PHYSICAL REVIEW D VOLUME 19, NUMBER 11 1 JUNE 1979

Elastic hadron-hadron scattering at ultrahigh energies and existence of many dips

T. T. Chou
Department of Physics, University of Georgia, Athens, Georgia 30602

Chen Ning Yang
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
(Received 20 February 1979)

Recent experiments on high-energy, large-t elastic pp scattering are discussed in the context of the geometrical picture. Detailed computations for elastic pp, πp, and $\pi\pi$ scatterings based on this picture are presented. It is emphasized that at very high energies there will be many dips in elastic scattering.

Recently there have been published a number[1,2] of new experiments on high-energy, high-t elastic pp scattering. The present paper is an effort to discuss these experiments in the context of a geometrical model[3] of high-energy elastic scattering first proposed in 1967.

I. GEOMETRICAL MODEL

In the geometrical model[3] one adopts the eikonal approximation for very small wavelengths and writes

$$\left(\frac{d\sigma}{dt}\right)_{\text{elastic}} = \frac{1}{4\pi}\left| \int e^{i\vec{k}\cdot\vec{b}}\{1-\exp[-\Omega(b)]\}d^2b\right|^2, \qquad (1)$$

where $\vec{b}$ is the two-dimensional impact parameter, $\vec{k}$ the two-dimensional momentum transfer, and $\Omega(\vec{b})$ the blackness at impact parameter $\vec{b}$. To relate the elastic differential cross section with the hadronic form factor, a further assumption is adopted; one assumes

$$\tilde{\Omega}(\vec{k}) = K[G_E(k)]^2 \qquad (2)$$

for pp scattering, where $\tilde{\Omega}$ is the two-dimensional Fourier transform of $\Omega(\vec{b})$, and $G_E(k)$ is the "electric" form factor of the proton.[4] The constant K in (2) is to be adjusted so that the total cross section σ_T is equal to the "imaginary" part of the forward scattering amplitude:

$$\sigma_T = 2\int [1-e^{-\Omega(b)}]d^2b \qquad (3)$$

since we have assumed $\Omega(b)$ to be real.

Originally it was thought[3] that at very high energies, σ_T approaches a numerical constant of about 40 mb. In that case K would be a constant. With the discovery[5] of increasing σ_T with increasing energy, it was pointed[6] out by Hayot and Sukhatme that the only way to generalize the geometrical model without destroying the underlying physical concepts is to make K dependent on the incoming energy.

The geometrical model incorporating the Hayot-Sukhatme generalization has been successful in the following comparisons with experiments, all of which contain no adjustable parameters other than to fit the constant K to the observed σ_T via Eq. (3).

(a) It produced[7] excellent agreement with high-energy pp elastic differential cross section, down to details in the $|t| \leq 1.2$-$(\text{GeV}/c)^2$ region.

(b) It predicted[8,9] the existence of a (first) minimum and a (second) maximum which were later found in CERN ISR experiments.[10] The calculated positions of the minimum and maximum[7] were in good agreement with the experimental results.

(c) It predicted[6] a shift of the position of the first minimum and a concurrent rise of the second maximum which are in general agreement with the newest experiments.[2]

It should be emphasized that no other model has had such predictive successes. In fact, no other model has quantitatively any predictive statements to make at all about the existence of a minimum. Furthermore, all other models involve several adjustable parameters. This does not mean that the other models are incorrect or irrelevant. They may be complementary to the geometrical model in certain aspects. In particular, the questions of the real part of the scattering amplitude, and the spin-dependent part[11] of the scattering amplitudes, oftentimes central problems in other models, are both neglected in the simple geometrical model outlined above. They must evidently be dealt with to complete the detailed physical description.

II. DISCUSSION OF RECENT EXPERIMENTS

As mentioned earlier, with the Hayot-Sukhatme generalization, the geometrical model predicts[6] the shift with increasing energy of the positions of the first dip and the second maximum and the rise of the second maximum. We have repeated the

Hayot-Sukhatme calculation using the dipole fit for $G_E(k^2)$. The result is exhibited in Table I. The sign of the change, with increasing energy, of the minimum and maximum positions and the height of the second maximum are all in qualitative agreement with experimental results. Quantitatively the agreement is not particularly impressive. As pointed out in Ref. 6, inclusion of the real part of the scattering amplitude is likely to produce considerable changes in these quantities.

Summarizing the above discussions we are of the opinion that the geometrical picture is one excellent way to understand elastic scattering. It contains no adjustable parameters, and is consistent with time-tested physical concepts for wave propagation at small wavelengths.

The most conspicuous disagreement[12] between experimental data and calculation based on the geometrical model is that the former does not exhibit so far a second minimum and a third maximum while the latter does. We believe the reason for this is simply that the present experimental data are not at high-enough energies. Table I and Fig. 1 indicate that, according to the geometrical picture, when the total cross section is sufficiently high there will be *not only a second minimum*, but *many additional ones*. It is not surprising that this should be the case, since increasing blackness would produce an effective black disc as the scattering center at sufficiently high energies. [This was already discussed by H. Cheng and T. T. Wu, in Phys. Rev. Lett. 24, 1456 (1970); and also H. Cheng, J. K. Walker, and T. T. Wu, in report submitted to the XVIth International Conference on High Energy Physics, Chicago, 1972 (unpublished).]

Are there already hints in the present experimental data of the existence of more than one dip in pp elastic scattering? In an earlier publication[13] we have compared the change of slope at two $|t|$ regions between the 200-GeV data and the 1500-GeV data[1,2] and suggested that there are already such hints. In the meantime, it was pointed out in Ref. 2 that for

$$|t| = 2 \text{ to } 6\ (\mathrm{GeV}/c)^2, \quad \sqrt{s} = 23 \text{ to } 62\ \mathrm{GeV}, \tag{4}$$

the $d\sigma/dt$ curve follows generally a geometrical-scaling[14] rule:

$$\sigma_T^{-2}\frac{d\sigma}{dt}\bigg|_{t=\sigma_T^{-1}a} = \text{independent of energy for fixed } a. \tag{5}$$

One implication of (5) is that the slope parameter

$$\beta = \left|\frac{d}{dt}\left(\ln\frac{d\sigma}{dt}\right)\right|$$

increases with energy proportionally with σ_T in the region (4). This tendency is in general agreement also with the data of Ref. 1. If this tendency remains valid for higher energies, then the slope parameter β will increase to higher values. This increase is indicative of a possible development of a second minimum, as emphasized in Ref. 13, since a comparison of the data of Refs. 1 and 2 indicates that in the higher-$|t|$ region of 7 to 10 $(\mathrm{GeV}/c)^2$, β *decreases* with increasing energy. Thus the general validity of the geometrical-scaling rule in the region (4) is consistent with the possible development of a second minimum.

In Ref. 15 the development of the *first* minimum in pp scattering as incident momentum increases from 30 to 260 GeV/c was plotted. It is a very interesting study and is reproduced here in Fig. 2. We also reproduce as Fig. 3 a plot from Ref. 2 for high-$|t|$ data at ISR energies. A comparison of the two curves in these two figures indicated by arrows suggests to us that the development of a second minimum

TABLE I. Parameters in elastic pp scattering at ISR and ultrahigh energies. The parameters include positions of the first dip and the second maximum, height of the second maximum, elastic cross section, the slope parameter β_0 at $t=0$, and the X, Y parameters.

σ_T (mb)	$(-t)_{\text{1st dip}}$ $[(\mathrm{GeV}/c)^2]$ cal.	expt.	$(-t)_{\text{2nd max}}$ $[(\mathrm{GeV}/c)^2]$ cal.	expt.	$(d\sigma/dt)_{\text{2nd max}}$ $[\mu\mathrm{b}\,(\mathrm{GeV}/c)^{-2}]$ cal.	expt.	σ_{el} (mb) cal.	$X=\frac{\sigma_{el}}{\sigma_T}$ cal.	β_0 $[(\mathrm{GeV}/c)^{-2}]$ cal.	$Y=\frac{\sigma_T}{16\pi\beta_0}$ cal.
38.9	1.46	1.44	1.85	1.97	1.5×10^{-2}	4.5×10^{-2}	6.46	0.165	12.3	0.161
40.2	1.38	1.42	1.76	1.93	2.8×10^{-2}	4.2×10^{-2}	6.87	0.170	12.4	0.166
41.7	1.30	1.36	1.67	1.92	5.4×10^{-2}	5.2×10^{-2}	7.30	0.175	12.4	0.171
42.5	1.26	1.34	1.63	1.81	7.3×10^{-2}	5.8×10^{-2}	7.53	0.177	12.5	0.173
43.0	1.23	1.31	1.61	1.81	8.3×10^{-2}	6.3×10^{-2}	7.70	0.179	12.5	0.175
60	0.78		1.10		4.5		13.6	0.226	13.6	0.225
80	0.55		0.83		5.4×10^{1}		21.5	0.269	15.1	0.270
100	0.42		0.65		2.7×10^{2}		30.0	0.300	16.9	0.303
120	0.34		0.54		8.2×10^{2}		38.8	0.323	18.9	0.324
200	0.19		0.32		8.6×10^{3}		74.7	0.373	29.6	0.344

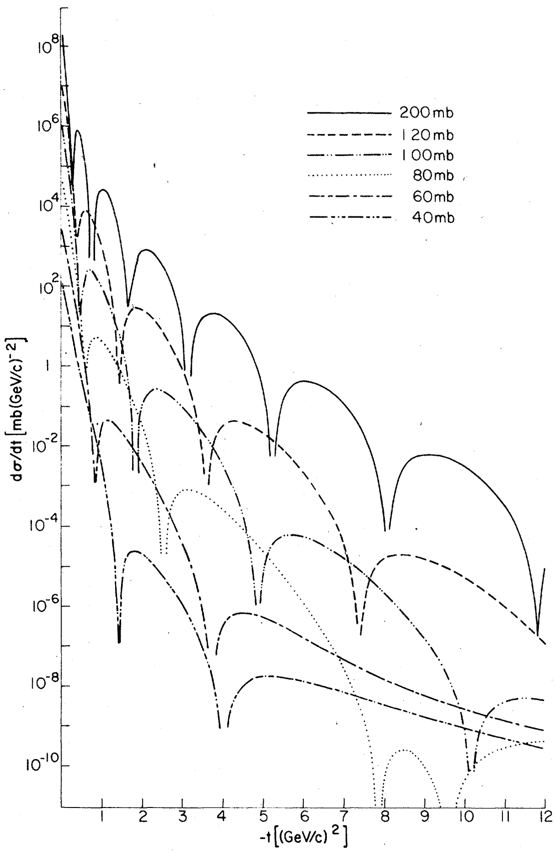

FIG. 1. Differential cross sections for proton-proton elastic scattering computed from the geometrical model for σ_T=40, 60, 80, 100, 120, and 200 mb. The vertical scale shown is for σ_T=40 mb. The other curves are displaced from each other by a factor of 10.

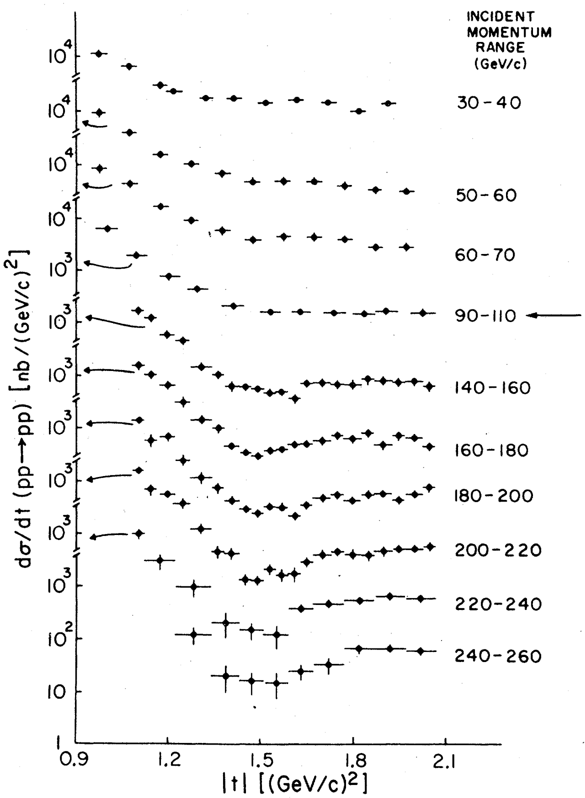

FIG. 2. Differential cross sections for proton-proton elastic scattering from 30 to 250 GeV/c, reproduced from Fig. 3 of Ref. 15.

at $|t| < 7$ (GeV/c)^2 at $\sqrt{s} > 53$ GeV may be an attractive hypothesis.

To test this hypothesis we have suggested in Ref. 13 ISR experiments to measure the energy dependence of $d\sigma/dt$ at fixed t, for $|t| = 4$ to 9 (GeV/c)^2. An alternative test would be to repeat the $\sqrt{s} = 53$-GeV measurement at $-t = 8$ to 10 (GeV/c)^2, and to extend it to higher $|t|$ values to see whether a "shoulder" [i.e., a point of inflection along the $\ln(d\sigma/dt)$ vs t curve] already exists for the $\sqrt{s} = 53$-GeV curve. We believe this test to be of *crucial importance* since the four points at $\sqrt{s} = 53$ GeV at $-t = 8$ to 10 (GeV/c)^2 reported first in Ref. 16 and replotted in Ref. 2 indicate the existence of a shoulder. If this is confirmed it is hard to escape the conclusion that a second minimum is in the process of formation. If on the other hand these four points turn out to be statistical fluctuations, all on the high side, then there is at present no experimental hints of any second minimum at all.

At what t values will the second minimum appear and at how high an energy will the development of a second minimum be evident? There is no clear way of answering this question. If the value of β remains proportional to σ_T as the geometrical-scaling rule states, then it will require very high values of $\sigma_T > 120$ mb before β increases to a value as large as 6 (GeV/c)$^{-2}$. The energy required to reach such a high value of σ_T would be enormous. We have computed from the geometrical model the $d\sigma/dt$ curve for several high values of σ_T and exhibit the results in Fig. 1 and Table I. The $|t|$ value at the second dip at such high values of σ_T is quite small, as is obvious from Fig. 1:

$$(-t)_{\text{second minimum}} = 1.39 \ (\text{GeV}/c)^2 \quad \text{for } \sigma_T = 120 \text{ mb}. \tag{6}$$

On the other hand, Fig. 2 clearly shows that during the formation of the first minimum, the slope parameter β for $|t|$ values below the first minimum increases quite rapidly over an energy region in which the total cross section σ_T does not change very much at all. If this pattern repeats itself, the geometrical-scaling rule would not re-

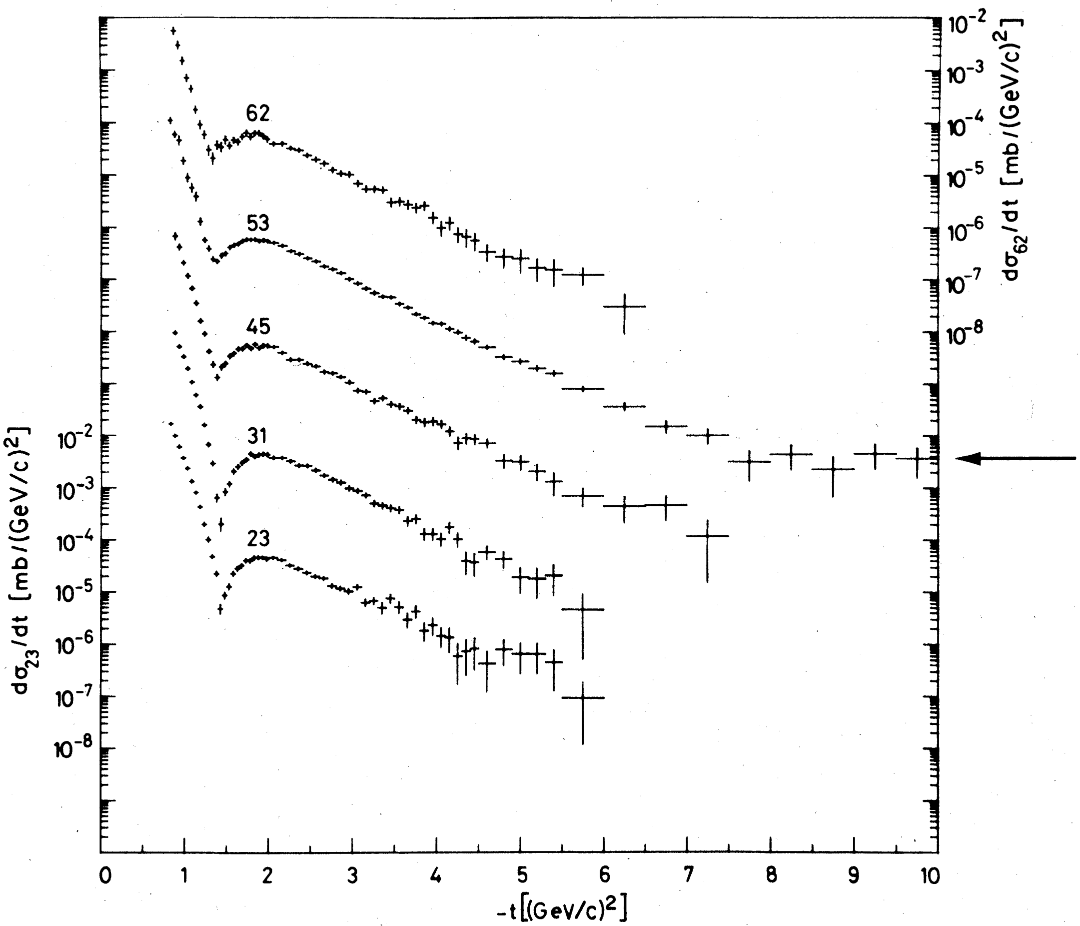

FIG. 3. Differential cross sections for proton-proton elastic scattering at $\sqrt{s}=23$, 31, 45, 53, and 62 GeV, reproduced from Fig. 14 of Ref. 2. The five different curves are displaced by a factor of 100 from each other.

main valid for higher energies, and the second dip may show up at a higher $|t|$ value and a lower total cross section σ_T than indicated in (6).

III. FURTHER COMPUTATIONS

We have also computed the value of σ_{el}/σ_T for various values of σ_T in pp scattering, using the geometrical model. The results are tabulated in Table I.

Application of the geometrical model to πp scattering has been used[17] to determine the pion radius. We applied the geometrical model to compute high-energy $\pi\pi$ scattering by using the pion form factor given in Ref. 18. The differential cross section for different values (11 mb and 17 mb) of the total cross section σ_T is plotted in Fig. 4. It is interesting that the two curves are very little different. The values of σ_{el}/σ_T are tabulated in Table II.

So far πp and Kp elastic scattering do not show any dips at all. For the same reason as for the case of pp scattering we believe many dips would

TABLE II. Parameters in elastic $\pi\pi$ scattering at ultrahigh energies.

σ_T (mb)	σ_{el} (mb)	$(d\sigma/dt)_{t=0}$ [mb (GeV/c)$^{-2}$]	β_0 [(GeV/c)$^{-2}$]	$X=\frac{\sigma_{el}}{\sigma_T}$	$Y=\frac{\sigma_T}{16\pi\beta_0}$
11	1.07	6.18	7.26	0.097	0.077
13	1.45	8.63	7.37	0.112	0.090
15	1.88	11.50	7.49	0.126	0.102
17	2.36	14.77	7.60	0.139	0.114

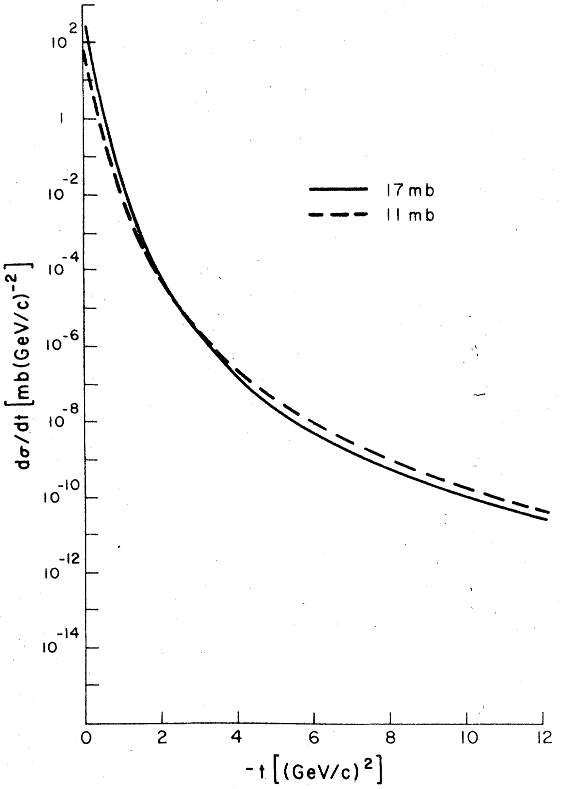

FIG. 4. Differential cross sections for pion-pion elastic scattering computed from the geometrical model for σ_T=11 and 17 mb.

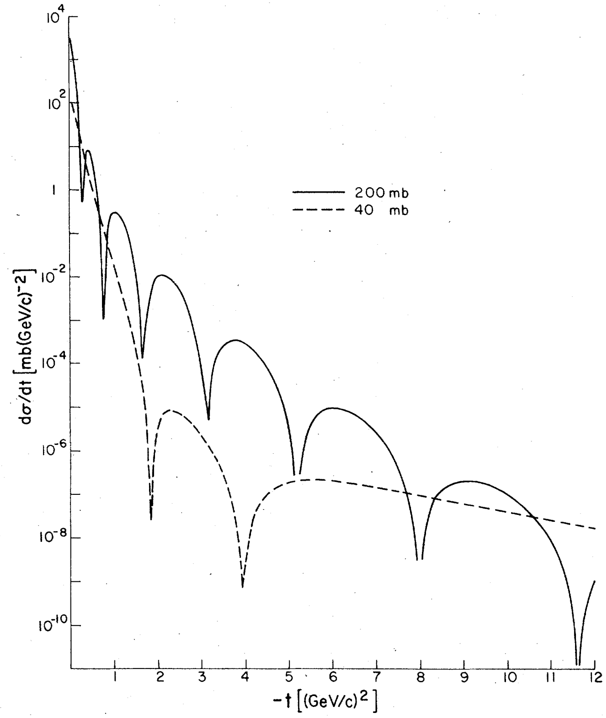

FIG. 5. Differential cross sections for pion-proton elastic scattering computed from the geometrical model for σ_T=40 and 200 mb.

develop for sufficiently high total cross sections in πp and Kp elastic scattering. We exhibit a sample calculation for πp scattering in Fig. 5 using[19] the geometrical model, taking $\sigma_T = 40$ and 200 mb. The elastic differential cross sections for πp and pp scatterings at $\sigma_T = 200$ mb are remarkably similar. In fact the locations of zeros for both cases are almost identical, and, to a good approximation, coincide with the zeros of scattering from a black disc.

ACKNOWLEDGMENT

We wish to thank Dr. S. L. Olsen for sending us the original graph used for reproduction of Fig. 2. The work of T.T.C. was supported in part by the Department of Energy under Contract No. EY-76-S-09-0946. The work of C.N.Y. was supported in part by the National Science Foundation under Grant No. PHY7615328.

[1]J. L. Hartmann *et al.*, Phys. Rev. Lett. 39, 975 (1977); S. Conetti *et al.*, *ibid.* 41, 924 (1978).

[2]E. Nagy *et al.*, Nucl. Phys. B150, 221 (1979).

[3]T. T. Chou and C. N. Yang, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland, Amsterdam, 1967), pp. 348–359; Phys. Rev. 170, 1591 (1968).

[4]T. T. Chou and C. N. Yang, Nucl. Phys. B107, 1 (1976).

[5]U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973); S. R. Amendolia *et al.*, *ibid.* 44B, 119 (1973); A. S. Carroll *et al.*, Phys. Rev. Lett. 33, 932 (1974). The increase of total cross sections was conjectured theoretically by H. Cheng and T. T. Wu, *ibid.* 24, 1456 (1970).

[6]F. Hayot and U. P. Sukhatme, Phys. Rev. D 10, 2183 (1974).

[7]M. Kac, Nucl. Phys. B62, 402 (1973).

[8]T. T. Chou and C. N. Yang, Phys. Rev. Lett. 20, 1213 (1968).

[9]L. Durand and R. Lipes, Phys. Rev. Lett. 20, 637 (1968).

[10]A. Böhm *et al.*, Phys. Lett. 49B, 491 (1974).

[11]Some aspects of the spin-dependent effect have been discussed in Ref. 4.

[12]U. P. Sukhatme, Phys. Rev. Lett. 38, 124 (1977); T. Maehara, T. Yanagida, and M. Yonezawa, Prog. Theor. Phys. 57, 1097 (1977); S. Wakaizumi and

M. Tanimoto, Phys. Lett. 70B, 55 (1977).
[13]T. T. Chou and C. N. Yang, Phys. Rev. D 17, 1889 (1978).
[14]J. Dias de Deus, Nucl. Phys. B59, 213 (1973); A. J. Buras and J. Dias de Deus, *ibid.* B71, 481 (1974).
[15]R. Rusack *et al.*, Phys. Rev. Lett. 41, 1632 (1978).
[16]H. De Kerret *et al.*, Phys. Lett. 68B, 374 (1977).
[17]T. T. Chou, Phys. Rev. D 11, 3145 (1975); 19, 3327 (1979).
[18]C. J. Bebek *et al.*, Phys. Rev. D 17, 1693 (1978).
[19]See also S. C. Chan *et al.*, Phys. Rev. D 17, 802 (1978).