# Simplifications of conformal supergravity

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We show that *all* dependent gauge fields of conformal supergravity follow from constraints on curvatures (rather than from a mixture of constraints and field equations). We also give a simplified form of the action and show that if regarded as a one-loop counterterm of Poincaré supergravity, it is (at least) quadratic in field equations.

## I. INTRODUCTION

The purpose of this paper is to introduce some simplifications of a previous article on conformal supergravity,[1] hereafter referred to as I, and some new results. In I we gave the action and transformation rules of the fields and proved complete invariance under both local supersymmetries, denoted by $Q$ and $S$. Crucial to our results was the fact that many of the gauge fields of the superconformal group were not independent but could be expressed as functions of the other physical fields either by constraints on group curvatures or by a nonpropagating field equation. In this paper we extend and simplify the results of I in the following ways:

(i) We show that *all* the dependent gauge fields can be found from solving constraints on curvatures. As well as being interesting in itself, this result is also useful for computational purposes.

(ii) We show that the conformal supergravity action can be written as an expression (at least) quadratic in field equations of Poincaré supergravity. We also give the action in a more explicit form than appears in I. Throughout we use the conventions and many of the results of I.

## II. THE CONSTRAINTS ON CURVATURES

The action of conformal supergravity can be written as an expression quadratic in the curvatures of the superconformal group. It is

$$I=\int d^4x\,\{\epsilon^{\mu\nu\rho\sigma}[R_{\mu\nu ab}(M)R_{\rho\sigma cd}(M)\epsilon^{abcd} - 8R_{\mu\nu}(Q)\gamma_5\bar{R}_{\rho\sigma}(S) + 4iR_{\mu\nu}(A)R_{\rho\sigma}(D)] - 8eg^{\mu\rho}g^{\nu\sigma}R_{\mu\nu}(A)R_{\rho\sigma}(A)\}\,. \tag{2.1}$$

[For conventions, see I. In particular, $\kappa=1$ and $\bar{R}_{\rho\sigma}(S)=\partial_\sigma\phi_\rho+\cdots$ while $R_{\mu\nu}(Q)=\partial_\nu\bar{\psi}_\mu+\cdots$.] The fields appearing in this action are the gauge fields $(\omega_{\mu ab}, e_{a\mu}, f_{a\mu}, b_\mu, A_\mu, \bar{\psi}^\alpha_\mu, \bar{\phi}^\alpha_\mu)$ corresponding to the superconformal generators $(M_{ab}, P_a, K_a, D, A, Q_\alpha, S_\alpha)$. But only $e^a{}_\mu$, $\psi_\mu$, $A_\mu$, and $b_\mu$ are independent fields; the remainder can be expressed as functions of these independent fields through the set of constraints

$$R_{\mu\nu a}(P)=0\,, \tag{2.2a}$$

$$R_{\mu\nu}(Q)\gamma^\nu=0\,, \tag{2.2b}$$

$$R_{\nu\lambda ab}(M)e^{a\lambda}e^b{}_\mu-\tfrac{1}{2}R_{\lambda\mu}(Q)\gamma_\nu\psi^\lambda+\tfrac{1}{4}ie^{-1}\tilde{R}_{\mu\nu}(A)=0\,. \tag{2.2c}$$

The algebraic meaning of these constraints is that they eliminate the lower spin parts of the curvatures; for their geometrical meaning we refer to I. These constraints also imply the following useful relations:

$$\begin{gathered}R_{\mu\nu}(Q)+\tfrac{1}{2}e^{-1}\tilde{R}_{\mu\nu}(Q)\gamma_5=0\,,\quad R_{\mu\nu}(Q)\sigma^{\mu\nu}=0\,,\\ R_{\mu\nu}(Q)\gamma_\lambda+R_{\lambda\mu}(Q)\gamma_\nu+R_{\nu\lambda}(Q)\gamma_\mu=0\,,\\ R_{\alpha\beta ab}(M)e^{a\beta}e^{b\alpha}=0\,.\end{gathered} \tag{2.3}$$

The first two constraints of Eq. (2.2) can be solved for $\omega_{\mu ab}$ and $\phi_\mu$. The solution is given in I. The third constraint of Eq. (2.2), which is new, can be solved for the proper conformal gauge field $f_{a\mu}$. The solution is

$$f_{\mu\nu}=-\tfrac{1}{4}(\hat{R}_{\nu\mu}-\tfrac{1}{6}g_{\mu\nu}\hat{R})+\tfrac{1}{8}R_{\lambda\mu}(Q)\gamma_\nu\psi^\lambda -(ie^{-1}/16)\,\hat{R}_{\mu\nu}(A)\,, \tag{2.4}$$

where $\hat{R}_{\mu\nu ab}$ is $R_{\mu\nu ab}(M)$ without the "$ef$" terms.[1] Since $\omega=\omega(e,\psi,A,b)$ and $\phi=\phi(e,\psi,A,b,\omega)$ and $f=f(e,\psi,A,b,\omega,\phi)$, it is irrelevant in which order one solves the constraints. We recognize Eq. (2.4) as Eq. (2.7) of I where it was found as the algebraic field equation of $f_{a\mu}$. So we have two ways of obtaining Eq. (2.4): (1) as a field equation of conformal supergravity; (2) as a constraint or superconformal group curvatures. This is reminiscent of the status of the equation

$$\omega_{\mu ab}=\tfrac{1}{2}[e_a{}^\nu(e_{b\nu,\mu}-e_{b\mu,\nu})+e_a{}^\lambda e_b{}^\rho e_{c\lambda,\rho}e^c{}_\mu] -(a\leftrightarrow b) \tag{2.5}$$

 

in pure Einstein gravity. (Similar remarks apply to Poincaré supergravity.) One can deduce this equation either (i) as the $\omega_{\mu ab}$ field equation or (ii) from the constraint $R_{\mu\nu a}(P)=0$ (the $P$ curvature of the Poincaré group).

It is apparently a coincidence that these two methods coincide in this simple case because if one considers other actions they no longer coincide. In fact, if one considers $R^2$-type actions, then $\omega_{\mu ab}$ propagates and *cannot* be eliminated algebraically by its field equation, and one can only obtain the desired relation for $\omega_{\mu ab}$ by imposing a constraint. This is one of several reasons why we regard the "constraint" (or second-order) formulation as preferable to the "field equation" (or first-order) formulation for $\omega_{\mu ab}$. Another reason is that only the second-order formalism arises naturally from superspace.[2] Specifically, the spin connection does not appear in the vector superfield $H_\mu(x,\theta)$. (Note also that if $\omega_{\mu ab}$ were an independent auxiliary field, it would require at least 18 more fermionic auxiliary fields as opposed to the six bosonic auxiliary fields of the second-order formalism.)[3]

This analogy with the spin connection leads us to prefer to deduce (2.4) from the constraint (2.2c), rather than from the $f_{a\mu}$ field equation. Since the results of the two methods coincide this is only a preference, not a necessity. But again the above analogy with $\omega_{\mu ab}$ suggests that there may be other theories containing the gauge field $f_{a\mu}$ but where $f_{a\mu}$ *cannot* be eliminated by its field equation. For example, we could consider gauge theories of higher groups containing the conformal group such as Sp(8). In such theories we would see that the constraint formulation was the more fundamental and that it just happened to coincide with a field equation in a particular model.

The meaning of the new constraint (2.2c) is that it is a supersymmetric generalization of the constraint

$$R_{\nu\lambda ab}(M)e^{a\lambda}\equiv R_{\nu b}(M)=0\,, \tag{2.6}$$

which simply requires all contractions of $R(M)$ to vanish. This ensures that $R(M)$ is just the Weyl curvature rather than the usual Riemann curvature. To see that (2.2c) is the supersymmetric generalization of (2.6), consider the behavior of these constraints under $S$ supersymmetry. Equation (2.6) is not $S$ invariant but (2.2c) *is* $S$ invariant. In fact all these constraints of (2.2) are invariant under $M$, $K$, $D$, $S$, and $A$ gauge transformations in that they rotate into each other. This implies, just as for $\omega_{\mu ab}$ and $\phi_\mu$, that

$$\delta'_S f_{a\mu}=0 \quad (\text{idem for } M,K,D,A)\,. \tag{2.7}$$

That is, under $S$ supersymmetry (or any of the other symmetries) $f_{a\mu}$ transforms according to the group prescription. However, under $Q$-supersymmetry *gauge* transformations, none of Eqs. (2.2) are invariant. This implies (again just as for $\omega_{\mu ab}$ and $\phi_\mu$) that $\delta'_Q f_{a\mu}\neq 0$. Using the methods of I applied to the new constraint, we can deduce $\delta'_Q f_{a\mu}$. We leave this calculation to the end of this section. [Recall that $\delta'_Q f'_{a\mu}$ was not needed for the proof of $Q$ invariance of the conformal supergravity action in I because its contribution to $\delta\mathcal{L}$ is multiplied by $\delta\mathcal{L}/\delta f_{a\mu}$, the $f_{a\mu}$ field equation. This field equation vanishes *identically* because of the constraint (2.2c).]

With the modified $\delta_Q$ transformations given by $\delta_Q=\delta_Q^{\text{gauge}}+\delta'_Q$, the three constraints of (2.2) are maintained. One can see why the $R(Q)\gamma\psi$ term is needed in (2.2c) from the fact that $R(M)$ contains derivatives of $\omega_{\mu ab}$ which will produce $\partial_\mu\epsilon$ terms when we include the contribution of $\delta'_Q\omega_{\mu ab}$ to $\delta_Q R(M)$. These are canceled by the $\delta_Q\psi_\mu=\partial_\mu\epsilon$ term in $R(Q)\gamma\psi$. Since no other $\partial_\mu\epsilon$ arise (because $\delta'_Q\omega_{\mu ab}$, $\delta'_Q\phi_\mu$, $\delta'_Q f_{a\mu}$ do not contain them) the supercovariantization of $R(M)$ is just

$$R^{\text{cov}}_{\mu\nu ab}(M)=R_{\mu\nu ab}(M)+\tfrac{1}{2}R_{ab}(Q)\gamma_\nu\psi_\mu - \tfrac{1}{2}R_{ab}(Q)\gamma_\mu\psi_\nu\,. \tag{2.8}$$

With this notation the constraint (2.2c) can be written in the particularly simple form

$$R_{\mu\nu}(M)^{\text{cov}}+R_{\nu\mu}(M)^{\text{cov}}=0\,,$$
$$R_{\mu\nu}(M)^{\text{cov}}-R_{\nu\mu}(M)^{\text{cov}}=\tfrac{1}{2}ie^{-1}\tilde{R}_{\mu\nu}(A)\,. \tag{2.9a}$$

One observation that may have significance is that if one defines a new spin connection

$$\omega'_{\mu ab}=\omega_{\mu ab}-\tfrac{1}{2}i\epsilon_{\mu abc}A^c$$

and defines a Lorentz curvature $R'_{\mu\nu ab}(M)$ in terms of it, then the constraint (2.2c) becomes simply

$$R'_{\mu\nu}(M)^{\text{cov}}=0\,. \tag{2.9b}$$

This connection $\omega'_{\mu ab}$ is surprisingly equal to the quantity that appears in the commutator algebra of Poincaré supergravity.[3]

To complete this section we will give some details of the calculation of $\delta'_Q f_{a\mu}$. We use the obvious theorem that $\delta'_Q f_{a\mu}$ must be such that the variation of the new constraint vanishes under the complete $Q$-supersymmetry transformation. In the variation of this constraint we use

$$\delta_Q(\text{constraint})=\delta_Q^{\text{gauge}}(\text{constraint}) +\delta'_Q(\text{constraint})\,. \tag{2.10}$$

$\delta_Q^{\text{gauge}}$(constraint) is determined straightforwardly from the covariant rotation of curvatures into each other, while $\delta'_Q$(constraint) is determined by substituting the known results for $\delta_Q' \omega_{\mu ab}$ and $\delta_Q' \phi_\mu$ (given in I) and the unknown $\delta_Q' f_{a\mu}$. This gives the following equation for $\delta_Q' f_{a\mu}$:

$$\delta_Q' f_{a\mu} = -\tfrac{1}{4} R_{b\mu}(S)\sigma_{ab}\epsilon_Q - \tfrac{1}{24} e_{a\mu}(R_{\alpha\beta}(S)\sigma^{\alpha\beta}\epsilon_Q) - \tfrac{1}{16}\tilde{R}_{a\mu}(S)\gamma_5\epsilon_Q + \tfrac{1}{8}(D_\rho R_a{}^\rho(Q))\gamma_\mu\epsilon_Q + (3i/32)(R_{ab}(Q)\gamma_\mu\gamma_5\epsilon_Q)A^b + \tfrac{1}{16}(R_{ab}(Q)\gamma_\mu\epsilon_Q)b^b + \text{explicit}\,\psi_\mu \text{ terms}. \tag{2.11}$$

We ignore terms with explicit *undifferentiated* $\psi_\mu$'s on the grounds that these will simply serve to supercovariantize the final result. Now we use the Bianchi identity for $R(Q)$, $\epsilon^{\mu\nu\rho\sigma}D_\nu R_{\rho\sigma}(Q) \equiv 0$, and (2.2b) to get

$$D_\mu R^{\mu\nu}(Q) - (3i/4)R^{\mu\nu}(Q)A_\mu\gamma_5 + \tfrac{1}{2}R^{\mu\nu}(Q)b_\mu = \tfrac{1}{2}\tilde{R}^{\mu\nu}(S)\gamma_\mu\gamma_5 + \text{explicit}\,\psi_\mu \text{ terms}. \tag{2.12}$$

This allows us to write (2.11) as

$$\delta_Q' f_{a\mu} = -\tfrac{1}{2}R_{\rho\mu}(S)\sigma_a{}^\rho\epsilon_Q - \tfrac{1}{8}\tilde{R}_{a\mu}(S)\gamma_5\epsilon_Q - \tfrac{1}{16}e_{a\mu}R_{\alpha\beta}(S)\sigma^{\alpha\beta}\epsilon_Q + \text{explicit}\,\psi_\mu \text{ terms}. \tag{2.13}$$

If one now takes $\gamma_\nu$ times Eq. (2.12), the left-hand side vanishes by (2.2b) except for explicit $\psi_\mu$ terms. Hence we have

$$R^{\alpha\beta}(S)\sigma_{\alpha\beta} = \text{explicit}\,\psi_\mu \text{ terms}. \tag{2.14}$$

So (2.13) becomes

$$\delta_Q' f_{a\mu} = -\tfrac{1}{2}R_{\rho\mu}(S)\sigma_a{}^\rho\epsilon_Q - \tfrac{1}{8}\tilde{R}_{a\mu}(S)\gamma_5\epsilon_Q + \text{explicit}\,\psi_\mu \text{ terms}. \tag{2.15}$$

We now introduce the supercovariantized $R_{\mu\nu}(S)^{\text{cov}}$ which is

$$R_{\mu\nu}(S)^{\text{cov}} = R_{\mu\nu}(S) - \tfrac{1}{4}i\bar{\psi}_\nu\gamma^\lambda[\gamma_5 R_{\lambda\mu}(A) - \tfrac{1}{2}\tilde{R}_{\lambda\mu}(A)] + \tfrac{1}{4}i\bar{\psi}_\mu\gamma^\lambda[\gamma_5 R_{\lambda\nu}(A) - \tfrac{1}{2}\tilde{R}_{\lambda\nu}(A)] \tag{2.16}$$

since there is only a derivative on $\delta'\phi_\mu$ in $\delta R(S)$. Now we may write $\delta_Q' f_{a\mu}$ finally as

$$\delta_Q' f_{a\mu} = -\tfrac{1}{2}R_{\rho\mu}(S)^{\text{cov}}\sigma_a{}^\rho\epsilon_Q - \tfrac{1}{8}R_{a\mu}(S)^{\text{cov}}\gamma_5\epsilon_Q\,. \tag{2.17}$$

As we pointed out previously, this result was not needed in I, but we find it convenient to give it here because we will need it in a later publication[4] dealing with new results on tensor calculus. The normalization is as in I.

## III. THE ACTION

Substituting the solution of the constraints for $f_{a\mu}$ into (2.1) gives the following expression:

$$I = 8\int d^4x\,\{e[\hat{R}_{\mu\nu}(M) - \tfrac{1}{2}R_{\lambda\nu}(Q)\gamma_\mu\psi^\lambda][\hat{R}^{\nu\mu}(M) - \tfrac{1}{2}R_\lambda{}^\mu(Q)\gamma^\nu\psi^\lambda] + \tfrac{1}{2}i[\hat{R}_{\mu\nu}(M) - \tfrac{1}{2}R_{\lambda\nu}(Q)\gamma_\mu\psi^\lambda]\tilde{R}^{\mu\nu}(A) - \tfrac{3}{4}eR_{\mu\nu}(A)R^{\mu\nu}(A) + 4\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\rho\gamma_5\gamma_\sigma D_\nu\phi_\mu + \tfrac{1}{2}i\bar{\psi}_\mu\phi_\nu\tilde{R}^{\mu\nu}(A) + 3i(\partial_\sigma A_\nu)\bar{\psi}_\rho\phi_\mu\epsilon^{\mu\nu\rho\sigma} - 3i(\bar{\phi}_\rho\gamma_\sigma\phi_\mu)A_\nu\epsilon^{\mu\nu\rho\sigma} - (\bar{\psi}_\mu\sigma_{ab}\phi_\nu)\bar{\psi}_\rho\gamma_5\sigma^{ab}\phi_\sigma\epsilon^{\mu\nu\rho\sigma} - \tfrac{1}{3}e\hat{R}(M)^2\}. \tag{3.1}$$

[This is Eq. (2.8) of I with misprints corrected.] Now the three equations

$$\hat{R}_{\mu\nu}(M) - \tfrac{1}{2}R_{\lambda\nu}(Q)\gamma_\mu\psi^\lambda \equiv \hat{R}_{\mu\nu}(M)^{\text{cov}} = 0\,, \tag{3.2a}$$

$$\phi_\mu = 0\,, \tag{3.2b}$$

$$A_\mu = 0 \tag{3.2c}$$

are equivalent to the $e_\mu^a$, $\psi_\mu$, and $A_\mu$ field equations of Poincaré supergravity. Thus, if (3.1) is regarded as a one-loop invariant of Poincaré supergravity, we see that it is quadratic in field equations (at least). That this is expected can be seen as follows: We should be able to write the conformal supergravity action as a linear combination of the $R_{\mu\nu}{}^2$ and $R^2$ invariants of Poincaré supergravity up to a total derivative (by a super-Gauss-Bonnet theorem[4,5]). But these invariants are constructed via the "tensor calculus"[5] by squaring two supermultiplets, each of which vanishes on-shell. Hence these invariants must be at least quadratic in field equations. This argument is verified explicitly by our action (3.1).

Finally we give another form of the action in which the dependence of the action on the Ricci tensor and the field strength tensor for $A_\mu$ is explicit:

$$
\begin{aligned}
\tfrac{1}{8}I = \int d^4x\, e\{ & R_{\mu\nu}(E)R^{\nu\mu}(E) - \tfrac{1}{3}R^2(E) - \tfrac{3}{4}[F_{\mu\nu}(A)]^2 + 4\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\rho\gamma_5\gamma_\sigma D_\nu\phi_\mu \\
& + 2R_{\nu\mu}(E)(\bar{\psi}_\lambda\sigma_{\lambda\nu}\phi_\mu - \bar{\psi}_\mu\sigma_{\lambda\nu}\phi_\lambda) - \tfrac{4}{3}R(E)(\bar{\psi}_\lambda\sigma_{\lambda\nu}\phi_\nu) - R_{\mu\nu}(E)(R_{\lambda\mu}(Q)\gamma_\nu\psi_\lambda) \\
& + [\tfrac{1}{4}i\bar{\psi}\cdot\gamma(\gamma_5F_{\alpha\beta} - \tfrac{1}{2}\tilde{F}_{\alpha\beta})S^{\alpha\beta} + \tfrac{1}{2}i\bar{\psi}^\lambda\gamma^\alpha(\gamma_5F_{\alpha\beta} - \tfrac{1}{2}\tilde{F}_{\alpha\beta})S^{\beta}{}_\lambda + i\bar{\psi}^\alpha\gamma_\lambda(\gamma_5F_{\alpha\beta} - \tfrac{1}{2}\tilde{F}_{\alpha\beta})S^{\beta\lambda}] \\
& + R_{\mu\nu}(E)\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\rho\gamma_5\phi_\sigma - \tfrac{4}{3}(\bar{\psi}_\mu\sigma^{\mu\nu}\phi_\nu)^2 - 3i\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\rho\gamma_\sigma\phi_\mu A_\nu \\
& + \tfrac{3}{2}(\bar{\psi}_\mu\gamma_5\phi_\nu)(\bar{\psi}^\mu\gamma_5\phi^\nu - \bar{\psi}^\nu\gamma_5\phi^\mu) + \bar{\psi}_\mu\phi_\nu\bar{\psi}_\rho\gamma_5\phi_\sigma\epsilon^{\mu\nu\rho\sigma} \\
& + (\bar{\psi}_\mu\sigma_{\lambda\nu}\phi^\lambda)(\bar{\psi}^\nu\sigma^{\tau\mu}\phi_\tau) + (\bar{\psi}_\lambda\sigma^{\lambda\nu}\phi_\mu)(\bar{\psi}_\tau\sigma^{\tau\mu}\phi_\nu) - (\bar{\psi}_\mu\sigma_{\lambda\nu}\phi^\lambda)(\bar{\psi}_\tau\sigma^{\tau\mu}\phi_\nu) - (\bar{\psi}_\lambda\sigma^{\lambda\nu}\phi_\mu)(\bar{\psi}_\nu\sigma^{\lambda\mu}\phi_\lambda) \\
& - (\bar{\psi}_\mu\sigma_{ab}\phi_\nu)(\bar{\psi}_\rho\gamma_5\sigma_{ab}\phi_\sigma)\epsilon^{\mu\nu\rho\sigma} + (\bar{\psi}_\lambda\sigma^{\lambda\nu}\phi_\mu)(\bar{\psi}_\rho\gamma_5\phi_\sigma)\epsilon^{\mu\nu\rho\sigma} \\
& - (\bar{\psi}_\mu\sigma_{\lambda\nu}\phi^\lambda)(\bar{\psi}_\rho\gamma_5\phi_\sigma)\epsilon^{\mu\nu\rho\sigma} - \tfrac{1}{4}(\bar{\psi}_\mu\gamma_5\phi_\nu)(\tilde{R}^{\mu\nu}(Q)\gamma\cdot\psi) \\
& + (\bar{\psi}_\mu\sigma_{\lambda\nu}\phi^\lambda - \bar{\psi}_\lambda\sigma^{\lambda\nu}\phi_\mu)R^{\tau\mu}(Q)\gamma_\nu\psi_\tau + \tfrac{1}{4}(R_{\tau\nu}(Q)\gamma_\mu\psi^\tau)(R_{\lambda\mu}(Q)\gamma_\nu\psi^\lambda)\}\,. \qquad (3.3)
\end{aligned}
$$

In this expression $R_{\mu\nu}(E)$ is the Ricci tensor (with torsion, so it is not symmetric), $F_{\mu\nu}$ is $\partial_\mu A_\nu - \partial_\nu A_\mu$, and $S_{\mu\nu}$ is the curl of $\psi_\mu$ defined in I. Finally we recall that the field $b_\mu$ may be set everywhere equal to zero in the action, since it cancels anyhow.

## IV. COMMENTS

We have shown that the proper conformal gauge field $f_{a\mu}$ may be expressed in terms of the other independent fields via a constraint on curvatures, just as for $\omega_{\mu ab}$ and $\phi_\mu$. This constraint happens to coincide with the $f_{a\mu}$ field equation of the conformal supergravity action so that the equation for $f_{a\mu}$ obtained here is the same as that in I, as it must be. But there may be gauge theories of higher groups such as Sp(8) for which a constraint is necessary if $f_{a\mu}$ is to be eliminated, just as $R(P)=0$ is needed in conformal supergravity (and pure Weyl gravity) if $\omega_{\mu ab}$ is to be eliminated. It is not excluded that a supersymmetric $R^2$-type action exists with $\omega_{\mu ab}$ as an independent field (although, since the auxiliary fields of first-order Einstein supergravity are unknown, one cannot obtain it as a one-loop counterterm), but to construct it we would need many more fermion fields in order to balance the boson and fermion degrees of freedom. This would be a very nonminimal approach and it seems much more natural to impose $R(P)=0$. Likewise we consider the constraint for $f_{a\mu}$ to be a more natural way to formulate the theory. Indeed, in the proof of $Q$ invariance of conformal supergravity in I, we needed to use the equation for $f_{a\mu}$ so that we were not able to really consider $f_{a\mu}$ at any time as an independent field. If $f_{a\mu}$ is regarded as an independent field (which is subsequently eliminated by its field equation), then the invariance of the action is unknown. Probably this invariance exists just as an invariance exists in both first- and second-order forms of Poincaré supergravity, but one can see at once that the algebra of gauge transformations on $e^a_\mu, \psi_\mu, A_\mu, f_{a\mu}, b_\mu$ cannot close in this case. To see this, one need only count the off-shell degrees of freedom. After subtracting for 6 local Lorentz, 4 general coordinate, and 1 scale invariances, there are 5 components of $e_{a\mu}$ left; after subtracting for 4$Q$ and 4$S$ invariances, there are 8 components of $\psi_\mu$ left; after subtracting 1 for chiral invariance there are three components of $A_\mu$ left. This already gives an equal number of boson and fermion fields so that no auxiliary fields are needed to close the algebra if these are the only fields. (This counting also applies if one includes the dilation field $b_\mu$, since it is eliminated by the proper conformal gauge invariances.) In fact it was shown in I that the algebra does close. However, if $f_{a\mu}$ is another independent field, one needs 16 more auxiliary fermion fields to close the algebra.


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