Comments and Addenda

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Absence of tachyons in supergravity and classical relativity

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The relation between energy and supercharge in supersymmetry and supergravity implies that tachyons have vanishing four-momentum there and consequently in classical Einstein gravity also.

The total energy in O(N) supergravity, with or without sources, is given as the sum of squares of the supercharges, which implies that it is nonnegative,[1] at least formally. This property can then be shown to hold also in the classical ($\hbar = 0$) limit when there are no on-shell fermions.[2] We note here an immediate corollary of these results which excludes tachyons in these theories. This is of interest because a recent rigorous proof of energy positivity in classical gravity[3] does not cover[4] certain initial data, in particular those corresponding to spacelike P^μ.

In the supergravity proof[1] it is shown that the fundamental supersymmetry relations

$$\{Q_i^\alpha, \bar{Q}_j^\beta\} = -2\delta_{ij}\gamma_a^{\alpha\beta}P^a \tag{1}$$

are valid also in supergravity. Here Q_i^α are the Hermitian spinor charges with internal and spinor indices (i, α); possible central charges do not change the argument and are omitted here. The energy is a sum of squares,

$$P^0 = \frac{1}{4N}\sum_{i,\alpha}(Q_i^\alpha)^2 \geq 0\,, \tag{2}$$

and vanishing of P^0 therefore implies that Q_i^α vanishes. Consequently, $P^0 = 0$ implies that $P^\mu = 0$. But, by definition, tachyonic solutions have spacelike P^μ, so that there exist suitable (asymptotic) Lorentz frames where $P^0 = 0$. Then the full P^μ must also vanish in these (and therefore in all) frames, establishing the desired result, which evidently holds in the classical limit as well. (Similar considerations for classical gravity were presented earlier[5] in connection with a different argument for positivity there.) Note that $P^\mu = 0$ is the vacuum (flat space) in classical gravity[3], and presumably also in supergravity.

It also seems likely that solutions with lightlike P^μ are forbidden. Physically, null solutions would correspond to non-normalizable plane waves which would therefore not have bounded energy. It should be possible to show that P^μ is not definable in such cases by considering the consequences[6] of $\gamma \cdot P$ vanishing in (1).

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[1] S. Deser and C. Teitelboim, Phys. Rev. Lett. 39, 249 (1977).
[2] M. T. Grisaru, Phys. Lett. 73B, 207 (1978).
[3] R. M. Schoen and S.-T. Yau, Math. Phys. 65, 45 (1979). (to be published).
[4] J. W. York, Jr., North Carolina report, 1979 (unpublished).
[5] S. Deser, Nuovo Cimento 55B, 593 (1968).
[6] From (1), it follows that $|\vec{P}| = (1/4N)\,|\mathrm{tr}\,Q\vec{\alpha}Q| \leq (1/4N)\mathrm{tr}\,QQ$ and the equality can only hold if $|\langle\vec{\alpha}\rangle| = 1$, characteristic of a single free plane wave.

