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Energy dependence of the eikonal in p-p elastic collisions

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A careful analysis of the elastic p-p scattering in the energy range $20 \lesssim \sqrt{s} \lesssim 60$ GeV is presented. Under the hypothesis of a pure imaginary amplitude with two zeros, the experimental data suggest that the opacity be parametrized as $\chi(s,b) = \chi_f(b) + \ln(s/s_0)\chi_0(b)$, where the energy-dependent term is much wider than $\chi_f(b)$, contrary to the factorization hypothesis and some earlier results. This conclusion is precisely the one predicted by the two-component model which has been proposed in our previous work.

I. INTRODUCTION

In analyzing high-energy hadron-hadron elastic scatterings, it is customary to employ an eikonal model, where the eikonal (or opacity) χ is somehow related to the matter distribution inside the incident particles. When Chou and Yang[1] first proposed this kind of model it was everybody's belief that the cross sections were tending to some constant asymptotic values, so that their proposal $\chi \sim \langle F_A F_B \rangle$ (here $\langle\ \rangle$ indicates the two-dimensional Fourier transform and F_A and F_B are the electromagnetic form factors of the two interacting hadrons) was perfectly natural and this relation was expected to be valid as $s \to \infty$.

As the CERN ISR and, more recently, the Fermilab accelerator have begun to work, that belief has dissipated rapidly and today we know that as the incident energy rises the total, inelastic, and elastic cross sections slowly but steadily increase. Also, the angular distribution at small $-t$ continues to shrink. These energy dependences of the cross sections presented a problem to the earlier Chou-Yang proposal, since now we do not know at just which energy the above identification (namely $\chi \sim \langle F_A F_B \rangle$) should be done. It seems possible to overcome this difficulty by making an assumption that the opacity factorizes,[2] that is,

$$\chi(s,b) \sim f(s)\,\chi_0(b)\,, \tag{1}$$

and saying that the space-dependent part of χ is related to the hadronic matter distribution of the incident particles. We do not know, however, any fundamental reason for this factorization, so it should be carefully examined by comparing with the existing experimental data.

The main object of the present paper is to report the results of a systematic and careful analysis of the published pp elastic data, to compare them with some earlier works, and finally to interpret them in terms of our previous work.[3]

The plan of presentation is the following. In the next section we collect all the experimental data which have been used in this analysis, which is described in Sec. III, together with the results. Comparisons with some earlier results and also with some models are carried out in Sec. IV, where we briefly delineate the previously proposed two-component model and then compare it with the results obtained here.

II. EXPERIMENTAL DATA

In the present analysis, we have considered essentially all the existing pp elastic data, obtained both at the ISR[4-11] and at Fermilab,[12-15] covering the energy range of $\sqrt{s} = 20-60$ GeV.

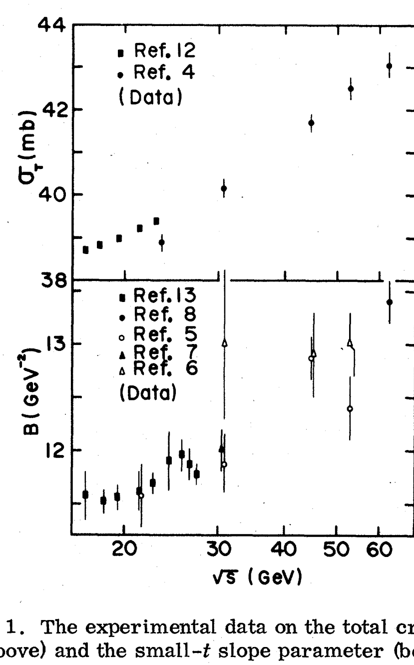

FIG. 1. The experimental data on the total cross section (above) and the small-t slope parameter (below) which have been used in the present analysis. The t interval where the slope parameter has been measured is not fixed, but depends on the energy and on the experiment.

These data may be grouped as follows.

(a) Total cross sections. We have used only the more recent accurate Fermilab data from Ref. 12 and the ISR data obtained by the CERN–Pisa–Rome–Stony Brook collaboration.[4] They are shown in Fig. 1, where one can see that they are approximately on the same curve (or perhaps there is a systematic deviation of ~0.5 mb between the two groups of data).

(b) Slope parameters for very small-t. There are five independent measurements[5-8,13] which are shown in Fig. 1. In treating these data, one must keep in mind which is the t interval where each point has been obtained. For instance, at $\sqrt{s}=30$ GeV, Ref. 6 gives the slope in the interval $0.015<-t<0.055$ GeV^2, whereas the data given by Ref. 5 refer to $0.046<-t<0.09$ GeV^2. This may be the main origin of their discrepancy as they appear in Fig. 1, since $B=13.0\pm0.7$ GeV^{-2} is compatible with the data points of Ref. 5 when the t interval is correctly chosen.

(c) Normalized angular distributions. There are four independent data[9,10,14,15] in different energy or t intervals, such that no direct consistency check is possible among them. We can see, however, that the data of Refs. 9 and 14 extrapolate correctly down to the small-t data mentioned in (a) and (b).

(d) Un-normalized angular distributions. There are two angular-distribution measurements at ISR energies which have not been normalized, but contain valuable information which is essential in determining the energy-dependent behavior of the amplitude. One is Ref. 5, which gives the angular distributions at small $-t$ for several energies. They have been normalized by extrapolating down to the optical point, using also the slopes mentioned in (b). The second group of un-normalized differential cross-section data is that of Ref. 11. When compared with the others, these data show some systematic tendency towards bending downward at the smallest t values. In normalizing them, we have taken this effect into account and, excepting these lowest-t points, we could see that they agree well with the other existing data (Fig. 6).

These data have been divided, according to their energy, into six distinct groups represented by the average center-of-mass energies $\sqrt{s}=19.5$, 23, 30.8, 44.8, 53, and 62 GeV. Each group of data is then parametrized as described in the next section and, after this, analyzed all together.

III. ANALYSIS AND RESULTS

In order to supply gaps in the available experimental information, the following assumptions have been made in the present analysis:

(1) As the energy increases, the amplitude varies smoothly.

(2) The amplitude is purely imaginary with two simple zeros.

The first assumption means the absence of any remarkable interaction mechanism, which abruptly changes the collision at definite energies. Since we do not know any such mechanism in the range considered here, and, since whenever data exist to test this requirement they do fulfill it, this assumption seems natural to us.

The second assumption is actually composed of two parts: (a) pure imaginary amplitude and (b) with two zeros. Direct information about the real part of the amplitude is only available in the forward direction,[8,16] where its contribution to the differential cross section is always less than 1% in the entire energy range we are considering. For nonforward direction, existing calculations[17,18] based on dispersion relations and derivative analyticity relations seem to indicate that, as far as $d\sigma/dt$ is concerned, the real part of the amplitude may be significant only in the neighborhood of the minima of $d\sigma/dt$. Thus, we believe that the omission of the real part does not seriously affect the results. Concerning the number of zeros, it has been pointed out by Durand and Lipes[19] that, in order that the Fourier transform of the opacity remain always positive, the amplitude should change its sign an even number of times. In that paper, this was fundamental since their proposal was $\chi\sim\langle F^2\rangle$ as in Ref. 1. Although we did not have such a restriction, we have nevertheless adopted the same criterion in determining χ. It is clear that if Eq. (1) is valid, with $\chi_0(b)\sim\langle F^2\rangle$, we must necessarily have such an amplitude. Having accepted the above assumption, we are still faced with the question of the exact number of zeros. The amplitude may have no zeros[18,20] remaining always positive. In this paper, we have taken a more orthodox view, changing its sign at the first minimum of $d\sigma/dt$. In this case, we must have at least one more zero somewhere. There is an indication of the second minimum[10] of $d\sigma/dt$ around $-t=8$ GeV^2 at $\sqrt{s}=53$ GeV, but in a similar measurement[15] at $\sqrt{s}=19.5$ GeV there is no such indication up to $-t=12$ GeV^2. In the present analysis, we have assumed the second zero of the amplitude to be at $-t=8$ GeV^2 for $\sqrt{s}=53$ GeV, whereas for $\sqrt{s}=19.5$ GeV, we assume it to be at $-t=12$ GeV^2. For the other values of energy, we have made a logarithmic interpolation as shown in Fig. 2. In the same figure the energy dependence of the position of the first minimum is also displayed, showing that it is consistent with a logarithmic decrease. Actually, the exact position of the second zero of the amplitude, as well as the possible ex-

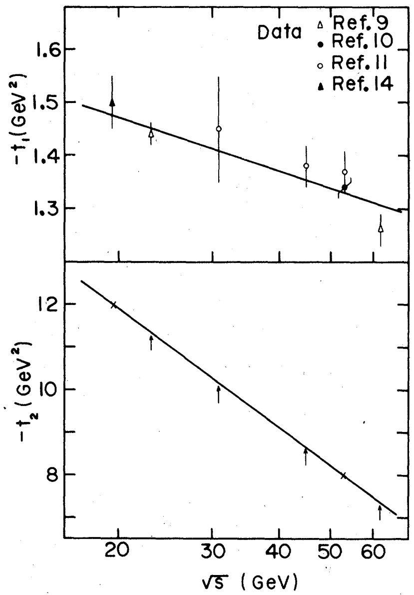

FIG. 2. In the upper part, we show the existing data on the position of the first minimum of $d\sigma/dt$ as a function of the center-of-mass energy. They are roughly on a straight line. In the lower part, the position of the second minimum, which has been used here, is displayed. At $\sqrt{s} = 53$ GeV. the cross indicates its value which appears in Ref. 10; at $\sqrt{s} = 19.5$ GeV, the cross indicates the highest $-t$ value where $d\sigma/dt$ has been measured (Ref. 15). An interpolation to other values of the energy is indicated by the straight line and the arrows.

istence of other zeros, has little practical importance because the experimental uncertainties at smaller values of t are too large to allow such refinements.

Besides the two assumptions mentioned above, we have also neglected the spin dependence, which may affect large-t data, but presumably has negligible affects in the final results on the opacity.

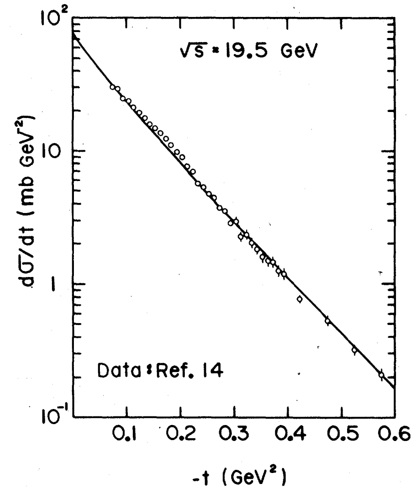

FIG. 3. The elastic cross section given by Eqs. (2) and (3) with the parameters listed in Table I, at $\sqrt{s} = 19.5$ GeV. Experimental data are also displayed for comparison.

The amplitude has then been written as a sum of exponentials, that is,

$$a(s,t) = a(s,0) \sum_{i=1}^{6} \alpha_i \exp(\beta_i t), \tag{2}$$

where $a(s,t)$ is related to $d\sigma/dt$ by

$$\frac{d\sigma}{dt} = \pi |a(s,t)|^2 \tag{3}$$

and the parameters $a(s,0)$, α_i, and β_i have been determined at each energy value by fitting the data listed in the preceding section. We summarize the results in Table I. Comparisons of our parametrization with data at two energies are made in Figs. 3, 4, 5, and 6.

Once the amplitude has been determined, we could compute the opacity following the conventional procedure, getting

TABLE I. The parameters defining the amplitude given by Eq. (2) and which have been fixed by fitting the published experimental data as explained in the text.

$\sqrt{s}$ (GeV)	19.5	23	30.7	44.9	53	62
$\|a(s,0)\|$ (GeV^{-2})	7.97	8.03	8.25	8.52	8.69	8.80
α_1	0.164 12	0.164 12	0.164 12	0.164 11	0.164 11	0.164 11
α_2	0.776 01	0.776 01	0.776 00	0.775 97	0.775 97	0.775 94
α_3	0.068 93	0.068 93	0.068 93	0.068 93	0.068 93	0.068 93
α_4	−0.008 38	−0.008 38	−0.008 38	−0.008 38	−0.008 38	−0.008 38
α_5	−0.000 76	−0.000 76	−0.000 76	−0.000 76	−0.000 76	−0.000 76
α_6	0.000 08	0.000 08	0.000 09	0.000 13	0.000 14	0.000 16
β_1	12.5	12.7	12.7	13.3	14.0	14.0
β_2	5.10	5.30	5.30	5.40	5.55	5.57
β_3	2.60	2.65	2.65	2.85	2.90	3.00
β_4	1.150	1.170	1.170	1.175	1.200	1.200
β_5	0.45	0.46	0.47	0.47	0.48	0.48
β_6	0.26	0.26	0.26	0.26	0.26	0.26

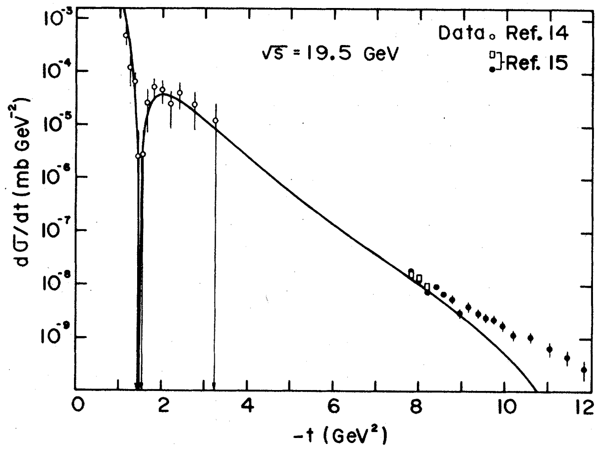

FIG. 4. The same as Fig. 3, but for large $-t$. The data indicated by □ correspond to measurements in the range $4.9 < -t < 8.3$ GeV^2 and lie on the curve shown above. Near the minima, the real part of the amplitude is expected to dominate.

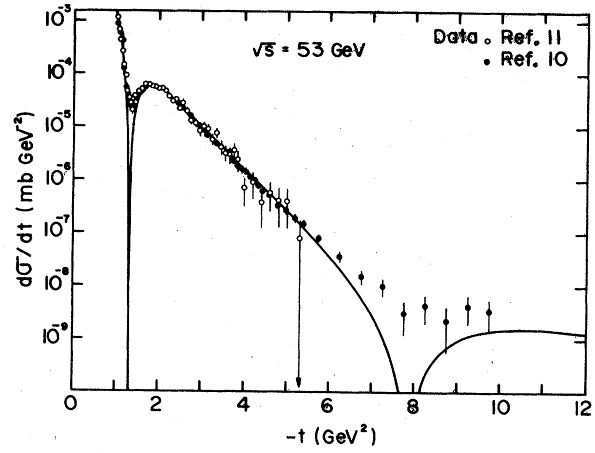

FIG. 6. Same as Fig. 5, but for large $-t$. Near the minima, the real part of the amplitude is expected to dominate.

$$\chi(s,b) = -\ln\left[1 + i\int \frac{d^2k}{2\pi} a(s, t = \vec{k}^2)\exp(i\vec{b}\cdot\vec{k})\right] \tag{4}$$

or, by using Eq. (2),

$$\chi(s,b) = -\ln\left[1 - |a(s,0)| \sum_{i=1}^{6} \frac{\alpha_i}{2\beta_i} \exp\left(\frac{-b^2}{4\beta_i}\right)\right]. \tag{5}$$

The results of this calculation at several values of b, using the parameters listed in Table I, are shown in Fig. 7. As can be seen there, a logarithmic increase of $\chi(s,b)$ is consistent with them in the entire range of b, suggesting a parametrization of the form

$$\chi(s,b) = \chi_f(b) + \ln(s/s_0)\chi_0(b), \tag{6}$$

where $\chi_f(b)$ and $\chi_0(b)$ are functions of the impact parameter alone, not necessarily identical to each other. s_0 in Eq. (6) is some scale factor, which can be taken, for instance, as the threshold value $s_0 = (2m_p + m_\pi)^2 \approx (2m_p)^2$. In a previous work,[3] where we studied a classical source model, this constant appeared naturally and with the same order of magnitude. We have indeed adopted this parametrization and, by using the least-squares method, fitted it to the points which have been obtained before, according to Eq. (5). The resulting curves for $\chi_f(b)$ and $\chi_0(b)$ are displayed in Fig. 8. The main features are the following: (i) $\chi_0(b)$ is much smaller than $\chi_f(b)$; (ii) $\chi_0(b)$ is much more peripheral compared with $\chi_f(b)$.

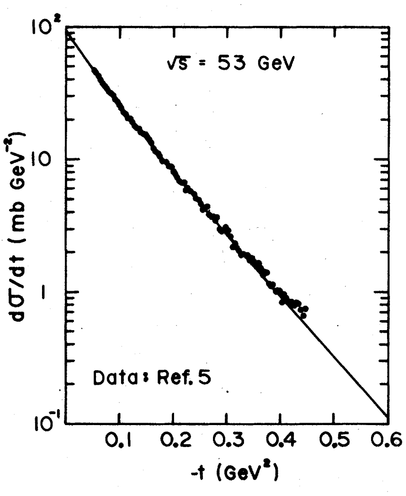

FIG. 5. The elastic cross section given by Eqs. (2) and (3) with the parameters listed in Table I, at $\sqrt{s} = 53$ GeV. Experimental data are also shown for comparison.

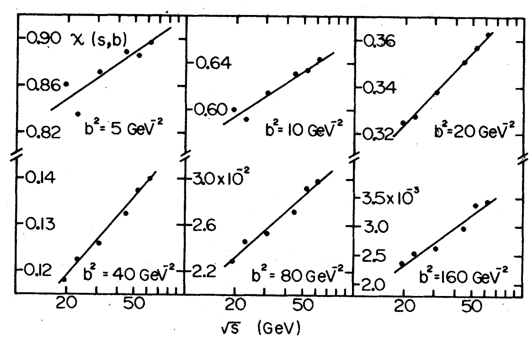

FIG. 7. Energy dependence of the eikonal $\chi(s,b)$, which has been calculated with the use of Eq. (5), for several values of the squared impact parameter. The straight lines are fits in terms of Eq. (6).

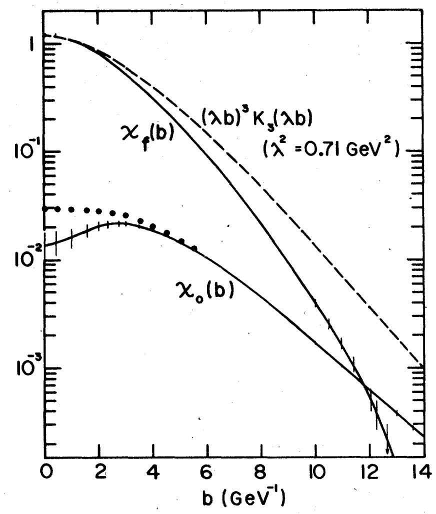

FIG. 8. $\chi_f(b)$ and $\chi_0(b)$ which appear in Eq. (6), as determined in the present analysis. For $\chi_0(b)$, we show two versions, one including all the data mentioned in Sec. II (solid line) and the other excluding the Fermilab data (dotted line). For comparison we also display the opacity predicted by the dipole formula with $\lambda^2 = 0.71$ GeV^2 (broken line). The error bars which appear in χ_f and χ_0 include just the fluctuations which appear in Fig. 7.

Going back to Fig. 7, one may feel that the lowest-energy points, those corresponding to the Fermilab data, are systematically in conflict when compared to the other points. A small (~0.5-mb) deviation which appears in the total cross-section data (Fig. 1) may be another indication of this discrepancy. Since these data have been obtained with a different machine, such a discrepancy is perfectly conceivable and, in order to avoid a possible distortion of the results, we have also performed a calculation omitting the Fermilab data. As seen in Fig. 8, the neglect of these data did not change the results stated above in any significant way. The only modification is that the central depression of χ_0 has now disappeared.

IV. COMPARISON WITH SOME OTHER RESULTS AND MODELS

We now proceed to compare the results obtained in the preceding section with some earlier results.

First of all, a factorized eikonal such as the one given by Eq. (1) and used by some authors[2] is discarded by our results. Yet, we observe here that if one would be contented with only a crude analysis of data the factorization hypothesis might be useful. The existence of some results which favor a factorized eikonal[21] as well as some other ones with a nonfactorized eikonal, but with the second conclusion at the end of Sec. III inverted,[22] is an indication of the above statement. We shall come back to the discussion of these works shortly. Anyway, let us emphasize that if all the existing experimental information is properly taken into account, it would tell us more than such a rough result.

In recent works,[22] Chou has analyzed mainly data from Ref. 9 and concluded that the energy-dependent part of the eikonal is more central than its constant part. In our opinion, however, the small-t cross section should be more carefully treated. In one of his analyses, the slope parameter B in the very-small-t intervals decreases with the incident energy, in evident opposition to the existing data shown in Fig. 1. For instance, according to his parametrization, the slope parameter in the interval $0.05 < -t < 0.1$ GeV^2 becomes $B = 12.50$, 11.92, and 11.88 GeV^{-2} for $\sqrt{s} = 23$, 53, and 62 GeV, respectively. This decrease of B, when combined with the increase of σ_T, clearly gives an eikonal which increases centrally. In the more recent version, he does consider a shrinking angular distribution, but it seems to us that the slope parameter for $\sqrt{s} = 62$ GeV has not been taken sufficiently large. It is not clear what has exactly been done there, but if the value $B = 12.40 \pm 0.30$ GeV^{-2} for $\sqrt{s} = 53$ GeV and interval $0.06 < -t < 0.11$ GeV^2, given by Ref. 5, has been extrapolated down to $t = 0$ and used for $\sqrt{s} = 62$ GeV, we think this value is definitely too low, as is clear in Fig. 1.

In some earlier works[18,23] a peripheral increase of the cross section has been pointed out. In all of them, however, the discussion goes around the overlap function and does not refer to the eikonal. It is clear that, even when the energy variation of the eikonal is central, the one corresponding to the overlap function may appear peripheral. Thus, the ambiguity should be removed by discussing directly the opacity, if one intends to consider the internal structure of the interacting hadrons.

Another hypothesis which has often been used in analyzing the impact structure of hadron-hadron collisions is geometrical scaling.[24] While this hypothensis becomes more reasonable (at least as an approximation) when stated in terms of the overlap function, since there is limiting value at the origin which is almost reached at the ISR energies, its validity also in terms of the eikonal has been claimed in Ref. 25, which is not obvious and must be carefully investigated. Our conclusion is that if one neglects the 1% increase in $\chi(b=0)$ in the interval $\sqrt{s} = 20-62$ GeV the geometrical scaling of the eikonal is quite reasonable. However, if one considers only the ISR data, omitting those at $\sqrt{s} = 19.5$ GeV, some systematic deviations are observed (see Figs. 9 and 10). Namely, in order to obtain the opacity at $\sqrt{s} = 62$ GeV the one cal-

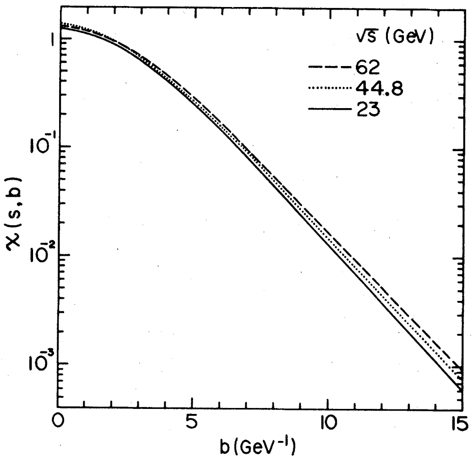

FIG. 9. The opacity, as a function of the impact parameter, for three ISR energies.

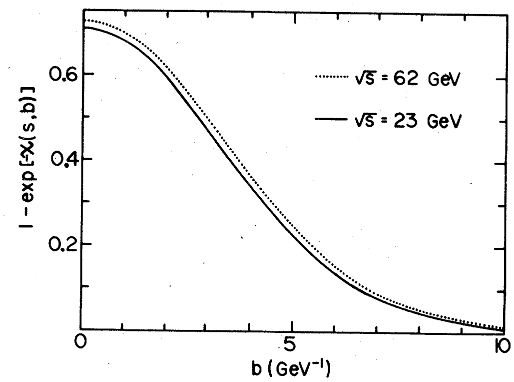

FIG. 10. The Fourier transform of the amplitude for the two extreme ISR energies.

culated at $\sqrt{s}=23$ GeV must be stretched, not uniformly as stated by the geometrical scaling, but in a way that the large-impact-parameter points are less expanded than the small-impact-parameter ones. Compared to the ratio $[\sigma_T(62/\sigma_T(23)]^{1/2}$, the expansion factor is about 7% larger at $b\simeq 0.5$ F and 2% smaller at $b\simeq 2$ F. It should also be noted that if we seriously consider the second minimum, which appears in the $\sqrt{s}=53$ GeV data[10] around $-t=8-9$ GeV^2, then the position of this minimum decreases too quickly compared with the geometrical scaling hypothesis ($t_2 \sim \sigma_T^{-1}$), since no such minimum has been observed at $\sqrt{s}=19.5$ GeV up to $-t=12$ GeV^2. In short, while the factorization hypothesis for the eikonal is definitely discarded, provided the assumptions introduced in Sec. III are valid, the soundness of the geometrical scaling hypothesis remains inconclusive, although there are indications of its breakdown.

In a previous paper,[3] we discussed a model for high-energy hadron-hadron collisions, where two independent interaction mechanisms were considered: (a) *pionization*, depicted as an excitation of the meson field induced by a classical source representing the incident particles in interaction and (b) *fragmentation*, described as a two-stage process consisting of an incident-particle excitation followed by its decay. We have shown that the first mechanism accounts for the $\chi_0(b)\ln(s/s_0)$ increase of the opacity, whereas the second mechanism has been assumed to give a constant background to these.

Let us then interpret the results exposed in Sec. III, in terms of our two-component model. First, let us tentatively identify, following Chou and Yang in the original version,[1] $\chi_f \sim \langle F^2\rangle$, that is, we are assuming that before the opacity begins to increase it is related to the matter distribution inside the proton in exactly the way proposed by those authors. Taking for F the dipole formula, we can readily obtain its Fourier transform[19]

$$\langle F^2\rangle \sim (\lambda b)^3 K_3(\lambda b)\,, \tag{7}$$

which is shown in Fig. 8.

It can be seen that our χ_f is slightly narrower than the result predicted with $\lambda^2=0.71$ GeV^2. Actually, it practically coincides with the curve so calculated with $\lambda^2\approx 1$ GeV^2, as in the Durand and Lipes paper. This means that, as far as the fragmentation is concerned, the matter distribution inside a proton is more compact than the charge distribution. (Here we are assuming the usual relation between the charge distribution and the form factor[26]; however, a somewhat different relation, which has been proposed recently,[27] gives an even more compact charge distribution, in relation to the one obtained here.)

Next, let us consider the pionization. If one neglects the depression which appears in the central region, $\chi_0(b)$ may well be approximated by a Gaussian $\exp(-0.0291\,b^2)$ which corresponds in its turn to a density $\rho(r)$ in Eq. (2.15) of Ref. 3 with the same Gaussian form above. In terms of the average radius, $\bar{r}\simeq 1.15$ F, so this distribution is broader in comparison with the part which is responsible for the fragmentation. That is, the pionization as depicted in our model is a more peripheral process.

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