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Dihadron spectrum, quarks, and Chao-Yang statistics

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By applying our model which introduces quarks into Chao-Yang statistics for "violent collisions," we obtain a pair-production spectrum of oppositely charged hadrons which is in qualitative agreement with the MIT and the Fermilab experiments. Predictions for other dihadron production spectra from proton-nucleon and pion-proton collisions are also given.

The dihadron production spectrum has been measured from 28.6 to 400 GeV over the large-mass range.[1] Theoretically, apart from the simple quark-counting rule,[2] this behavior has not been explained properly. In this paper we are using the standard quark[3] assumption together with Chao-Yang statistics[4] to study this dihadron spectrum. In brief, Chao-Yang statistics is a statistical solution for the "violent collision" in closed forms for the multiplicities and the two-particle correlation functions, when the final state consists of a definite number of hadrons. By combining quarks with Chao-Yang statistics we are able to obtain results for single-particle ratios which are consistent with experiment in general.[5] Therefore it is natural for us to investigate this approach in the dihadron spectrum.

We consider u, d, and s quarks. For a collection of l quarks of types u, d, s, $\bar{u}$, $\bar{d}$, and $\bar{s}$, the quantum state of the collection is denoted by (a, b, c) which is equivalent to a quantum state of $a\,u$, $b\,d$, and $c\,s$ quarks. We define the distribution function $N^l_{a,b,c}$, which is the number of possible ways to distribute the quantum number (a, b, c) over the l quarks. We have the following relations:

$$\begin{aligned} &n(u)+n(\bar{u})+n(d)+n(\bar{d})+n(s)+n(\bar{s})=l\,,\\ &n(u)-n(\bar{u})=a\,,\\ &n(d)-n(\bar{d})=b\,,\\ &n(s)-n(\bar{s})=c\,, \end{aligned} \qquad (1)$$

where $n(u)$, $n(\bar{u})$, $n(d)$, $n(\bar{d})$, $n(s)$, and $n(\bar{s})$ denote the number of the respective quarks in the $N^l_{a,b,c}$ configurations. The binomial distribution for $N^l_{a,b,c}$ with the suppression factor γ for the strange quark is

$$\left(x+y+\gamma z+\frac{1}{x}+\frac{1}{y}+\frac{\gamma}{z}\right)^l=\sum_{a,b,c} N^l_{a,b,c}\,x^a y^b z^c\,, \qquad (2)$$

which implies[6]

$$\sum_{a,b,c} N^l_{a,b,c}=(4+2\gamma)^l\,.$$

For a large value of l, the asymptotic expansion of $N^l_{a,b,c}$ with a, b, and c fixed is

$$\begin{aligned} N^l_{a,b,c}=(4+2\gamma)^l\left(\frac{2+\gamma}{2\pi l}\right)^{3/2}\frac{1}{\sqrt{\gamma}}\Bigg(1&-\frac{1}{4l}\left[(4+2\gamma)\left(a^2+b^2+\frac{c^2}{\gamma}\right)-\frac{\gamma^2-5\gamma+1}{\gamma}\right]\\ &+\frac{1}{128l^2}\left[(2+\gamma)^2\left(20+\frac{9}{\gamma^2}+\frac{4}{\gamma}\right)-70(2+\gamma)\left(2+\frac{1}{\gamma}\right)+385\right]\\ &+\frac{1}{48l^2}\Bigg\{(2+\gamma)^2\left[6\left(a^2+b^2+\frac{c^2}{\gamma}\right)^2-3\left(6a^2+6b^2+\frac{5c^2}{\gamma^2}+\frac{a^2+b^2+2c^2}{\gamma}\right)\right]\\ &+105(2+\gamma)\left(a^2+b^2+\frac{c^2}{\gamma}\right)\Bigg\}\Bigg)\,. \end{aligned} \qquad (3)$$

TABLE I. The equations for all possible hadron-pair production for p-p collisions are grouped by the charges of the produced hadrons.

	h^+h^-
π^-p	$\propto 1-\frac{8.17}{l}+\frac{27.73}{l^2}+\cdots$
$\pi^+\pi^-$	$\propto 1-\frac{19.72}{l}+\frac{167.57}{l^2}+\cdots$
$p\bar{p}$	$\propto 1-\frac{19.72}{l}+\frac{128.12}{l^2}+\cdots$
K^-p	$\propto 1-\frac{19.72}{l}+\frac{48.62}{l^2}+\cdots$
$K^-\pi^+$	$\propto 1-\frac{35.47}{l}+\frac{462.88}{l^2}+\cdots$
$\pi^+\bar{p}$	$\propto 1-\frac{41.77}{l}+\frac{747.61}{l^2}+\cdots$
$K^+\pi^-$	$\propto 1-\frac{27.07}{l}+\frac{221.59}{l^2}+\cdots$
K^+K^-	$\propto 1-\frac{19.72}{l}+\frac{167.57}{l^2}+\cdots$
$K^+\bar{p}$	$\propto 1-\frac{44.92}{l}+\frac{773.75}{l^2}+\cdots$
	h^-h^-
$\pi^-\pi^-$	$\propto 1-\frac{36.52}{l}+\frac{597.23}{l^2}+\cdots$
π^-K^-	$\propto 1-\frac{48.07}{l}+\frac{957.12}{l^2}+\cdots$
$\pi^-\bar{p}$	$\propto 1-\frac{54.37}{l}+\frac{1308.62}{l^2}+\cdots$
K^-K^-	$\propto 1-\frac{82.72}{l}+\frac{2837.19}{l^2}+\cdots$
$K^-\bar{p}$	$\propto 1-\frac{70.12}{l}+\frac{2133.92}{l^2}+\cdots$
$\bar{p}\bar{p}$	$\propto 1-\frac{82.72}{l}+\frac{3068.64}{l^2}+\cdots$
	h^+h^+
pp	$\propto 1+\frac{1.27}{l}+\frac{29.95}{l^2}+\cdots$
π^+p	$\propto 1-\frac{3.97}{l}+\frac{17.12}{l^2}+\cdots$
K^+p	$\propto 1-\frac{11.32}{l}-\frac{51.96}{l^2}+\cdots$
π^+K^+	$\propto 1-\frac{22.87}{l}+\frac{127.41}{l^2}+\cdots$
K^+K^+	$\propto 1-\frac{49.12}{l}+\frac{707.80}{l^2}+\cdots$
$\pi^+\pi^+$	$\propto 1-\frac{19.72}{l}+\frac{167.57}{l^2}+\cdots$

We follow the assumption of the Chao-Yang statistics that, for fixed l, each of the state $N^l_{a,b,c}$ has an equal probability and the probabilities of finding u, $\bar{u}$, d, $\bar{d}$, s, and $\bar{s}$ are as follows:

$$P_u(l)=\frac{N^{l-1}_{a-1,b,c}}{N^l_{a,b,c}}\,,\quad P_d(l)=\frac{N^{l-1}_{a,b-1,c}}{N^l_{a,b,c}}\,,\quad P_s(l)=\frac{N^{l-1}_{a,b,c-1}}{N^l_{a,b,c}}\,,\quad P_{\bar{u}}(l)=\frac{N^{l-1}_{a+1,b,c}}{N^l_{a,b,c}}\,,\quad P_{\bar{d}}(l)=\frac{N^{l-1}_{a,b+1,c}}{N^l_{a,b,c}}\,,\quad P_{\bar{s}}(l)=\frac{N^{l-1}_{a,b,c+1}}{N^l_{a,b,c}}\,. \tag{4}$$

The probability of finding k quarks can be generalized as follows:

$$P_{q_1,q_2,\cdots q_k}(l)=\frac{N^{l-k}_{a-\alpha,b-\beta,c-\delta}}{N^l_{a,b,c}}\,, \tag{5}$$

where (α,β,δ) is the quantum state of the collection of k quarks which is equivalent to αu quarks, βd quarks, and δs quarks. We also have the following relation:

$$\sum_{q_k} P_{q_1,q_2,\cdots q_k}(l)=P_{q_1 q_2,\cdots q_{k-1}}(l)\,, \tag{6}$$

and $P_{q_1,q_2\cdots q_k}(l)$ is invariant under any interchange of k quarks.

We now consider the hadron-pair (h_1h_2) production from the following reaction:

$$p+p\to h_1+h_2+X\,. \tag{7}$$

In order to find the probability of h_1h_2 production, we make the following assumption:

$$h_1h_2\propto\frac{N^{l-\lambda}_{A-a,B-b,C-c}}{N^l_{A,B,C}}\,, \tag{8}$$

where λ is the number of quarks of h_1h_2 pair with $a u$ quarks, $b d$ quarks, and $c s$ quarks, whereas A, B, C denote $A u$ quarks, $B d$ quarks, and $C s$ quarks of the two colliding protons. We assumed a suppression factor $\gamma=0.1$ for each strange quark created in the final state. This is due to the fact that the strange quark is much heavier than the u, d quarks. All the possible h^+h^- pairs for proton-proton collisions are listed in Table I.

The pair production for p-neutron reactions can similarly be obtained. In order to compare our result with Ref. 1 which measured the hadron pairs from the reactions

$$p+\mathrm{Be}\to h^++h^-+X\,, \tag{9}$$

TABLE II. The equations for all possible hadron-pair production for p–Be collisions are grouped by the charges of the produced hadrons.

	h^+h^-
π^-p	$\propto 1-\frac{5.84}{l}+\frac{21.83}{l^2}+\cdots$
$\pi^+\pi^-$	$\propto 1-\frac{18.55}{l}+\frac{148.76}{l^2}+\cdots$
$p\bar{p}$	$\propto 1-\frac{18.55}{l}+\frac{111.64}{l^2}+\cdots$
K^-p	$\propto 1-\frac{18.55}{l}+\frac{30.97}{l^2}+\cdots$
$K^-\pi^+$	$\propto 1-\frac{35.47}{l}+\frac{462.88}{l^2}+\cdots$
$\pi^+\bar{p}$	$\propto 1-\frac{41.77}{l}+\frac{747.61}{l^2}+\cdots$
$K^+\pi^-$	$\propto 1-\frac{24.74}{l}+\frac{169.27}{l^2}+\cdots$
K^+K^-	$\propto 1-\frac{18.55}{l}+\frac{148.76}{l^2}+\cdots$
$K^+\bar{p}$	$\propto 1-\frac{43.75}{l}+\frac{650.14}{l^2}+\cdots$
	h^-h^-
$\pi^-\pi^-$	$\propto 1-\frac{30.69}{l}+\frac{429.67}{l^2}+\cdots$
π^-K^-	$\propto 1-\frac{43.40}{l}+\frac{764.27}{l^2}+\cdots$
$\pi^-\bar{p}$	$\propto 1-\frac{49.70}{l}+\frac{1091.04}{l^2}+\cdots$
K^-K^-	$\propto 1-\frac{79.22}{l}+\frac{2567.93}{l^2}+\cdots$
$K^-\bar{p}$	$\propto 1-\frac{66.62}{l}+\frac{1911.93}{l^2}+\cdots$
$\bar{p}\bar{p}$	$\propto 1-\frac{79.22}{l}+\frac{2806.05}{l^2}+\cdots$
	h^+h^+
pp	$\propto 1+\frac{0.10}{l}+\frac{24.38}{l^2}+\cdots$
π^+p	$\propto 1-\frac{6.30}{l}+\frac{23.01}{l^2}+\cdots$
K^+p	$\propto 1-\frac{12.49}{l}-\frac{41.66}{l^2}+\cdots$
π^+K^+	$\propto 1-\frac{25.20}{l}+\frac{179.73}{l^2}+\cdots$
K^+K^+	$\propto 1-\frac{50.29}{l}+\frac{763.36}{l^2}+\cdots$
$\pi^+\pi^+$	$\propto 1-\frac{23.22}{l}+\frac{238.71}{l^2}+\cdots$

TABLE III. The equations for all possible hadron-pair production for π^-–p collisions are grouped by the charges of the produced hadrons.

	h^+h^-
π^-p	$\propto 1-\frac{1.27}{l}+\frac{28.67}{l^2}+\cdots$
$\pi^+\pi^-$	$\propto 1-\frac{3.97}{l}+\frac{21.10}{l^2}+\cdots$
$p\bar{p}$	$\propto 1-\frac{3.97}{l}+\frac{13.15}{l^2}+\cdots$
K^-p	$\propto 1-\frac{10.27}{l}-\frac{59.57}{l^2}+\cdots$
$K^-\pi^+$	$\propto 1-\frac{19.72}{l}+\frac{68.35}{l^2}+\cdots$
$\pi^+\bar{p}$	$\propto 1-\frac{19.72}{l}+\frac{147.85}{l^2}+\cdots$
$K^+\pi^-$	$\propto 1-\frac{11.32}{l}-\frac{40.63}{l^2}+\cdots$
K^+K^-	$\propto 1-\frac{3.97}{l}+\frac{21.10}{l^2}+\cdots$
$K^+\bar{p}$	$\propto 1-\frac{22.87}{l}+\frac{104.53}{l^2}+\cdots$
	h^-h^-
$\pi^-\pi^-$	$\propto 1-\frac{8.17}{l}+\frac{35.90}{l^2}+\cdots$
π^-K^-	$\propto 1-\frac{19.72}{l}+\frac{68.35}{l^2}+\cdots$
$\pi^-\bar{p}$	$\propto 1-\frac{19.72}{l}+\frac{167.57}{l^2}+\cdots$
K^-K^-	$\propto 1-\frac{54.37}{l}+\frac{966.10}{l^2}+\cdots$
$K^-\bar{p}$	$\propto 1-\frac{35.47}{l}+\frac{427.41}{l^2}+\cdots$
$\bar{p}\bar{p}$	$\propto 1-\frac{41.77}{l}+\frac{705.83}{l^2}+\cdots$
	h^+h^+
pp	$\propto 1-\frac{8.17}{l}+\frac{19.55}{l^2}+\cdots$
π^+p	$\propto 1-\frac{7.12}{l}+\frac{23.42}{l^2}+\cdots$
K^+p	$\propto 1-\frac{14.47}{l}-\frac{22.51}{l^2}+\cdots$
π^+K^+	$\propto 1-\frac{19.72}{l}+\frac{68.35}{l^2}+\cdots$
K^+K^+	$\propto 1-\frac{45.97}{l}+\frac{566.05}{l^2}+\cdots$
$\pi^+\pi^+$	$\propto 1-\frac{16.57}{l}+\frac{118.43}{l^2}+\cdots$

we take the weight of proton and neutron targets in the ratio 4/5, and the results are listed in Table II. For the convenience of comparing with experiment, we plot $[(h_1h_2)/C-1]l$ for $\gamma=0.1$ (Ref. 7) and $l=100$ (Ref. 8) on the reactions $p+\mathrm{Be}\to h_1+h_2+X$, where (h_1h_2) is the probability of finding the hadron pairs at high transverse momentum, and c is the proportionality constant.[9] A comparison of the MIT experiment[1] in Fig. 1 indicates that our model is useful in the sense that we are able to reproduce the qualitative features of the so called "band structure". In the case of K^+K^- pair[2] the calculated value lies a bit higher as compared to the Fermilab experiment.[1] This is due to the fact that we have not taken into account the suppression effect due to the strange quark-antiquark production in the final state. Work on extending our model to include this effect is in progress and the result will be published in separate paper.

We also carry out our calculations for hadron-pair production by using incident π beam instead of proton beam. These productions can be tested by experiment in the near future. Let us first take the following reactions:

$$\pi^-+p\to h_1+h_2+X. \quad (10)$$

The results are listed in Table III. (See Fig. 2.) For the reactions of

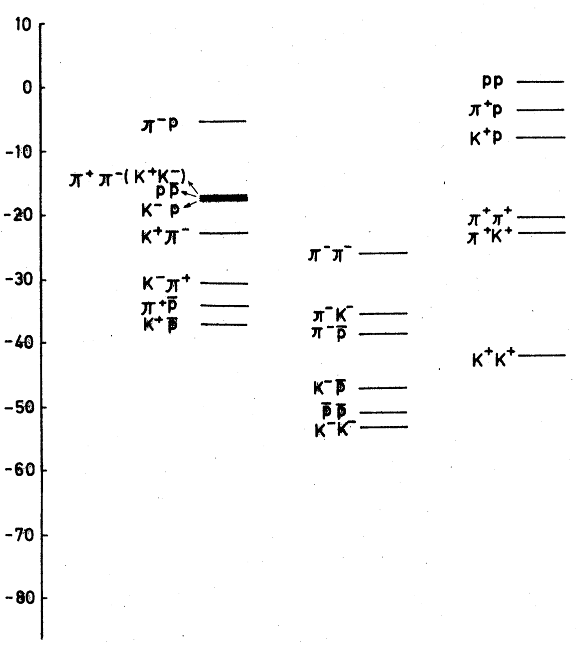

FIG. 1. A plot of $[(h_1h_2)/c-1)]l$ for $\gamma=0.1$ and $l=100$ on the reactions $p+\mathrm{Be}\to h_1+h_2+X$. The first column contains the spectrum of $+-$ hadron pairs, the second column the $--$ pairs, and the third column the $++$ pairs.

TABLE IV. The equations for all possible hadron-pair production for π^+-p collisions are grouped by the charges of the produced hadrons.

h^+h^-	
π^-p	$\propto 1-\frac{7.12}{l}+\frac{23.42}{l^2}+\cdots$
$\pi^+\pi^-$	$\propto 1-\frac{8.17}{l}+\frac{35.90}{l^2}+\cdots$
$p\bar{p}$	$\propto 1-\frac{8.17}{l}+\frac{19.55}{l^2}+\cdots$
K^-p	$\propto 1-\frac{14.47}{l}-\frac{22.51}{l^2}+\cdots$
$K^-\pi^+$	$\propto 1-\frac{19.72}{l}+\frac{68.35}{l^2}+\cdots$
$\pi^+\bar{p}$	$\propto 1-\frac{19.72}{l}+\frac{147.85}{l^2}+\cdots$
$K^+\pi^-$	$\propto 1-\frac{19.72}{l}+\frac{68.35}{l^2}+\cdots$
K^+K^-	$\propto 1-\frac{8.17}{l}+\frac{35.90}{l^2}+\cdots$
$K^+\bar{p}$	$\propto 1-\frac{27.07}{l}+\frac{194.52}{l^2}+\cdots$
h^-h^-	
$\pi^-\pi^-$	$\propto 1-\frac{29.17}{l}+\frac{374.53}{l^2}+\cdots$
π^-K^-	$\propto 1-\frac{36.52}{l}+\frac{498.01}{l^2}+\cdots$
$\pi^-\bar{p}$	$\propto 1-\frac{36.52}{l}+\frac{560.71}{l^2}+\cdots$
K^-K^-	$\propto 1-\frac{66.97}{l}+\frac{1698.47}{l^2}+\cdots$
$K^-\bar{p}$	$\propto 1-\frac{48.07}{l}+\frac{909.04}{l^2}+\cdots$
$\bar{p}\bar{p}$	$\propto 1-\frac{54.37}{l}+\frac{1254.25}{l^2}+\cdots$
h^+h^+	
pp	$\propto 1-\frac{3.97}{l}+\frac{13.15}{l^2}+\cdots$
π^+p	$\propto 1+\frac{1.27}{l}+\frac{28.67}{l^2}+\cdots$
K^+p	$\propto 1-\frac{10.27}{l}-\frac{59.57}{l^2}+\cdots$
π^+K^+	$\propto 1-\frac{11.32}{l}-\frac{40.63}{l^2}+\cdots$
K^+K^+	$\propto 1-\frac{41.77}{l}+\frac{392.48}{l^2}+\cdots$
$\pi^+\pi^+$	$\propto 1-\frac{3.97}{l}+\frac{21.10}{l^2}+\cdots$

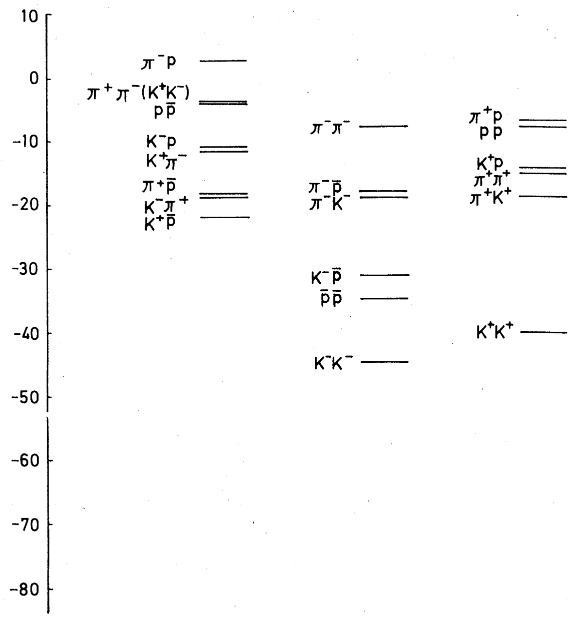

FIG. 2. A plot of $[(h_1h_2)/c-1)]l$ for $\gamma=0.1$ and $l=100$ on the reactions $\pi^-+p\to h_1+h_2+X$. The first column contains the spectrum of $+-$ hadron pairs, the second column the $--$ pairs and the third column $++$pairs.

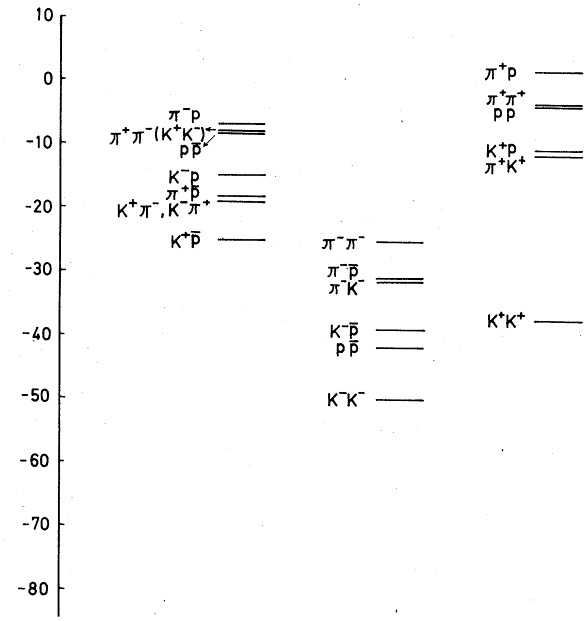

FIG. 3. A plot of $[(h_1h_2)/c-1)]l$ for $\gamma=0.1$ and $l=100$ on the reactions $\pi^++p\to h_1+h_2+X$. The first column contains the spectrum of $+-$ hadron pairs, the second column the $--$ pairs and the third column the $++$pairs.

$$\pi^++p\to h_1+h_2+X\,, \qquad (11)$$

the results are listed in Table IV. (See Fig. 3.)

In brief, we have shown that by combining Chao-Yang statistics with quarks, it is possible to obtain good results in the dihadron spectrum in large transverse momentum. Perhaps the most important point in our calculation is that we have obtained "band structure" without making any dynamical assumption.

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[6]x is the variable for u, $1/x$ for $\bar{u}$, y for d, $1/y$ for $\bar{d}$, z for s, and $1/z$ for $\bar{s}$. When $a=1$, $b=-1$, $c=2$, $N^l_{1,-1,2}$ is the distribution for one u, one $\bar{d}$, and two s quarks.

[7]Consider the reactions $p+p\to K^++X$ and $p+p\to\pi^++X$. In order to satisfy the experimental result $(K^+/\pi^+)<1$, the suppression factor γ has to be less than 0.2. The variation of γ will only change hadron-pair productions involving strange particles. If one changes γ slightly around 0.1 value, the overall pattern remains the same.

[8]The number of quarks in the state $l=100$ is arbitrary and of course depends on the energy of bombarding particles. If one varies l, of course it will affect the ratios of (h_1h_2) pair productions, but so long as one keeps on large value, the overall pattern remains the same.

[9]Since we are interested in the relative probability of the hadron pairs, c never shows up in the calculations.