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Polarization asymmetries in pion-proton radiative scattering

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A Lagrangian isobar model is used to calculate polarization asymmetries for pion-proton radiative scattering. Numerical results are presented for incident pions of 298 MeV kinetic energy and various coplanar and noncoplanar kinematical situations. It is found that the asymmetry calculations are sensitive to the structure of the interaction even at low photon energies, and thus may present a better way of studying structure effects than has been the case with differential cross sections.

INTRODUCTION

For the last several years, bremsstrahlung has been studied in depth as a possible way to learn the off-shell behavior of strong scattering amplitudes, with limited success. The particular process of radiative pion-nucleon scattering has been considered, in addition, to determine the magnetic dipole of the $\Delta^{++}(1232)$ resonance.[1,2] The motivation for this effort has been that the resonance plays an important part in pion-nucleon scattering at intermediate energies, and therefore its magnetic moment should play an important part in the radiative process. Unfortunately, the off-shell effects which come into play in the inelastic radiative process are still not well understood, as pointed out in various publications,[3-5] and this uncertainty has made it difficult to extract any information about the electromagnetic properties of the Δ^{++} resonance. A value of the magnetic moment of the resonance has been calculated based on the absence of any resonant structure in the data.[6] However, there is no comprehensive theoretical description of the radiative process which agrees with the data obtained by the only experimental group that has attempted this rather difficult experiment.[7-9]

The study of cross sections may in fact be a rather poor way to look for off-shell effects, since they consist of sums of squares of amplitudes and as such are insensitive to possibly small off-shell variations. This point was stressed by Moravcsik[1] in a model-independent study of off-shell measurements, with the conclusion that polarization experiments can provide a cleaner approach to the problem.

To see the significance of this point in pion-nucleon radiative scattering, consider the polarization asymmetries defined by $A_i = (d\sigma_+ - d\sigma_-)/(d\sigma_+ + d\sigma_-)$, where $d\sigma_\pm$ are the cross sections with the target nucleon spin parallel or antiparallel to $\hat{i}$. Parity conservation requires that the asymmetry vanish when the target-nucleon polarization is in the scattering plane, both for elastic pion-nucleon scattering and for the coplanar radiative process. As a result, any polarization asymmetry which exists in noncoplanar pion-nucleon bremsstrahlung, with target nucleon polarized in the scattering plane, must be strongly influenced by the off-shell part of the interaction.

Thus, it is clear that any future pion-nucleon bremsstrahlung experiments should include both cross-section and polarization-asymmetry measurements. This is a task which is particularly suitable for the meson facilities' intense pion beams and polarized targets.

The purpose of this paper is to present a calculation of polarization asymmetries for pion-nucleon radiative scattering based on a Lagrangian isobar model. The model is similar to that used by Pascual and Tarrach[6] in their calculation of the magnetic moment of the Δ^{++} and by other authors in the past.[3,11]

MODEL AND CALCULATION

Several types of theoretical calculations of πN bremsstrahlung have appeared in the literature. They range from models in which radiation from the external particles dominate[12,13] to elaborate applications of the soft-photon theorem[2] and gauge-invariant Lagrangian models.[3,11] The latter calculations for energies near the $\Delta(1232)$ predict a clear manifestation of the Δ resonance as a bump in the photon spectrum. However, the thorough experimental study conducted by the UCLA group[7-9] near the resonance region failed to demonstrate the existence of the bump for any reasonable value of the Δ magnetic moment.

The Lagrangian model presented here is similar to those used in the past[3,6,11] and, as will be seen, it predicts cross sections which are similar to earlier theoretical calculations. At the same time it will serve to illustrate the sensitivity of the polarization asymmetries to changes in the structure of the interaction even at low photon energies, where the cross-section predictions of the model are reliable and agree with experiment.

The diagrams which contribute to the process

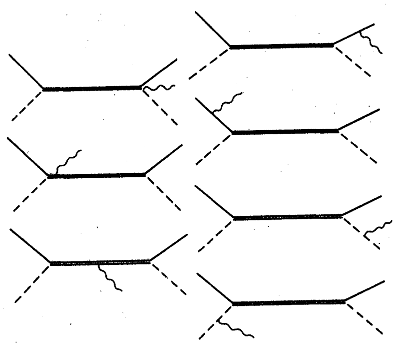

FIG. 1. Diagrams contributing to πN radiative scattering.

are shown in Fig. 1. Diagrams which involve $\Delta\Delta\pi$ vertices have been shown to be negligible.[1,2] The momenta of the initial and final pions and protons are denoted by q, q', p, and p', respectively; k and ϵ are the photon momentum and polarization; m, M, and M_Δ are the pion, nucleon, and Δ masses, respectively. The $\Delta^{++}p\pi$ vertex is described by the Hamiltonian

$$\mathcal{H}_{\Delta N\pi} = g_\Delta\{\bar{\psi}_\Delta^\mu \psi_p i\partial_\mu \phi_\pi + h.c.\},$$

with the coupling constant g_Δ determined from the decay width of the Δ^{++},

$$\Gamma = \frac{g_\Delta{}^2}{192\pi M_\Delta{}^5}\lambda^{3/2}(M_\Delta{}^2, M^2, m^2)[(M_\Delta + M)^2 - m^2], \tag{1}$$

and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$. The proton's electromagnetic vertex is

$$\Gamma_\mu^p = \gamma_\mu - \frac{i}{2M}\lambda^+\sigma_{\mu\nu}k^\nu, \tag{2}$$

where λ^+ is the proton's anomalous magnetic moment, and the analogous electromagnetic vertex of the Δ^{++} is

$$\Gamma_\mu^\Delta = \frac{(P+P')_\mu}{2M_\Delta} - \frac{i}{2M_\Delta}\mu_\Delta\sigma_{\mu\nu}k^\nu, \tag{3}$$

where P and P' are the initial and final momenta of the Δ^{++}, respectively, and μ_Δ is its magnetic moment. As predicted by SU(6), μ_Δ is twice the magnetic moment of the proton. This value was found by Pascual and Tarrach[6] to be consistent with the lack of a resonant structure in the UCLA data,[7-9] and is the value that is used for the earlier part of the calculation.

The spin-$\frac{3}{2}$ propagator is given by

$$S_{\mu\nu}(P) = -id_{\mu\nu}(P)/D(P). \tag{4}$$

As pointed out in Ref. 6, the full numerator to be used is given by

$$d_{\mu\nu}(P) = P_{\mu\nu}(P) + \frac{2}{3M_\Delta{}^2}(P^2 - M_\Delta{}^2) \times[\gamma_\mu P_\nu - \gamma_\nu P_\mu + (\not{P} + M_\Delta)\gamma_\mu\gamma_\nu], \tag{5}$$

with the spin-$\frac{3}{2}$ projector

$$P_{\mu\nu}(P) = (\not{P} + M_\Delta)\left[g_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M_\Delta}(\gamma_\mu P_\nu - \gamma_\nu P_\mu) - \frac{2}{3M_\Delta{}^2}P_\mu P_\nu\right]. \tag{6}$$

The denominator $D(P)$ is given by

$$D(P) = [P^2 - M_\Delta{}^2 + i\Gamma(Q/Q_R)^3(E_R/E)^2]/(E_R/E)^2, \tag{7}$$

where Q and E are the pion momentum and energy in the center-of-mass π-N system, and Q_R, E_R their values at resonance. This energy-dependent width gives a unitary πN amplitude which gives very good agreement with pion-proton scattering data.[3,14]

The total matrix element is the sum of the contributions coming from radiation by the external lines, the contact terms, and the internal emission:

$$M_{fi} = M_{\rm ext} + M_c + M_\Delta,$$

$$M_{\rm ext} = -ieg_\Delta{}^2\bar{u}(p')\left[q'_\mu S^{\mu\nu}(p'+q')q_\nu\frac{\not{p} - \not{k} + M}{-2p\cdot k}\epsilon^\alpha\Gamma_\alpha^p + \epsilon^\alpha\Gamma_\alpha^p\frac{\not{p}' + \not{k} + M}{2p'\cdot k}q'_\mu S^{\mu\nu}(p+q)q_\nu + q'_\mu S^{\mu\nu}(p'+q')(q-k)_\nu\frac{\epsilon\cdot(2q-k)}{-2q\cdot k} + \frac{\epsilon\cdot(2q'+k)}{2q'\cdot k}(q'+k)_\mu S^{\mu\nu}(p+q)q_\nu\right]u(p), \tag{8}$$

$$M_c = ieg_\Delta{}^2\bar{u}(p')[\epsilon_\mu S^{\mu\nu}(p+q)q_\nu + q'_\mu S^{\mu\nu}(p'+q')\epsilon_\nu]u(p),$$

$$M_\Delta = 2eg_\Delta{}^2\bar{u}(p')q'_\mu S^{\mu\nu}(p'+q')\epsilon^\alpha\Gamma_\alpha^\Delta S^{\nu\rho}(p+q)q_\rho u(p).$$

This expression for the matrix element is gauge-invariant in the two leading terms of an expansion in powers of k. Also, since the πN interaction that was used faithfully reproduces the πN scattering data in this energy region, the model satisfies the Low soft-photon theorem.[15]

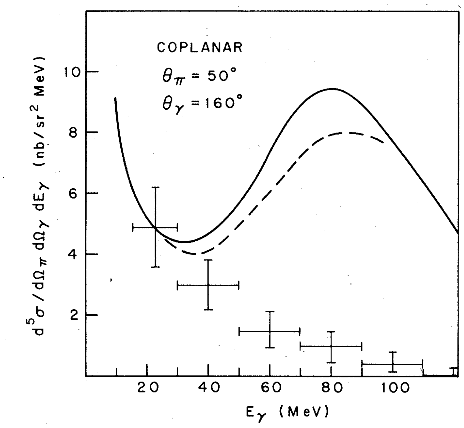

FIG. 2. Cross section predicted by the model (solid line) for a typical geometry, compared to the data of Refs. 7–9 and to an earlier calculation by Fischer and Minkowski (Ref. 2) (dashed line). Similar comparisons result for other choices of pion and photon angles.

The ambiguities involved in making a model of this type gauge-invariant to all orders in k have been discussed in Ref. 3. As such, our model is not reliable at large values of k, although the model used successfully by Pascual and Tarrach to calculate the Δ^{++} magnetic moment suffered from a similar difficulty. As seen below, the cross sections predicted by this model in fact agree quite well with other gauge-invariant predictions to large values of k. However, the strength of our results lies in the fact that asymmetry calculations are sensitive to the model used even at low values of k, where the reliability of the model is not in question.

Numerical results were obtained by constructing spinors and matrices in the computer and letting the computer do the matrix arithmetic shown in Eq. (8). The reliability of the calculation was tested by comparing the calculated cross sections to experimental data[7-9] and to a previous calculation by Fischer and Minkowski.[2] This last calculation was a "finite-difference" approach to Low's theorem which used a phase-shift parametrization of the πN amplitudes and a Lagrangian term for the inclusion of Δ-magnetic-moment radiation, and as such should predict cross sections which are similar to ours. The results are shown in Fig. 2 for a typical geometry at incident pion energy of 298 MeV, where the solid line is the prediction of this model and the dashed line is that of Fischer and Minkowski. As can be seen, the results (both for $\mu_\Delta = 2\mu_p$) are quite similar, and

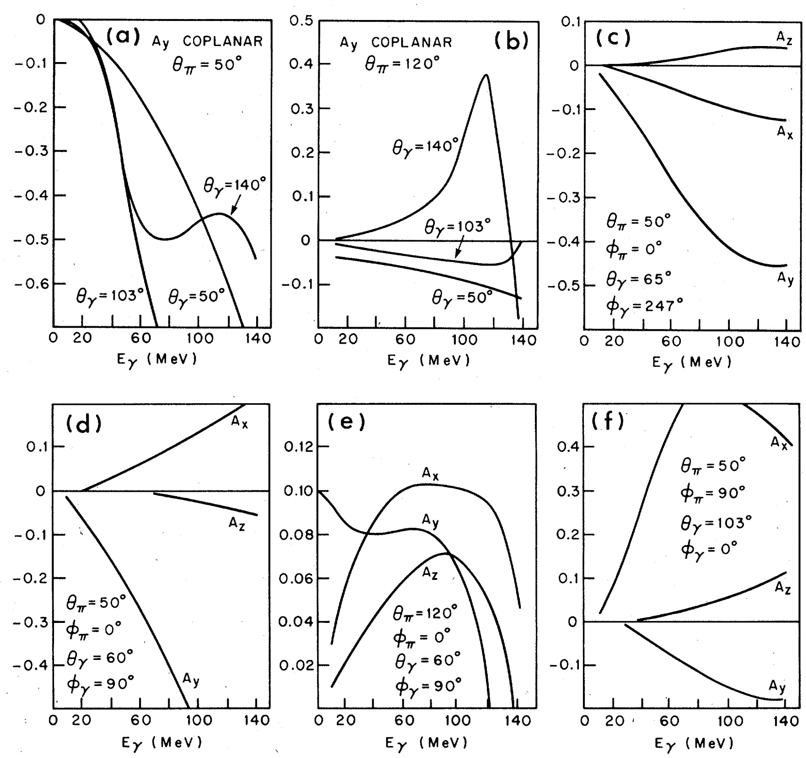

FIG. 3. Polarization asymmetries for incident pions of 298-MeV kinetic energy at various coplanar and noncoplanar laboratory angles. $A_i = (d\sigma_+ - d\sigma_-)/(d\sigma_+ + d\sigma_-)$, where $d\sigma_\pm = d\sigma/d\Omega_\pi d\Omega_\gamma dk$ are cross sections with the target nucleon spin along $\pm\hat{i}$.

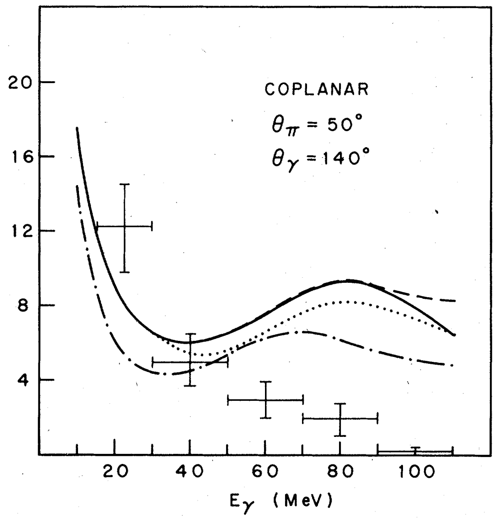

FIG. 4. Cross section predicted by the model (solid line) compared to the off-shell variation described in the text (dash-dot line) and to a change of Δ magnetic moment $\mu_\Delta = 4\mu_p$ (dotted line). Fischer and Minkowski's prediction for this geometry is also included (dashed lint). Similar comparisons result for other choices of pion and photon angles.

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RESULTS AND DISCUSSION

The polarization asymmetries calculated with this model are shown in Fig. 3 for an incident-pion energy of 298 MeV. Several kinematical situations are presented, including some which correspond to the geometry of the UCLA experiment.[7-9] The z axis has been chosen to be along the incident beam direction.

In his work on the possible use of polarization in off-shell studies, Moravcsik[10] states that noncoplanar polarization asymmetries in the scattering plane must come only from off-shell contributions, since parity conservation requires the on-shell elastic asymmetries to vanish in that plane. Actually, in noncoplanar kinematics it is not possible to define a scattering plane, and there is no unambiguous way to identify a direction perpendicular to the scattering plane. It seems, therefore, that polarization asymmetries in all directions contain both off-shell and on-shell contributions. A unique scattering plane is defined in the noncoplanar case only in the limit $k \to 0$, and in our choice of $d\sigma/d\Omega_\pi d\Omega_\gamma dk$ that plane is determined by the angular coordinates of the pion (e.g., $\phi_\pi = 0°$ corresponds to a xz scattering plane, $\phi_\pi = 90°$ to a yz scattering plane, etc.). Therefore, in the limit $k \to 0$, parity conservation requires polarization asymmetries to vanish in that particular plane, i.e., they are of order k or higher, while the asymmetries in the direction perpendicular to that plane may approach a nonvanishing limit, as can be seen in Fig. 2(e) in which the xz plane is the $k \to 0$ scattering plane.[16]

To test the sensitivity of the polarization asymmetries to the structure of the interaction, the calculations were redone first for a different value of the Δ magnetic moment ($\mu_\Delta = 4\mu_p$) and then for an energy-dependent width as a function of a virtual-pion mass, instead of a real one. This latter way of testing the model's off-shell behavior is similar to using Selleri form factors[17] and was discussed in Ref. 3. As can be seen in Fig. 4, the effect of these changes on the cross section is not very large at small photon energies. On the other hand, the fact that polarization asymmetries are sensitive to small relative changes in spin amplitudes can be seen in Fig. 5, where large deviations occur even for small values of photon energy, i.e., where the radiative calculation can

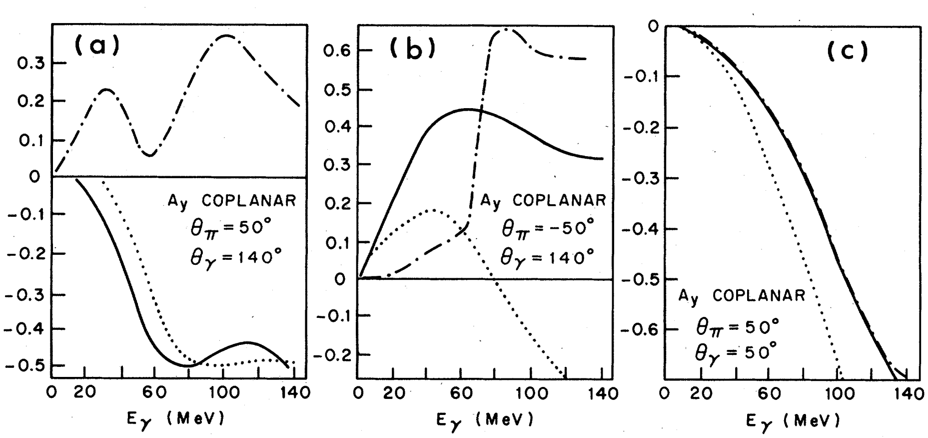

FIG. 5. Polarization asymmetries predicted by the model (solid line) compared to the off-shell variation described in the text (dash-dot line) and to a change of Δ magnetic moment $\mu_\Delta = 4\mu_p$ (dotted line).

be expected to be more reliable. Note also that the magnitude of this effect depends on the geometry and is not as pronounced when the photon is emitted near the pion's direction.

In conclusion, it is clear that asymmetry measurements with polarized targets in radiative pion-nucleon scattering will provide additional useful information in analyzing the off-shell pion-nucleon interaction. It is hoped that this conclusion will add to the motivation for further experimental efforts.

ACKNOWLEDGMENT

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