. Then, because of spin complications, these dibaryons would lie on trajectories that could be depressed one more unit of angular momentum $-\frac{1}{2}$ unit of angular momentum for each unit of baryon number. This would suggest that a leading dibaryon trajectory would be at $\alpha(u)-3$, at least before SU(3) splitting. Hence the first manifestation in nucleon-nucleon of such states would be near the threshold, with total spin 1, near the mass of the spin-4 $h(2040)$ meson. The next recurrence would be a spin 2 object near 2.4 GeV/c^2. Whether or not these correspond to some recently reported dinucleon resonances[30] remains to be explored.

That dibaryons appear via breaking of exchange degeneracy in the $\bar{B}B$ system, and that exotic mesons and baryons emerge from the same mechanism in the MM and MB systems, suggests a generalization to more exotic multiquark states that is reminiscent of string models and dual models.[3] That is, if we consider duality constraints applied to reactions with exotic external particles, exchange-degeneracy breaking will require yet more exotic exchanges on more depressed trajectories. These new exotics will appear as normally coupled objects in yet more complicated exotic reactions through duality in the exact exchange-degeneracy limit. Such an algorithm for multiquark states will never terminate—giving rise to an infinite family of more and more massive exotics. For the present, however, this is highly speculative. Further study of such a scheme requires a more careful treatment of the $\bar{B}B$ system.

V. CONCLUSION

We will now summarize the essential results that have emerged from our scheme for determining the properties of exotic states and compare these features to other approaches. To begin with, exchange-degeneracy breaking in the MM system requires exotic mesons on daughter trajectories. As suggested by Chew[6] and Veneziano and Rossi,[7] this gives the M_4 trajectory an intercept of $\sim -\frac{1}{2}$ and a normal slope. This intercept also arises in a multiperipheral approximation for exotic exchanges[28] and in phenomenological fits to exotic-exchange reactions.[19]

The FWR-rule-violating decay widths of the M_4 states into normal mesons are proportional to δ, the effective exchange-degeneracy-breaking parameter for the normal vector and tensor nonets. This connection makes sense in terms of dual unitarization since M_4 states are expected to arise in MM dual amplitudes through some of the same planarity-breaking corrections in the topological expansion that break exchange degeneracy.[7,15,16,17] That is, the cylinder (torus) correction, that splits opposite-signature singlets (nonets) in one channel, has exotic flavor in the crossed channel, and thereby contributes to building up crossed-channel exotic states, in the sense of finite-energy sum rules. It should be noted, however, that the approximate calculation of the cylinder correction[16] gives rise to the conjectured f-Pomeron identity, [15] and so the calculated breaking parameter for the f-ω trajectory is mass dependent. Our constant δ can only be a rough approximation to this dynamical effect. Furthermore, since we do not distinguish different sources of breaking, our δ represents the average splitting of the SU(3)-degenerate nonets. Even if the f-ω splitting is indeed large, as it would be at $t=0$ for the f-Pomeron identity $[\alpha_f(0)=\alpha_{\text{Pomeron}}(0)=1,\ \alpha_\omega(0)\approx\frac{1}{2}]$, our parameter δ would be on the order of $\frac{1}{10}$ (taking an average over the full nonet and assuming small breaking of $\sim\frac{1}{10}$ for the other nonet members). If the f-Pomeron identity is not correct, as some phenomenological[33] and theoretical considerations[34] suggest, δ will be somewhat smaller. Another measure of the splitting is the difference between the ρ and A_2 intercepts of $\sim\frac{1}{10}$ as determined from $\pi^- p\rightarrow\pi^0 n$, ηn, respectively.[29]

The direction of the splitting was chosen so that the tensor nonet trajectory was elevated by $\delta/2$, while the vector trajectory was suppressed by $\delta/2$. This direction is not arbitrary in our MM model. Positivity constraints on the exotic daughters can not be satisfied with the other choice, unless more degenerate multiplets are introduced. This directionality agrees with the observation that the f trajectory is raised.

The positivity conditions also require the decoupling of the $(10+\overline{10})$ from the MM system, assuming a minimal set of exotic poles. Yet these states must couple to $\overline{B}B$ via duality. Why the FWR rule is maintained for the $(10+\overline{10})$ system in our scheme is not clear, but implies that these exotics are the best candidates for narrow states.

As a result of factorization for M_4 exchanges in the MM and MB systems, and duality for $\overline{B}B$, the effective splitting parameter for the baryon trajectories, ϵ, is considerably larger than that for the mesons, δ, since $\epsilon\sim\sqrt{\delta}$. This is an intriguing relation that follows from our scheme in a natural way, and is probably more general than implied by this context. The actual splittings of the baryon trajectories are roughly of this order ($\sim 30\%$), but the precise orderings depend on dynamics.[18] The actual spectrum of observed baryon states is not settled as yet, either.[12,35] We have used the simplest spectrum for implementing duality, [14] but more general forms can be treated in a similar manner. The method we have used for studying the relation between M_4 states and the broken spectrum of baryons is suggested by some of the conjectured topological corrections for the baryons, [17,13] although a consistent scheme for incorporating baryons into dual unitarization is still being sought.[15]

The spectrum of M_4 states that emerges from our MM, MB, and $\overline{B}B$ duality constraints is rich, but not complete, since we have only considered the leading natural-parity states. However, these states are similar to those natural-parity states predicted in the diquark color-confining model of Chan and Hogaasen.[9] The states that couple strongly to $\overline{B}B$ must be a nearly exchange-degenerate 27-plet (of both signatures), two $(10+\overline{10})$ multiplets, and at least two nonets that mix with normal daughters. These correspond to the T states of Chan and Hogaasen contained in the nearly degenerate $[2,36]$, $[1,18]$ $[1,\overline{18}]$, and $[0,9]$ (where the first number refers to the spin of the diquark-antidiquark system and the second numbers are the degenerate flavor representations $36=1+8+27$, $18=8+10$, $\overline{18}=8+\overline{10}$, $9=1+8$). The exotic 27-plet (k type) and $(10+\overline{10})$ couple weakly to mesons as in the T states. We have an additional 27-plet trajectory (h type), however, that couples strongly to mesons and weakly to $\overline{B}B$ and would correspond to broad meson resonances. Since we have not incorporated color into our scheme, we do not know if including color would suppress the meson couplings so that this $h(27)$ would become a Chan-Hogaasen M state. The M states presumably couple weakly to both mesons and $\overline{B}B$ since they consist of color-antisymmetric diquarks and antidiquarks.[9] On the other hand, it has been argued that M_4 states that couple strongly to mesons emerge naturally in the bag model[8] for low relative orbital angular momenta.[36] The small-angular-momentum $h(27)$ poles and the mixed nonets may fall into that category. Settling these questions will require a careful consideration of the couplings and a comparison with known candidates for the states. Broad, low-mass exotic state coupling to many mesonic channels may be difficult to disentangle phenomenologically, [8] although there is evidence for cryptoexotic low-mass mesonic states.[8,36]

With our spectrum of M_4 states on daughter trajectories with normal slopes of $\sim 1\ \text{GeV}^{-1}$, we expect narrow nonet, $(10+\overline{10})$ and $k(27)$ states near the $\overline{B}B$ thresholds. In particular we expect, in order of increasing widths, spin 3 resonances near 1.9 GeV, spin 4 near 2.1 GeV, spin 5 near 2.4 GeV, and spin 6 near 2.6 GeV. There is plenty of activity in the nonstrange-meson-resonance data around these values.[10] The narrow S resonance at 1.935 GeV may have spin 3, although spin 2 may be preferred, and probably consists of at least two states to account for its suppression in $pp\rightarrow\overline{n}n$ relative to $\overline{p}p\rightarrow\overline{p}p$. The $T(2190)$ and $U(2350)$ are broad and have spin assignments at 3 and 4, one unit below our expectation, but these spins are not firmly established. At the moment the phenomenology of nonstrange baryonium candidates is confused by a plethora of possible states,[10,11] and we only await the determination of some spins to pin down the trajectory. For strange states there is one striking candidate, the I meson at 2.6 GeV with $\Gamma \simeq 20$ MeV, seen in the decay mode $K_S^0 \pi^{\pm}\pi^{+}\pi^{-}$.[37] Even with the strangeness ± 1 trajectories split from the nonstrange M_4 trajectory, in our scheme such a state would have to be at least spin 4. Many more strange states must be seen, or course, for any multiquark model to be believable. More urgent is the necessity of seeing unambiguous exotic flavor states. A determination of the unspecified parameters in our scheme will allow us to predict some of the exotic widths, branching ratios, and production cross sections.

Exchange-degeneracy breaking in the MB system also gives rise to exotic baryon daughters, with couplings to ordinary MB states of order $\sqrt{\epsilon}$, which is $\sim\frac{1}{2}$ of normal strength. Hence the FWR violating decays of B_5 states may have broad widths. The $\overline{10}$ and 27 plets would be difficult to disentangle from background, although possible Z^* resonances in the KN system have often been conjectured.

The dibaryon states that appear at the second daughter level as a result of exchange-degeneracy breaking in the $\overline{B}B$ system may contain the resonances that are reported in pp phase-shift analyses.[30] More definitive statements about the dibaryon spectrum require a consistent treatment of the invariant amplitudes, so that the spin is properly taken into account. Such dibaryon states are predicted in string[4] and color bag models[38] as well and in various potential models[39] for quark clusters.[40] The coupling of dibaryons to ordinary baryons will violate the FWR rule and will be proportional to $\sqrt{\delta}$, just as in the M_4 coupling to ordinary mesons. Allowed decays of dibaryons (with sufficient mass) will be into BB_5 states, since dibaryons will be dual to ordinary meson exchanges in the BB_5 scattering amplitudes.

It is obvious from the last statement that the generation of more and more complicated exotic states arises by using exotic states as external particles in dual systems. For BB_5, dibaryons ($6q$) arise in the s channel dual to normal mesons in the t channel and to M_4 states in the u channel. But M_6 ($3q3\overline{q}$) states also arise in the u channel dual to normal mesons in the t channel. And these same M_6 states will appear in the MM_4 system when exchange degeneracy is broken, at the second daughter level, since M_4 is at the first daughter level. This procedure, which is illus-

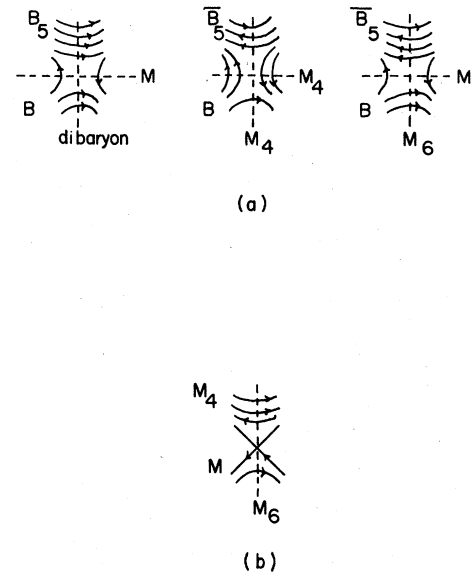

FIG. 5. Generation of more exotic mesons. (a) B_5B and $\overline{B}_5B$ allowed diagrams with M, M_4, M_6 intermediate states. (b) u-t dual diagram that enters s-channel pole residue at second daughter level due to exchange-degeneracy breaking.

trated in Fig. 5, can continue indefinitely, with M_{2n} states appearing on $\alpha - n + 1$, coupling with strength $\sim\sqrt{\delta}$ to MM_{2n-2}, and with normal strength to $\overline{B}B_{2n-1}$. Similar rules can be obtained for states with baryon number 1, 2, 3, etc. We are led thereby to a dual scheme for constructing metastable multiquark systems on lower trajectories which is similar to string,[4] color bag,[38] and cluster models,[40] but in which violation of FWR rules[3] through exchange-degeneracy breaking determines the daughter trajectories on which the multiquark states lie. How such a scheme relates to previous dual models and dual unitarization with color[7] remains to be explored.

At a more pedestrian level, completeness requires that we consider the full SU(6) system of external and internal states; i.e., both pseudoscalar and vector mesons and spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons as external particles, along with lower-lying pseudoscalar and axial-vector mesons as internal lines. Then the question of whether the baryon spectrum is in the form (56) $\longleftrightarrow$ (70) for all L or (56) L even $\longleftrightarrow$ (70) L odd can be considered,[35] by using a more general spectrum as the starting point for breaking exchange degeneracy in the meson-baryon system.

Secondly, the parameters for the M_4 states must be determined by a phenomenological fit to the existing baryonium candidates. This will be somewhat arbitrary at this time, and will require some *ad hoc* procedure for SU(3) breaking, but will thereby determine decay widths and branching ratios for the exotic states and will enable estimates of production cross sections. Furthermore, the B_5 spectrum will be constrained as well by this procedure. To pin down the dibaryon states a more thorough treatment of spin in the $\bar{B}B$ system must be undertaken. Considerable work remains for our scheme to make contact with experimental data. We believe that because of the many desirable features that emerge from the calculation presented herein, additional efforts will be rewarded by a deeper understanding of the connection between duality, exchange-degeneracy breaking, and multiquark states.

ACKNOWLEDGMENT

The authors appreciate the experimental interest of Tony Mann, Jack Schneps, and Fu Tak Dao in baryonium states that provided the initial impetus for this undertaking. One of us (G.R.G.) has benefited by conversations with Professor Carl Rosenweig.

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