Dynamical chiral symmetry breaking in quantum chromodynamics

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We examine both dynamical chiral symmetry breaking and explicit breaking due to current quark masses in quantum chromodynamics (QCD). The renormalized current and constituent quark mass are defined. The quark self-energy $\Sigma(p) = \Sigma_D(p) + \Sigma_E(p)$ has unique contributions from dynamical and explicit symmetry breaking. We determine the asymptotic behavior of $\Sigma_D(p)$ and $\Sigma_E(p)$ as $-p^2 \to \infty$. Although the explicit symmetry breaking dominates in the region controlled by perturbation theory, the dynamical term, which receives contributions from instantons, dominates in the subasymptotic region. The dynamical term is often ignored in the calculations. We also discuss the possibility of a phase transition in QCD for massive quark systems. The structure of the chiral perturbation expansion for light quarks is found to have not only an essential singularity in the gauge-field coupling constant for $g^2 \leq 0$ but also a cut for $g^2 \leq 0$ in amplitudes for which the essential singularity is absent. We also calculate the second-order axial-vector renormalization for a quark with the result $g_A = 1 - g^2/6\pi^2$.

I. INTRODUCTION AND SYNOPSIS

We adopt quantum chromodynamics[1] (QCD) with $N=3$ colors and n flavors as our model of the strong interactions. In the absence of explicit symmetry breaking due to current quark mass terms the chiral $SU(n)\times SU(n)$ symmetry of QCD is assumed to be realized dynamically by a (n^2-1)-plet of Goldstone bosons. The purpose of this article is to discuss both explicit and dynamical chiral symmetry breaking in QCD.

The structure of the quark propagator in the presence of both explicit and dynamical symmetry breaking is analyzed. The renormalized quark propagator is $S^{-1}(p,g,\mu,m)$, where g is the renormalized coupling at renormalization mass μ and m is the renormalized current quark mass representing explicit symmetry breaking. The quark propagator is related to the renormalized vertex for the scalar operator $\bar{q}q$ by

$$\Gamma(p,p,g,\mu,m) = -\frac{\partial S^{-1}(p,g,\mu,m)}{\partial m}.$$

Integrating this equation we have

$$S^{-1}(p,g,\mu,m) = S_E^{-1}(p,g,\mu,m) + S_D^{-1}(p,g,\mu),$$

$$S_E^{-1}(p,g,\mu,m) = -\int_0^m \Gamma(p,p,g,\mu,m')dm'.$$

Here $S_D^{-1}(p,g,\mu)$, an integration constant, is the contribution of dynamical symmetry breaking and is independent of the current quark mass m. $S_E^{-1}(p,g,\mu,m)$ is the contribution of explicit symmetry breaking and vanishes if the current quark mass $m=0$. Several recent treatments[2,3] of the quark mass, using the renormalization group, ignore the dynamical quark mass term completely and hence violate partial conservation of axial-vector current (PCAC). The dynamical term can have an important effect on hadron processes.

Extensive use will be made of Weinberg's[4] version of the renormalization-group equations in which masses are treated as coupling constants. This renormalization prescription has the advantage that the β function is independent of m/μ and α, the gauge parameter. We examine the asymptotic behavior of the quark propagator and consider separately the explicit and dynamical terms. For the explicit symmetry-breaking term we find for the part of $S_E^{-1}(p)$ that anticommutes with γ_5, $\Sigma_E(p)$,

$$\Sigma_E(p) \underset{p^2\to-\infty}{\sim} m[\ln(-p^2/\mu^2)]^{-c/b},$$

where $c/b = 12/(33-2n)$. This result follows unambiguously from the use of the renormalization group and the boundary condition on the scalar vertex Γ.

For the dynamically generated quark self-energy $\Sigma_D(p)$ the renormalization-group equations offer no clue as to the asymptotic behavior because the boundary condition on this amplitude is unknown. There is no mass renormalization because there is no mass. Instead $\Sigma_D(p)$ must be obtained from an integral equation. Using asymptotic freedom for the appropriate kernel one finds that if $\Sigma_D(p)$ does not vanish identically it can have only two kinds of asymptotic behavior corresponding to the regular (Neumann series) and irregular solutions for the bound-state Goldstone-boson wave function. We argue in favor of the regular solution since it implies rapidly falling electromagnetic form factors for the bound-state meson, while the irregular solution yields nonfalling form factors corresponding to elementary mesons.[5] Further for the irregular solution the usual operator-product expansion fails[6] as does the Bĕg-Shei theorem[7] on the merger of Nambu-Goldstone and Wigner-Weyl chiral realizations on the light cone. For the regular solution, then, one has

$$\Sigma_D(p) \underset{p^2\to-\infty}{\sim} \frac{\mu^3}{-p^2}[\ln(-p^2/\mu^2)]^{+c/b}.$$

Although $\Sigma_D(p)$ is small relative to $\Sigma_E(p)$ in the asymptotic region controlled by perturbation theory for moderate $p^2 \simeq -\mu^2$, $\Sigma_D(p)$ becomes large because of nonperturbative instanton interactions.[8] This large increase of the quark mass in the subasymptotic region could be an important effect of instantons on hadron dynamics.

As a byproduct of this discussion we have calculated g_A, the axial-vector renormalization constant, for a quark to first order in perturbation theory. The result is infrared finite and given by

$$g_A = 1 - \frac{g^2}{8\pi^2}C_2(N), \quad C_2(3) = \tfrac{4}{3}.$$

In the additive quark model $g_A^{\text{nucleon}} = \frac{5}{3}g_A^{\text{quark}}$, where $g_A^{\text{nucleon}} = 1.25 \pm 0.02$. With $g^2/8\pi^2 \sim \frac{3}{16}$ the result comes out right. While quantitative significance cannot be attached to this result the coupling comes out reassuringly small.

We have also examined the question of a phase transition in QCD as the renormalized current quark mass m approaches a critical value m^*. For vanishing current quark mass m the meson decay constant $f(g,\mu,m)$ is nonvanishing $f(g,\mu,0) \neq 0$ signaling a Goldstone realization of the chiral symmetry. For small m/μ the meson decay constant is known to increase like $f(g,\mu,m) = f(g,\mu,0) + F_2(g)m\ln(\mu/m) + \cdots$ with $F_2(g) > 0$ a numerical constant. So for light quarks u, d, and s one can do perturbation theory in m/μ. However, for heavy quarks like $c, b, \ldots$, $m/\mu \sim O(1)$ and it is not possible to study symmetry breaking in a perturbative way. Further there may exist a critical value $m = m^*$ for which $f(g,\mu,m^*) = 0$ signaling a phase transition. At this transition there is no chiral symmetry but for $m > m^*$ the bound states may no longer be smoothly connected to the Goldstone mode. For example, if $m_c > m^*$ then $D^{\pm,0}$ mesons are not "almost Goldstone bosons" like the collective $K^{\pm,0}$ and $\pi^{\pm,0}$ states but presumably atomic states.[9] We have been unable to verify the existence of this transition in QCD but have examined some constraints imposed on the problem by the renormalization group.

In the SU(3)×SU(3) linear Σ model in the tree approximation[10] one can show that this transition does indeed take place when the symmetry is explicitly broken to SU(2)×SU(2). At the critical value of the symmetry-breaking parameter $f_\pi/f_K = 0$, $m_\pi^2 = m_\sigma^2 = 0$, and $m_K^2 = m_\kappa^2$, where m_K is the kaon mass and m_κ is the kappa mass. This can also be understood in terms of the domain structure of chiral symmetry breaking.[11] Since in the real world $m_K^2/m_\kappa^2 \sim 0.2$, we are far from this phase transitions at least for light quarks.

Finally we have examined the structure of the perturbation series in the light-quark mass. For the meson decay constant f and ground-state pseudoscalar mass M, chiral perturbation theory implies[12]

$$f(g,\mu,m) = \mu F_1(g) - m\ln(m/\mu)F_2(g) + mF_3(g) + \cdots,$$

$$M^2(g,\mu,m) = m[\mu H_1(g) + m\ln(m/\mu)H_2(g) + mH_3(g) + \cdots].$$

Using the renormalization group one can show that as $g \to 0$

$$F_1(g) = f_1 g^{-4d/b} e^{-1/bg^2},$$
$$F_2(g) = f_2 g^{-2c/b},$$
$$H_1(g) = h_1 g^{-2(c+2d)/b} e^{-1/bg^2},$$
$$H_2(g) = h_2 g^{-4c/b},$$

where f_i and h_i are numerical constants undetermined by this analysis and $c = 1/2\pi^2$, $b = (11 - \frac{2}{3}n)/8\pi^2$, $d = (19n/3 - 51)/2(8\pi^2)^2$. We also show that $f_1 f_2/h_1 = 3/64\pi^2$. While it is well known that the dynamical generation of mass requires an essential singularity in the complex g^2 plane like e^{-1/bg^2}, it is clear from this analysis that in some symmetry-breaking amplitudes [like $H_2(g)$ and $F_2(g)$] the essential singularity is absent. However, there is a cut for $g^2 < 0$. So perturbation theory in g for symmetry-breaking amplitudes is not possible.

II. DYNAMICAL SYMMETRY BREAKING IN QCD

A. Renormalization prescription and Ward identity

Here we define our renormalization prescription allowing for both explicit and dynamical γ_5 symmetry breaking. The SU(n)×SU(n) symmetry of QCD is presumed to be broken by a quark mass term given by

$$\sum_{i=1}^{n} m_0^i \bar{q}_i^0 q_i^0, \tag{1}$$

where the zero indicates an unrenormalized cutoff-dependent quantity. In what follows we will drop the flavor index i as the algebraic structure in flavor can easily be recovered. In expression (1) m_0 is the bare current quark mass. The renormalized current quark mass m is specified by

$$m = Z_m^{-1}(\Lambda)m_0(\Lambda) \tag{2}$$

as $\Lambda\to\infty$. Here Z_m is the renormalization constant for the $\bar{q}^0q^0$ vertex. To be precise let $\Gamma^0(p',p,g_0,m_0,\alpha_0,\Lambda)$ be the unrenormalized vertex corresponding to the scalar vertex $\bar{q}^0q^0$. Here g_0, m_0, α_0 are the bare coupling, mass, and gauge parameters. The renormalized vertex is defined by

$$\Gamma(p',p,g,m,\alpha,\mu)=Z_m Z_2\Gamma^0(p',p,g_0,m_0,\alpha_0,\Lambda)\,, \tag{3}$$

where g, m, and α are the renormalized constants and μ is the renormalization mass. We normalize Γ as in Weinberg's[4] zero-mass renormalization scheme. This requires first setting $m_0=0$ in (3) so that the normalization

$$\Gamma(p',p,g,0,\alpha,\mu)|_{p'^2=p^2=(p'-p)^2=-\mu^2}=1 \tag{4}$$

defines the Z's to be functions of Λ/μ, g, and α but not m/μ. Similarly the quark wave-function renormalization constant Z_2 is defined by

$$\begin{aligned} &Z_2 S(p,g,m,\alpha,\mu)=S_0(p,g,m_0,\alpha_0,\Lambda)\,,\\ &S^{-1}(p,g,m,\alpha,\mu)=A(p^2,g,m,\alpha,\mu)\not{p}\\ &\qquad -\Sigma(p^2,g,m,\alpha,\mu)\,, \end{aligned} \tag{5}$$

with the normalization $A(-\mu^2,g,0,\alpha,\mu)=1$. In normalizing amplitudes at $m_0=0$ we have adopted the usual prescription that amplitudes that do not vanish if $m_0=0$, $g_0=0$ are normalized at μ by their zeroth-order value. The normalization of $\Sigma(p)$ we will discuss in the sequel.

This procedure defines the normalization constants Z_m and Z_2 and anomalous dimensions

$$\begin{aligned} &\gamma_m(g)=\frac{\partial \ln Z_m}{\partial \ln\mu}\,,\\ &\gamma_F(g,\alpha)=\frac{1}{2}\frac{\partial \ln Z_2}{\partial \ln\mu}\,, \end{aligned} \tag{6}$$

which are independent of m/μ since we have used zero-mass renormalizations. $\gamma_m(g)$ is gauge independent and

$$\begin{aligned} &\gamma_m(g)=cg^2+fg^4+\cdots\,,\\ &c=\frac{3}{8\pi^2}\left(\frac{N^2-1}{2N}\right)=\frac{1}{2\pi^2}\,, \end{aligned}$$

while (7)

$$\begin{aligned} &\gamma_F(g,\alpha)=hg^2+lg^4+\cdots\,,\\ &h=\frac{\alpha}{16\pi^2}\left(\frac{N^2-1}{2N}\right)=\frac{\alpha}{12\pi^2}\,. \end{aligned}$$

In the Landau gauge $\alpha=0$ and $\gamma_F(g,0)=O(g^4)$.

Similarly we can define the pseudoscalar vertex corresponding to $\bar{q}^0\gamma_5q^0$. By an argument of Adler and Bardeen[13] the pseudoscalar vertex renormalization is the same as the scalar vertex so

$$\Gamma_5(p',p,g,m,\alpha,\mu)=Z_mZ_2\Gamma_5^0(p',p,g_0,m_0,\alpha_0,\Lambda)\,, \tag{8}$$

and we normalize according to

$$\Gamma_5(p',p,g,0,\alpha,\mu)|_{p'^2=p^2=(p'-p)^2=-\mu^2}=\gamma_5\,. \tag{9}$$

Finally the axial-vector–vector vertex corresponding to $\bar{q}\gamma_5\gamma_\mu q$ gets renormalized according to

$$Z_A{}^5\Gamma_\mu^0(p',p,g_0,m_0,\alpha_0,\Lambda)={}^5\Gamma_\mu(p',p,g,m,\alpha,\mu)\,. \tag{10}$$

If in the $m=0$ theory the γ_5 symmetry is dynamically broken then the Ward identity implies a Goldstone-boson pole in ${}^5\Gamma_\mu$ at $q^2=0$. Taking this possibility into account we write

$$\begin{aligned} {}^5\Gamma_\mu(p',p,g,0,\alpha,\mu)={}&{}^5\Gamma_\mu^R(p',p,g,0,\alpha,\mu)\\ &+\frac{q_\mu}{q^2}G(p',p,g,0,\alpha,\mu)\,, \end{aligned} \tag{11}$$

where G, the residue at the pole, is the bound-state wave function. In a theory such as QCD with dynamical γ_5 breaking, G vanishes to every order in perturbation theory; it is of $O(e^{-1/bg^2})$. The regular piece of the axial-vector vertex is normalized as in perturbation theory

$${}^5\Gamma_\mu^R(p',p,g,0,\alpha,\mu)|_{p'^2=p^2=(p'-p)^2=-\mu^2}=\gamma_\mu\gamma_5\,. \tag{12}$$

We now have all the ingredients to write down the axial-vector Ward identity for the n^2-1 axial-vector currents that are free of anomalies. The $U_A(1)$ axial-vector current has an anomaly and requires special treatment. The unrenormalized amplitudes satisfy

$$\begin{aligned} &q^{\mu\,5}\Gamma_\mu^0(p',p)=2m_0\Gamma_5^0(p',p)+S_0{}^{-1}(p')\gamma_5\\ &\qquad +\gamma_5S_0{}^{-1}(p)\,,\\ &q=p'-p\,. \end{aligned}$$

Upon renormalization as specified above one obtains with $\bar{g}_A=Z_2/Z_A$

$$\begin{aligned} q^{\mu\,5}\Gamma_\mu(p',p)=\bar{g}_A{}^{-1}[&2m\Gamma_5(p',p)+S^{-1}(p')\gamma_5\\ &+\gamma_5S^{-1}(p)]\,. \end{aligned} \tag{13}$$

The pseudoscalar (and scalar) vertex $\Gamma_5(p',p)$ can be shown to satisfy the renormalized integral equation

$$\Gamma_5=\int S\Gamma_5SK\,, \tag{14}$$

where K is the renormalized quark-quark two-particle irreducible (2PI) scattering kernel, renormalized according to $K=Z_2{}^2K_0$.

For a finite cutoff Λ or in a dimensionally regularized theory for $n \neq 4$, Eq. (14) has an inhomogeneous term $Z_m Z_2 \gamma_5$. Order by order in perturbation theory this term is divergent as $\Lambda \to \infty$ and cancels a divergence in the integral. However, if we first sum the perturbation series using the renormalization group then, in the Landau gauge, letting $\Lambda \to \infty$, one finds $Z_m Z_2 = 0$ and there results the homogeneous Eqs. (14). The solution to the homogeneous integral equation (14) is necessarily nontrivial because of the boundary condition (9) which requires $\Gamma_5 = \gamma_5$ at the normalization point.

The Goldstone alternative is evident from this Ward identity. If $m_0(\Lambda) \equiv 0$ then the renormalized current quark mass $m = 0$ and we have a chiral invariance. Then either (i) $[S^{-1}(p), \gamma_5]_+ = 0$ and there is no Goldstone pseudoscalar boson or (ii) $[S^{-1}(p), \gamma_5]_+ \neq 0$ indicating a dynamically generated mass and the Ward identity (13) implies ${}^5\Gamma_\mu(p', p) \sim q_\mu / q^2$ the Goldstone-boson pole term. In either case in the symmetric theory renormalized amplitudes of QCD are parametrized by only two numbers, g and μ (related by the renormalization group). If $m_0(\Lambda) \neq 0$ then $m \neq 0$ and QCD has its chiral symmetry explicitly broken. The theory is described in terms of the parameters g, μ, and m. If the symmetry in the $m = 0$ theory was realized by Goldstone bosons in the $m \neq 0$ theory the quark propagator now acquires an explicit as well as a dynamically generated mass and the Goldstone boson becomes massive.

There has been some confusion due to the fact that $m_0(\Lambda)$ vanishes as $\Lambda \to \infty$. This does not imply that there is a chiral symmetry for the associated flavor. Only if the renormalized current quark mass $m = Z_m^{-1} m_0$ vanishes is there a chiral symmetry. The symmetric theory with Goldstone bosons has $m = 0$.[14]

The axial-vector renormalization constant $\bar{g}_A = Z_2 / Z_A$ as defined by the above normalization conditions, can be shown by an application of the Ward identity (13) to satisfy $\bar{g}_A = 1$ to every order in perturbation theory. This is because G and the dynamically generated quark mass must vanish to every order in perturbation theory. However, if there is dynamical symmetry breaking we have

$$\bar{g}_A = 1 + O(g^2 e^{-1/bg^2}),$$

and $\bar{g}_A \neq 1$. (If $\bar{g}_A = 1$ then the symmetry is realized in the Wigner-Weyl mode and there is no dynamical breaking.) This result one obtains by calculating to lowest order in g^2 the ratio $\bar{g}_A = Z_2 / Z_A$ using the fact that the dynamically generated quark mass in the loop is $O(\mu e^{-1/bg^2})$. This definition of $\bar{g}_A$ is dependent on the gauge parameter α so that no physical significance can be attached to this number.

If instead of normalizing the vertex function and propagator at an unphysical point, we normalized them on the constituent quark mass shell $p\!\!\!/' = p\!\!\!/ = m_c$, then the corresponding renormalization constants Z_2' and Z_A' have the property that $g_A = Z_2'/Z_A'$ is gauge independent.[15] This quantity can then be interpreted as the axial-vector renormalization constant for a quark. If one calculates this g_A to lowest order in g^2 with $m_c \neq 0$, the result is infrared and ultraviolet finite,

$$g_A = 1 - \left(\frac{N^2 - 1}{2N}\right)\frac{g^2}{8\pi^2} + \cdots = 1 + \frac{g^2}{6\pi^2} + \cdots .$$

This calculation is given in Ref. 21, and as remarked in the introduction the correction has the right sign to account for the nucleonic g_A^N. It is a special property of our definition of g_A and the normalization conditions that we obtain a nontrivial result in perturbation theory.

B. Renormalization-group equations

The renormalization-group equations for the quark propagator (5) reads

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_F(g,\alpha) + \delta(\alpha, g)\frac{\partial}{\partial\alpha} - \gamma_m(g) m \frac{\partial}{\partial m}\right] S^{-1}(p) = 0 , \quad (15)$$

where

$$\beta(g) = -\tfrac{1}{2} b g^3 + d g^5 + \cdots ,$$
$$\delta(\alpha, g) = -2\alpha\gamma_V(g,\alpha) ,$$
$$\gamma_V(g,\alpha) = f g^2 + \cdots ,$$
$$b = (11 - 2n/3)/8\pi^2 ,$$
$$d = (-51 + 19n/3)/2(8\pi^2)^2 ,$$
$$f = -(13 - 3\alpha - 4n/3)/32\pi^2 .$$

For the scalar vertex one has

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_F(g,\alpha) + \delta(\alpha, g)\frac{\partial}{\partial\alpha} - \gamma_m(g)\left(1 + m\frac{\partial}{\partial m}\right)\right]\Gamma(p', p) = 0 . \quad (16)$$

Evidently

$$\Gamma(p, p) = -\frac{\partial S^{-1}(p)}{\partial m} , \quad (17)$$

a well-known relation[13] which is consistent with differentiating (15) with respect to m and using (16).

If we integrate (17) we obtain

$$S^{-1}(p,g,m,\alpha,\mu)=S_E^{-1}(p,g,m,\alpha,\mu)+S_D^{-1}(p,g,\alpha,\mu),$$
$$S_E^{-1}(p,g,m,\alpha,\mu)=-\int_0^m\Gamma(p,p,g,m',\alpha,\mu)dm'. \tag{18}$$

Here $S_D(p)$ the dynamical quark propagator is an integration constant independent of m, the current quark mass. $S_E(p)$ is the contribution of explicit symmetry breaking to the quark propagator which vanishes if $m=0$. Since we have used zero-mass renormalization $S_D^{-1}(p)$ is normalized as is $S^{-1}(p)$ in Eq. (5). $S_E^{-1}(p)$ is also normalized through its representation in terms of Γ. Γ is specified as the solution to the homogeneous integral equation

$$\Gamma=\int S\Gamma SK \tag{19}$$

with the nontrivial boundary conditions (4). This requires

$$\frac{\partial S_E^{-1}}{\partial m}\bigg|_{p^2=-\mu^2,\, m=0}=-1.$$

With

$$S_E^{-1}(p)=A_E(p^2,m)\not p-\Sigma_E(p^2,m),$$
$$S_D^{-1}(p)=A_D(p^2)\not p-\Sigma_D(p^2), \tag{20}$$

the representation (18) provides an unambiguous separation of the quark mass into a current (Σ_E) and constituent (Σ) part

$$\Sigma(p)=\Sigma_E(p)+\Sigma_D(p). \tag{21}$$

Recent articles,[2,3] beginning with a paper of Georgi and Politzer,[2] have examined the behavior of the quark mass as function of μ. The normalization these authors use is, with $m\neq 0$,

$$S^{-1}(p)\,|_{p^2=-\mu^2}=\not p-m. \tag{22}$$

Consequently, $[S^{-1}(p),\gamma_5]_+=0$ if $m=0$ and from the Ward identity (13) there are no Goldstone bosons violating PCAC. The point is that the normalization condition (22) precludes the presence of a dynamically generated term in $S^{-1}(p)$. This omission can have serious consequences for the phenomenological analysis of hadron processes since the dynamical term dominates for moderate momenta.

The constituent quark mass can be defined as the position of the pole in the quark propagator. This would be the solution M^2 to the equation

$$M^2A^2(M^2)=\Sigma^2(M^2) \tag{23}$$

if it exists. It is easy to show that the solution to (23) is necessarily a renormalization-group invariant, that is, it satisfies

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma_m(g)m\frac{\partial}{\partial m}\right)M=0. \tag{24}$$

Next we examine the asymptotic behavior of the quark propagator. We will choose the Landau gauge $\alpha=0$ as this simplifies the analysis. From the renormalization-group equation (15) and the boundary condition (5), $A(-\mu^2,g,0,\alpha,\mu)=1$, one finds from the standard analysis

$$A(p^2,g,m,0,\mu)\underset{p^2\to-\infty}{\sim}C(g)=\exp\left(\int_0^g\frac{2\gamma_F(x,0)}{\beta(x)}dx\right), \tag{25}$$

since we are in the Landau gauge, where $\gamma_F(x,0)/\beta(x)\sim O(x)$ as $x\to 0$.

From the renormalization-group equation for the scalar vertex (16) and the normalization condition (4), we find

$$\Gamma(p,p)\underset{p^2\to-\infty}{\sim}C(g)[b\ln(\sqrt{-p^2}/\mu)]^{-c/b}. \tag{26}$$

Using (18) and (20) we have for the explicit symmetry-breaking contribution to $\Sigma(p)$

$$\Sigma_E(p)\underset{p^2\to-\infty}{\sim}mC(g)[b\ln(\sqrt{-p^2}/\mu)]^{-c/b}. \tag{27}$$

Notice that this vanishes if $m=0$ as it must.

C. Dynamical mass generation

Next we turn to the dynamically generated mass term $\Sigma_D(p)$ which is independent of the current quark mass m. We can set $m=0$ in discussing this amplitude. The renormalization-group equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-2\gamma_F(g,0)\right)\Sigma_D(p)=0$$

is satisfied if

$$\Sigma_D(p)=\sqrt{-p^2}H(\bar g(t,\mu))\times\exp\left(-\int_g^{\bar g(t,g)}\frac{2\gamma_F(x,0)}{\beta(x)}dx\right), \tag{28}$$

where $\bar g(t,\mu)$ is given by

$$t=\ln(\sqrt{-p^2}/\mu)=\int_g^{\bar g(t,g)}\frac{dx}{\beta(x)},$$
$$\left(\frac{\partial}{\partial t}-\beta(g)\frac{\partial}{\partial g}\right)\bar g(t,g)=0. \tag{29}$$

The arbitrary function $H(g)$ is completely unspecified. The point is that for $\Sigma_D(p)$ there is no boundary condition for the simple reason that there is no mass to be renormalized. In the absence of a boundary condition the renormalization-group equation cannot specify the asymptotic behavior of $\Sigma_D(p)$.

Instead we must turn to the integral equation satisfied by $\Sigma_D(p)$. This is obtained as follows. The bound-state amplitude $G(p+q,p)$ for the ground-state Goldstone boson is given by

$$\Gamma^5_\mu(p+q,p) \underset{q^2\to 0}{\sim} \frac{q_\mu}{q^2} G(p+q,p)\,. \tag{30}$$

$G(p+q,p)$ satisfies the Bethe-Salpeter equation shown in Fig. 1 or

$$G(p+q,p)=\int d^4k S_D(k+q)G(k+q,k)S_D(k)K(p,k,q)\,. \tag{31}$$

From the Ward identity (13), $G(p,p)\sim\gamma_5\Sigma_D(p)$ and there results the nonlinear integral equation for $\Sigma_D(p)$

$$\gamma_5\Sigma_D(p)=\int d^4k S_D(k)\gamma_5\Sigma_D(k)S_D(k)K(p,k,0)\,. \tag{32}$$

The question of whether dynamical symmetry breaking can occur at all is tantamount to constructing a nontrivial solution to (32). This question we do not address here.

The 2PI kernel of the integral equation (31) can be written as

$$K(p,k,q)=K_{\rm P}(p,k,q)+K_{\rm NP}(p,k,q)\,, \tag{33}$$

where $K_{\rm P}(p,k,q)$ corresponds to the usual perturbative skeleton expansion and $K_{\rm NP}(p,k,q)$ incorporates nonperturbative effects. The latter are known to exist coming from the flower graph of the instanton interaction shown in Fig. 2. The instanton interactions are presumably the dominant part of the kernel for nonasymptotic momenta $-p^2\sim\mu^2$. It has been suggested by Caldi[16] and Callen, Dashen, and Gross[8] that these instanton interactions render the γ_5-invariant vacuum unstable. This implies a nontrivial $\Sigma_D(p)$ and hence the Goldstone realization. In certain approximations to these instanton interactions one can show that the integral equations have a solution for $\Sigma_D(p)$ providing the number of flavors is not too large.[16,17] The typical behavior of $\Sigma_D(p)$ in p^2 coming from instanton interactions in the kernel is that it is asymptotically negligible, but rises rapidly for $-p^2\sim\mu^2$.

For large momenta we can ignore the nonperturbative contribution to the kernel, and the perturbative term is dominant. This feature of asymptotic freedom, emphasized by Lane,[6] allows us to determine the large-momentum behavior of the complete kernel exactly and consequently the exact large-momentum behavior of $\Sigma_D(p)$, providing $\Sigma_D(p)$ does not vanish identically. We will not repeat this analysis here. It turns out that there are two solutions which are specified by their asymptotic behavior,

$$\Sigma_D^{(+)}(p) \underset{p^2\to-\infty}{\sim} \mu[\ln(\sqrt{-p^2}/\mu)]^{-c/b} \quad \text{(irregular)}\,, \tag{34}$$

$$\Sigma_D^{(-)}(p) \underset{p^2\to-\infty}{\sim} \frac{\mu^3}{-p^2}[\ln(\sqrt{-p^2}/\mu)]^{+c/b} \quad \text{(regular)}\,. \tag{35}$$

The terms regular and irregular refer to the behavior of the Bethe-Salpeter amplitude $G(p+q,p)$, related to $\Sigma_D(p)$ by the Ward identity, near the origin. Both of these solutions are consistent with the renormalization-group equation. Without examining the nonasymptotic regime of the integral equation, a formidable task, we cannot determine which of the two solutions, if either, is correct. So unlike the explicit term $\Sigma_E(p)$, the asymptotic behavior of the dynamically generated term is not uniquely determined.

To attempt to determine the asymptotic behavior of the dynamically generated mass, use has been made of the operator-product expansion (OPE). This argument was first proposed by Lane[6] and subsequently by Politzer.[18] If one examines the short-distance behavior of $\bar{q}(x)q(y)$ using the OPE, one concludes that the regular solution is the correct behavior. This procedure for determining the asymptotic behavior of a solution of the Bethe-Salpeter equation, however, begs the question since the OPE is itself proven by assuming the regular solution to the Bethe-Salpeter equation. This logical lacuna in the OPE argument was pointed out by Langacker.[19] The only way that I

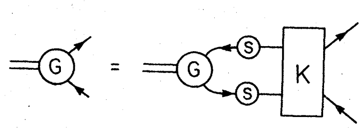

FIG. 1. Bethe-Salpeter equation for the bound-state amplitude G.

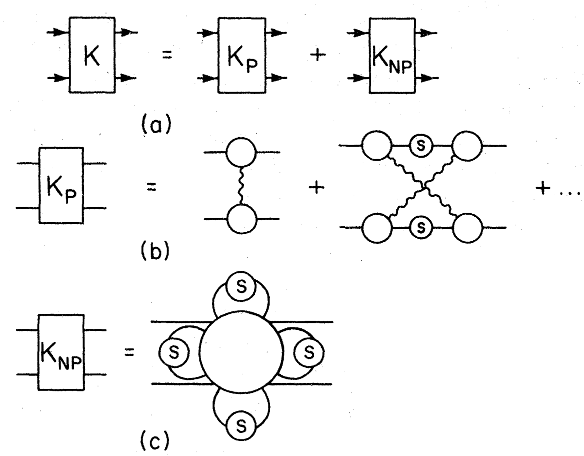

FIG. 2. (a) Decomposition of the 2PI kernel into a perturbative and nonperturbative part. (b) Skeleton expansion of perturbative kernel. (c) Flower graph contribution due to instanton interactions. There are $n-2$ petals on the flower.

know that one can settle this question on the basis of theory alone is to know more about the sub-asymptotic kernel—a difficult task.

However, we can argue in favor of the regular solution *and* the usual OPE on experimental grounds. The regular solution corresponds to a typical bound-state behavior, while the irregular solution corresponds to an elementary pointlike ground-state pseudoscalar meson. Applequist and Poggio[5] have examined the pion electromagnetic form factor $F_\pi(q^2)$ as $q^2 \to -\infty$ in the asymptotically free $(\phi^3)_6$ theory in the triangle-graph approximations shown in Fig. 3. For the regular solution $F_\pi(q^2) \sim O(1/q^4)$, while for the irregular solution $F_\pi(q^2) \sim O(1)$ up to logarithms as $q^2 \to -\infty$. Simple power counting in the vector-gluon theory with the regular solution implies $F_\pi(q^2) \sim O(1/q^2)$ as $q^2 \to -\infty$, the experimentally observed behavior. For the irregular solution $F_\pi(q^2) \sim O(1)$. We would conclude that the usual bound-state physics of experimentally falling form factors, transverse-momentum distributions, etc. could not be maintained with the irregular behavior of the wave function. So we conclude that experiment favors the regular solution.

Bég and Shei[7] have shown that the Nambu-Goldstone and Wigner-Weyl realizations of chiral symmetry merge at short distances. This attractive result is predicated upon the usual OPE and would fail if we have the irregular solution.

Langacker[19] has argued in favor of the irregular solution. He, however, assumed that $\Sigma(p)$ was normalized according to $\Sigma(p^2 = -\mu^2) = m$, where m is the current quark mass. Then the renormalization group unambiguously implies $\Sigma(p) \sim m[\ln(\sqrt{-p^2}/\mu)]^{-c/b}$, and we must have the irregular solution. However, we have seen there is no such boundary condition for a dynamically generated mass—the boundary condition applies only to the explicit mass term. Consequently, there is no reason to have the irregular solution. Experiment favors the regular solution for the dynamical term.

We conclude that as $p^2 \to -\infty$

$$\Sigma_D(p) \sim \frac{\mu^3}{-p^2}[\ln(\sqrt{-p^2}/\mu)]^{+c/b}, \qquad (36)$$
$$\Sigma_E(p) \cong m[b\ln(\sqrt{-p^2}/\mu)]^{-c/b}.$$

The full mass term $\Sigma(p) = \Sigma_D(p) + \Sigma_E(p)$ is shown in Fig. 4 with the large enhancement of $\Sigma_D(p)$ at moderate momenta coming from instantons as estimated by Callen, Dashen, and Gross.[8]

III. EXPLICIT SYMMETRY BREAKING IN QCD

A. Bound-state wave function and the decay constant

We have argued that if the symmetry is dynamically broken in the $m = 0$ theory, the Goldstone-boson bound-state wave function is the regular solution. If this is so then in the explicitly broken theory with $m \neq 0$ the solution to the Bethe-Salpeter equation will continue to have regular behavior asymptotically. This follows because the effect of explicit symmetry breaking is to alter the quark propagator according to $S^{-1}(p) \to S^{-1}(p) - m$ up to logarithms. Such mass insertions in the Bethe-Salpeter equation do not alter the leading asymptotic behavior of the kernel and hence the asymptotic behavior of the solution. This regularity of the bound-state wave function in the presence of explicit symmetry breaking has implications for the meson decay constant which we now examine.

If $|\pi(k)\rangle$ is the Goldstone-boson state, then the decay constant is defined by

$$\langle 0 | A_\mu(0) | \pi(k)\rangle = ifk_\mu, \quad k^2 = M^2 \qquad (37)$$

where M is the bound-state mass. The exact integral representations for f^2 given by Jackiw and Johnson[20] is shown in Fig. 5. Here $\tilde{f} = g_A^{-1} f$ and ${}^5\Gamma_\mu^R(p+q, p)$ and the bound-state wave function $G(p+q, p)$ appearing in Fig. 5 are defined by

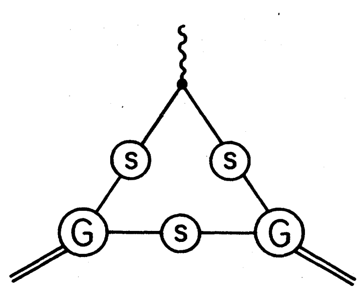

FIG. 3. Pion form factor in the triangle-graph approximation.

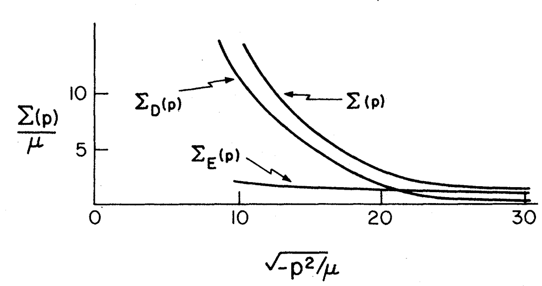

FIG. 4. Contribution of dynamical and explicit symmetry breaking to quark self-energy. Dynamical symmetry-breaking contribution taken from Ref. 8.

$$(q^2-M^2)^5\Gamma_\mu(p+q,p)\underset{q^2\to M^2}{\sim} q_\mu G(p+q,p)\,,$$
$$^5\Gamma_\mu(p+q,p)=\frac{q_\mu}{q^2-M^2}G(p+q,p)+{}^5\Gamma^R_\mu(p+q,p)\,. \tag{38}$$

In an approximation to the exact representation for $\tilde{f}^2$ in which only the dominant high-momentum terms are kept,[21] one obtains

$$\tilde{f}^2\cong\frac{2iN}{(2\pi)^4}\int\frac{d^4k\Sigma(k)\gamma_5G(k,k)}{[k^2-\Sigma^2(k)]^2}\,. \tag{39}$$

If we have no explicit breaking the Ward identity implies $G(k,k)=-\bar{g}_A^{-1}2\gamma_5\Sigma_D(k)$, and then

$$\tilde{f}^2\cong\frac{-4iN}{\bar{g}_A(2\pi)^4}\int\frac{d^4k\Sigma_D{}^2(k)}{[k^2-\Sigma^2(k)]^2}$$
$$\sim\frac{N}{\bar{g}_A(2\pi)^2}\int^\infty\frac{dk^2}{k^2}\Sigma_D{}^2(-k^2)>0\,. \tag{40}$$

The integral is convergent since $\Sigma_D(-k^2)\sim O(1/k^2)$ up to logarithms. The necessary condition[21] to be in the Nambu-Goldstone phase $\tilde{f}^2>0$ is satisfied by (40). Of course nonasymptotic contributions which were dropped in writing (39) could be important, especially instanton interactions.

If we include explicit symmetry breaking then the asymptotic behavior of $\Sigma(k)=\Sigma_D(k)+\Sigma_E(k)$ is dominated by the explicit term

$$\Sigma(k)\underset{-k^2\to\infty}{\sim} m[\ln(-k^2/\mu^2)]^{-c/b}\,. \tag{41}$$

However, since the bound-state wave function still has regular behavior $G(k,k)\sim O(1/k^2)$ the integral (39) for $\tilde{f}^2$ continues to converge safely. We would conclude that nonasymptotic terms control the actual value of f^2.

B. Perturbation theory for light quarks

For light quarks u, d, and s the explicit current quark mass m is presumably small relative to $\mu\sim1$ GeV. This suggests that one can do chiral perturbation theory in m/μ for these flavor-breaking amplitudes. Chiral perturbation theory in m/μ has known nonanalytic behavior.[12] Using the pseudoscalar meson mass M and the decay constant f as examples, one finds for the chiral perturbation expansion

$$f(g,\mu,m)=\mu F_1(g)+F_2(g)m\ln(\mu/m)+F_3(g)m+\cdots\,,$$
$$M^2(g,\mu,m)=m[\mu H_1(g)+H_2(g)m\ln(\mu/m)+H_3(g)+\cdots]\,. \tag{42}$$

Next we use the fact that $f(g,\mu,m)$ and $M(g,\mu,m)$ are renormalization-group invariants so they satisfy

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma_m(g)m\frac{\partial}{\partial m}\right]f=0\,,$$
$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-\gamma_m(g)m\frac{\partial}{\partial m}\right]M^2=0\,. \tag{43}$$

Substituting (42) into (43) and equating terms of the same order in m/μ one obtains

$$\left[1+\beta(g)\frac{\partial}{\partial g}\right]F_1(g)=0\,,$$
$$\left[1-\gamma_m(g)+\beta(g)\frac{\partial}{\partial g}\right]H_1(g)=0\,,$$
$$\left[-\gamma_m(g)+\beta(g)\frac{\partial}{\partial g}\right]F_2(g)=0\,,$$
$$\left[-2\gamma_m(g)+\beta(g)\frac{\partial}{\partial g}\right]H_2(g)=0\,. \tag{44}$$

These equations imply $F_1(g)F_2(g)/H_1(g)$ and $F_1{}^2(g)H_2(g)/H_1{}^2(g)$ are constants independent of g. Solving (44) we obtain the singular parts of these functions as $g\to0$,

$$F_1(g)=f_1g^{-4d/b}e^{-1/bg^2}\,,$$
$$F_2(g)=f_2g^{-2c/b}\,,$$
$$H_1(g)=h_1g^{-2(c+2d)}e^{-1/bg^2}\,,$$
$$H_2(g)=h_2g^{-4c/b}\,, \tag{45}$$

where f_i and h_i are integration constants and b, c, and d are given by Eqs. (7) and (15). These equations exhibit the known[22] essential singularity for $g^2\le0$ associated with dynamical symmetry breaking. In addition we learn that some explicit symmetry-breaking amplitudes which vanish as $m\to0$ have no essential singularity, but a cut in the complex g^2 plane for $g^2\le0$. Even for such amplitudes the radius of convergence of perturbation theory in g is zero.

The numerical constants f_i and h_i cannot be determined by the homogeneous renormalization-group equations. However, if we consider two flavors corresponding to the π and K channels then we can use the chiral limit theorem[12]

$$f_\pi/f_K=1+\frac{3(M_K{}^2-M_\pi{}^2)}{64\pi^2f^2}\ln(M^2/\mu^2)\,. \tag{46}$$

Using the expansions (42) one finds

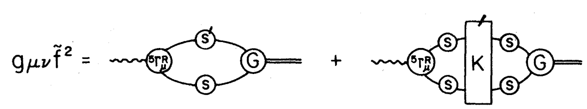

FIG. 5. Exact integral representation for the meson decay constant. The slash denotes differentiation with respect to momentum and then setting the momentum to zero.

$$f_\pi/f_K = 1 + \frac{M_K{}^2 - M_\pi{}^2}{f^2}\frac{F_1(g)F_2(g)}{H_1(g)}\ln(M^2/\mu^2), \quad (47)$$

and comparing one has for the constants in (45) the relation

$$f_1 f_2/h_1 = 3/64\pi^2. \quad (48)$$

C. Phase transition for heavy quarks

1. In QCD

For heavy quarks like c, t, b presumably $m/\mu \sim O(1)$, and chiral perturbation theory is not applicable. For these quarks an interesting possibility of a phase transition exists. Let $f(g,\mu,m)$ be the decay constant, which is the order parameter, for a heavy meson and consider m to be variable. Since $f(g,\mu,0)$ exists and is nonvanishing, we have a Goldstone realization. As m increases so does $f(g,\mu,m)$ as indicated by chiral perturbation theory. But for very large m, $f(g,\mu,m)$ could decrease and eventually vanish, $f(g,\mu,m^*)=0$ as shown in Fig. 6. This does not entail parity doubling of hadron states because the γ_5 symmetry is explicitly broken by the large current quark mass m^*. If $m_c > m^*$, then the heavy pseudoscalars like the $D^{\pm,0}$ may not be smoothly interpolated as "almost Goldstone bosons" like the $\pi^{\pm,0}$, $K^{\pm,0}$. Presumably an atomic model rather than a collective model[9] is appropriate for the description of states with quarks with $m > m^*$.

The renormalization group places some (but not much) constraint on the problem of a heavy-quark-mass phase transition. We write

$$f(g,\mu,m) = \mu F(g,t), \quad t = \ln(m/\mu), \quad (49)$$

where $F(g,-\infty)$ exists and is given by

$$F(g,-\infty) = \exp\left(\int_g^{g_0} dx/\beta(x)\right). \quad (50)$$

Here g_0 is a number and we presume g_0 and g are in the domain of attraction of the origin. $F(g,-\infty)$ is just the general solution to the renormalization-group equation (43) when $m=0$. For $m \neq 0$ we can substitute (49) into the Eq. (43) which has the general solution

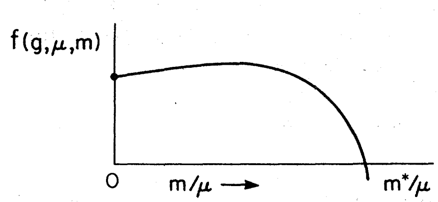

FIG. 6. Possible behavior of the decay constant indicating a phase transition.

$$F(g,t) = \exp\left(\int_g^{G(\bar{g}'(t,g))} dx/\beta(x)\right), \quad (51)$$

where $G(g)$ is an arbitrary function satisfying

$$G(\bar{g}'(-\infty,g)) = g_0, \quad (52)$$

and $\bar{g}'(t,g)$ is implicitly given by

$$t = \int_g^{\bar{g}'(t,g)} dx[1+\gamma_m(x)]/\beta(x). \quad (53)$$

It follows that

$$\frac{f(g,\mu,m)}{f(g,\mu,0)} = \exp\left(\int_{G(\bar{g}'(-\infty,g))}^{G(\bar{g}'(t,g))} dx/\beta(x)\right)\Bigg|_{t=\ln(m/\mu)}. \quad (54)$$

Suppose we know the function $G(g)$ and that as m increases that we find a value $t^* = \ln(m^*/\mu)$ such that $G(\bar{g}'(t^*,g))=0$. Then the integral in (54) behaves like $-1/G^2$ and $f(g,\mu,m^*)=0$. This is the condition for the phase transition.

We can analyze other quantities such as a hadron mass M. Then the same above analysis implies

$$\frac{M(g,\mu,m)}{M(g,\mu,0)} = \exp\left(\int_{H(\bar{g}'(-\infty,g))}^{H(\bar{g}'(t,g))} dx/\beta(x)\right), \quad (55)$$

where $H(g)$ is another arbitrary function. We assume that if the phase transition does occur at $m = m^*$ that $H(\bar{g}'(t^*,g)) \neq 0$, otherwise (55) requires the hadron mass to vanish there—a disaster for the physical spectrum. Of course without knowledge of the functions $G(g)$, $\gamma(g)$, and $\beta(g)$ we can only speculate on the existence of the phase transition.

2. In linear σ model

It is interesting to look at the linear SU(3)×SU(3) σ model in the tree approximation in which the symmetry is realized by an octet of Goldstone bosons.[23] Of course this theory has nothing to do with QCD, but it has the advantage that we can control the phase-transition problem completely. We will explicitly break the SU(3)×SU(3) invariance of the Hamiltonian to SU(2)×SU(2) by adding the linear term $\epsilon\kappa(x)$, where $\kappa(x)$ is the 0^+ field (transforming like $\bar{s}s$ in the quark model), and ϵ is the explicit symmetry-breaking parameter. For small ϵ the kaon gets a mass $m_K{}^2 \sim O(\epsilon)$ and $f_\pi/f_K = 1 + O(\epsilon)$, while the triplet of pions remains massless because of the remaining chiral SU(2) ×SU(2) invariance. As one increases ϵ there is a critical value $\epsilon = \epsilon^*$ at which point $m_K{}^2 = m_\kappa{}^2$, $f_\pi/f_K = 0$, $m_\pi{}^2 = m_\sigma{}^2 = 0$ where m_κ is the κ-meson mass and σ is the 0^+ meson. What has happened is that at the critical value the chiral SU(2)×SU(2) is being realized by the Wigner-Weyl realization and the spectrum is parity doubled. For $\epsilon > \epsilon^*$

this realization is presumably maintained.

Although the above remarks are verified in the SU(3)×SU(3) σ model, in the tree approximation the possibility of the phase transition is independent of the tree approximation. To see this we consider the domain structure of the model as considered by Mathur and Okubo.[11] This is shown in Fig. 7. As one increases the ϵ parameter from 0 to ϵ^* one can move from the Goldstone realization of SU(2)×SU(2) to the Wigner-Weyl realization as shown in Fig. 7.

The domains are consequences of unsubtracted spectral representations and spectral positivity for the correlation function for the explicit symmetry-breaking operators. For scalar fields of dimension 1 this assumption is all right, but in QCD the relevant operator is $\bar{q}q$ with dimension 3 in perturbation theory. So subtractions are required, and the naive argument on the basis of symmetry-breaking domains given for the σ model does not apply to QCD without modification.

D. Light quark mass ratios

The quark mass ratios of different flavored quarks $R_{ij}=m_i/m_j$ are a renormalization-group invariant so that $[\mu(\partial/\partial\mu)+\beta(g)\partial/\partial g]R_{ij}=0$. This is true in Weinberg's renormalization prescription in which one renormalizes at zero current quark mass, even for heavy quarks. Instead one can renormalize at finite current quark mass. This has the advantage, from a phenomenological standpoint, of explicitly decoupling the heavy quarks.[2] However, the anomalous dimensions are now dependent on the current quark mass and the gauge parameter α : $\beta(g,m/\mu)$, $\gamma_m(g,\alpha,m/\mu)$. The lowest-order calculations of these anomalous dimensions and their gauge dependence have been done by Nachtmann and Wetzel.[3] In this finite-mass renormalization scheme the usual approximations will yield gauge-dependent results for physical amplitudes. No physical significance can be attached to a gauge-dependent quantity. Of course, if one did not approximate in the usual way of retaining just the lowest-order terms in the anomalous dimension etc., the gauge dependence must disappear. But going beyond the standard approximations to obtain gauge-invariant results seems a formidable undertaking. For this reason zero-mass renormalization may be preferred even for heavy quarks. Then the standard approximations give gauge-invariant results for physical amplitudes.

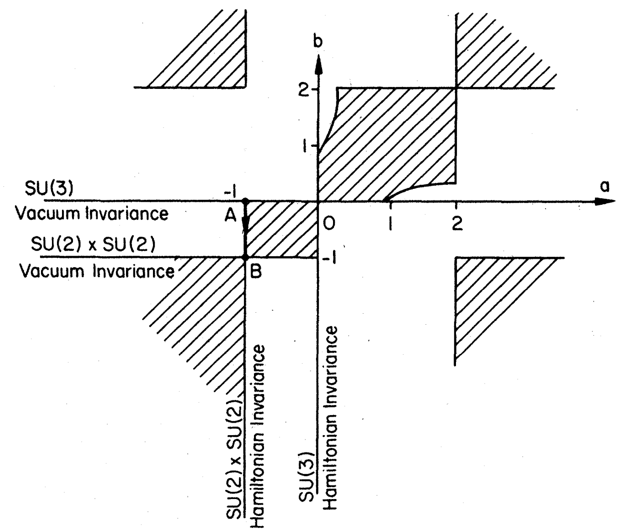

FIG. 7. Domain structure of chiral symmetry breaking taken from Ref. 11. The axis b labels the vacuum symmetry, while a labels the Hamiltonian symmetry. At the point A the vacuum symmetry is SU(3) and the Hamiltonian symmetry SU(3) × SU(3). As one breaks the SU(3) × SU(3) Hamiltonian symmetry to SU(2) × SU(2), one moves along the line AB. At B the vacuum becomes SU(2) ×SU(2) invariant, and the states are parity doublets.

Finally we remark that in QCD the ratios of the light quarks have been calculated[24,25] with the result

$$m_u/m_d=0.38\pm0.13,$$

$$m_d/m_s=0.045\pm0.011.$$

The principal input into this calculation is the assumption that the usual photonic contribution to electromagnetic mass shifts is correctly estimated by the low-lying states and that chiral perturbation theory in SU(3)×SU(3) is all right.

ACKNOWLEDGMENTS

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