Flavor unification, τ decay, and b decay within the six-quark–six-lepton Weinberg-Salam model

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The leptons ν_e, e, ν_μ, μ, ν_τ, τ, and analogously the quarks u, d, c, s, t, b, are unified within the Weinberg-Salam $SU_2 \times U_1$ gauge model without enlarging the gauge group. The result is a theory in which the familiar leptons, quarks, and gauge bosons, plus some extra Higgs bosons necessary for unification, all carry a new multiplicatively conserved quantum number $\pi = \pm 1$. The most striking results of this unification are (1) π conservation forbids $\mu \rightarrow e\gamma$ but allows the Higgs-boson-mediated decays $\tau \rightarrow l\gamma$ ($l = e$ or μ) and $\tau \rightarrow \mu ee$ or $e\mu\mu$, at a calculable rate with a calculable lower limit; (2) for quarks, two of the three Cabibbo angles must be zero, so that the b quark (assumed lighter than t) decays only via Higgs-boson exchange, *always* semileptonically and *always* with lepton-number violation, e.g., $b \rightarrow de^+\mu^-$. This singular prediction will confirm or exclude the model as soon as b-flavored mesons are discovered. These and other phenomenological consequences of this unification are explained, and rates are estimated.

I. INTRODUCTION

The old puzzle of e-μ similarity has recently been exacerbated by the discovery of the τ lepton[1] with properties apparently similar to its lighter relatives. If there is an independent neutrino ν_τ associated with it, gauge theories of weak interactions must now deal with at least six leptons (ν_e, e), (ν_μ, μ), and (ν_τ, τ), all with identical gauge interactions, and whose only distinguishing feature is the different masses of the charged leptons e, μ, and τ. In this paper I discuss the consequences of unifying these leptons via a simple permutation symmetry that treats all leptons identically before spontaneous symmetry breaking. I also show that the six quarks (u,d), (c,s), and (t,b) may be similarly unified in analogy with the leptons. Some of these results were previously reported.[2] Here the model is elaborated upon, the extension to quark unification examined, and the phenomenology of leptons and hadrons discussed.

Previous attempts[3,4] to unify fundamental fermions have usually involved enlarging the gauge group of the theory so as to group formerly unrelated fermions in multiplets of the larger group. This necessarily leads to many new gauge bosons, mediating peculiar and unobserved decays, with typical gauge coupling strength; they must therefore be made very heavy by somewhat contrived symmetry breaking. Here instead I adopt the more cautious approach of assuming that all gauge bosons, leptons, and quarks are already known (consistent with the apparent success of the standard $SU_2 \times U_1$ gauge theory of weak and electromagnetic interactions[5,6]). The unification of the three lepton doublets of $SU_2 \times U_1$ is then achieved by assigning them to one representation of a group G' that is not gauged. The three quark doublets are unified separately but analogously.

A natural choice for G' is S_3, the permutation group of three objects acting upon both the three known lepton flavors and the three quark doublets. This mathematically expresses the observed similarity of the different flavored leptons (as well as the different quark doublets) with regard to their interactions with gauge bosons, since these are already flavor permutation invariant in the standard model.[5] S_3 flavor invariance is therefore the simple discrete symmetry I shall investigate as a means of unifying different fermions prior to spontaneous symmetry breaking. Technically, I assume that both the leptons of flavor $i = 1, 2, 3$, the three up quarks of charge $\frac{2}{3}$, and the three down quarks of charge $-\frac{1}{3}$ all belong to the three-dimensional reducible representation of S_3 in which their flavors simply permute under six group operations.[7] (This proposed appending of the discrete symmetry S_3 to the gauge group $SU_2 \times U_1$ can obviously be used to unify similar fermion multiplets in models with larger gauge groups too, and has recently been considered as a way of relating different multiplets in SU_5 grand unified models[3] by Glashow and collaborators.[8])

The physically motivated assumption of S_3 flavor invariance appended to the standard model has the following consequences for leptons. First, the dual requirements of spontaneously broken e-μ-τ S_3 symmetry and nondegenerate lepton masses demand the existence of Higgs fields which carry lepton flavor. I shall assume there exists only one Higgs boson for each lepton flavor. Assuming T invariance for leptons, I shall then demonstrate that only one vacuum of the theory can generate an e-μ-τ mass matrix with three nondegenerate eigenstates. This uniquely determined vacuum still preserves a residual S_2 symmetry for the total Lagrangian and this ensures multiplicative conservation of a new quantum number π of eigenvalue ± 1 assigned to all particles. π is a generalization of the multiplicative-lepton-number scheme of Feinberg and Weinberg.[9] The apparent prohibition of $\mu \rightarrow e\gamma$ then constrains the assignment of physical leptons to the theory's eigenstates: One finds that τ and either e or μ must carry $\pi = +1$, the other (μ or e) must carry $\pi = -1$. In the unitary gauge the extra Higgs bosons mediate the violation of additive τ, μ, and e number, but still conserve π. They have large Yukawa couplings $\sim\sqrt{G_F}\, m_\tau$ to all leptons, and thus cause relatively large Higgs-boson-exchange even among light leptons. For example, as discussed below, for a typical reasonable Higgs-boson-mass of 5 GeV, the branching ratios for $\tau \rightarrow \mu ee$ (or $e\mu\mu$) is $\sim 7\times 10^{-4}$, not too far below the current upper limit of 6×10^{-3}. Similarly the expected branching ratio for $\tau \rightarrow \mu\gamma$ or $e\gamma$ is about 4×10^{-4}, smaller than the present upper limit of 0.02 but still not negligible. All these results, as well as lower bounds on various rare τ decays that serve as tests of the model, and other phenomenological consequences, are discussed in the main part of the paper starting in Sec. II. In particular, if the model is correct, the branching ratio $B(\tau \rightarrow \mu\gamma \text{ or } e\gamma) \gtrsim 2\times 10^{-9}$ and could be much larger.

There are several important features of the multiplicative π conservation law emerging from the model. Firstly, additive e, μ, and τ number is calculably violated. In contrast, such violations and the ensuing rare decays of leptons in the Feinberg-Weinberg scheme[9] could only be said to be not forbidden, but in no way quantitatively estimated. The present model gives the only known dynamical realization of multiplicative lepton-number conservation. Secondly, since all additive lepton-number violations and other unorthodoxies occur because of Higgs bosons, their effects (even for light Higgs-boson-mass) are naturally smaller than those of the usual gauge-mediated interactions and thus do not spoil the good agreement with experiment of the standard model. Nevertheless, since the S_3 symmetry ensures that the lepton mass relevant to determining Yukawa couplings is m_τ rather than m_μ or m_e, Higgs-boson couplings to leptons are suppressed only by m_τ/m_W compared to W-boson couplings, and thus still lead to nonnegligible effects.

The analogous unification scheme for the quark SU_2 doublets (u,d), (c,s), and (t,b) has equally interestingly structured consequences. I shall show that t and b quarks must carry $\pi = -1$, whereas u, d, c, and s have $\pi = 1$. Assuming b to be the heavy quark hypothetically responsible for the Υ structure in hadron-hadron scattering,[10] so that b is lighter than t, the b quark is therefore unable because of π conservation to decay via a $W^\pm$ into a light quark, with $\pi = +1$, and so is stable under gauge-mediated interactions. This provides a natural explanation of the apparent universality of gauge weak interactions in the (u,d) and (c,s) sector alone, despite the existence of two heavier quarks, since the π assignments for quarks naturally necessitate that two of the Cabibbo mixing angles for quark charged currents are strictly zero. The model does nevertheless predict that b decays, *always* semileptonically via Higgs-boson interactions which violate additive lepton number, e.g. $b \rightarrow de^+\mu^-$. This singular signature of b-flavored particles will allow a conclusive (and certainly the best) test of the model as soon as b-containing hadrons are found, say in e^+-e^- collisions. If a nonleptonic decay mode is observed, the model is wrong.

Real Higgs-boson production is not considered here. The $\pi = 1$ Higgs bosons will be shown to allow K^0-$\bar{K}^0$ mixing as well as $\pi \rightarrow e\nu$ via scalar currents, and must therefore have masses of several hundred GeV to ensure experimental compatibility. The $\pi = -1$ bosons responsible for most unorthodox effects can in principle be as light as 5–10 GeV.

The results of this paper are presented as follows. Section II is a diversion summarizing the idea of a multiplicatively conserved quantum number in field theory and its relation to the permutation group. Section III defines the flavor-permutation-invariant $SU_2\times U_1$ model postulated here, and shows that flavored Higgs bosons must exist to guarantee nondegenerate lepton masses. Section IV examines the properties of the Higgs vacuums of the theory. Section V discusses Yukawa couplings and the lepton mass matrix in different vacuums, concluding that the condition $m_e \neq m_\mu \neq m_\tau$ chooses a unique vacuum. The physical leptons are then assigned to the mass eigenstates so as to be compatible with $\mu \not\rightarrow e\gamma$ and $m_\tau > m_\mu > m_e$. Section VI derives the total unitary gauge Lagrangian, in preparation for the phenomenology of Sec. VII. In Sec. VIII hadrons (quarks) are incorporated into the model, and some experimental consequences analyzed.

II. MULTIPLICATIVELY CONSERVED LEPTON NUMBER AND PERMUTATION SYMMETRY

Multiplicatively conserved lepton number was first introduced by Feinberg and Weinberg,[9] who assigned a quantity called muon parity which was -1 for μ, ν_μ and their antiparticles, $+1$ for e, ν_e and all other particles. They postulated that muon parity for systems of particles was given by the product of individual particle values, and was conserved—hence the name multiplicative conservation law. Their motivation was to "explain" the absence of $\mu\to e\gamma$, $3e$, and $\mu p\to ep$, all forbidden by this assignment of muon parity. This scheme *allows* $\mu^+e^-\to\mu^-e^+$ for example but makes no prediction for rate or mechanism.

Feinberg, Kabir, and Weinberg[11] and Cabibbo and Gatto[12] then showed that such a scheme follows from assuming invariance of all interactions under permutation of two primitive leptons e_1 and e_2. Their argument in essence was that if the Hamiltonian $H(e_1, e_2)$ is invariant under the permutation P: $e_1 \leftrightarrow e_2$, then $PHP^\dagger = H$, with P a unitary symmetry operator, and $P^2=1$. Thus P is Hermitian and commutes with H, and is therefore conserved and has real eigenvalues $\pi=\pm 1$. All physical states are simultaneous eigenstates of H and P, with π multiplicatively conserved. For example, $|\mu\rangle = \frac{1}{2}|e_1-e_2\rangle$ and $|e\rangle = \frac{1}{2}|e_1+e_2\rangle$, with $\pi_\mu=-1$ and $\pi_e=+1$.

This simple invariance scheme was extended to weak interactions,[12] where it predicts the existence of two neutrinos, as can be seen by considering π conservation in $\mu\to e\nu\nu'$. Since $\pi_\mu=-1$ and $\pi_e=1$, π conservation necessitates $\pi_\nu=-\pi_\nu'$, so that $\nu\neq\nu'$.

This old (pre-gauge-theory) scheme of π assignments does not constitute a complete model, since it cannot predict rates for multiplicatively allowed processes (such as $\mu^-e^+\to\mu^+e^-$) that are not mediated by standard weak gauge interactions. In a sense, the main point of this paper is to extend leptonic permutation invariance also to the Higgs-boson–lepton couplings that exist in modern weak-interaction theories.[5] Analogously to the deduction of two neutrinos, one can deduce the existence of extra Higgs bosons. These Higgs bosons dynamically mediate processes such as $\mu^+e^-\to\mu^-e^+$ and now allow their calculation. This paper is an extension (partially discussed in Ref. 2) of the principle of lepton-permutation invariance to *three* leptons e_1, e_2, and e_3 in the standard gauge theory. I shall show below that a residual $1\leftrightarrow 2$ permutation invariance still survives in the theory after spontaneous symmetry breaking, and so leads to π conservation in the whole theory, as explained above.

III. $(SU_2\times U_1)_{gauge}\times(S_3)_{flavor}$ MODEL AND THE NEED FOR NEW HIGGS BOSONS

As outlined in Sec. I, I shall unify the leptons by requiring invariance of the Lagrangian $\mathcal{L}$ under lepton flavor permutation. The standard $SU_2\times U_1$ model[5] for three lepton flavors contains the usual $\vec{A}_\mu$ and B_μ gauge bosons, three left-handed (LH) lepton doublets $l_i=(\nu_i, e_i^-)_L$, three right-handed (RH) lepton singlets $r_i=(e_i^-)_R$, and Higgs doublets of the form $\phi_j=(\phi_j^+,\phi_j^0)$. The couplings of $\vec{A}_\mu$ and B_μ to all three leptons are identical:

$$\mathcal{L}_{lW} = i\bar{l}_k\gamma^\mu\left(\partial_\mu + \frac{ig\vec{\tau}\cdot\vec{A}_\mu}{2} - \frac{ig'B_\mu}{2}\right)l_k + i\bar{r}_k\gamma^\mu(\partial_\mu - ig'B_\mu)r_k\,. \tag{3.1}$$

$\mathcal{L}_{lW}$ is invariant under all six S_3 permutations acting on the lepton flavors $i=1$ to 3.

Consider now the Yukawa couplings of an arbitrary number N of Higgs doublets to leptons:

$$\mathcal{L}_Y = \sum_{i,j=1}^{3}\sum_{k=1}^{N}\bar{l}_i\phi_k r_j (A^k)_{ij} + \text{H.c.} \tag{3.2}$$

Theorem. Let $\mathcal{L}_Y$ also be flavor-permutation (S_3) invariant. Then if the Higgs doublets carry no lepton flavor, $\mathcal{L}_Y$ cannot describe the leptons e, μ, and τ.

Proof. If ϕ_k are all S_3 singlets, then S_3 invariance in (3.2) (with S_3 operators acting on l_i and r_j alone) implies $(A^k)_{ij}$ has three equal diagonal elements and another six equal off-diagonal elements. The general 3×3 mass matrix $M = \sum_k (A^k)_{ij}\langle\phi_k^0\rangle_{vac}$ has identical structure, and always has two degenerate eigenvalues, inconsistent with $m_e\neq m_\mu\neq m_\tau$.

Some Higgs doublets ϕ_i must therefore carry lepton flavor.[13] I shall assume there exist only three Higgs doublets $\phi_i=(\phi_i^+,\phi_i^0)$, one for each lepton flavor. S_3 invariance requires the total Lagrangian $\mathcal{L}$ be invariant under S_3 permutations on the flavor labels of leptons and Higgs bosons simultaneously. One easily then sees that the 27 A_{ij}^k (for $N=3$) in (3.2) reduce to five independent coupling constants such that

$$\mathcal{L}_Y = a\bar{l}_i\phi_i r_i + b\bar{l}_i\phi_i(r_j+r_k) + c\bar{l}_i(\phi_j+\phi_k)r_i + d(\bar{l}_j+\bar{l}_k)\phi_i r_i + e\bar{l}_i\phi_j r_k + \text{H.c.} \tag{3.3}$$

Here i,j,k in each S_3-symmetric term range from 1 to 3 subject to the constraint $j\neq k\neq i$ and a,b,c,d,e are independent. [To be specific, $\bar{l}_i\phi_i r_i \equiv \bar{l}_1\phi_1 r_1 + \bar{l}_2\phi_2 r_2 + \bar{l}_3\phi_3 r_3$ and $\bar{l}_i\phi_i(r_j+r_k) \equiv \bar{l}_1\phi_1(r_2+r_3) + \bar{l}_2\phi_2(r_1+r_3) + \bar{l}_3\phi_3(r_1+r_2)$.] Only the Higgs potential itself need now be specified to ensure a totally S_3 flavor-invariant $SU_2\times U_1$ model. This is done below.

IV. HIGGS-BOSON POTENTIAL AND VACUUM SYMMETRY BREAKING

In this section I analyze the S_3-invariant Higgs-boson potential and prove some results about the symmetry structure of its vacuums.

The most general $SU_2\times U_1\times S_3$ Higgs-boson potential for the three flavored Higgs doublets is

$$\begin{aligned}V(\phi)=&\sum_i[-\lambda\phi_i^\dagger\phi_i+A(\phi_i^\dagger\phi_i)^2]\\&+\sum_{i<j}\{\tfrac12\gamma(\phi_i^\dagger\phi_j+\text{H.c.})+C(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j)+\bar C|\phi_i^\dagger\phi_j|^2+\tfrac12 D[(\phi_i^\dagger\phi_j)^2+\text{H.c.}]\}+\sum_{i\neq j}[\tfrac12 E_1(\phi_i^\dagger\phi_i)(\phi_i^\dagger\phi_j)+\text{H.c.}]\\&+\sum_{i\neq j\neq k,j<k}\{\tfrac12 E_2[(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_i)+\text{H.c.}]+\tfrac12 E_3[(\phi_i^\dagger\phi_i)(\phi_k^\dagger\phi_j+\text{H.c.})]+\tfrac12 E_4[(\phi_i^\dagger\phi_j)(\phi_i^\dagger\phi_k)+\text{H.c.}]\},\end{aligned}\tag{4.1}$$

where $\lambda,\gamma,A,C,\bar C,D,E_i$ are real by requiring T invariance *ab initio*, with $\lambda>0$ for spontaneous symmetry breaking.[14]

At the minimum of $V(\phi)$, let ϕ_i be written

$$\langle\phi_i\rangle_{\min}=2^{-1/2}\begin{pmatrix}\sigma_i e^{i\alpha_i}\\ \rho_i e^{i\theta_i}\end{pmatrix},\tag{4.2}$$

where σ_i,ρ_i for $i=1$ to 3 are real and positive, and the angles θ_i and α_i are real. The global $SU_2\times U_1$ symmetry always allows the minimum to be chosen such that $\sigma_1=0$ and $\alpha_2=\alpha_3=0$. In order to then obtain a naturally T-invariant theory (spontaneous T violation will be considered elsewhere), the minimum must occur for all θ_i identical so that all relative phases are zero. Rewriting (4.1) using the angular variables $\theta=\theta_1-\theta_2$ and $\theta'=\theta_2-\theta_3$, one can straightforwardly observe that $V(\phi)$ has a local minimum at $\theta=\theta'=0$ provided $\gamma,\bar C+D,D,E_i$ are all negative, so that T invariance is naturally obtained. Furthermore, writing $\rho_i=r_i\cos\omega_i$, $\sigma_i=r_i\sin\omega_i$ for $i=2,3$ in (4.1), one can show that the negativeness of γ, $\bar C+D$, D, and E_i guarantees a minimum at $\omega_i=0$, so that the charged fields ϕ_i^+ all naturally have zero vacuum expectation values to ensure charge conservation.

In terms of the neutral fields ρ_i the potential $V(\phi)$ takes the S_3-invariant form

$$\begin{aligned}4V=&-2\lambda(\rho_1^2+\rho_2^2+\rho_3^2)+2\gamma(\rho_1\rho_2+\rho_2\rho_3+\rho_3\rho_1)+A(\rho_1^4+\rho_2^4+\rho_3^4)\\&+c'(\rho_1^2\rho_2^2+\rho_2^2\rho_3^2+\rho_3^2\rho_1^2)+E_1[\rho_1^3(\rho_2+\rho_3)+\rho_2^3(\rho_3+\rho_1)+\rho_3^3(\rho_1+\rho_2)]+E'\rho_1\rho_2\rho_3(\rho_1+\rho_2+\rho_3),\end{aligned}\tag{4.3}$$

where $C'=C+\bar C+D$ and $E'=E_2+E_3+E_4$.

Theorem. The S_3-invariant potential V for the neutral fields has no natural spontaneously broken minimum with $\rho_1\neq\rho_2\neq\rho_3$.

Proof. The proof consists of assuming all ρ_i different, and showing this leads to an unnatural constraint on the independent coupling constants.

Consider the extremum conditions $V_i\equiv\partial V/\partial\rho_i=0$. Equation (4.3) then yields

$$\begin{aligned}V_1-V_2&=(\rho_1-\rho_2)F_{12}=0,\\V_2-V_3&=(\rho_2-\rho_3)F_{23}=0,\end{aligned}\tag{4.4}$$

where F_{ij} are functions of ρ_1, ρ_2, and ρ_3. Since $\rho_1\neq\rho_2\neq\rho_3$ by assumption, $F_{ij}=0$. The equation $F_{12}-F_{23}=0$ then becomes

$$(\rho_1-\rho_3)(\rho_1+\rho_2+\rho_3)(4A-2C'-E_1+E')=0,$$

and since $\rho_1\neq\rho_3$, cannot be satisfied unless $4A-2C'-E_1+E'=0$, an unnatural condition.

One can now see that there are only two possible types of local minima of V, both of which maintain some residual permutation symmetry. The first is vacuum I with $\rho_i=[(\lambda-\gamma)/(A+C'+2E_1+E')]^{1/2}$ for all i. This vacuum is still S_3 symmetric. The second is vacuum II with $\rho_1\equiv\rho_2\equiv\rho'$ and $\rho_3=\rho$, where the values of ρ and ρ' are discussed below. The important feature of vacuum II is that though it has broken S_3 invariance, it still maintains an *unbroken* S_2 *invariance* under $1\leftrightarrow 2$ flavor exchange, and will therefore lead to a *multiplicative conservation law* as explained in Sec. II.

ρ and ρ' in vacuum II are complicated functions of the coupling constants λ, γ, A, C, $\bar C$, D, and E_i. However, the crucial and interesting feature of multiplicative lepton-number conservation (to emerge below) depends only upon the equality $\rho_1\equiv\rho_2$, and not on the value of ρ'. In order to illustrate this feature in the remainder of this paper, without undue algebraic complication, I shall proceed in the approximation

$$\epsilon \equiv -E_1/A \ll 1\,,\quad \epsilon' \equiv -E'/A \ll 1\,,\quad \delta \equiv -\gamma/\lambda \ll 1\,. \tag{4.5}$$

In this case one can easily extremize (4.3) to obtain the approximate solution

$$\rho \simeq (\lambda/A)^{1/2}\,,\quad \rho' \simeq \frac{\sqrt{\lambda A}}{2(C'-2A)}(\epsilon + 2\delta)\,, \tag{4.6}$$

with corrections $\sim\epsilon^2$, ϵ'^2, and δ^2. By examining $\partial^2 V/\partial\rho_i\partial\rho_j$ this extremum can be shown to be a true local minimum for small ϵ, ϵ', and δ, provided $C' > 2A > 0$. Furthermore, in this approximation, $C' > 2A$ also guarantees that vacuum II lies lower than the S_3-symmetric vacuum I.

Having shown the existence of a true minimum of type II for a continuous range of small parameters ϵ, ϵ', and δ, I shall henceforth proceed in the limit $\epsilon, \epsilon', \delta = 0$, so that the vacuum expectation values in vacuum II are

$$\langle\phi_1^0\rangle = \langle\phi_2^0\rangle = 0\,,\quad \langle\phi_3^0\rangle = 2^{-1/2}\rho\,,\quad \rho = (\lambda/A)^{1/2}\,. \tag{4.7}$$

This greatly simplifies obtaining explicit expressions for the Lagrangian in the unitary gauge, but causes no essential changes in the structure of the theory as regards the multiplicative conservation law.

Which of these vacuums, I or II, can describe the world in which e, μ, and τ exist is determined in Sec. V below. Once the unique vacuum is specified, the Higgs mass eigenstates and their interactions will be listed in Sec. VI.

V. YUKAWA COUPLINGS, LEPTON STATES, AND UNIQUENESS OF THE VACUUM

The lepton mass eigenstates are determined by the general S_3-invariant Yukawa Lagrangian of (3.3):

$$\mathcal{L}_Y = a\bar{l}_i\phi_i r_i + b\bar{l}_i\phi_i(r_j + r_k) + c\bar{l}_i(\phi_j + \phi_k)r_i + d(\bar{l}_j + \bar{l}_k)\phi_i r_i + e\bar{l}_i\phi_j r_k + \text{H.c.}\,, \tag{5.1}$$

where a, b, c, d, e are assumed real from T invariance. One can now compare the mass eigenvalues of $\mathcal{L}_Y$ in vacuums I and II with those of the e-μ-τ system.

Vacuum I, with all $\langle\phi_i^0\rangle$ equal for all i, produces an e-μ-τ mass matrix identical in structure to the one discussed in the theorem of Sec. III, where all Higgs bosons were S_3 singlets. This leads to two degenerate lepton mass eigenvalues and so cannot describe the physical leptons. (Since even the spontaneously broken Lagrangian is still S_3 invariant in vacuum I, this mass degeneracy persists to all orders in perturbation theory.)

Vacuum II with $\langle\phi_1^0\rangle = \langle\phi_2^0\rangle = 0$, $\langle\phi_3^0\rangle = \rho/\sqrt{2}$, is therefore the only candidate, and leaves the Lagrangian $\mathcal{L}$ still invariant under S_2 transformations on flavor indices 1 and 2. ρ is later related to G_F. In this vacuum the charged-lepton mass matrix between left-handed (LH) and right-handed (RH) e_i states is

$$M = \frac{\rho}{\sqrt{2}}\begin{bmatrix} c & e & d \\ e & c & d \\ b & b & a \end{bmatrix}, \tag{5.2}$$

with a manifest $1 \leftrightarrow 2$ symmetry which allows M to be explicitly diagonalized such that $U_L M U_R^\dagger$ has the three diagonal mass eigenvalues

$$m_{\text{I}} = 2^{-1/2}F\rho\,,\quad m_{\text{II}} = 2^{-1/2}G\rho\sin(\chi_0/2)\,,\quad m_{\text{III}} = 2^{-1/2}G\rho\cos(\chi_0/2)\,, \tag{5.3}$$

with

$$G = [a^2 + 2b^2 + 2d^2 + (c+e)^2]^{1/2}\,,\quad F = |c - e|\,,\quad \sin\chi_0 = \left|\frac{2a(c+e) - 4bd}{G^2}\right|\,. \tag{5.4}$$

Note that $\sin^2\chi_0 < 1$, and χ_0 can be chosen in $[0, \pi/2]$, with $m_{\text{II}} < m_{\text{III}}$.

The three masses determine F, G, and χ_0, and thus three combinations of the fundamental a, b, c, d, and e. The remaining two may be defined in terms of two angles $\beta^\pm$, with

$$\tan\beta^\pm = \frac{\sqrt{2}\,(d \pm b)}{(a \mp c \mp e)}\,. \tag{5.5}$$

The linear combinations

$$\phi = \tfrac{1}{2}(\beta^+ + \beta^-)\,,\quad \phi' = \tfrac{1}{2}(\beta^+ - \beta^-) \tag{5.6}$$

determine the left and right diagonalizing matrices U_L and U_R, which are given by

$$U_L = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 \\ \cos\phi & \cos\phi & -\sqrt{2}\sin\phi \\ \sin\phi & \sin\phi & \sqrt{2}\cos\phi \end{bmatrix} \tag{5.7}$$

with U_R given by replacing ϕ by ϕ'. a, b, c, d, and e can thus be explicitly found in terms of three masses and the two angles ϕ and ϕ'.

The left and right lepton eigenstates corresponding to (5.3) in the e_i basis are determined purely in terms of ϕ and ϕ' from (5.7):

$$|\text{I}\rangle_L = 2^{-1/2}(1, -1, 0), \quad \pi_{\text{I}} = -1$$

$$|\text{II}\rangle_L = 2^{-1/2}(\cos\phi, \cos\phi, -\sqrt{2}\sin\phi), \quad \pi_{\text{II}} = 1 \tag{5.8}$$

$$|\text{III}\rangle_L = 2^{-1/2}(\sin\phi, \sin\phi, \sqrt{2}\cos\phi), \quad \pi_{\text{III}} = 1.$$

The $|\rangle_R$ eigenstates have ϕ replaced by ϕ'. The π value of each state is its eigenvalue under the residual $1 \leftrightarrow 2$ permutation operator which commutes with the total Hamiltonian. π will be the multiplicatively conserved generalized lepton number.

The lepton eigenstates of (5.8) must now be identified with $|e\rangle$, $|\mu\rangle$, and $|\tau\rangle$. Since $m_{\text{III}} > m_{\text{II}}$, e cannot be assigned to $|\text{III}\rangle$. The only possibilities for $[\text{I}, \text{II}, \text{III}]$ are then (1) $[e, \mu, \tau]$, (2) $[\mu, e, \tau]$, and (3) $[\tau, e, \mu]$. I shall show in the next section that π is multiplicatively conserved, and that gauge bosons carry $\pi = 1$. The decay $|\text{III}\rangle \to |\text{II}\rangle + \gamma$ is allowed and so assignment (3) predicts the unobserved decay $\mu \to e\gamma$ as well as $\mu \to 3e$; this assignment is therefore excluded. [In Sec. VII I shall discuss more carefully to what extent assignment (3) is actually forbidden by the small upper limit on $\mu \to e\gamma$.] Assignments (1) and (2) differ only in which one of the light leptons, e or μ, has the same π value as τ. Present information about the τ is insufficient to distinguish (1) from (2); possible distinguishing features will be discussed in Sec. VII. In what follows I shall for definiteness use assignment (1) with

$$|e\rangle_L = 2^{-1/2}(1, -1, 0), \quad m_e = 2^{-1/2}F_\rho, \quad \pi_e = -1$$

$$|\mu\rangle_L = 2^{-1/2}(\cos\phi, \cos\phi, -\sqrt{2}\sin\phi),$$

$$m_\mu = 2^{-1/2}G\rho\sin\frac{\chi_0}{2}, \quad \pi_\mu = 1$$

$$|\tau\rangle_L = 2^{-1/2}(\sin\phi, \sin\phi, \sqrt{2}\cos\phi),$$

$$m_\tau = 2^{-1/2}G\rho\cos\frac{\chi_0}{2}, \quad \pi_\tau = 1 \tag{5.9}$$

with $|\rangle_R$ states given by replacing ϕ by ϕ', but always recall that e and μ can be interchanged.

VI. INTERACTIONS IN THE UNITARY GAUGE

The nondegeneracy of leptonic masses has selected vacuum II of Sec. IV. It is now possible to evaluate the total Lagrangian in the unitary gauge and to explicitly demonstrate π conservation. Below the Higgs-boson threshold the main interactions of experimental interest are the Yukawa couplings, but all interactions are listed below for completeness, using the approximation of (4.7) for the vacuum expectation values.

Higgs-boson self-couplings. The Higgs doublets ϕ_i of (4.1) can be perturbed about vacuum II of (4.7), and the resulting boson mass matrix diagonalized to yield the physical Higgs eigenstates. The resultant fields, tabulated by charge Q and multiplicative π number (their eigenvalue under $\phi_1 \leftrightarrow \phi_2$ exchange), are

	$Q=1$	$Q=0$
$\pi=1$	Ω	η, ζ, χ .
$\pi=-1$	H	K, L

(6.1)

Their independent masses are determined by the coefficients λ, γ, A, C, $\bar{C}$, D, and E_i of (4.1), and by the vacuum expectation value ρ and ρ' of vacuum II. The approximation $\epsilon, \epsilon', \delta \to 0$ of (4.5) leads to $\rho' \simeq 0$, and thus $M_\Omega = M_H$, $M_\eta = M_K$, and $M_\zeta = M_L$, but this is an accidental degeneracy due to the approximation scheme, and in general all masses are different.[15]

In terms of these fields

$$\phi_1 = 2^{-1/2}\begin{pmatrix} \Omega + H \\ \dfrac{\eta + K + i(\zeta + L)}{\sqrt{2}} \end{pmatrix},$$

$$\phi_2 = 2^{-1/2}\begin{pmatrix} \Omega - H \\ \dfrac{\eta - K + i(\zeta - L)}{\sqrt{2}} \end{pmatrix}, \tag{6.2}$$

$$\phi_3 = \begin{pmatrix} 0 \\ \dfrac{\rho + \chi}{\sqrt{2}} \equiv \dfrac{\chi'}{\sqrt{2}} \end{pmatrix}.$$

Clearly ϕ_3 behaves like the one Higgs field in the standard model,[5] but ϕ_1 and ϕ_2 are new. The π values of all fields can easily be read off by examining their sign change under $\phi_1 \leftrightarrow \phi_2$. ($\pi = 1$ fields are denoted by Greek letters, $\pi = -1$ by Roman.) In terms of the physical Higgs bosons $V(\phi)$ in (4.1) takes the form

$$
\begin{aligned}
V(\phi) = & -A\rho^2[\Omega^\dagger\Omega + H^\dagger H + \tfrac{1}{2}(\eta^2+\zeta^2+\chi'^2+K^2+L^2)] + \tfrac{1}{4}(2A+C)[\Omega^\dagger\Omega + H^\dagger H + \tfrac{1}{2}(\eta^2+\zeta^2+K^2+L^2)]^2 \\
& + \tfrac{1}{4}(\bar{C}+D)[\Omega^\dagger\Omega - H^\dagger H + \tfrac{1}{2}(\eta^2+\zeta^2-K^2-L^2)]^2 + \tfrac{1}{4}(2A-C)(\Omega^\dagger H + H^\dagger\Omega + \eta K + \zeta L)^2 \\
& + \tfrac{1}{4}(\bar{C}-D)[-i(H^\dagger\Omega - \Omega^\dagger H) + \zeta K - \eta L]^2 + \tfrac{1}{4}A\chi'^4 + \tfrac{1}{2}C\chi'^2(\Omega^\dagger\Omega + H^\dagger H) \\
& + \tfrac{1}{4}(C+\bar{C})\chi'^2(\eta^2+\zeta^2+K^2+L^2) + \tfrac{1}{4}D\chi'^2(\eta^2+K^2-\zeta^2-L^2)\,.
\end{aligned} \tag{6.3}
$$

Since $V(\phi)$ is an even function of all the $\pi=-1$ fields H, K, and L, π is multiplicatively conserved, as guaranteed by the discussion of Sec. II.

Gauge coupling of Higgs bosons. ϕ_3 behaves like the usual single Higgs doublet of the standard model,[5] and produces the standard $W^\pm$, Z, and γ physical gauge bosons, with $M_W=\rho e/(2\sin\theta_W)$ and $M_W/M_Z=\cos\theta_W$, where θ_W is the Weinberg angle. γ, Z, and W carry no lepton index and are therefore permutation invariant, with $\pi_\gamma=\pi_Z=\pi_W=1$. Their self-interactions are those of the standard model.

S_3 invariance requires that all ϕ_i couple identically to the $\vec{A}^\mu$ and B^μ gauge bosons. From (6.2) and the usual definitions of W, Z, and γ in terms of $\vec{A}_\mu$ and B_μ, one obtains the unitary gauge couplings of Higgs bosons to gauge bosons:

$$
\begin{aligned}
\mathcal{L}_{\text{Higgs-gauge}} = & \, ie(A_\mu + \cot 2\theta_W Z_\mu)(\Omega\partial_\mu\Omega^\dagger - \Omega^\dagger\partial_\mu\Omega + H\partial_\mu H^\dagger - H^\dagger\partial_\mu H) + \frac{eZ_\mu}{\sin 2\theta_W}(\zeta\partial_\mu\eta - \eta\partial_\mu\zeta + L\partial_\mu K - K\partial_\mu L) \\
& + \frac{ieW_\mu^-}{2\sin\theta_W}[\Omega\partial_\mu(\eta-i\zeta) - (\eta-i\zeta)\partial_\mu\Omega + H\partial_\mu(K-iL) - (K-iL)\partial_\mu H] + \text{H.c.} \\
& + \frac{eM_W\chi}{2\sin\theta_W}(2W_\mu^+W_-^\mu + Z_\mu{}^2\sec^2\theta_W) + e^2(\Omega^\dagger\Omega + H^\dagger H)\left[(A_\mu + \cot 2\theta_W Z_\mu)^2 + \frac{W_\mu^+W_-^\mu}{2\sin^2\theta_W}\right] \\
& + \frac{e^2}{2\sin\theta_W}(A_\mu - \tan\theta_W Z_\mu)\{W_\mu^-[\Omega(\eta-i\zeta) + H(K-iL)] + \text{H.c.}\} \\
& + \frac{e^2}{8\sin^2\theta_W}(2W_\mu^+W_-^\mu + Z_\mu{}^2\sec^2\theta_W)(\chi^2+\eta^2+\zeta^2+K^2+L^2)\,.
\end{aligned} \tag{6.4}
$$

It is clear that the $\pi=-1$ fields H, K, and L always appear bilinearly so that π is again conserved.

Gauge couplings of leptons. The S_3-invariant gauge couplings of leptons are given by (3.1). Using (5.8) and (5.9) to transform from the primitive e_i basis to the physical $e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$ basis, one straightforwardly obtains the standard model's[5] gauge couplings to physical leptons. (The massless ν_e, ν_μ, and ν_τ are defined in terms of ν_i by their charged gauge couplings to e, μ, and τ, respectively.) As is usual, the Higgs vacuum expectation value of (4.7) is determined to be

$$\rho^2 = 2^{-1/2}G_F{}^{-1}\,. \tag{6.5}$$

Yukawa couplings of leptons to Higgs bosons. Below the Higgs-boson threshold, the crucial difference between this S_3-invariant model and the standard model lies in the unorthodox additive lepton-number-violating interactions to be displayed below. Using (5.4), (5.5), (5.6), (5.9), and (6.2), $\mathcal{L}_Y$ in (5.1) can be rewritten in terms of physical lepton and Higgs states, with all coupling strengths expressed in terms of m_e, m_μ, m_τ and the observable angles ϕ and ϕ'. The resulting expression becomes particularly simple and transparent in the physically good approximation $m_e/m_\tau = m_\mu/m_\tau = 0$ when $F, \chi_0 \to 0$ in (5.9). Then in terms of the neutrino and charged lepton vectors

$$\nu = (\nu_e, \nu_\mu, \nu_\tau)\,, \quad \epsilon = (e, \mu, \tau)\,, \tag{6.6}$$

one finds

$$
\mathcal{L}_Y = (m_\tau/\rho)\bar{\tau}\tau\chi + \frac{3m_\tau}{2\sqrt{2}\rho}\left[\begin{matrix}(\sqrt{2}\Omega\,\bar{\nu}\,\mathfrak{F}^+\epsilon + \text{H.c.} + \eta\bar{\epsilon}\,\mathfrak{F}^{(+)}\epsilon + i\zeta\,\bar{\epsilon}\gamma_5\mathfrak{F}^+\epsilon \\ +\sqrt{2}H\bar{\nu}\,\mathfrak{F}^-\epsilon + \text{H.c.} + K\bar{\epsilon}\,\mathfrak{F}^-\epsilon + iL\,\bar{\epsilon}\gamma_5\mathfrak{F}^-\epsilon)\end{matrix}\right]. \tag{6.7}
$$

Here $\mathfrak{F}^+$ and $\mathfrak{F}^-$ are 3×3 Hermitian matrices in (e,μ,τ) flavor space, with $\mathfrak{F}^+$ linking only states of similar π (so that $\mathfrak{F}_{e\mu}^+ = \mathfrak{F}_{e\tau}^+ = \mathfrak{F}_{\tau e}^+ = \mathfrak{F}_{\tau\mu}^+ = 0$), and $\mathfrak{F}^-$ linking only states of opposite π (so that only $\mathfrak{F}_{e\mu}^-, \mathfrak{F}_{e\tau}^-, \mathfrak{F}_{\tau e}^-, \mathfrak{F}_{\mu e}^-$ are nonzero). [Recall that assignment (1) for the leptons is used here.] The nonzero elements of both $\mathfrak{F}^+$ and $\mathfrak{F}^-$ are given by

$$2\mathfrak{F}_{ij}^\pm = f_i f_j'(1+\gamma_5) + f_j f_i'(1-\gamma_5)\,, \tag{6.8}$$

where i and j range over e, μ, τ and

$$f_e = \sin\psi\,,\quad f_\mu = \sqrt{3}\,\sin\psi\cos\psi\,,\quad f_\tau = \sqrt{3}\,\sin(\psi+z)\sin(\psi-z)\,,$$
$$f'_e = \sin\psi'\,,\quad f'_\mu = \sqrt{3}\,\sin\psi'\cos\psi'\,,\quad f'_\tau = \sqrt{3}\,\sin(\psi'+z)\sin(\psi'-z) \tag{6.9}$$

with $z=\tan^{-1}\sqrt{2}$ and $\psi=\phi-z$, $\psi'=\phi'-z$.

Since the $\pi=1$ Higgs fields Ω,η,ζ appear only with $\mathcal{F}^+$ which links similar π states, and the $\pi=-1$ H,K,L fields with $\mathcal{F}^-$ which links opposite π, π is explicitly conserved. [This is of course true even when the approximations $m_e=m_\mu=0$ and $\langle\phi_1^0\rangle=\langle\phi_2^0\rangle=0$ in (4.7) are not made.]

χ in (6.7) has only flavor-conserving couplings, as in the standard model. However, the unorthodox Higgs bosons H,K,L,Ω,η,ζ all have unusually large couplings $\sim G_F^{1/2}m_\tau\simeq 4.5\times10^{-3}$ to all leptons, even to the light μ and e. These Higgs-boson–lepton interactions, while naturally small compared to gauge interactions of strength $G_F^{1/2}M_W$, are nevertheless at least twenty (m_τ/m_μ) and sometimes thousands (m_τ/m_e) of times larger than in the standard model, and can lead to large systematic violations of additive lepton number.

VII. EXPERIMENTAL CONSEQUENCES: UNORTHODOX LEPTONIC INTERACTIONS

In this section I assume assignment (1) with $\pi_\tau=\pi_\mu=-\pi_e$ for leptons, but recall the possibility of $e\leftrightarrow\mu$ interchange.

Below Higgs threshold the most dramatic consequences of the model arise from the fact that Higgs exchange between leptons conserves only π, but not μ, e, or τ number separately. Thus although $\mu\not\to e\gamma$, eee, so that there are no easily observable rare μ decays, rare τ decays (unexpected in the usual sequential lepton scheme) such as

$$\tau\to3\mu\,,\quad \tau\to\mu\gamma,\quad \tau\to\mu ee \tag{7.1}$$

are predicted to proceed via virtual Higgs-boson exchange, with amplitudes only $\sim(m_\tau/m_{\rm Higgs})^2$ smaller than usual gauge-mediated decays.[15] The superficially similar decays

$$\tau\to3e\,,\quad \tau\to e\gamma\quad \tau\to e\mu\mu \tag{7.2}$$

are, however, strictly forbidden. Note than an experimental choice between lepton assignments (1) and (2) is made by observing whether (7.1) or (7.2) are the observed rare τ decays. In either case an unavoidable prediction is that τ must decay to $l\gamma$ and $3l$, where $l=e$ or μ.

Other heterodox Higgs-boson-mediated reactions are $\mu^+e^-\to\mu^-e^+$, $e^-e^-\to\tau^-\tau^-$, and $\mu^-\to\nu_e\bar{\nu}_\mu e^-$, all of which must occur in any assignment scheme. These Higgs bosons of large coupling strength could also produce anomalies (relative to the usual charged- and neutral-current expectations) in processes such as $\nu_\mu+e\to\nu_\mu+e$, provided they were not too heavy. Rates for some normally forbidden processes are estimated below, and compared with current experimental limits.

A. $\tau\to\mu\gamma$

I shall assume $M\gg m_\tau\gg m_\mu$, where M represents a typical Higgs mass. The dominant diagrams for $\tau\to\mu\gamma$ are shown in Fig. 1, where only neutral Higgs boson emission and reabsorption is considered, since amplitudes involving charged bosons are relatively suppressed by factors $\sim(\ln M^2)^{-1}$. The dominant amplitudes lead to a predicted branching ratio

$$B(\tau\to\mu\gamma)\simeq\frac{3^5\alpha}{2^7\pi}\left(\frac{m_\tau}{M}\right)^4\ln^2\left(\frac{M^2}{m_\tau^{\,2}}\right)r\,, \tag{7.3}$$

where r, a function of the angles ϕ and ϕ' and of mass ratios of Higgs bosons, is generally of order unity.

For $m_\tau\simeq2$ GeV and $M\simeq5$ GeV (a not excluded value of the Higgs-boson mass), $B\simeq4\times10^{-4}$. This is appreciably below the present upper limit[16] of 0.026 for $\tau\to e\gamma$ and 0.013 for $\tau\to\mu\gamma$, but much larger than the upper limit[17] of 1.1×10^{-9} for $\mu\to e\gamma$.

I shall show in the next section, where I extend this S_3-invariant model to the quark sector, that the b quark decays solely via Higgs-boson exchange to a semileptonic final state, and that the apparent instability of b-flavored mesons[18] necessitates that at least one Higgs boson must be lighter

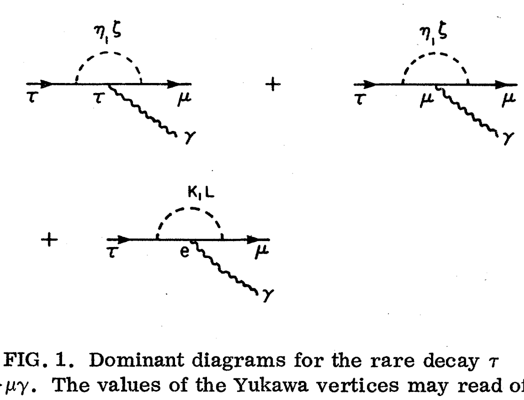

FIG. 1. Dominant diagrams for the rare decay $\tau\to\mu\gamma$. The values of the Yukawa vertices may read off from Eq. (6.7) in the text.

than $M\simeq 250$ GeV. From (7.3) this yields a *lower limit*

$$B(\tau\to\mu\gamma)\gtrsim 2\times 10^{-9}. \quad (7.4)$$

The important point to emphasize is that, for Higgs-boson masses in the range of current prejudice (say 5–20 GeV), one naturally expects $B(\tau\to\mu\gamma$ or $e\gamma)$ to lie two to three orders of magnitude below the current upper limit, so that the sensitivity of present experiments on rare τ decays is just beginning to approach the values suggested by this model. Secondly, *this model can be disproved,* albeit by the difficult means of observing no radiative τ decay down to the branching ratio (7.4).

A note about the possibility that $\pi_\mu=\pi_e=1$ [i.e., assignment (3) of Sec. V] is appropriate here. If this were correct, the decay $\mu\to e\gamma$ would occur via diagrams similar to those of Fig. 1, with a branching ratio

$$B(\mu\to e\gamma)=\frac{3^3\alpha}{2^{10}\pi}\frac{m_\tau^{\,6}}{m_\mu^{\,2}}\left(\frac{1}{M_K^{\,2}}\ln\frac{M_K^{\,2}}{m_\tau^{\,2}}-\frac{1}{M_L^{\,2}}\ln\frac{M_L^{\,2}}{m_\tau^{\,2}}\right)\times[\sin^2(\psi+\psi')+\sin^2(\psi-\psi')]\,, \quad (7.5)$$

assuming $M_{K,L}\gg m_\tau\gg m_{\mu,e}$. In order to suppress this below the observed upper limit[17] 1.1×10^{-9} without contrived and unnatural cancellations requires $M_{K,L}\gtrsim 500$ GeV. I shall not consider this possibility here, assuming instead that $\mu\to e\gamma$ is strictly forbidden. The (not necessarily prohibitive) difficulties and peculiarities arising from such large Higgs-boson masses have been discussed, for example, by Veltman.[19]

B. $\tau\to 3\mu$

This process proceeds via exchange of $\pi=1$ Higgs bosons, as illustrated in Fig. 2. Using the Yukawa vertices of (6.7) in the limit $M_{\eta,\zeta}\gg m_\tau\gg m_\mu$, one obtains

$$B(\tau\to 3\mu)=\left(\frac{3m_\tau}{4}\right)^4 f_\mu^{\,2}f_\mu'^{\,2}(f_\mu^{\,2}f_\tau'^{\,2}+f_\tau^{\,2}f_\mu'^{\,2})\times\left(\frac{3}{M_\eta^{\,4}}+\frac{3}{M_\zeta^{\,4}}+\frac{2}{M_\eta^{\,2}M_\zeta^{\,2}}\right). \quad (7.6)$$

f_i, M_ζ, and M_η are experimentally unknown, but assuming $M_\eta\simeq M_\zeta\simeq M$ and $\psi\simeq\psi'\simeq 45°$ implies

$$B(\tau\to 3\mu)\simeq 0.42M^{-4}. \quad (7.7)$$

The experimental limit[16] is $B<6\times 10^{-3}$, which only sets the mild bound $M_{\eta,\zeta}\gtrsim 3$ GeV. For $M\simeq 5$ GeV, $B\simeq 7\times 10^{-4}$, not far below the current limit.

C. $\tau\to\mu ee$

Similar estimates apply to this mode, which unlike $\tau\to 3\mu$ is mediated by $\pi=1$ *and* $\pi=-1$ bosons. Thus even if all the $\pi=1$ bosons were very heavy (see Sec. VIII where this condition is deduced from the quark sector of the model) this decay could still proceed via light $\pi=-1$ bosons. For $M\simeq 5$ GeV, $B(\tau\to\mu ee)\sim 7\times 10^{-4}$, about ten times smaller than the current limit.[16]

Such a decay would produce a striking signature in $e^+e^-\to\tau^+\tau^-\to\mu eee+$ missing energy. The main background would be from electromagnetic e^+e^- brehmsstrahlung, suppressed relative to $e^+e^-\to\tau^+\tau^-\to\mu e+$ neutrinos by $\alpha^2\simeq 5\times 10^{-5}$. This rare $\tau\to\mu ee$ decay mode would allow for genuine reconstruction of the τ resonance from decay momenta because of its neutrinoless final state. Analogous to (7.4), $B(\tau\to\mu ee)\gtrsim 10^{-10}$.

Rare τ decays to hadrons are discussed at the end of Sec. VIII and shown to be purely $\pi=1$ mediated, and therefore negligible.

D. $e^+\mu^-\to e^-\mu^+$

Muonium to antimuonium conversion occurs in this model via K or L exchange with amplitude $\sim G_F m_\tau^{\,2}/M_{K,L}^{\,2}$. Current experiments[20] are sensitive to amplitudes $\sim G_F$, so that for a Higgs-boson mass of 5 GeV, the model's predictions are only about 6 times smaller than present sensitivity.

E. $\mu^+\to e^+\bar\nu_e\nu_\mu$

This decay of the muon to "wrong" neutrinos occurs via H exchange, as shown in Fig. 3. Since $\pi_H=-1$, this still proceeds with non-negligible rate even if all $\pi=+1$ bosons are very heavy (see Sec. VIII). The amplitude is $\sim G_F m_\tau^{\,2}/M_H^{\,2}$. The upper bound on this decay comes from limits on

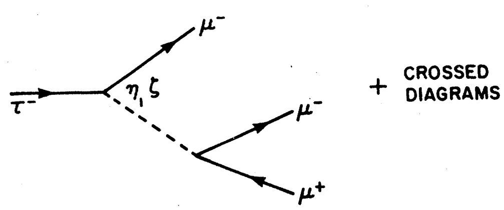

FIG. 2. Feynman diagrams for rare decay $\tau\to 3\mu$.

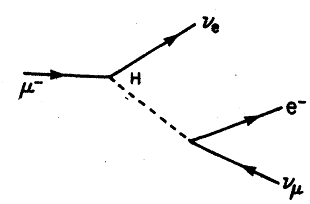

FIG. 3. μ decay to "wrong" neutrinos.

the additional $\bar{\nu}_e$ flux that such unorthodox decays contribute to $\bar{\nu}_e$ beams at accelerators[21]; present experiments are not sensitive to amplitudes much smaller than G_F.

As shown in Sec. VIII, $\pi=-1$ Higgs bosons can be light, though $\pi=1$ Higgs bosons must have masses of hundreds of GeV. If some $\pi=-1$ bosons were in the 5–20 GeV range, many strange effects discussed above would lie only one or two orders of magnitude below current upper bounds, simply because these effects are Higgs-boson-mediated rather than W-mediated. Since this scheme with π conservation emerges so naturally from the attempt to discretely unify different lepton (and quark) flavors, it seems worthwhile to test it by searching more carefully for rare τ decays.

VIII. INCORPORATION OF HADRONS

As discussed in Sec. I, the quark weak gauge interactions themselves seem to be flavor-permutation-invariant. Assuming the existence of three physical quark doublets (u,d), (c,s), and (t,b), with b lighter than t, I shall outline here how S_3 flavor symmetry may be extended to the quarks. I shall show that this leads naturally to two Cabibbo-type angles being exactly zero. Assuming $m_b<m_t$, the b quark will then be seen to decay only via Higgs-boson exchange, *always* semileptonically and *always* with lepton-number violation, so that b-flavored mesons and baryons have unique and striking weak decay signatures in this model, which will be conclusively tested the instant they are observed.

The zero value of the two extra Cabibbo angles in this model provides a natural explanation of the universality of the (u,d) and (c,s) gauge interactions, despite the existence of two extra quarks.

I shall also show that the apparent instability of b-flavored mesons[18] provides an upper bound on the mass of $\pi=-1$ Higgs bosons; the small K_L-K_S mass difference similarly yields a lower bound on the mass of $\pi=1$ Higgs bosons. These bounds yield better estimates for previously discussed rare τ decays.

A. The model

Since the only substantial difference between the leptons and quarks here is that all quarks have both LH and RH states, the mathematical treatment of the quark sector is exactly like the lepton one with the addition of RH neutrinos. I shall therefore rely on this analogy in describing the model for quarks, and thus avoid repeating similar calculations.

Let $\psi_i=(p_i,n_i)_L$, with quark flavors $i=1$ to 3, denote the usual[5] weak isodoublet of up and down LH quarks before spontaneous symmetry breaking. Similarly let p_{i_R}, n_{i_R} denote the RH weak isosinglet states, and let ϕ_i be the same Higgs bosons as coupled to leptons previously. The quark–Higgs-boson interaction must then also be invariant under S_3 flavor permutations on quark and Higgs-boson labels simultaneously. Analogously to (5.1), this gives the general Yukawa interaction

$$\begin{aligned}\mathcal{L}_{q-H}=&A_d\bar{\psi}_i\phi_i n_{i_R}+B_d\bar{\psi}_i\phi_i(n_j+n_k)_R+C_d\bar{\psi}_i(\phi_j+\phi_k)n_{i_R}+D_d(\bar{\psi}_j+\bar{\psi}_k)\phi_i n_{i_R}+E_d\bar{\psi}_i\phi_j n_{k_R}+\text{H. c.}\\&+A_u\bar{\psi}_i\tilde{\phi}_i p_{i_R}+B_u\bar{\psi}_i\tilde{\phi}_i(p_j+p_k)_R+C_u\bar{\psi}_i(\tilde{\phi}_j+\tilde{\phi}_k)p_{i_R}+D_u(\bar{\psi}_j+\bar{\psi}_k)\tilde{\phi}_i p_{i_R}+E_u\bar{\psi}_i\tilde{\phi}_j p_{k_R}+\text{H.c.},\end{aligned}\tag{8.1}$$

where $\tilde{\phi}_i$ denotes the conjugate spinor $i\sigma_2\phi_i^*$, and there are ten independent coefficients, real from assuming T invariance.

Determination of quark mass eigenstates now proceeds analogous to Sec. V, except that the p_i quarks play the role of massive neutrinos. Physical quark eigenstates also carry the conserved multiplicative "lepton" number π, and as in (5.9), there are two $\pi=1$ eigenstates and one $\pi=-1$ state for both up (charge $\frac{2}{3}$) and down (charge $-\frac{1}{3}$) quarks. The $W^\pm$ bosons carry $\pi=1$, and thus couple the two up $\pi=1$ quarks to the two down $\pi=1$ quarks, but not to the $\pi=-1$ quarks. This forces the identification of $\pi=1$ eigenstates with the light quarks u, d, c, and s, which are known to involve Cabibbo mixing. The physical t and b quarks must therefore be the $\pi=-1$ states and couple only to each other via $W^\pm$.

In terms of the ten coefficients of (8.1), analogous to (5.3) to (5.7) the mass eigenvalues are

$$\begin{aligned}&m_b=2^{-1/2}F_d\rho,\quad \pi=-1\\&m_s=2^{-1/2}G_d\rho\cos(\chi_d/2),\quad \pi=1\\&m_d=2^{-1/2}G_d\rho\sin(\chi_d/2),\quad \pi=1\end{aligned}\tag{8.2}$$

where

$$\begin{aligned}&G_d=[A_d^2+2B_d^2+2D_d^2+(C_d+E_d)^2]^{1/2}\\&F_d=|C_d-E_d|\\&\sin\chi_d=\left|\frac{2A_d(C_d+E_d)-4B_dD_d}{G_d^2}\right|.\end{aligned}\tag{8.3}$$

The up quark masses are given by similar formulas with d subscripts replaced by u subscripts. The mass eigenstates are given by formulas such as (5.8) in terms of four angles $\beta_d^\pm, \beta_\mu^\pm$ defined analogously to (5.5) and (5.6) for leptons.

B. Quark–Higgs-boson interactions and selection rules

I shall not explicitly display the unitary-gauge quark–Higgs-boson couplings here. It suffices to state that in the limit $m_t, m_b \gg m_c, m_u, m_d, m_s$, all quark–Higgs-boson couplings allowed by the multiplicatively conserved π assignment

$$\begin{aligned} &H,K,L:\pi=-1\,,\\ &\chi,\Omega,\eta,\zeta:\pi=1\,,\\ &t,b:\pi=-1\,,\\ &u,d,c,s,:\pi=1 \end{aligned} \tag{8.4}$$

actually do occur, with a structure similar to that of (6.7)–(6.9), but involving the four hadronic angles $\beta_{u,d}^\pm$ in the coefficients. *All* these couplings have large strength $\sim\sqrt{G_F}m_t$ or $\sqrt{G_F}m_b$ (modulo functions of the $\beta_{u,d}^\pm$ which I assume not to drastically change their order of magnitude).

C. The Cabibbo angle

The most remarkable prediction of this model, emerging from the lepton-quark analogy, is that since u,d,c,s carry $\pi=1$ and t, b have $\pi=-1$, it is natural for only one nonzero Cabibbo-type angle to occur, since only u and d can decay into c and s via the $\pi=1$ W bosons. Six quark models generally contain three Cabibbo-type angles (and one phase), and so this model provides a natural explanation of Cabibbo universality in the light-quark sector alone. The actual value of the one nonzero Cabibbo angle is

$$\theta_C=\tfrac{1}{2}(\beta_u^+ + \beta_u^- - \beta_d^+ - \beta_d^-)\,, \tag{8.5}$$

not calculable in the model in its present form.

D. Large mass of $\pi = 1$ Higgs bosons

The $\pi=+1$ Higgs bosons Ω,η,ζ can mediate the processes

$$\pi\to\Omega\to e\nu_e\,, \tag{8.6a}$$

$$K^0\to(\eta,\zeta)\to\bar K^0\,, \tag{8.6b}$$

$$K^0\to(\eta,\zeta)\to e^+e^- \tag{8.6c}$$

with amplitudes $\sim G_F m_t m_\tau/M_+^2$ where M_+ is a typical $\pi=+1$ Higgs boson mass. All of the amplitudes in (8.6) must be highly suppressed—(8.6a) because the good agreement[22] of $\Gamma(\pi\to e\nu_e)/\Gamma(\pi\to\mu\nu_\mu)$ with $V-A$ theory is spoiled by even small scalar contributions which are not suppressed in the $m_e=0$ limit, (8.6b) because the K_L-K_S mass difference is consistent with second-order weak effects, and (8.6c) because of the restrictive bounds on the existence of strangeness-changing neutral currents. Assuming that the angles $\beta_{u,d}^\pm$ that enter the Yukawa couplings in (8.6) do not take on values that somehow drastically suppress the typical order of magnitude of these processes, the requirement that Higgs-boson contributions not spoil compatibility with experiment requires that M_+ *be greater than several hundred GeV*.

There is no such bound on the mass of $\pi=-1$ bosons, since they mediate processes involving t and b quarks which have not yet been carefully measured. Since $\tau\to\mu\gamma$, $\tau\to\mu ee$, $e^+\mu^-\to e^-\mu^+$, and $\mu^+\to e^+\bar\nu_e\nu_\mu$ all receive contributions from diagrams involving $\pi=-1$ bosons, and since there is no reason why these bosons should not be as light as 5 to 10 GeV, these processes could occur with fairly large rates. $\tau\to3\mu$, however, is mediated solely via $\pi=1$ Higgs bosons in tree approximation, and therefore has the negligible branching ratio of less than 10^{-9} [cf. Eq. (7.7)] if this extension to hadrons is correct.

E. Remarkable weak decays of the b quark

The π assignments in (8.4) lead to the charged weak current involving t and b quarks

$$J_\mu(t,b)\sim\bar t\gamma_\mu(1-\gamma_5)b\,. \tag{8.7}$$

Assuming b to be lighter than t, the b is stable against weak (gauge-mediated) decay, thus exemplifying dynamically a theory with stable hadrons as considered by Cahn.[23] However, the b can decay via virtual $\pi=-1$ Higgs bosons (I assume here that b is lighter than any Higgs boson), which necessarily *always* lead to a final state with different flavored leptons, since all the light quarks have $\pi=1$. Thus, for example,

$$b\to de^+\mu^-\,,\ se^+\mu^-\,,\ ue^-\bar\nu_\mu\,,\ c\mu^-\bar\nu_e\,,\ de^+\tau^-\,,\ \text{etc.}, \tag{8.8}$$

are all π-allowed. A typical b-decay diagram is shown in Fig. 4.

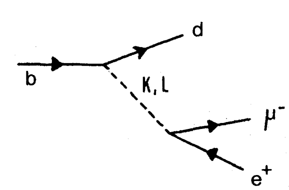

FIG. 4. $b\to de^+\mu^-$, a typical semileptonic decay of the b quark mediated by Higgs bosons.

b-flavored hadrons are therefore only *semi-stable*, with decay amplitudes $\sim G_F m_b m_\tau / M_-^2$, where M_- is the typical $\pi = -1$ Higgs mass. There are no truly stable hadrons, only ones with lifetime $\sim M_-^4/(m_b^2 m_\tau^2)$ greater than that expected from ordinary weak decays. This lifetime is consistent with the upper bound of 5×10^{-8} seconds for long-lived hadrons[18] provided at least one $\pi = -1$ boson has mass

$$M_- < 250 \text{ GeV}. \qquad (8.9)$$

It is this bound that leads to the lower limit (7.4) on radiative τ decay.

Equation (8.8) shows that the lowest-mass b-flavored hadrons always undergo semileptonic decays, characterized by easily observable lepton-number violation, e.g.,

$$(\bar b u) \to (\bar d u) e^+ \mu^- \equiv \pi^+ e^+ \mu^-$$
$$(\bar b u) \to (\bar d u) e^+ \tau^- \equiv \pi^+ e^+ \tau^- , \qquad (8.10)$$
$$(\bar b d) \to e^+ \mu^-$$

and in particular have large branching ratios to final states involving τ leptons. The $e\mu$ mode in (8.10) would allow exact reconstruction of the meson and would be easy to search for. Since this model allows no tree level nonleptonic b decays, the model can be tested the instant heavy b-flavored hadrons are observed. This is by far the best test of the model (and of S_3 flavor symmetry) in the near future, since all b decays are to peculiar states like those of (8.10), whereas tests involving τ decay require looking for rare decay modes against a large background of normal gauge-mediated decays which are absent for the b.

F. Weak production of b and t quarks

The current (8.7) ensures t and b will not be produced via W exchange in νN scattering. They will be produced via $\pi = -1$ Higgs-boson exchange, with flavor violation at both leptonic and hadronic vertices, e.g. $\nu_\mu d \to \tau^- t$, etc., with cross section

$$\sigma \sim G_F^2 s \left(\frac{m_\tau m_t}{M_-^2}\right)^2 ,$$

which could be appreciable if $m_t \simeq M_-$.

G. τ decay to hadrons

Energy conservation only allows τ to decay to states containing light quarks with $\pi = 1$, so that such decays are mediated by $\pi = 1$ bosons which were shown to be heavy. Thus

$$\tau^- \to l^- + (\text{hadrons})^0 ,$$

where l denotes e or μ (whichever carries $\pi = 1$) is expected to be much rarer than purely leptonic decays such as $\tau \to \mu e e$ which can proceed via light $\pi = -1$ Higgs bosons.

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[14] I thank D. R. T. Jones for pointing out an error in an earlier manuscript on this work, in which I had excluded the terms in (4.1) with coefficients γ and E_i by incorrectly using a reflection symmetry. The inclusion of these terms here does not all all affect the emergence of the multiplicatively conserved lepton number. I am grateful to Dr. Jones for correspondence on this and other matters.
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$\langle\phi_1^0\rangle=\langle\phi_2^0\rangle=\rho'\to 0$, which leads to degeneracies among Higgs-boson masses, I shall nevertheless treat the Higgs-boson masses independent nondegenerate quantities when making phenomenological estimates of rates, so as not to arrive at incorrect relations based upon an arbitrary approximation scheme.

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