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Deuteron electromagnetic form factor: Data analysis and asymptotic behavior

M. K. Parida
P. G. Department of Physics, Sambalpur University, Jyoti Vihar, Burla 768 017, Orissa, India
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Available data on the deuteron electromagnetic form factor are analyzed with a view to obtaining information on its asymptotic behavior and extrapolating into the timelike region. For data analysis we adopt an N/D method where the N and the D functions are assumed to represent the anomalous and the two-pion cut contributions, respectively. The D function is represented by an effective-range-type formula and the N function by optimized polynomial expansion in Laguerre polynomials in terms of a parabolic conformally mapped variable. Contrary to the earlier cases of data analysis on the proton and the pion form factor by such representation, the presence of the exponential weight function for Laguerre-polynomial expansion of the N function provides a very effective method of parametrizing the data with economy of parameters. Existing data on $A(t)$ are consistent with an asymptotic behavior $\exp[-0.931(\ln t)^2]/t^3$. The deuteron charge radius is computed to be 2.02 fm. The formula smoothly extrapolates into the timelike region without showing any evidence of resonance peaks. The magnitude of the form factor near threshold of the process $e^+e^- \to \bar{d}d$ is found to be $|A(14\text{ GeV}^2)| = 1.765\times 10^{-9}$.

I. INTRODUCTION

The asymptotic behavior of an electromagnetic form factor of a hadron is an important subject of current interest. The simplicity and importance of this subject have resulted in an enormous amount of work in this field.[1] Apart from supplying information on the compositeness or elementarity of a hadron, the asymptotic behavior of its electromagnetic form factor reveals, to a certain extent, the dynamics of interactions at short distances. Although there exist some models and model-independent results, it is very important that meaningful results be obtained from data analysis. The deuteron is the simplest of the definitely known composite systems, and its electromagnetic form factor provides an ideal illustration of the continuity between nuclear and particle physics at the microscopic level.[2] The successful description of the deuteron form factor in a manner similar to the hadron form factors would also indicate the possibility of applying the techniques of particle physics in nuclear physics. Further, values of the form factor in the timelike region below the threshold of the process $e^+e^- \to h^+h^-$ are not directly accessible to currently known experiments. Therefore it is extremely useful that analytic parametrization of the spacelike data yields magnitudes of the form factor in the timelike region on extrapolation. It is the purpose of the present paper to show that a similar scheme of parametrization, previously adopted for the proton[3] and the pion[4] form factors, succeeds in parametrizing the data on the deuteron electromagnetic form factor and supplying useful information.

In applying the theory of analytic approximations to describe the proton form-factor data, a modified N/D method was proposed[3] incorporating correct analyticity in t, correct threshold behavior, the lowest inelastic branch point, and leading to an asymptotic behavior $(\ln t)^m/t^n$ with m and n being integers. Although analysis of the spacelike data could not distinguish between two types of fits with $m=2$ and $n=2$ or 3, extrapolation to the timelike region revealed a better ρ signal and reproduction of the Frascati datum point in the case of the fit with $m=2$ and $n=3$, thus indicating a falloff faster than the traditional dipole. Strong support, in evidence of faster falloff than the dipole fit, has been advanced by many authors.[5-8] Using such a technique an analytic formula was proposed for the pion electromagnetic form factor including ρ-ω interference which succeeded in describing both the spacelike and timelike data. Data analysis indicated asymptotic falloff faster than the single pole. From the best fit to the data, meaningful information on the pion's charge radius and the ρ-ω interference parameter was obtained. Such a formula for data analysis has been found useful elsewhere.[9]

Although experimental data on the deuteron form factor $A(t)$ are available in the spacelike region only up to $|t|=6\text{ GeV}^2$, it is possible to obtain information on its asymptotic behavior from data analysis since the deuteron is known to be a much more loosely bound system than the proton or the pion. Our analysis shows that the anomalous and the two-pion cut can accout for almost all the t dependence of the data on $A(t)$. The data are well fitted with a formula having an asymptotic behavior of the form $\exp[-\alpha(\ln t)^2]/t^3$ as opposed to the $(\ln t)^m/t^n$ type of behavior observed in the case of the proton and the pion form factor, and the power falloff t^{-10} predicted by the dimensional quark-counting rule (DQCR).[2,10] Such exponential modifications of the asymptotic behavior are supported by asymptotic freedom and quantum chromodynamics (QCD). From the best fit to the data on $A(t)$ we compute the charge radius of the deuteron to be $\langle r_d \rangle = 2.02$ fm. Unlike the case of the proton and the pion, extrapolation into the timelike region yields no signals for meson resonances. The magnitude of the form factor near the threshold of the process $e^+e^- \to \bar{d}d$ is found to be 1.765×10^{-9} The result of our extrapolation can be tested from the results of future colliding-beam experiments.

We plan the paper as follows: In Sec. II we summarize some results relevant to the present work. In Sec. III we develop the formula for parametrization. Section IV deals with results of data analysis and extrapolation of the form factor into the timelike region. Conclusions from this analysis are summarized in Sec. V.

II. SOME RESULTS ON THE DEUTERON FORM FACTOR

A. Definition of form factors

In our notation the four-momentum squared of the photon $t = q^2 = -Q^2$ is positive (negative) for the timelike (spacelike) region. In the one-photon-exchange approximation the covariant decomposition of the elastic form factor of the deuteron is written as[11]

$$\begin{aligned}\Gamma^\mu(t) &= \frac{1}{(2D^0 2D'^0)^{1/2}} \langle D' | j^\mu | D \rangle \\ &= -e\Big\{ G_1(t)(\epsilon' \cdot \epsilon) d^\mu \\ &\quad + G_2(t)[\epsilon^\mu(\epsilon' \cdot q) - \epsilon'^\mu(\epsilon \cdot q)] \\ &\quad - G_3(t) \frac{(\epsilon \cdot q)(\epsilon' \cdot q)}{2M^2} d^\mu \Big\} . \end{aligned} \tag{1}$$

In (1) ϵ and ϵ' are polarization vectors for the incoming and outgoing deuterons of momenta D and D' satisfying the conditions

$$\epsilon \cdot D = \epsilon' \cdot D' = 0, \quad \epsilon^2 = \epsilon'^2 = -1$$

and (2)

$$d^\mu = D'^\mu + D^\mu, \quad q^\mu = D'^\mu - D^\mu .$$

In our notation M, m, and m_π are the deuteron, proton, and pion masses, respectively, and B is the binding energy of the deuteron.

We assume the Lorentz-scalar functions $G_1(t)$, $G_2(t)$, and $G_3(t)$ are analytic in the complex t plane with right-hand cuts only. The anomalous cut starts from $t_a = 16mB$ and the two-pion cut from $t_R = 4m_\pi^2$. Besides these, there are other inelastic cuts lying farther away from the origin in the t plane. The physical form factors, which are called the charge form factor $G_c(t)$, the quadrupole moment form factor $G_Q(t)$, and the magnetic form factor $G_M(t)$, are defined in terms of G_1, G_2, and G_3 as[11]

$$\begin{aligned} G_c(t) &= G_1(t) - \frac{t}{6M^2} G_Q(t) , \\ G_Q(t) &= G_1(t) - G_2(t) + \left(1 - \frac{t}{4M^2}\right) G_3(t) , \\ G_M(t) &= G_2(t) . \end{aligned} \tag{3}$$

From experimental values these three form factors are normalized as

$$G_c(0) = 1 ,$$

$$G_M(0) = 1.71 \quad (\text{in units of } e/2M) , \tag{4}$$

and

$$G_Q(0) = 25.84 \quad (\text{in units of } M^{-2}) .$$

The differential cross section for elastic e-d scattering is given by the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[A(t) + B(t) \tan^2 \frac{\theta_L}{2} \right] , \tag{5}$$

where the elastic form factors $A(t)$ and $B(t)$ are related to G_c, G_Q, and G_M in the following manner[12]:

$$A(t) = G_c^2 + \frac{t^4 G_Q^2}{18M^4} - \frac{t}{6M^2}\left(1 - \frac{t}{4M^2}\right) G_M^2 , \tag{6}$$

$$B(t) = -\frac{t}{3M^2}\left(1 - \frac{t}{4M^2}\right) G_M^2 . \tag{7}$$

It is obvious that $A(t)$ and $B(t)$ also possess the same analytic structure in the t plane as any one of the form factors G_1, G_2, and G_3. Extensive data on $A(t)$ are available[13] in the spacelike region extending up to 6 GeV^2. We will develop a formula for parametrizing $A(t)$ in the next section.

B. Some theoretical models and asymptotic behavior

During the past few years some attractive models have been proposed for the deuteron form factor. For a hadron with n constituents the dimensional quark-counting rule (DQCR)[10] predicts an asymptotic behavior of the type

$$F_n(t) \propto \frac{1}{t^{n-1}} . \tag{8}$$

Taking the deuteron to be a six-quark system the form factor $A(t)$ falls off at large t like t^{-10} in this model. Recently Brodsky and Chertok[2] have plotted the function $f(t) = t^5 [A(t)]^{1/2}$ against $|t|$ using experimental data[13] on $A(t)$. Existing data[13], on the deuteron form factor do not show constancy of the function $f(t)$ for large $|t|$. Schmidt and Blankenbecler[14] have proposed a parton-model description of the form factor having the same asymptotic behavior as DQCR. Although the two-parameter formula proposed by the authors agrees well with the data for low- and high-Q^2 values, agreement for the intermediate Q^2 values is poor. Recently[15] a covariant model has been proposed including spin which contains four parameters and yields a fit similar to that of Ref. 14. To the best of our knowledge there has not been any work yet in the literature which deals with data analysis and reports values of χ^2/DOF for the deuteron form factor.

Besides these works we summarize below some asymptotic results relevant for the present analysis. Although Broadhurst[16] proved asymptotic upper bounds of the form $(\ln t)^c/t^{(p+1)/2}$ for the Dirac form factor of the proton, where p is the exponent in the threshold behavior of the structure function νW_2, the same result can be derived for any composite object if one ignores spin.[17] The Drell-Yan-West relation[18] saturates this upper bound without factors involving powers of logarithm. In earlier works form-factor parametrization was proposed by formulas which asymptotically saturate such bounds. West[6] has demonstrated using data on $A(t)$ for $|t| \geq 2$ GeV^2 that $A(t) \propto t^{-12}$ for large $|t|$, falling off faster than the prediction of DQCR. An asymptotic falloff of the type

$$A(t) \propto \left[\frac{\ln t}{t}\right]^{12} \tag{9}$$

has been obtained in the dynamical model of factorizing quarks.[19] Repetition of the same calculations as that adopted for the proton using local duality and asymptotic freedom[7] would yield the asymptotic behavior

$$A(t) \propto \exp[-\text{const} \times (\ln t)^2]\,. \tag{10}$$

Quantum chromodynamics (QCD) predicts such type of modifications to DQCR asymptotic behavior arising out of quark form factors. In particular duplication of the analysis of Coquereaux and Rafael[8] would yield the asymptotic behavior

$$A(t) \propto t^{-10(1+\epsilon)} \exp[-\tfrac{16}{15}(\ln t)(\ln \ln t)]\,, \tag{11}$$

where ϵ is an unknown constant stemming from corrections introduced by binding of the quarks. All the asymptotic results mentioned here are consistent with the lower bound[20]

$$A(t) > \exp(-b|t|^{1/2}) \tag{12}$$

derived as early as 1965.

To summarize, we have noted that although DQCR predicts asymptotic behavior t^{-10} for $A(t)$, there is enough theoretical evidence in support of a faster falloff for large $|t|$. In the next section we propose a formula for parametrizing $A(t)$ which has the potentialities of yielding most of the asymptotic behaviors cited in this section.

III. FORM-FACTOR PARAMETRIZATION

Following earlier works[2,3] we propose

$$A(t) = N(t)/D(t)\,, \tag{13}$$

where $D(t)$ represents the two-pion cut and has the form

$$D(t) = L(t) + h(t) + \frac{m_\pi^{\,2}}{\pi} \tag{14}$$

with[21]

$$L(t) = \sum_n a_n t^n\,, \tag{15}$$

$$h(t) = \frac{2}{\pi}\frac{k^3}{\sqrt{t}} \ln[(t/t_R)^{1/2} + (t/t_R - 1)^{1/2}] - i\frac{k^3}{\sqrt{t}} \tag{16}$$

for $n \geq 2$ and

$$k = (\tfrac{1}{4}t - m_\pi^{\,2})^{1/2}\,. \tag{17}$$

Justification for this choice of the D function has been discussed earlier.[3,4] Unlike the proton and the pion cases where the other cuts in the t plane are farther than the two-pion cuts, in the deuteron case there is an anomalous cut which is closer to the origin and extends from $t_a = 16mB$ to $t = \infty$. We assume that the N function represents the anomalous cut. In the absence of our knowledge about the function on the anomalous cut we adopt the ideas of analytic approximation theory[3,4] of data analysis[22-24] and map the anomalous cut onto the boundary of a parabola[24] with focus at the origin, in the Z plane where

$$Z(t) = \{\ln[(-t/t_a)^{1/2} + (-t/t_a + 1)^{1/2}]\}^2\,. \tag{18}$$

We note that for $|t| \to \infty$

$$Z(t) \sim (\ln|t|)^2\,. \tag{19}$$

In the Z plane the image of the physical region is the entire right half of the real axis, a physical region appropriate for Laguerre-polynomial expansion and the image of the cut coincides with the figure of convergence of Laguerre polynomials. We can now represent the N function by Laguerre-polynomial expansion with the appropriate exponential weight function[23,24] $\exp(-\alpha Z)$,

$$N(t) = \exp(-\alpha Z) \sum_m C_m L_m(2\alpha Z)\,. \tag{20}$$

In (20) α is a constant which fixes the size of the parabolic figure of convergence. Further, since the entire cut t plane is squeezed into the interior of the parabola in the Z plane, the series (20) converges at the fastest rate.[22] Now we can truncate[25] the series for the purpose of data analysis by retaining only the first N significant terms and rewrite (20) as

$$N(t)=\exp(-\alpha Z)\sum_{m=0}^{N} e_m Z^m . \qquad (21)$$

The presence of the exponential weight function has been emphasized in the context of diffraction scattering.[23] Recently it has been shown that such an expansion of the scattering amplitude yields a unified description of slopes of diffraction scattering for all energies.[26] When one takes into account energy dependence of α in scattering processes, such a representation has been shown to describe scaling of the cross-section-ratio data remarkably well.[27] In earlier works[3,4] on form factors the experimental data were consistent with this weight function being unity ($\alpha \simeq 0$). In the present case, however, we will see in the next section that the presence of this weight function is essential for the economic parametrization of the form-factor data. With the formulas (14)–(17) for the D function, the subtraction procedure,[21] and the representation (21) for the N function we observe that for any m and n the formula (13) has the potentiality of satisfying the asymptotic behavior of the type

$$A(t)\propto \frac{(\ln t)^m}{t^n}\exp[-\alpha(\ln t)^2]. \qquad (22)$$

For $\alpha=0$ formula (22) saturates Broadhurst-type bounds, but for $m=n=0$ it yields asymptotic behavior (10) obtained by using local duality and asymptotic freedom.[7] For $m=0$ and $\alpha=0$ it yields asymptotic behavior predicted by DQCR for $n=10$. In view of these potentialities it will be interesting to see how this formula fits the data. In the next section we report our result of unbiased data analysis using the formula (13).

IV. RESULTS AND DISCUSSION

A. Data analysis and asymptotic behavior

We have collected 59 data points on $A(t)$ from the literature.[13] At first we tried to make a data analysis with a bias from DQCR and our earlier analysis[3,4] taking $\alpha=0$. It may be noted here that to attempt to fit the data on $A(t)$ with formula (13)–(21) with $\alpha=0$ would require at least 11 parameters for the D function if DQCR is correct, while it requires at least 13 parameters to be consistent with asymptotic behavior[19] of the type t^{-12}. In addition, other parameters are required in the N function to account for the possible presence of logarithmic terms. Using a search program it appeared to be almost a formidable task to find the unknown parameters a_i's and e_i's with correct signs[28] with an IBM 1130 computing system. We found that inclusion of as many as six parameters in the D function yielded[28] a total χ^2 value more than four times the accepted value for a good fit and for a good fit the data required more parameters. We next carried out an unbiased data analysis with α as a free parameter. To our surprise, a good fit was obtained with

$$\begin{aligned}\alpha&=0.931\,,\\ e_0&=a_0=1.0\ \text{GeV}^2\,,\\ a_1&=-6.507\,,\\ a_2&=74.289\ \text{GeV}^{-2}\,,\\ a_3&=0.192\ \text{GeV}^{-4}\end{aligned} \qquad (23)$$

with only four parameters in the D function. We note that unlike the other case[28] the series in the D function in the present case starts converging after the third term. For this fit $\chi^2/\text{DOF}=1.58$ for 54 degrees of freedom.[28] The fit is shown in Fig. 1. Inclusion of other parameters e_i with $i \geq 1$ did not improve the fit significantly. The asymptotic behavior for the fit (23) is

$$A(t)\propto \exp[-0.931(\ln t)^2]/t^3 . \qquad (24)$$

From the present analysis it is very clear that the presence of the exponential weight function for the Laguerre-polynomial expansion in the N function serves as an important factor for the economic parametrization of the form-factor data. As has been already pointed out, the presence of this weight function gets strong support from predictions of asymptotic freedom[7] and QCD.[8]

B. Extrapolation into the timelike region, computation of charge radius, and $\sigma_{e^+e^-\to \bar{d}d}$ near threshold

The theory of analytic approximation by conformal mapping ensures stability of extrapolations from interior points onto the boundary.[22] Although the D function has not been approximated previously by conformal mapping, we assume that it is a good approximation and stable against extrapolation, from our earlier experience, particularly from the extrapolation of the proton form factor.[3] For extrapolation it is necessary to state that in the timelike region

$$Z(t)=-\left[\tan^{-1}\left(\frac{t_a}{t_a-t}\right)^{1/2}\right]^2, \text{ for } t<t_a \qquad (25)$$

and

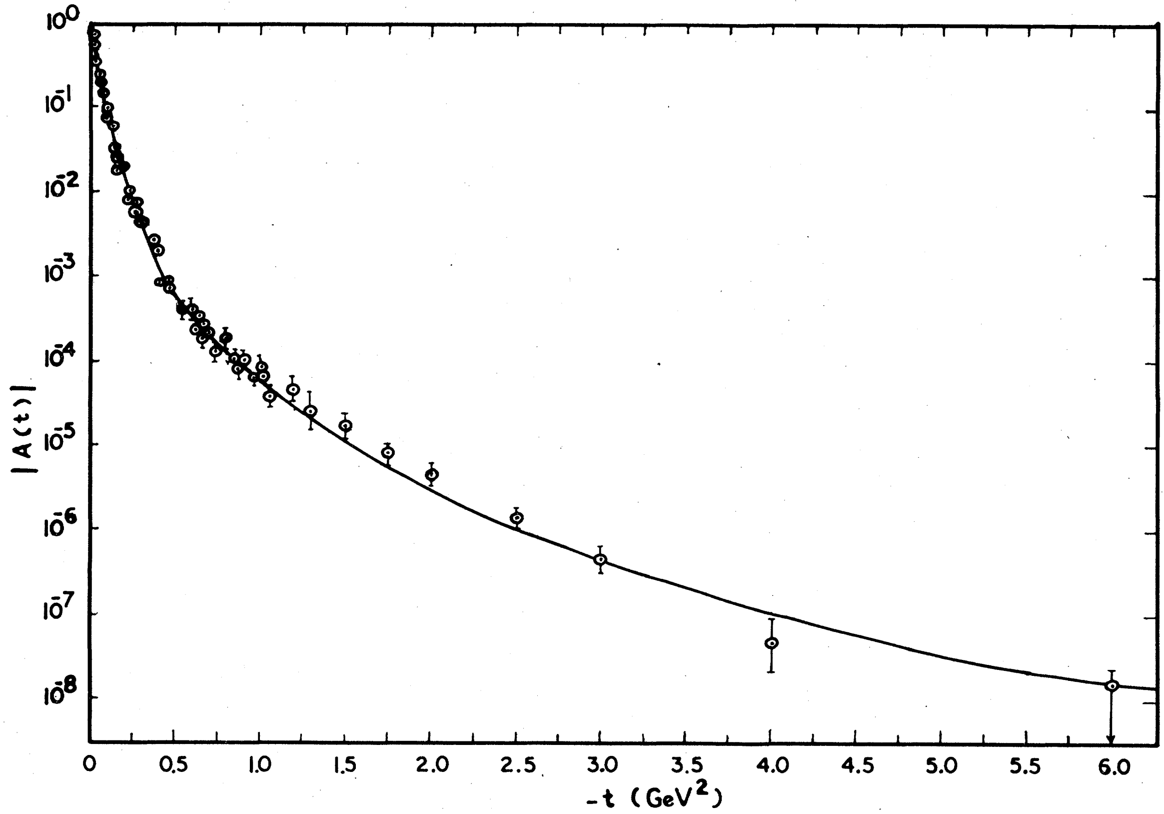

FIG. 1. Fit to the deuteron form-factor data on $A(t)$ in the spacelike region. The data points are from Ref. 13.

$$Z(t)=\frac{-\pi^2}{4}+i\pi\cosh^{-1}(t/t_a)^{1/2}+[\cosh^{-1}(t/t_a)^{1/2}]^2,\ \text{for } t>t_a. \quad (26)$$

Thus the magnitude of the N function increases up to $t=t_a$ where it reaches a maximum value of $\exp(\alpha\pi^2/4)$ and decreases thereafter. Extrapolation onto the region with $t<t_a$ and up to $t=0.11$ GeV^2 is shown in Fig. 2. We find that the behavior around the origin is very smooth. From the parameters given in (23) we obtain the derivative at the origin to be

$$A'(0)=34.8\ \text{GeV}^{-2}. \quad (27)$$

Now using the relation (6) and this value of $A'(0)$ we obtain the deuteron charge radius

$$\langle r_d\rangle=[6G_c'(0)]^{1/2} =2.02\ \text{fm}. \quad (28)$$

To our knowledge there does not exist any computation of the deuteron's charge radius from data analysis.[1] As early as 19 year ago, Frazer and Fulco[29] conjectured that the large value of the proton's charge radius may be due to the exchange of a *pion-pion* p-wave resonance.

In the present case the anomalous cut is closer to the origin than the two-pion cut and has a dominant contribution to the charge radius. From our calculation we find that the anomalous cut contributes nearly 81% to $\langle r_d^2\rangle$ and the rest 19% is due

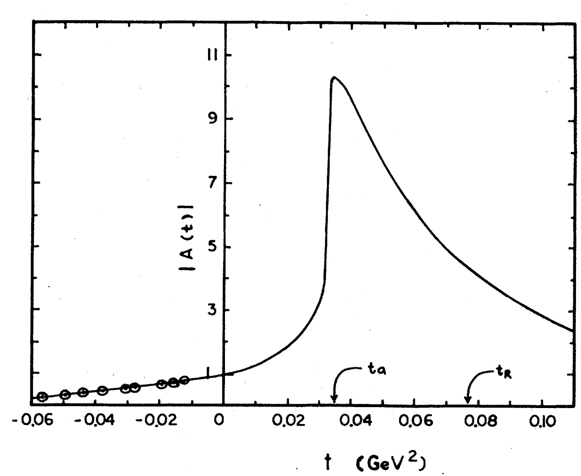

FIG. 2. Smooth extrapolation of the deuteron form factor from the spacelike into the timelike region. The peak in the timelike region appears at the start of the anomalous cut.

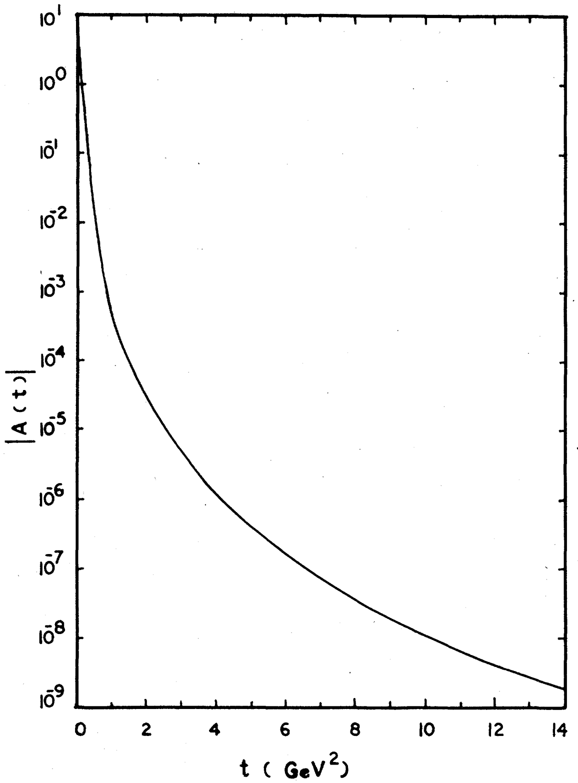

FIG. 3. Extrapolation of the deuteron form factor into the timelike region for large t and for $t>0.1$ GeV^2.

to the two-pion cut.

Extrapolation of the form factor $A(t)$ into the timelike region up to the threshold of the process $e^+e^- \to \bar{d}d$ is shown in Fig. 3. We note that unlike the case of the pion[4] and the proton,[3] there are no resonance peaks in this case. This occurs because of the large value of a_2 relative to a_1 and a_3 as demanded by the best fit to the data. $|A(t)|$ decreases rapidly for increasing t and near threshold it becomes

$$|A(14\ \text{GeV}^2)| = 1.765 \times 10^{-9}. \tag{29}$$

From this value we calculate the cross section for the process $e^+e^- \to \bar{d}d$ near threshold to be $\sigma \simeq 1.1 \times 10^{-8}$ nb. Recently the proton form factor has been measured[31] in the timelike region near the threshold of the $\bar{p}p$ production from the process $\bar{p}p \to e^+e^-$. Our extrapolated results on the deuteron form factor can be verified by the results of future experiments. Questions may be raised about the accuracy of these extrapolations. But we believe these extrapolated results are accurate at least in order of magnitude. Such a belief gets strong support from the extrapolated results on the proton form factor[3] and their agreement with experimental values.[30,31] The absence of vector-meson signals in this case may be attributed to the small coupling of these mesons to the deuteron. Such small couplings may be due to the smallness of the amplitudes for $\bar{d}d \to h^+h^-$ where h^+h^- is a charged-hadron pair occurring as intermediate states in the form-factor diagram.[32]

The results quoted in this section are the extrapolated results for which it may be necessary to specify the error corridors. Owing to exigencies of programming techniques, we have not been able to specify errors in the parameters. The errors in the experimental data contribute to the errors in the parameters. An estimation of errors in the parameters in such type of analysis was carried out for the pion form factor[4] for which the data both in the timelike and spacelike regions were contaminated with relatively more errors than the present case. But the errors in the parameters relevant for extrapolation were found to be small. In the present case we hope the errors are still smaller.

V. CONCLUSION

Including the two-pion cut contribution by an effective-range type of formula in the D function and the anomalous cut contribution in the N function by means of conformal mapping and optimized polynomial expansion, we have accounted for almost all the t dependence of the data on the deuteron form factor in an effective manner. The presence of the exponential weight function for the Laguerre-polynomial expansion has yielded an effective method of parametrizing the data with an economy in parameters. The present data are consistent with an asymptotic behavior $t^{-3}\exp[-0.931(\ln t)^2]$ which falls off faster than the prediction of DQCR. Such type of exponential modifications of the asymptotic behavior is expected from asymptotic freedom[7] and QCD.[8] The charge radius of the deuteron is found to be 2.02 fm. Such a large value of the charge radius is due to the anomalous cut. Unlike the earlier result on the proton form factor,[3] extrapolation into the timelike region in the present case yields no resonance peaks. Extrapolation of the form factor onto the threshold of the process $e^+e^- \to \bar{d}d$ yields a small magnitude for the form factor implying a small cross section for this process.

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[28]When the number of parameters in the denominator is large it is not possible to fix the parameters uniquely. Several sets of parameters may yield the same fits. Fitting the data with the bias $\alpha=0$ required at least six parameters: $e_0=a_0=0.5\ \mathrm{GeV}^2$, $a_1=-2.14$, $a_2=558.6\ \mathrm{GeV}^{-2}$, $a_3=0.89\ \mathrm{GeV}^{-4}$, $a_4=2120.5\ \mathrm{GeV}^{-6}$, $a_5=-8562\ \mathrm{GeV}^{-8}$ yielding the total $\chi^2=242.6$. It is clear that the series in the D function has not converged even after taking six terms. Inclusion of more parameters is necessary to obtain a good χ^2 value.
[29]W. R. Frazer and J. R. Fulco, Phys. Rev. Lett. 2, 365 (1959); Phys. Rev. 117, 169 (1960).
[30]In Fig. 4 of Ref. 3 curve II gives a better ρ signal and yields the value at $t=4.42\ \mathrm{GeV}^2$ which agrees well with the Frascati datum point. Similarly at threshold this curve yields the value $|G_M^p|=|G_E^p|\simeq 0.34$ which is not far away from the recently quoted experimental value in Ref. 31.
[31]G. Bassompierre *et al.*, Phys. Lett. 68B, 477 (1977). These authors have measured the values of the proton form factor to be

$$|G|=\begin{cases} 0.51\pm 0.08, \text{ at } t=3.52\ \mathrm{GeV}^2, \\ 0.46\pm^{0.15}_{0.09}, \text{ at } t=3.61\ \mathrm{GeV}^2, \end{cases}$$

using the relation $G=G_M^p=G_E^p$.
[32]The deuteron form factor $G_E^d(t)$ can be written as

$$|G_E^d(t)|\ \alpha\ |F_h(t)|\ |T(t)|\ ,$$

where $F_h(t)$ is the form factor of the hadron h^+ and $T(t)$ is the amplitude for the process $\bar{d}d\rightarrow h^+h^-$. The resonance signals in $G_E^d(t)$ are either very small or have zeros around the resonance region. Such an argument has been advanced in connection with the extrapolation of the proton form factor. See for example H. Pfister, Nucl. Phys. B5, 320 (1969); B5, 327 (1970); and Ref. 3.