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Radii of the pions and the kaons

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The radii of the pions and the kaons are computed with the geometrical model of hadron-hadron collisions using the recently published meson-proton scattering data at 200 GeV.

We report in this article some new results for the radii of the pions and the kaons calculated with the geometrical model[1] of hadron-hadron collisions. The present calculation is based on the final published data[2] for elastic meson-proton scattering performed by the Michigan-Fermilab-Argonne-Indiana group. Similar analysis of meson form factors using unpublished preliminary curves from the same experiment has been reported[3] earlier, which led to the speculation that the matter distributions for different mesons might have the same shape. Since the input experimental meson-proton data used in this paper are more accurate, which differ considerably from the preliminary version, we believe our new analysis is useful and results are more reliable. Comparison of theoretical values for the meson radii with experiments is now possible. A direct measurement[4] of the pion radius by electron scattering has been published by the UCLA–Notre Dame–Pittsburgh–Fermilab–Dubna group. The same collaboration has been carrying out at Fermilab a kaon experiment and a remeasurement of the pion radius at a higher energy. Their preliminary result[5] on the kaon experiment was recently reported at the Tokyo conference. It is worth noting that our newly calculated values[6] for meson radii are compatible with results of the electron scattering experiments, but are in disagreement with the electroproduction experiments.[7] The disparity of results in direct and indirect measurements is disturbing and points to the need for more precise experiments in the future.

In the geometrical model the differential cross section for elastic scattering of hadrons A and B is given, in the eikonal approximation, by

$$\frac{d\sigma}{dt} = \pi |a_{AB}|^2 ,$$

$$a_{AB} = \{1 - S_{AB}(b)\}, \tag{1}$$

$$= (2\pi)^{-1} \int (1 - S_{AB}) e^{i\vec{q}\cdot\vec{b}} d^2 b ,$$

where the curly bracket denotes the Fourier transform from the two-dimensional impact-parameter $\vec{b}$ space to the two-dimensional momentum-transfer $\vec{q}$ space.

The elastic S-matrix element $S_{AB}(b)$ is related[1] to the opaqueness $\Omega_{AB}(b)$ at impact parameter b by

$$S_{AB}(b) = \exp[-\Omega_{AB}(b)]. \tag{2}$$

It has been further postulated[1] that the opaqueness Ω is proportional to the convolution of the compressed hadronic matter density functions D of the colliding particles. Thus,

$$\Omega_{AB} = (\text{constant}) \times D_A * D_B$$

or, in the momentum-transfer space,

$$\{\Omega_{AB}\} = (\text{constant}) \times F_A F_B , \tag{3}$$

where F_A and F_B are the hadronic form factors for particles A and B, respectively.

Expanding form factors in powers of q^2,

$$F(q^2) = 1 - \tfrac{1}{6}\langle r^2 \rangle q^2 + \cdots , \tag{4}$$

we obtain

$$\{\Omega_{AB}\} = (\text{constant}) \times [1 - \tfrac{1}{6}(\langle r_A{}^2\rangle + \langle r_B{}^2\rangle) q^2 + \cdots]. \tag{5}$$

Hence

$$\langle r_A{}^2\rangle + \langle r_B{}^2\rangle = \frac{3}{2} \frac{\int \Omega_{AB} b^2 d^2 b}{\int \Omega_{AB} d^2 b} . \tag{6}$$

If Ω is assumed to be real, and the spin-dependent part of the scattering amplitude is neglected, we can use (1) and (2) to evaluate the opaqueness of the AB system from experimental differential-cross-section data. The sum of mean-square radii for the colliding particles is then obtainable from (6). It should be remarked that the calculation up to this stage involves no uncertainties and has no adjustable parameters.

The experimental pp, $\pi^{\pm}p$, and $K^{\pm}p$ data at 200 GeV used in our calculation were taken from Ref. 2 by Akerlof *et al.* To avoid possible bias in our analysis, we adopt the parametrization and numerical fits to the differential-cross-section data provided by these authors in their published paper.

TABLE I. Parameters for a fit to 200-GeV differential-cross-section data.

	A (mb GeV^{-2})	b (GeV^{-2})	c (GeV^{-4})
π^+p	28.77 ± 0.22	9.25 ± 0.12	1.97 ± 0.14
π^-p	30.12 ± 0.15	9.26 ± 0.06	2.00 ± 0.05
K^+p	20.11 ± 0.20	8.13 ± 0.37	1.47 ± 0.48
K^-p	22.19 ± 0.19	8.79 ± 0.53	2.21 ± 0.72
pp	77.31 ± 0.24	10.73 ± 0.12	0.82 ± 0.17

The $d\sigma/dt$ in the region of $0 < |t| < 1.3$ GeV^2 has been fitted[2] in the form

$$\frac{d\sigma}{dt} = A \exp(bt + ct^2), \tag{7}$$

where parameters A, b, and c for various collisions were given in Ref. 2 and are reproduced here in Table I. For $-t > 1.3$ GeV^2, where no experimental information is available for meson-proton scattering, we assume an exponential fall for $d\sigma/dt$. In the case of pp scattering, since large-t experiments have indicated the existence of a dip near $-t = 1.5$ GeV^2 at 200-GeV incident energy, we shall assume the pp scattering amplitude changes sign at the dip position and then falls off exponentially.

Following the procedures outlined above, the sums of mean-square radii for particles involved in various collisions can be numerically calculated. Results are exhibited in Table II.

To extract the meson radii we need to know the proton radius. There are two alternative values we can use for the radius of proton:

(i) We may take the value computed from the 200-GeV pp data, i.e., from the last line in Table II,

$$\langle r_p^2 \rangle = 0.541 \pm 0.009 \text{ F}^2.$$

The calculated meson radii are exhibited in Table III.

(ii) If, instead, the charge radius of the proton as measured by ep scattering is used to separate the meson radius, i.e., we adopt

$$\langle r_p^2 \rangle = \frac{12}{0.71} \text{ GeV}^{-2} = 0.658 \text{ F}^2$$

TABLE II. Sums of mean-square radii for colliding hadrons.

AB collisions	$\langle r_A^2 \rangle + \langle r_B^2 \rangle$ (F^2)
π^+p	0.973 ± 0.018
π^-p	0.970 ± 0.009
K^+p	0.862 ± 0.057
K^-p	0.928 ± 0.084
pp	1.081 ± 0.019

TABLE III. Mean-square radii and rms radii of the proton, π, and K mesons. The proton radius is obtained from 200-GeV pp data.

	$\langle r^2 \rangle$ (F^2)	$\langle r^2 \rangle^{1/2}$ (F)
π^+	0.432 ± 0.020	0.657 ± 0.015
π^-	0.429 ± 0.013	0.655 ± 0.010
K^+	0.321 ± 0.058	0.567 ± 0.051
K^-	0.387 ± 0.085	0.622 ± 0.068
p	0.541 ± 0.009	0.735 ± 0.006

evaluated from the dipole form of the proton form factor, then we obtain the result listed in Table IV.

The first alternative appears logically more consistent, yet the results of the second approach are in better agreement with experimental values measured in direct electron scattering experiments. For a comparison of theoretical predictions and experimental values, see Table V.

We now make a few observations:

(1) Our calculation shows that π^+ and π^- have about the same size. The equality of K^+ and K^- mean-square radii also holds within the errors,

$$\langle r_{K^-}^2 \rangle - \langle r_{K^+}^2 \rangle = 0.065 \pm 0.102 \text{ F}^2.$$

(2) The average kaon radius appears to be slightly smaller than the pion. From Table II, we have

$$\langle r_{\pi^-}^2 \rangle - \langle r_{K^-}^2 \rangle = 0.042 \pm 0.085 \text{ F}^2,$$

$$\langle r_{\pi^+}^2 \rangle - \langle r_{K^+}^2 \rangle = 0.111 \pm 0.060 \text{ F}^2.$$

However, because of the error bars for $K^\pm$, the interesting possibility[3] that the pions and the kaons have equal radius may still be valid.

(3) The meson root-mean-square radii given in Table III are uniformly larger than that in Table IV by about 0.1 F. This discrepancy is of course due to the different values used for the proton radius in deducing the meson radius. At CERN ISR energies, the discrepancy would disappear, because previous investigations[8] have shown that the

TABLE IV. Mean-square radii and rms radii of the proton, π, and K mesons. The proton radius is obtained from the ep scattering.

	$\langle r^2 \rangle$ (F^2)	$\langle r^2 \rangle^{1/2}$ (F)
π^+	0.314 ± 0.018	0.561 ± 0.016
π^-	0.312 ± 0.009	0.559 ± 0.008
K^+	0.204 ± 0.057	0.451 ± 0.063
K^-	0.270 ± 0.084	0.520 ± 0.081
p	(0.658)	(0.811)

TABLE V. Comparison of theoretical and experimental values for meson radius.

	Theory (i)	Theory (ii)	Mean theor. value	Expt.
π^+	0.66 ± 0.02 F	0.56 ± 0.02 F	0.61 ± 0.03 F	0.56 ± 0.04 F (Ref. 4) [a]
π^-	0.66 ± 0.01 F	0.56 ± 0.01 F		0.71 ± 0.02 F (Ref. 7) [b]
K^+	0.57 ± 0.05 F	0.45 ± 0.06 F	0.54 ± 0.14 F	0.51 ± 0.07 F (Ref. 5) [a]
K^-	0.62 ± 0.07 F	0.52 ± 0.08 F		

[a] Direct measurement.
[b] Electroproduction experiments.

proton radius computed from *pp* elastic scattering at these energies is in excellent agreement with that from the *ep* experiment. In the absence of any higher-energy data on meson-proton scattering, accurate theoretical prediction about meson radii is certainly impossible. Nevertheless, we tend to conjecture that the true value for meson radius may fall in the range with two solutions given in Table V as its limits. If an average is taken over the two alternative solutions for both charge states, we obtain the following mean theoretical values:

$$r_\pi = 0.61 \pm 0.03 \text{ F},$$

$$r_K = 0.54 \pm 0.14 \text{ F}.$$

(4) It appears that the second solution and the mean theoretical values shown in Table V are closer to the results of direct experimental measurements. The disagreement between the electroproduction experiments and the electron scattering experiments is rather distrubing. The main difficulty with the determination of radius from electroproduction has to do with theoretical models used in data analysis. Hence, results of such experiments are subject to large theoretical uncertainties. On the other hand, the most precise and model-independent value of meson radius can only be obtained from direct measurements by electron scattering. Thus it would be interesting to see what value for the pion radius the 200-GeV pion-electron scattering experiment currently in progress at Fermilab will yield.

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