PHYSICAL REVIEW D VOLUME 19, NUMBER 11 1 JUNE 1979

Unification of linear Regge trajectories for all $Q\bar{Q}$ families

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(Received 30 June 1978)

We report on a simple mass formula for the relativistic bound state $Q\bar{Q}$ system which describes remarkably well the family of mesons ($\rho,\omega,\phi,J/\psi,\Upsilon$; π,K,K^*,D,D^*,F,F^*):

$$M^2 = (m_1 + m_2)^2 + [2m_1m_2/(m_1^2+m_2^2)](m_1 + m_2)\Omega(n + 2) - c(m_1 + m_2)^2/(m_1^2+m_2^2),$$

where m_1,m_2 are the constituent quark masses, Ω is a universal constant (= 0.6624 GeV), n is the quantum number for the state, and c is an effective constant which measures how far off-shell the quarks are in their bound state, i.e., $\langle p_1^2+ m_1^2+ p_2^2+m_2^2\rangle = c$. c depends weakly on the spin-triplet or singlet nature of the $Q\bar{Q}$ system. The parameters found in our fit are (in GeV units) $m_u = m_d = 0.83869$, $m_s = 0.87988$, $m_c = 1.50967$, $m_b = 4.39312$, c(triplet) = 2.77690, c_π(singlet) = 2.50840, c_K(singlet) = 2.49276. A derivation based on a parton picture of a constituent-bound-state system of $Q\bar{Q}$ is given. Implications of this mass formula for higher-mass states are discussed.

I. INTRODUCTION

In this report we present a simple mass formula for the family of linear meson trajectories associated with the $Q\bar{Q}$ bound-state system, where Q can be any one of the u, d, s, c, b "discovered" so far. The mass formula is given by

$$M^2 = (m_1+m_2)^2 + \frac{2m_1m_2}{m_1^2+m_2^2}(m_1+m_2)\Omega(n+2) - c\frac{(m_1+m_2)^2}{m_1^2+m_2^2}, \qquad (1)$$

where m_1, m_2 are the constituent quark masses, Ω is the level spacing of an internal relativistic oscillator assumed to be flavor and color independent, n is the quantum number for the state (see Sec. II), and c is an effective constant which measures how far off-shell the quark and antiquark are off their individual mass shell, viz.,

$$\langle p_1^2+m_1^2+p_2^2+m_2^2\rangle = c. \qquad (2)$$

The parameters used in our fit[1] are

$$\begin{aligned} &m_u=m_d=0.838\,69,\ m_s=0.879\,88,\\ &m_c=1.509\,67,\ m_b=4.393\,12,\\ &c(\text{triplet})=2.776\,90,\\ &c_\pi(\text{singlet})=2.508\,40, \quad c_K(\text{singlet})=2.492\,76 \end{aligned} \qquad (3)$$

in GeV units.

Equation (1) summarizes the vast store of knowledge concerning the meson trajectories made up of noncharmed quarks and antiquarks. It includes the masses of the well-known vector mesons ρ, ω, ϕ, K^*, and the Regge recurrences and daughters where known. It also includes the pesudoscalar mesons π and K and their presumed Regge recurrences. For the J/ψ family of trajectories, a set of linear trajectories with slope $=\frac{1}{2}$ is known[2] to give a good accounting of the psions including the χ intermediate states. For the Υ family, the trajectories have a Regge slope of $1/5.82 \simeq \frac{1}{6}$ and the masses of the three lowest 1^{--} mesons in the family are then (in GeV)

Υ	9.440 vs 9.46 ± 0.01 (Refs. 2 and 3),
$\Upsilon'_{A,B}$	10.037 vs 10.0 (Ref. 2),
$\Upsilon''_{A,B,C}$	10.601 vs 10.4 (Ref. 2).

The generic spectrum associated with Eq. (1) is shown in Fig. 1 with J^{PC} values for natural-parity states. This figure also displays, for the Υ family, a comparison of calculated values versus experimental.[3] Figures 2–5 enable similar easy comparisons[4,5] for psion, ρ, ω, and K^* families. Other $Q\bar{Q}$ families of experimental interest, such as the unnatural-parity families, currently all have less than three daughter candidates. The situation for these will be discussed later on in the text.

In this simple picture, charmed meson trajectories are also included. With the parameters given in Eq. (3), we find the following predictions (ignoring electromagnetic mass differences):

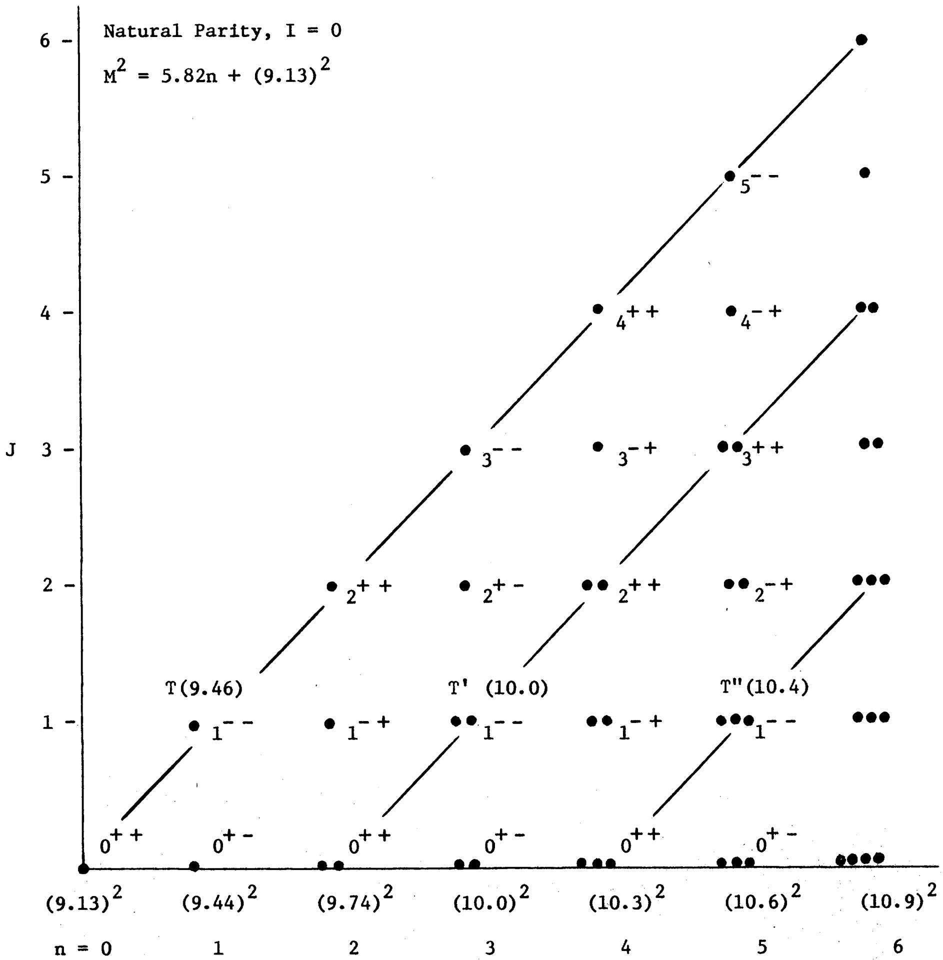

FIG. 1. Natural-parity states of the Υ family described by a simple mass formula with b-quark mass 4.393 12 GeV. Calculated masses in GeV lie below the horizontal axis for comparison with experimental values listed in parentheses following each assigned state. Predicted Regge slope is $1/5.82 \cong \frac{1}{6}$. Note that the approximate equality $M[\Upsilon'] - M[\Upsilon] \simeq M[\psi'] - M[\psi]$ in mass differences, not in mass-squared, for heavy-quark families is a consequence of the fact that Ω, the oscillator frequency, is universal. The procedure for fixing parameters of the model is explained in Ref. 1.

D 1.876 vs 1.866 (Ref. 6),

D^* 2.084 vs 2.007 (Ref. 6),

F 1.950 vs 2.040 ± 0.001 (Ref. 7),

F^* 2.156 vs 2.140 ± 0.060 (Ref. 7).

The corresponding mesons formed from the b quark with u, d, s, c can also be predicted:

$$\text{singlet}\left\{\begin{array}{ll}(b\bar{u})=(b\bar{d}) & =5.147\\ (b\bar{s}) & =5.200\\ (b\bar{c}) & =5.969\end{array}\right\}\text{pseudoscalar mesons}$$

$$\text{triplet}\left\{\begin{array}{ll}(b\bar{u})=(b\bar{d}) & =5.235\\ (b\bar{s}) & =5.291\\ (b\bar{c}) & =6.129\end{array}\right\}\text{vector mesons.}$$

Our mass formula[8] arose in a study of the parton picture[9] of composite particles. We argue that a quark in a bound state is no longer on-shell, and introduce an effective constant to describe how far the quark and the antiquark are off their individual mass shell, Eq. (2). Assuming an internal oscillator[10] binding force (see Sec. V), the constraint leads immediately to Eq. (1).

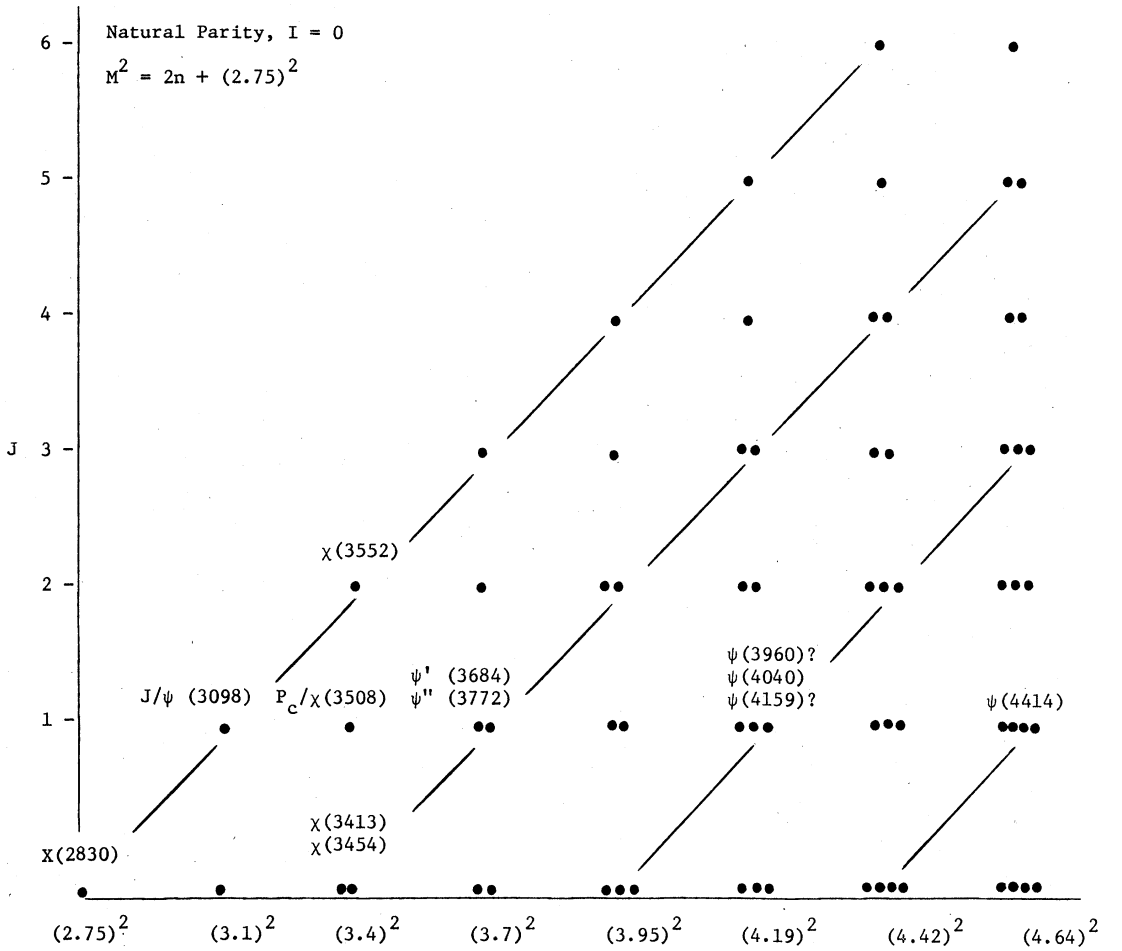

FIG. 2. Natural-parity states of the psion family described by simple mass formula with c-quark mass 1.509 67 GeV. Assignment of P_c/χ (3508) is not unique; it is shown as a 1^{-+} state but a 1^{++} assignment in unnatural-parity $c\bar{c}$ family is also allowed. Question marks denote states not well established.

II. Υ AND J/ψ FAMILIES

We begin with the phenomenology of our mass formula and reserve for Sec. V a discussion of its derivation. We turn first to the natural-parity states of the psion family because there exists here the largest number of well established candidates for daughter states, and because there is here remarkably good agreement[2] ($\Delta M^2/M^2 < 8\%$) with a simple description of these states by mass-squared linear Regge trajectories, with Regge slope equal to $\frac{1}{2}$, and anchored at J/ψ mass (compare Fig. 2). These equally spaced, mass-squared linear Regge trajectories with the existence of the $X(2830)$ psion are suggestive of the level scheme of an O(4) harmonic oscillator. The J^{PC} assignment and the increasing multiplicity of the lower-lying daughter states for the members of a family (compare Fig. 1), are based on O(4) wave functions which are symmetric, but are not necessarily traceless. The experimental discoveries of two approximately degenerate 1^{--} levels near 3.7 GeV, of two possibly scalar levels near 3.4, and of the multipeaked structure of R in the ~3.8 to ~4.2 region are suggestive of the larger family of symmetric O(4) tensor fields.

For example, with symmetric O(4) tensor fields for the natural-parity states displayed in Figs. 1 and 2, the multiplicities of the vector and scalar levels increase with mass according to

$$\begin{aligned} J^{PC} = 1^{--}&: \text{ multiplicity} = (n+1)/2\,, \\ J^{PC} = 0^{++}&: \text{ multiplicity} = (n+2)/2\,, \end{aligned} \tag{4}$$

where n is the oscillator quantum number for the level. It is easy to exhibit the associated O(4) wave functions:

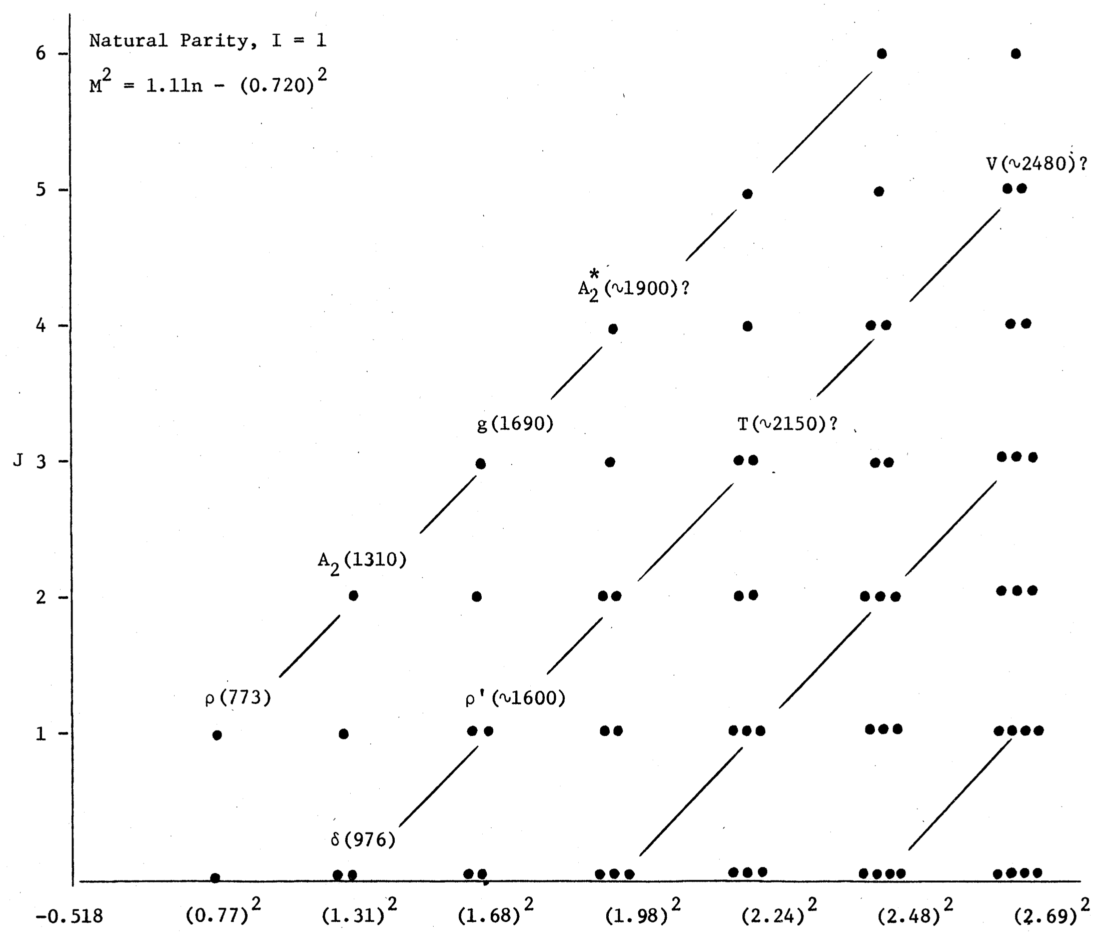

FIG. 3. Natural-parity, $I=1$ states of the ρ-A_2 family described by a simple mass formula with degenerate u and d quark mass 0.838 69 GeV. Question marks (see text) in this and the following graphs denote states not listed in "Meson Tables" of the 1976 compilation of Particle Data Group, Ref. 4.

$$\begin{aligned}
n=2,\quad &\phi_{2A}(\xi)=\frac{1}{2\sqrt{2\pi}}(\xi_\mu\xi_\nu-\tfrac{1}{4}\xi^2\delta_{\mu\nu})e^{-(1/4)\xi^2},\\
&\phi_{2B}(\xi)=\frac{1}{2\sqrt{2\pi}}\delta_{\mu\nu}(\tfrac{1}{4}\xi^2-2)e^{-(1/4)\xi^2},\\
n=3,\quad &\phi_{3A}(\xi)=\frac{1}{2\sqrt{6\pi}}[\xi_\mu\xi_\nu\xi_\lambda-\tfrac{1}{6}\xi^2(\delta_{\mu\nu}\xi_\lambda+\delta_{\mu\lambda}\xi_\nu+\delta_{\nu\lambda}\xi_\mu)]e^{-(1/4)\xi^2},\\
&\phi_{3B}(\xi)=\frac{1}{6\sqrt{6\pi}}(6-\tfrac{1}{2}\xi^2)(\delta_{\mu\nu}\xi_\lambda+\delta_{\mu\lambda}\xi_\nu+\delta_{\nu\lambda}\xi_\mu)e^{-(1/4)\xi^2}.
\end{aligned}\tag{5}$$

Experimentally,[11] very little is known about the $\chi(3454)$ since to date it has only been observed in the double-γ-ray cascade from the $\psi'(3684)$ to the J/ψ. As for the $X(2830)$, we wish to strongly emphasize the importance of a measurement of its parity.[12] Both of these states are somewhat problematic in the nonrelativistic charmonium approach, but both are naturally suggested by mass-squared linear Regge trajectories for a relativistic, $Q\overline{Q}$ composite system. For the ~3.8 to ~4.2 region, R is multipeaked and, ignoring the question of relative normalizations, the experiments' error bars are too large for a clean systematic determination of the number of states

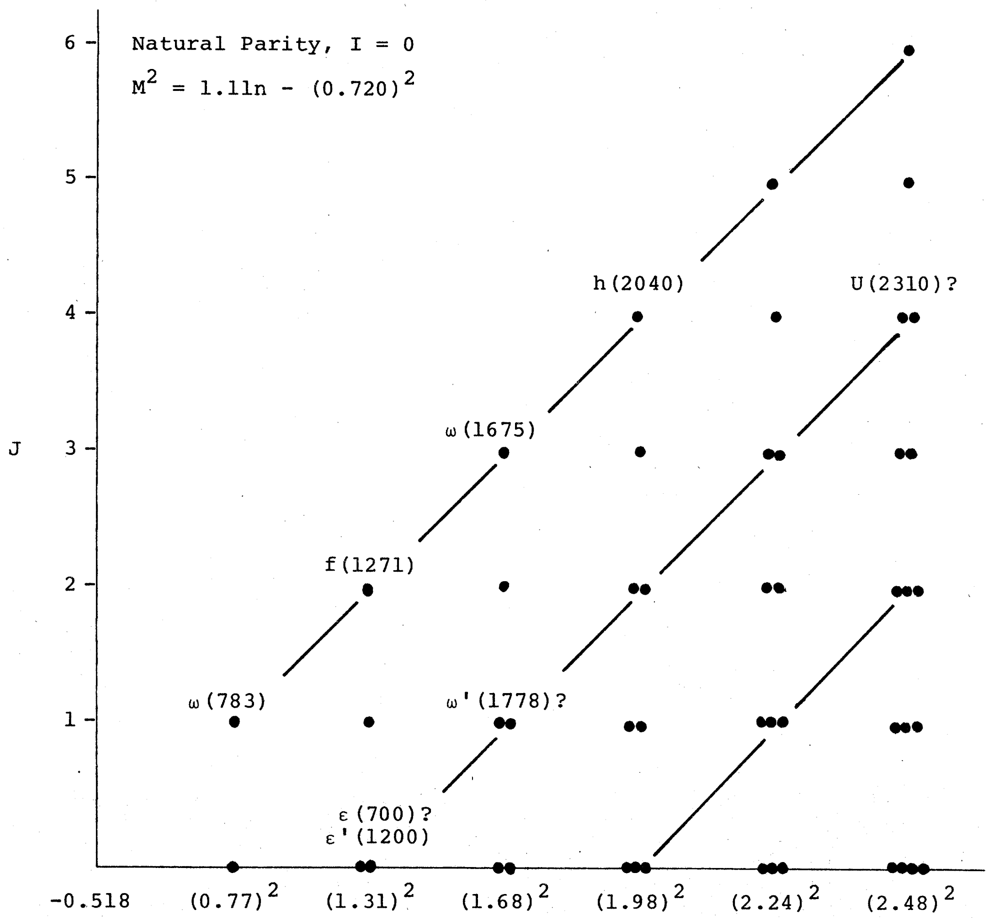

FIG. 4. Natural-parity, $I=1$ states of ω family.

present by combining the SLAC-LBL, PLUTO, and DASP data.[13] The $\psi(4.03)$ and $\psi(4.16)$ structures are consistent with the data points from all three groups. The $\psi(4.16)$ is most clearly resolved in the DASP data but, nevertheless, more structures might be present in this region. Only the SLAC-LBL data have many points from ~3.8 to ~4.0 and they suggest a $\psi(3.96)$. The question marks in Fig. 2 are to emphasize that one should wait before making any strong statements regarding the quantitative aspects of R from ~3.8 to ~4.2.

Next we return to the region of the χ states near 3.4. The assignment of the $P_c/\chi(3508)$ based on Eq. (1) is not unique, for although its charge-conjugation quantum number is +1, its parity has not yet been measured. Therefore, it could be either a 1^{-+} and assigned to an $n=2$ level of the natural-parity psion family as shown in Fig. 2, or be a 1^{++} and assigned to an $n=1$ level of an unnatural-parity psion family with $CP=+1$ for the leading trajectory. A 1^{++} state would be analogous to that for the A_1 family. Unfortunately, although the A_1 now seems better established,[14] the A_1 is broad with a width ~400 and with an associated uncertainty in its central position ~(1100–1200). Equation (1) is consistent with such a mass (see Sec. III), but because of the inherent uncertainties, we think one cannot reliably use Eq. (1) to make any quantitative prediction for the mass of the analogous $(c\bar{c})$, $J^{PC}=1^{++}$, except to note that such an assignment of the $\chi(3508)$ would not be inconsistent with Eq. (1). It would be prudent for experimentalists to measure, rather than infer in any theoretical model, the parity of the $\chi(3508)$. *Both* 1^{-+} and 1^{++} forbid decay into 2π, $K\bar{K}$. Both allow decay into $\pi K\bar{K}$. But 1^{++} allows decay into $(K^*\bar{K}+\bar{K}^*K)$ and $\delta\rho$ (where δ is $I=1$, $J^{PC}=0^{++}$), whereas 1^{-+} forbids these modes. Note that $K^*\bar{K}^*$ is allowed by both 1^{-+} and 1^{++}; its decay distribution could be used for a parity determination.[12]

There could, of course, also be another unnatural-parity family with $J^{PC}=0^{-+}, 1^{+-}, 2^{-+}, \ldots$ for the leading trajectory which would be the $(c\bar{c})$

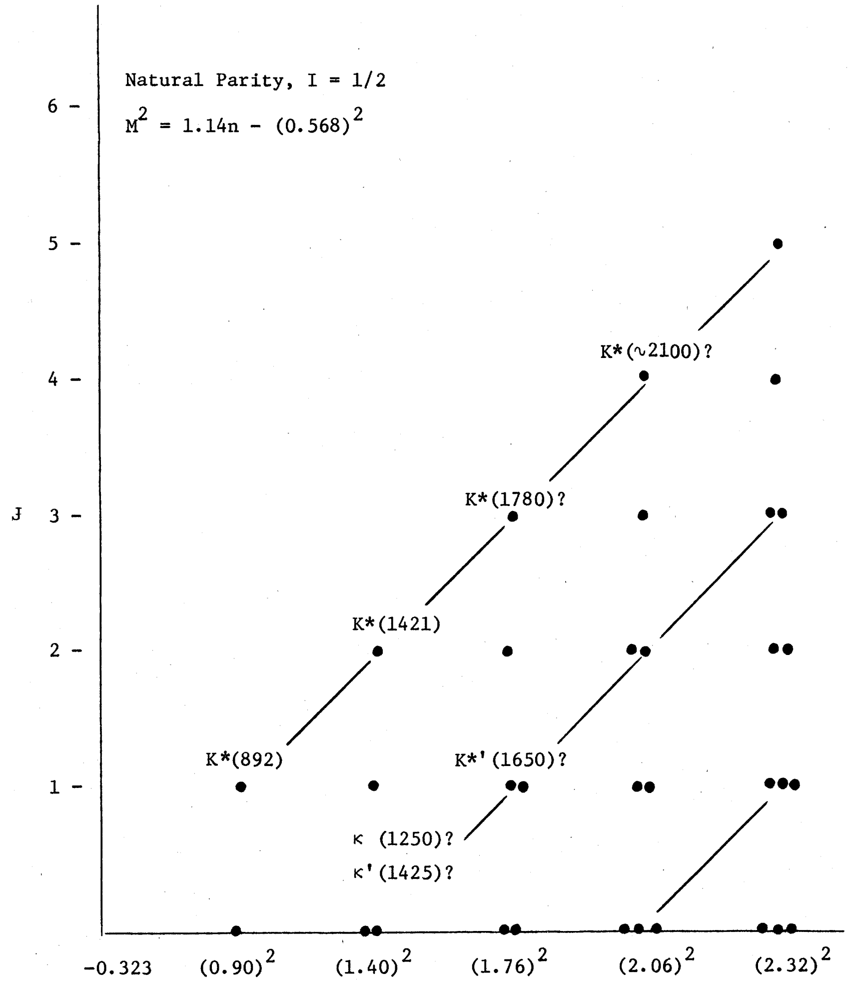

FIG. 5. Natural-parity, $I=\frac{1}{2}$ states of K^* family. States with question marks following are from Ref. 5.

analog of the η-η' family. Here Eq. (1) cannot be used to make quantitative predictions for anchoring ($c\bar{c}$) levels because of the large η-η' splitting and the related mixing ambiguity. However, the $n=0$, 0^{+-} and two $n=2, 0^{-+}$ may, respectively, lie near the J/ψ and $\psi'(3684)$. The 1^{+-} would be hard to observe.

The straightforward extrapolation to the Υ family is shown in Fig. 1. Here, because of unknown dynamics concerning higher-order splittings not included in Eq. (1), we think one must wait for more data from production of the resonances in this region in $e\bar{e}$ annihilation before drawing conclusions about aspects which are very sensitive to the interplay of different features in the dynamics and of the precise values of the parameters. Most of these 1^{--} states are not expected to have been seen to date. In particular, as in the psion family, e.g., compare the $\psi(3772)$ and $\psi'(3684)$, increasing hadronic widths and/or decreasing leptonic partial widths can obscure the presence of massive-vector-meson levels in hadroproduction and photoproduction. [While our study of the phenomenology of Eq. (1) was in progress, the fit of $m_b \simeq 3m_c$ and the lack of motivation for b in the present knowledge of the structure of weak processes, suggested to us (and see Ref. 15) that we interpret m_b not as a new fundamental quark of nature, but as a colorless block of three c quarks. However, an immediate numerological consequence would be a colorless (ccu) block and (cuu), (uuu) blocks which would give rise to new states at 8.15, 6.55, and 4.89 GeV. Since the $\Upsilon(9.48 \pm 0.01)$ width is presumably less than 10

MeV, these lower-lying states with such narrow widths would presumably have shown up if they were to exist.]

III. ORDINARY $Q\bar{Q}$ FAMILIES

We turn next to the ordinary $Q\bar{Q}$ families composed of u, d, and s quarks. It is well known that fits to the experimental data have been claimed by models based on nonrelativistic quark binding and also by linear Regge trajectories. Until the data with respect to higher excited states are clarified, the possibility remains that the low-lying mesons are relativistic bound states. Our mass formula belongs to this latter school of thought, and explores the fit among the low-lying states.

Frankly, we find no candidates for odd daughters. But, since it was not until 1976 that data for the ordinary mesons have established that even daughters exist, odd-daughter states—if they should indeed exist—may show up first in the new meson spectroscopy through γ-ray cascades from C-odd states.

A. Natural-parity-trajectory families

ρ and ω trajectories. In our picutre, the two trajectories are degenerate. The $Q\bar{Q}$ system is now in a triplet spin state. c_ρ, appropriately, is different from c_π [see Eq. (3)]. As we have pointed out in the Introduction, c is a measure of the binding effect on the spin nature, triplet or singlet, of the $Q\bar{Q}$ system. However, it should not depend on the flavor of the quarks.

The family of ρ trajectories here have a slope of $\alpha'_\rho = 0.9$, with the leading trajectory passing through $J=1$ at the ρ mass ($n=1$). By exchange degeneracy, the next state which occurs for $n=2$ has four states $J=2$, 1 and two $J=0$. The leading state, with $J=2$, can be identified with the A_2 meson, $J^{PC}=2^{++}$. The $\delta(976)$, with $I=1$, $J^{PC}=0^{++}$, is in the Meson Table of the Particle Data Group.[4] In Ref. 16, a structure has been recently reported in s-wave K^-K^0 10-GeV production data at ~1300 MeV with a width ~250 MeV. This candidate for the other 0^{++} is not displayed in Fig. 3. [This new structure is *not* the same as the $\delta'(1255)$ of Ref. 17 which was shown by Ref. 18 and by reanalysis to have $I=0$, and hence be consistent with the $\epsilon'(1200)$.] At the $n=3$ level, both the g and ρ' are well established. At still higher mass, there are the $A_2^*(1950)$, $K^G=1^-$, $J^P=4^+$, with a width of ~200 MeV,[19] and the resonances[20] suggested by using both polarization and differential-cross-section measurements in $p\bar{p}\to\pi^-\pi^+$, i.e., $\Upsilon(2150 \pm 30)$ with $\Gamma=200\pm 25$ MeV and $J^{PC}=3^{--}$, $I^G=1^+$ and $V(2480\pm 30)$ with $\Gamma=280\pm 25$ MeV, $J^{PC}=5^{--}$, $I^G=1^+$. Other high-mass states also may have been seen such as the bump[21] in diffractive 6π photoproduction which might be the $\rho''(\sim 2200)$.

The leading ω trajectory states in Fig. 4 for $n=1$ to 4 are from the Meson Tables, as is the $\epsilon'(1200)$ with $\Gamma\sim 200$ MeV. We have shown, despite its controversy, the $\epsilon(700)$ having a very broad Γ ~700 MeV which is suggested by the $\pi\pi$ phase shift which passes slowly through 90° in this region. We have listed this state in part because of the apparently similar phenomena[5] in the $K\pi$ system (see below). The recently discovered[22] $\omega'(1778)$ is the $I=0$ analog of the $\rho'(\sim 1600)$, and the $U(2310 \pm 30)$ with $\Gamma=210\pm 25$ MeV, $J^{PC}=4^{++}$, $I^G=0^+$ is another of the structures from the study[20] of $p\bar{p}\to\pi^+\pi^-$.

ϕ trajectory. Here we used the ϕ mass to determine m_s and find $M^2 = 1.165n - 0.1285$ in GeV^2 so the Regge slope is $\alpha' = 0.858$ GeV^{-2}. At the $n=2$ level, we find a mass of 1.49 GeV versus the $f'(1516)$; the scalar $S^*(993)$ which apparently couples dominantly to $K\bar{K}$ is assigned as one of the spin-zero daughters with the other yet to be found. The ϕ' is predicted to lie near 1.84 GeV and the ϕ'', near 2.39 GeV.

K^* trajectory. This is the last of the natural-parity trajectories for the ordinary $Q\bar{Q}$ mesons. The states shown with question marks in Fig. 5 are from Ref. 5. The $K^{*\prime}(1650\pm 50)$ has a width of 275 ± 50 MeV and is a $J^P=1^-$ in $K\pi$ phase-shift analysis of $K^\pm p$ collisions at 13 GeV/c. As expected, based on symmetric O(4) wave functions, there is an interesting double multiplicity of scalars in this s wave $K\pi$ phase-shift analysis with a very broad $\kappa(1250\pm 100)$, $\Gamma\sim 450$ MeV, and at higher mass another $\kappa'(1425\pm 10)$, $\Gamma\sim(250\pm 50$ MeV).

B. Unnatural-parity trajectory families

π and K trajectories. In our fit, we took as input, the pion (kaon) mass and used it to determine c_π (c_K). As our Eq. (3) reveals, the numerical fit to c_π and c_K shows a good but not exact agreement between them. For the π-B family we find $M^2 = 1.1111n + (0.138)^2$ in GeV^2, so $\alpha'_\pi = 0.9$ GeV^{-2} and for the K-Q_B family we find $M^2 = 1.137n + (0.495)^2$, so $\alpha'_K = 0.88$ GeV^{-2}. Thus, for the π trajectory we find 0.138, 1.06 [1.127], and 1.50 [1.587] GeV vs $\pi(138)$, $B(1228\pm 10)$ with $\Gamma=125\pm 10$ MeV and $A_3(1640)$ with $\Gamma\sim 300$ MeV. The values in the square brackets are for $\alpha'_\pi = 0.8$ GeV^{-2}, which would correspond to a value of Ω of 0.7452 GeV, instead of 0.6624, for the unnatural-parity states. This would imporve the agreement, but an extra variable is not clearly warranted since the number of unnatural-parity states with Regge recurrences are few and these are generally broad. For the K

trajectory, we find 0.495, 1.176 [1.235], and 1.587 [1.674] GeV to be compared with $K(495)$, $Q_B(1350)$, and $L(1765\pm 10)$ with $\Gamma = 140\pm 50$ MeV. Again from the isobar analysis of the $K\pi\pi$ final state,[5] there is a $J^P=0^-$ in $K(\pi\pi)_{s\,\text{wave}}$ at $K(1405\pm 15)$ with $\Gamma = 230\pm 20$ MeV, which would be one of the two 0^{-+} states predicted at the $n=2$ level.

A_1 trajectory. To avoid a light $J^{PC}=0^{--}$ state we anchor the family at zero mass so $M^2=1.111n$ GeV^2. This predicts an A_1 mass of 1.054 GeV versus[14] $A_1(\sim 1100-1200)$ with a broad $\Gamma\sim 400$ MeV. With a slope of $\alpha'=0.8$ GeV^{-2} and the observed mass, the 0^{--} mass-squared is negative. Clearly, here too little is known to draw any firm conclusions regarding implications for A_1-like members of the new meson families.

IV. CHARMED AND OTHER NEW-FLAVORED $Q\bar{Q}$ FAMILIES

In the Introduction we discussed the least massive pseudoscalar and vector composites containing a single c or a single b quark. Note that a progression of successively flatter Regge slopes for more massive new quark flavors does not follow from Eq. (1), except for $Q\bar{Q}$ families of hidden flavor. In fact, already $[\alpha'(u\bar{b})=0.783$ GeV$^{-2}]>[\alpha'(D^*)=0.757$ GeV$^{-2}]$ and $[\alpha'(s\bar{b})=0.743$ GeV$^{-2}]>[\alpha'(F^*)=0.726$ GeV$^{-2}]$. The tensor levels should lie at about

$$D^{**}\sim 2.380,\quad F^{**}\sim 2.455,$$
$$(u\bar{b})^{**}\sim 5.355,\quad (s\bar{b})^{**}\sim 5.417$$

(in GeV).

By lepton-quark symmetry, one might naively entertain the possibility of even more massive states such as a nominal $(t\bar{t})$ composite, $J^{PC}=1^{--}$, near 30 GeV (nominal input). By Eq. (3) we find $m_t=14.6$ GeV, $1/\alpha'_t=19.3$ GeV^2, and approximately *equal-mass spaced* $J^{PC}=1^{--}$ excitations. For the flavored levels, the pseudoscalar $(t\bar{b})_P=15.38$ GeV and the vector $(t\bar{b})_V=15.4$ GeV are approximately degenerate. The Regge slope is again greater with $\alpha'(t\bar{u})=0.854$ GeV^{-2}. This slope approaches the limiting value of $(\frac{1}{2}m_u\Omega)=\alpha'_\rho$ as the mass of m_t is increased. The universality of Ω, the oscillator frequency, again implies an approximate equality in mass differences, $M[(t\bar{t})']-M[(t\bar{t})]\simeq M[\psi']-M[\psi]$. The mass difference in this approximation is simply $2\Omega m(\text{quark})/M[(Q\bar{Q})]$ which by our mass formula approaches Ω (0.6624) GeV as $m(\text{quark})\to\infty$.

The apparent systematics of ρ, ϕ, psion, and Υ trajectories suggest that the progression of trajectory slopes is $\Gamma(1)$, $\Gamma(2)$, $\Gamma(3)$, $\Gamma(4),\ldots$, so by our mass formula, the m_t mass would then be about 18 GeV with the lowest $(t\bar{t})$ vector state at about 37 GeV $[\Gamma(n+1)=n!]$.

V. SIMPLE MASS FORMULA FOR $Q\bar{Q}$ FAMILIES

We turn now to the derivation of the mass formula. Consider the wave function of a particle with structure

$$\psi=e^{iP\cdot x}\,\phi_n(\xi)\,,$$

where $(m\equiv m_1+m_2)$

$$x_\mu=\frac{m_1}{m}x_{1\mu}+\frac{m_2}{m}x_{2\mu}\,,$$
$$\xi_\mu=(x_1-x_2)_\mu\,. \tag{6}$$

$\phi_n(\xi)$ is the complete set of solutions for the reduced mass problem with the potential taken here to be an internal oscillator. The equation for ϕ_n reads $(\mu\equiv m_1m_2/m)$

$$\left(-\frac{1}{2\mu}\frac{\partial}{\partial\xi}\cdot\frac{\partial}{\partial\xi}+\frac{\mu\Omega^2}{2}\,\xi\cdot\xi\right)\phi_n(\xi)=(n+2)\Omega\,\phi_n(\xi)\,, \tag{7}$$

where $\xi\cdot\xi\equiv\vec{\xi}\cdot\vec{\xi}+(\xi_4)^2$.

In Eqs. (6) and (7), ξ is an internal coordinate. As is well known, keeping $\xi_0(\equiv -i\xi_4)$ real leads to non-normalizable wave functions. The conventional way out is to impose constraints on Eq. (7) which suppress the timelike modes. The resulting spectrum and degeneracy of energy levels for such a conventional approach usually conform with the general pattern of energy levels of a nonrelativistic quark model and is certainly different from the psion family assignment we have made.

We have taken advantage of the fact that ξ_μ is *not* an observable and simply assert that ξ_μ appearing in Eqs. (6) and (7) is an internal coordinate, with $\vec{\xi}$ and ξ_4 all real. The quark time coordinates t_1 and t_2 are also not observables and in our formalism are in fact not Hermitian. As we now show, this does not contradict the observability of parton distributions.

In a bound state, the quark and antiquark must be off-shell. With respect to our overall wave function, we have $(m\equiv m_1+m_2)$

$$\frac{1}{i}\frac{\partial}{\partial x_{1\mu}}\psi=\frac{m_1}{m}P_\mu\psi+\frac{1}{i}e^{iP\cdot x}\frac{\partial}{\partial\xi_\mu}\phi_n(\xi)\,.$$

Therefore,

$$(p_1\cdot p_1+p_2\cdot p_2)\psi=\frac{m_1^{\,2}+m_2^{\,2}}{m^2}(P\cdot P)\psi+[4(n+2)\mu\Omega-2\mu^2\Omega^2\xi\cdot\xi]\psi\,. \tag{8}$$

For an oscillator, endowed with a proper Hilbert space,

$$\langle \mu\Omega\xi\cdot\xi\rangle = (n+2). \tag{9}$$

Therefore, Eq. (8) becomes

$$\frac{m_1^2+m_2^2}{m^2}(-M^2)+2(n+2)\frac{(m_1m_2)}{m}\Omega+m_1^2+m_2^2=c \tag{10}$$

and our mass formula results.

The Hilbert space, with respect to which $\phi_n(\xi)$ is normalizable, is the O(4) ξ_μ space (i.e., ξ_i and ξ_4 all real) with measure

$$d^4\xi \equiv d\xi_1 d\xi_2 d\xi_3 d\xi_4$$

and the inner product is, as usual,

$$\int d^4\xi\, \phi_f^*(\xi)\phi_i(\xi).$$

As a consequence, while the individual quark three-momentum is Hermitian, the quark energy operator is strictly not Hermitian. The non-Hermitian part has precisely to do with the binding effects. In the infinite-momentum limit

$$p_0^{(1)} = -\frac{1}{i}\frac{\partial}{\partial t^{(1)}} = \frac{m_1}{m}E - \frac{\partial}{\partial\xi_4} \to \frac{m_1}{m}E,$$

the quark energy becomes proportional to p_z and is the "measured" quantity in the parton picture.

Strictly speaking, what is measured in deep-inelastic scattering is not directly the individual quark momenta $p_\mu^{(1)}$, $p_\mu^{(2)}$, but the expectation values

$$\sum_n \langle i|j_\mu(0)|n\rangle\delta(p_i+q-p_n)\langle n|j_\nu(0)|i\rangle,$$

and the only requirement is that these quantities be real and Lorentz covariant. As we have shown in our work[2] on psions, current matrix elements with respect to the O(4) Hilvert space are both real and Lorentz covariant.

The approach we have taken here, of an imaginary ξ_0, is somewhat akin in spirit to the Wick-rotated Bethe-Salpeter approach of Bohm, Joos, and Krammer (see Ref. 10), who have also made use of O(4) classification of levels. In our assignment the meson family belongs to the complete set of symmetric O(4) tensors, which are to be distinguished from other spinorial representations of O(4). The very simple alternating J^{PC} quantum numbers (0^{++}, 1^{--}, 2^{++}, ..., along the leading trajectory) fit very nicely in this symmetric tensor representation. In this regard, the spin-parity determination for $X(2830)$ is absolutely vital, if nothing more than to confirm the nonrelativistic nature of charmonium.

ACKNOWLEDGMENTS

The work of one of us (N.P.C.) was partially supported by the National Science Foundation under Grant No. 77-01350. The work of (C.A.N.) was partially supported by a SUNY Research Foundation Award and partially by the National Science Foundation under Grant No. 78-09617.

APPENDIX A

In the language of nonrelativistic physics, it is well known that $J^{PC}=0^{--}$, and the series 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, ..., cannot be formed out of a quark-antiquark pair. For relativistic bound-state systems, however, this is no longer true, as Thirring[22] and Low[23] have already observed. Because this is apparently not so well known, we include a detailed argument here pointing out how this is possible.

For purposes of comparison, first we review the argument for the selection rule in nonrelativistic physics. Consider the $Q\overline{Q}$ bound state, in the center-of-mass frame, with, say, spin $J=0$ for simplicity. Rotational invariance implies that

$$|\psi(J=0),\vec{P}=0\rangle = \int d^3p\,d^3p'\,\delta(\vec{p}+\vec{p}\,')[f(\vec{p},\vec{p}\,')+\vec{\sigma}\cdot(\vec{p}-\vec{p}\,')g(\vec{p},\vec{p}\,')]_{st'}C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}\,',t)|0\rangle, \tag{A1}$$

where f, g are rotationally invariant functions of the argument

$$(\vec{p}-\vec{p}\,')^2.$$

Under $\mathfrak{C}$, the state transforms into

$$\mathfrak{C}|\psi(J=0),\vec{P}=0\rangle = \int d^3p\,d^3p'\,\delta(\vec{p}+\vec{p}\,')[f(\vec{p}\,',\vec{p})+\vec{\sigma}\cdot(\vec{p}-\vec{p}\,')g(\vec{p}\,',\vec{p})]_{st'}C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}\,',t)|0\rangle, \tag{A2}$$

where we have used the anticommuting property of $b^\dagger$, $d^\dagger$ and

$$C\vec{\sigma}C^\dagger = -\vec{\sigma}^*. \tag{A3}$$

Now the crucial point of the argument in nonrelativistic physics is that f, g are functions only of $(\vec{p}-\vec{p}\,')^2$

and hence are automatically symmetric under exchange of $\vec{p}$ with $\vec{p}'$ and we have the cases

$$\begin{aligned}&\text{(I) } f=0:\ J=0,\ L=1,\ S=1,\ C=+,\ P=+\\&\text{(II) } g=0:\ J=0,\ L=0,\ S=0,\ C=+,\ P=-.\end{aligned} \tag{A4}$$

For relativistic physics, the Hamiltonian contains particle-creation terms and the state vector for a bound state consists of, in addition to the original $Q\overline{Q}$ state, an infinite sequence of $Q\overline{Q}$ plus gluons, as well as $Q\overline{Q}$ with a virtual sea of $Q\overline{Q}$ pairs. The $Q\overline{Q}$ state by itself is not an eigenvector of H, but in current folklore, it is a good choice as a first approximation. We write

$$|\psi(J=0),\vec{P}=0\rangle=\int\frac{d^3p\,d^3p'}{\omega\omega'}\,\delta(\vec{p}+\vec{p}')[f(\vec{p},\vec{p}',\omega,\omega')+\vec{\sigma}\cdot(\vec{p}-\vec{p}')g(\vec{p},\vec{p}',\omega,\omega')]_{st'}C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}',t)|0\rangle+\cdots, \tag{A5}$$

where $\cdots$ represents the other virtual states, and f, g are now functions of the arguments

$$(\vec{p}-\vec{p}')^2,\ \omega,\omega'.$$

Under $\mathcal{C}$, the state transforms into

$$\mathcal{C}|\psi(J=0),\vec{P}=0\rangle=\int\frac{d^3p\,d^3p'}{\omega\omega'}\,\delta(\vec{p}+\vec{p}')[f(\vec{p}',\vec{p},\omega',\omega)+\vec{\sigma}\cdot(\vec{p}-\vec{p}')g(\vec{p}',\vec{p},\omega',\omega)]_{st'}C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}',t)|0\rangle+\cdots. \tag{A6}$$

The point is, since f and g depend on additional variables ω, ω', symmetry under $\vec{p},\vec{p}'$ exchange no longer automatically follows. (See Thirring, Ref. 22.) Thus, for relativistic physics, we can have, in addition to the states (I) and (II), the states

$$\begin{aligned}&\text{(III) } f=0,\ g \text{ antisymmetric}:\ J=0,\ L=1,\ S=1,\ C=-,\ P=+\\&\text{(IV) } g=0,\ f \text{ antisymmetric}:\ J=0,\ L=0,\ S=0,\ C=-,\ P=-.\end{aligned} \tag{A7}$$

To illustrate further, consider a $J=1$ state,

$$\begin{aligned}|J=1,\ \vec{P}=0,\ m_J=\lambda\rangle=\epsilon_i(\lambda)\int\frac{d^3p\,d^3p'}{\omega\omega'}\delta(\vec{p}+\vec{p}')[\sigma_iF+(p-p')_iG+(p-p')_i\vec{\sigma}\cdot(\vec{p}-\vec{p}')H+\epsilon_{ijk}(p-p')_j\sigma_kI]_{st'}\\ \times C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}',t)|0\rangle+\cdots,\end{aligned} \tag{A8}$$

$$\begin{aligned}\mathcal{C}|J=1,\ \vec{P}=0,m_J=\lambda\rangle=\epsilon_i(\lambda)\int\frac{d^3p\,d^3p'}{\omega\omega'}\,\delta(\vec{p}+\vec{p}')[-\sigma_i\tilde{F}-(p-p')_i\tilde{G}-(p-p')_i\vec{\sigma}\cdot(\vec{p}-\vec{p}')\tilde{H}+\epsilon_{ijk}(p-p')_j\sigma_k\tilde{I}]_{st'}\\ \times C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}',t)|0\rangle+\cdots,\end{aligned} \tag{A9}$$

where F, $G,\ldots$ are functions of $(\vec{p}-\vec{p}')^2$, ω, ω' and $\tilde{F}$, $\tilde{G},\ldots$ denote the functions obtained by swapping $\vec{p}$ and $\vec{p}'$. Under $\mathcal{P}$, parity inversion, we have

$$\begin{aligned}\mathcal{P}|J=1,\ \vec{P}=0,m_J=\lambda\rangle=\epsilon_i(\lambda)\int\frac{d^3p\,d^3p'}{\omega\omega'}\,\delta(\vec{p}+\vec{p}')[-\sigma_iF+(p-p')_iG-(p-p')_i\vec{\sigma}\cdot(\vec{p}-\vec{p}')H+\epsilon_{ijk}(p-p')_j\sigma_kI]_{st'}\\ \times C_{t't}b^\dagger(\vec{p},s)d^\dagger(\vec{p}',t)|0\rangle+\cdots.\end{aligned} \tag{A10}$$

For relativistic physics, the exotic 1^{-+} state can arise in either of the following cases:

$$\begin{aligned}&\text{(I) } G=H=I=0,\ F \text{ antisymmetric}:\ J=1,\ L=0,\ S=1,\ C=+,\ P=-\\&\text{(II) } F=G=I=0,\ H \text{ antisymmetric}:\ J=1,\ L=2,\ S=1,\ C=+,\ P=-.\end{aligned} \tag{A11}$$

Generalization of these arguments to higher spins is self-evident, and it is clear how the exotic states can be obtained in $Q\overline{Q}$ dynamics in relativistic physics.

APPENDIX B

A careful distinction must be made between the bound-state $Q\overline{Q}$ system and a *free* $Q\overline{Q}$ state vector. The state vectors (A5) and (A8) are not manifestly Lorentz covariant. Under a boost, the virtual gluon and $Q\overline{Q}$ sea content of the state is affected. On the other hand, a free $Q\overline{Q}$ state, under a boost, transforms into itself.

To any finite order in perturbation theory, there exists the following remarkable theorem:

Theorem. Free $Q\overline{Q}$ states do not couple to exotic mesons (0^{--}, 0^{+-}, $1^{-+}, \ldots$). We shall prove this explicitly for 1^{-+}, although the generalization to other exotic J^{PC} is obvious.

A free $Q\overline{Q}$ state is a scattering state. The $J=1$ projection of the state may be written

$$\begin{aligned} |J=1,\ P_\mu\rangle = \int \frac{d^3p\, d^3p'}{\omega\omega'}\, \delta^{(4)}(p+p'-P)\overline{u}(p,s) \\ \times [\gamma_\lambda F_1 + \sigma_{\lambda\rho}P_\rho F_2 + i(p-p')_\lambda F_3] \\ \times v(p',t) b^\dagger(\vec{p},s) d^\dagger(\vec{p}',t)|0\rangle . \end{aligned} \tag{B1}$$

Note that, in contrast with (A8), the energy-conserving δ function appears in Eq. (B1). The $C=+$ quantum number for the state would result if

$$F_i(-p^2, -p'^2, -(p+p')^2) = -F_i(-p'^2, -p^2, -(p+p')^2). \tag{B2}$$

Clearly, no point coupling of 1^{-+} to a free $Q\overline{Q}$ state can satisfy Eq. (B2).[23] The question then is whether induced couplings,[24] through higher-order effects, can satisfy Eq. (B2).

To any finite order in perturbation theory, the domain of holomorphy for the vertex functions considered as a function of the three complex variables

$$\begin{aligned} z_1 &= -p^2 , \\ z_2 &= -p'^2 , \\ z_3 &= -P^2 \end{aligned} \tag{B3}$$

includes the region[25]

$$\mathrm{Im} z_i > 0, \quad i=1,2,3 . \tag{B4}$$

Condition (B2) is valid within the domain of holomorphy of the function, and therefore, holds as an equation for the analytic function

$$F_i(z_1, z_2, z_3) = -F_i(z_2, z_1, z_3) . \tag{B5}$$

Then for $z_1 = z_2$, F_i must vanish in the upper half z_1, z_3 planes. Since the physical vertex function is the limit from the upper half planes, the on-shell vertex function must also vanish. Q.E.D.

We remark that the domain of holomorphy, based on axiomatic field theory, as derived by Källen and Wightman,[26] is actually smaller than that given by perturbation theory. For fixed y_1, $y_2 > 0$ ($z_k = x_k + iy_k$, $k=1,2,3$), the domain of holomorphy given in (B4) is reduced by a new boundary ($\mathfrak{F}$) above the z_3 real positive axis, where

$$\begin{aligned} \mathfrak{F}:\ z_1z_2 + z_2z_3 + z_1z_3 - \rho(z_1+z_2+z_3) + \rho^2 = 0 , \\ \rho_1 < \rho < \infty, \quad y_1y_2 \geq 0, \quad y_1y_3 \geq 0 , \end{aligned} \tag{B6}$$

with

$$\rho_1 = \frac{(x_1^2+y_1^2)y_2 + (x_2^2+y_2^2)y_1}{x_1y_2 + x_2y_1} . \tag{B7}$$

For $\rho = \rho_1$, $\mathfrak{F}$ crosses the real positive z_3 axis at the point P'.

For the point $z_1 = z_2 = m^2 + i\epsilon$, the point P' is

$$\mathrm{Re}\ z_3 = \epsilon^2/m^2, \quad \mathrm{Im}\ z_3 = 0 , \tag{B8}$$

and the point on the $\mathfrak{F}$ curve where Re $z_3 = M^2$ (the 1^{-+} meson mass squared) will have

$$\mathrm{Im}\ z_3 = m^4/2\epsilon \tag{B9}$$

[the parameter ρ at that point is $\rho = 2m^2 + (4(M^2-m^2)/m^4)\epsilon^2$]. From axiomatics alone, therefore, the limit $z_1 = z_2 = m^2 + i\epsilon$, $z_3 = M^2 + i\epsilon$, as $\epsilon \to 0$, is *not* in their domain of holomorphy. The theorem, then, does not follow. It is an open question whether a sum over an infinite series of perturbation graphs can produce a singularity on the $\mathfrak{F}$ curve, invalidating the theorem.

For free $\pi^+\pi^-$, $K^+K^-, \ldots$ states there is a similar result to any finite order in perturbation theory:

Theorem. Free $\pi^+\pi^-$, $K^+K^-, \ldots$ states do not couple to exotic meson series 0^{+-}, 1^{-+}, $2^{+-}, \ldots$.

APPENDIX C

Based on the perturbation-theory theorem, we can conclude that exotic J^{PC} states cannot be observed in a formation experiment in $N\overline{N}$ scattering:

$$N\overline{N} \not\to \text{exotic } J^{PC} \to \text{anything.}$$

This signature is also shared by the so-called M-baryonium states[27] in which the diquarks are in the color 6 representation. Their argument for the decoupling of M-baryonium states from $N\overline{N}$ relies on the relative difficulty of producing three $q\overline{q}$ pairs from vacuum polarization. However, if the J^{PC} quantum numbers of the M-baryonium states are in the sequence (0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, $3^{-+}, \ldots$) the decoupling is in fact rigorous, within the context of finite-order perturbation theory.

The decoupling theorem does not imply that exotic J^{PC} states *totally* decouple from all hadronic physics. They couple to other free decay channels, e.g., in the case of 1^{-+}, we have the allowed point couplings ($\mathfrak{V}_\mu$ is the 1^{-+} field)

$$\begin{aligned} &\mathfrak{V}_\mu \eta \overleftrightarrow{\partial}_\mu \eta' , \\ &\partial_\nu \mathfrak{V}_\mu (\overline{K}_\nu^* K_\mu^* + \overline{K}_\mu^* K_\nu^*) , \\ &\text{etc.} \end{aligned} \tag{C1}$$

While these exotic states cannot be formed in $N\overline{N}$ or $e\overline{e}$ collisions, they can either be produced in association with other mesons or, as in the case of our identification of $P_c/\chi(3.51)$ with the 1^{-+} state, it can be produced as a result of the cascade decay of a parent nonexotic meson.

APPENDIX D

Finally we comment again on the relativistic $Q\bar{Q}$ bound-state system in the exotic J^{PC} series, particularly in view of the decoupling theorem for free $Q\bar{Q}$ states. As the theorem implies, a point coupling such as

$$\mathcal{V}_\mu \bar{\psi}\sigma_{\mu\nu}\overleftrightarrow{\partial}_\nu \psi \tag{D1}$$

vanishes on-shell, since for *free* Dirac fields it can be shown that the coupling is equivalent to

$$\partial_\mu \mathcal{V}_\mu \bar{\psi}\psi \, . \tag{D2}$$

However, for *bound* Dirac fields, the proof fails because Ψ no longer satisfies the free Dirac equation. The 1^{-+} does not decouple from a bound $Q\bar{Q}$ system.

To conclude, it must be pointed out that all our remarks in these Appendixes go towards showing that exotic states are *not* forbidden by present theoretical understanding. We do not have an example of a bona fide relativistic bound-state field theory which exhibits it, although exotic states have been found in earlier studies of Bethe-Salpeter equations.[28]

In the model of Bohm, Joos, and Krammer, they found the exotic $1^{-+}, \ldots$ solutions which explicitly decouple *on shell*, but are present in the bound-state Bethe-Salpeter amplitude.[29]

In the S-matrix theory, the decoupling theorem for these anomalous states does not of course mean a total decoupling from physical Hilbert space. They can couple to on-shell $Q\bar{Q}$ + gluon, $Q\bar{Q}Q\bar{Q}$ states, etc. Examples of such are the $\eta\eta'$ decay modes already cited in Appendix C.

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[1] The seven parameters in our fit were determined as follows: The ρ and J/ψ trajectories $M(\rho)^2 = (1/0.9)n - 0.518$ and $M(J/\psi)^2 = 2n + (2.75)^2$ in GeV units fix m_c, m_u, Ω, and c(triplet). The Υ mass-squared splitting $M(\Upsilon')^2 - M(\Upsilon)^2 = 11.64$ fixes m_b and $\phi(1.0197)$ mass fixes m_s. The K and π masses fix c_K and c_π. Only for the very light π with $M(\pi)^2 = 0.019$ is the 0.6% distinction between c_π and c_K relevant; otherwise this distinction should be ignored.

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