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Simple semiclassical model for the rotational states of mesons containing massive quarks

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We present a semiclassical relativistic model for the orbital spectra of mesons, based on the assumption of a universal, flavor-independent linear confining interaction. Flavor dependence of the spectra arises from the quark masses.

I. INTRODUCTION

Most hadrons which consist of light quarks have been grouped in rotational families[1] where

$$J=\alpha' M^2+\alpha_0 \tag{1}$$

is the relation between spin (J) and mass squared (Chew-Frautschi plot). In the case of mesons (which we shall discuss here) all integer J values are included with alternating parity for the mesons as J increases (exchange degeneracy). The slope parameter α' varies only slightly from family to family (by about 10% or less). In some cases the assignment to a particular family is not certain, or the formula (1) fails. This failure is most acute for the rotational family to which the pseudoscalar mesons (π and K) belong.

The quark model suggests a unique assignment (Table I) and also indicates that the family is based upon orbital angular momentum.

The mass spectrum (1) is obtained in any relativistic model where the orbital angular momentum is carried by a rotating "linear" field (constant rest energy per unit length) such as in the dual string model[2] or (approximately) in the bag model with massless colored quarks and gluons.[3] The bag model relates the slope parameter α' to parameters which are flavor independent (B and α_c). Thus in this model one expects that flavor variations in the form of the spectrum are governed by the quark mass. We shall explore this in what follows.

It has already been pointed out[4] how (1) becomes modified in the same kind of model when equal-mass particles are attached at the ends of the string or are the sources of stretched color-electric flux lines. Here we would like to extend this trivially to allow unequal masses at the ends. We shall treat the system as a string, but the same results will be obtained when quarks of various masses are treated classically in the colored-quark bag model. We have in mind in particular, application to the spectrum of rotationally excited D's (and D^*'s) and F's (and F^*'s). The results will of course also apply to mesons containing still heavier quarks.

Our model is classical, but we shall argue that a slightly modified version of it still could give an accurate representation of the mass spectrum in a full quantum theory. For example, the classical string model gives (1) without the intercept α_0. The full quantum treatment provides only the "quantum defect" α_0 in J.[2] In situations where a full quantum-mechanical treatment is difficult or impossible, it may also be true that a "quantum-defect" correction to the classical relation between angular momentum and mass is quite accurate even though not exact. It has been suggested[3] that in the absence of a complete theory a useful approximation might be obtained by calculating the ground-state mass and using that result together with (1) to determine α_0. We can test this suggestion in the limit opposite to the massless relativistic string. Consider two equal-mass quarks moving nonrelativistically in a linear potential with slope $1/2\pi\alpha'$. The classical (Bohr) model with a "quantum defect" in l gives

$$l=l_0+\frac{4\pi}{3^{3/2}}\alpha'\sqrt{m}(M-2m)^{3/2}, \tag{2}$$

where M is the particle mass and m is the quark mass. We shall compare (2) with the predictions of the Schrödinger equation. We determine the "defect" l_0 by fitting (2) to the exact ground state ($l=0$). The Schrödinger equation was solved numerically for several low values of l. The comparison is displayed in Table II. The agreement is impressive. (We are aware of the fact that a quantum "defect" determined in this way will not in general agree with the leading corrections to the classical formula which can be calculated with the WKB approximation. This is evident in Table II by the fact that the difference between the exact eigenvalue and the approximate one is not decreasing with l.)

What should we infer from this? We guess that such a quantum-defect formula can give a reasonable representation of the excited states of a system which corresponds to the balancing of an infinitely rising long-range attraction against a cen-

TABLE I. Light-quark mesons grouped in rotational families (Regge trajectories). Only the states with maximum J consistent with a given l and S are numerous enough to allow comparisons.

$l=0$ State	Quarks	$l=0$	$l=1$	$l=2$	$l=3$
$I^G J^P$					
1^-0^-		$\pi(0.140)$	$B\,(1.228)_{J=1^+}$	$A_3(1.64)_{J=2^-}$	...
0^-0^-	$u\bar{u}+d\bar{d}-2s\bar{s}$	$\eta\,(0.549)$	...	...	...
	$u\bar{u}+d\bar{d}+s\bar{s}+\,?$	$\eta'\,(0.958)$	...	...	...
1^+1^-		$\rho\,(0.773)$	$A_2(1.310)_{J=2^+}$	$g\,(1.690)_{J=3^-}$	...
			$A_1(1.10)_{J=1^+}$	...	...
			$\delta\,(0.980)_{J=0^+}$ [a]	...	...
0^-1^-	$u\bar{u}+d\bar{d}$	$\omega\,(0.783)$	$f\,(1.271)_{J=2^+}$	$(1.667)_{J=3^-}$	$h\,(2.040)$
			$D\,(1.285)_{J=1^+}$	...	...
			$\epsilon\,(1.300)_{J=0^+}$	...	...
	$s\bar{s}$	$\phi\,(1.020)$	$f'\,(1.516)_{J=2^+}$	...	...
			$E(1.420)_{J=1^+}$	...	...
			$S^*(0.980)_{J=0^+}$ [a]	...	...
$\frac{1}{2}\,0^-$		$K\,(0.495)$	$Q_B\,(1.355)$	$L\,(1.765)$	...
$\frac{1}{2}\,1^-$		$K^*(0.892)$	$K^*(1.421)_{J=2^+}$	$K^*(1.783)_{J=3^-}$	...
			$Q_A(1.335)_{J=1^+}$	...	...
			$\kappa\,(1.40)_{J=0^+}$ [a]	...	...

[a] The identification of the observed scalar flavor nonet as the $J=0$ state $L=1$, $S=1$ quark-antiquark state is not necessarily the best from a phenomenological point of view. It may be a diquark-diantiquark state (Ref. 7).

trifugal repulsion, since in this case the quantum wave function is well localized about a classical orbit. When would we expect such a guess to fail? It should fail in those situations where in addition to a long-range attraction there are strong attractive short-range interactions. In states with $l\neq 0$, these could be relatively unimportant, but in the $l=0$ state they would have an important effect. In such a case we would not expect that the energies of the excited states would be simply determined by the quantum defect approximation with l_0 obtained from the $l=0$ mass. The wave function would first have to climb out of the attractive hole before finding itself in the long-range rising potential whose dominance for finite l is the physics of the quantum-defect approximation. On the other hand, if the short-range interactions are repulsive then the particles are already apart in the ground state and orbital excitation would just move them farther into the long-range part of the interaction. Hence one might anticipate that the classical formula with the quantum defect could work quite well even in going from $l=0$ to $l=1$.

In the case where the relationship fails for the $l=0$ state, one might still expect that the excited states are well represented by such a formula since short-range effects are greatly reduced when $l\neq 0$. In this circumstance the defect would be better determined by fitting the mass of the $l=1$ state, for example.

We would like to remark that the meson spectrum indicates that this kind of dynamics may be operating. The states π and K which do not lie on linear Chew-Frautschi plots are ones where the short-range quark spin-spin interaction (which is proportional to $\sigma_1\cdot\sigma_2$) is strong and attractive ($\sigma_1\cdot\sigma_2=-3$). In contrast, in the case of the ρ and K^* states, the short-range interaction is repulsive and weaker ($\sigma_1\cdot\sigma_2=+1$), and the ρ and K^* belong to families where the linear dependence is quite accurate. It is amusing to note that if one draws a straight line through the B and A_3, which we take as the $l=1$ and $l=2$ cousins of the π, the line has almost the same slope ($\alpha'=0.85$) as for the ρ family and passes through $l=0$ at $m^2=(0.57)^2$. According to the above discussion, this would be the mass

TABLE II. Comparisons between the eigenvalues obtained from a numerical solution of the Schrödinger equation and the semiclassical quantum-defect formula given by Eq. (2.1). To put (2) in a convenient dimensionless form we have set $M-2m=(1/2\pi\alpha')^{2/3}(1/m^{1/3})e$, so $l=l_0+(2/3^{3/2})e^{3/2}$. The "defect" l_0 is equal to -1.376.

l	e Schrödinger	e Eq. (2)	Difference
0	2.338	fit	
1	3.361	3.365	0.004
2	4.248	4.253	0.005
3	5.051	5.056	0.005
4	5.794	5.800	0.006

of the π with the short-range interaction removed. It is interesting that it is much closer to the ρ mass, consistent with the dynamics which is believed to govern the spin-spin interaction (short range). An equivalent way of making the comparison is to notice that as l increases, the π and ρ families become approximately spin degenerate; that is, the states which differ by one unit in J (equal l) have nearly equal masses ($A_3=1640$, $g=1690$).

The same exercise can also be applied to the K with the $Q_B(1.35)$ and $L(1.77)$ identified as the $l=1$ and $l=2$ partners. In this case we find the straight line goes through $l=0$ at $m^2=(0.72)^2$, closer to degeneracy with the $K^*(0.89)$. [Also, $L(1.765)$ and $K^*(1.783)$, the $J=2^-$, and $J=3^-$ states have nearly the same mass.]

Of course the classical model which we shall investigate, as well as being incomplete in regard to the spin-spin interaction, also is not capable of incorporating the effects of spin-orbit splittings. Just as the spin-spin interaction is repulsive in the maximum-spin states, the spin-orbit interaction is observed to be repulsive in the states of maximum total angular momentum, which (with one exception) are the states for which the most information is available. This is also consistent with quantum chromodynamics, at least in the nonrelativistic limit.[5] Hence in applying our model we shall restrict ourselves to quark states with maximum spin and total angular momentum.

The paper will be organized as follows. In Sec. II we shall present the classical formula. In Sec. III we shall apply it to the various cases of mesons which contain massive quarks: K's, K^*'s, ϕ's, D's, D^*'s, F's, and F^*'s.

II. SEMICLASSICAL MASS FORMULA

A classical relativistic string with masses m_1 and m_2 attached at its extremities, and moving in its own rest system with the maximum angular momentum consistent with a given total mass M, is straight and rotates rigidly with mass and angular momentum

$$M=\frac{m_1}{(1-v_1^2)^{1/2}}+\frac{m_2}{(1-v_2^2)^{1/2}}+\frac{1}{2\pi\alpha'\omega}\int_{-v_2}^{v_1}\frac{dv}{(1-v^2)^{1/2}},$$
$$J=\frac{m_1v_1^2}{\omega(1-v_1^2)^{1/2}}+\frac{m_2v_2^2}{\omega(1-v_2^2)^{1/2}}+\frac{1}{2\pi\alpha'\omega^2}\int_{-v_2}^{v_1}dv\,\frac{v^2}{(1-v^2)^{1/2}},\qquad(2.1)$$

where ω is the rotational frequency. The speeds v_1 and v_2 of the ends are determined so that the masses m_1 and m_2 move under the tension at their respective ends of the string. This gives the relation between the ends speeds v_1 and v_2 and the rotational frequency ω,

$$v_i=(1+m_i^2\pi^2\alpha'^2\omega^2)^{1/2}-m_i\pi\alpha'\omega.\qquad(2.2)$$

$1/2\pi\alpha'$ is the energy per unit length in the rest system of a point along the string, that is, the invariant tension. With the speeds determined by (2.2), (2.1) may be regarded as a parametric (ω) representation of the dependence $J=J(M)$.

The results should be the same when two colored point particles move classically in the bag formed from the balance of the classical color field energy carried by the particles against the bag confinement energy. The special case with massless quarks was worked out earlier.[3]

We suggest as a semiclassical version of (2.1) the same formulas with J in (2.1) replaced by $J-J_0$, where J_0 is taken as a quantum defect. We will now discuss various limiting cases of (2.1) modified by the inclusion of the "defect". As m_1 and m_2 tend to zero, the ends move at the velocity of light [but $m/(1-v^2)^{1/2}=0(\sqrt{m})\to 0$] and (2.1) reduces to the linear trajectory

$$M=\frac{1}{2\alpha'\omega}\quad J-J_0=\frac{1}{4\alpha'\omega^2},\quad\text{or}\quad J=\alpha'M^2+J_0.$$

When m_1 and m_2 are large and equal and α' is such that $v_1=v_2\ll 1$, we obtain (1.2).

Since both (1.1) and (1.2) give a good representation of the observed spectrum of mesons, (in one case those which contain two light quarks and in the other those which contain two heavy quarks) we expect with some confidence that the mesons which contain one massless quark and one heavy quark should also be well represented by (2.1) with $J\to J-J_0$. Although (2.1) is perfectly suited to actual calculations, we shall conclude this section with some alternative expressions which might be useful in special cases. If we apply (2.1) to the case of one massless quark and one heavy quark, it can be written in the simpler parametric form based upon the velocity of the heavy quark,

$$J=\pi m^2\alpha'\left[\frac{v^3}{(1-v^2)^{3/2}}+\frac{v^2}{(1-v^2)^2}(\pi/2+\sin^{-1}v)\right],$$
$$M=m\left[\frac{1}{(1-v^2)^{1/2}}+\frac{v}{(1-v^2)}(\pi/2+\sin^{-1}v)\right].\qquad(2.3)$$

In the limit when $M-m/m\ll 1$ and the velocity $v\ll 1$, we obtain the approximate form

$$J=2\alpha'(M-m)^2\left[1-\left(\frac{2}{\pi}\right)^2\frac{M-m}{m}+\cdots\right].\qquad(2.4)$$

In the opposite limit $v\to 1$ ($M\gg m$) we of course recover the linear (in M^2) dependence, $J\to\alpha'M^2$.

TABLE III. Linear (in M^2) fits of the ρ trajectory, for various values of the slope α'. $\alpha'=0.877$ is obtained by fitting the A_2 and g; $\alpha'=0.894$ by fitting the ρ and A_2. $\alpha'=0.886$ is obtained from a least squares fit to the three states. The intercept α_0 is calculated from (1) with $J=l+1$. Masses are given in GeV.

	ρ $l=0$	A_2 $l=1$	g $l=1$	α_0
$\alpha'=0.877$	0.759	...	...	0.495
$\alpha'=0.894$	...	...	1.684	0.466
$\alpha'=0.886$	0.770	1.312	1.688	0.520
Observed mass	0.773 ± 0.003	1.310 ± 0.005	1.690 ± 0.020	

In the case of the D's, the relativistic dependence is sufficiently important that (2.3) is about as convenient to use as the approximate form (2.4). In the case of the heaviest quarks discovered so far ($m\simeq 5$ GeV), the simple form $J-J_0=2\alpha'(M-m)^2$ will be sufficiently accurate for the lowest angular excited states of mesons with one heavy quark and one light quark.

III. APPLICATIONS

We can test the mass formula by comparing its predictions with the mass spectrum of the K^* family, which has three fairly well measured members, the $K^*(892)$, $K^*(1434)$, and $K^*(1784)$. These masses are accurately represented by a linear plot (see Table III) but with a somewhat lower slope parameter than that of the ρ family. However, if we use (2.1) and in accordance with the expected flavor independence, we choose for α' the *same* parameter used for the ρ states (see Table IV), and attempt to fit the spectrum by allowing the strange quark mass to be different from zero, we get an even more accurate representation (Table III) of the spectrum when $m_s=0.29$ GeV. Although this great accuracy is probably fortuitous, we note that in a study of the ground states of the light hadrons using the MIT bag model with massless up and down quarks, $m_s=0.28$ GeV was obtained. As a further test of the flavor independence of α' and validity of (2.1) we can compute the mass of the f' meson, if we assume that it is the rotationally excited version of the ϕ which contains two strange quarks. In this case we determine the defect from the mass of the $\phi(1.020)$ and use the strange-quark mass $m_s=0.29$ GeV obtained from the K^* system and universal parameter $\alpha'=0.886$ GeV^{-2}. We find the mass to be 1.540, which is to be compared to the experimentally determined mass 1.516. Because (2.1) works so well in this case, we propose to use the same parameter α' ($\simeq 0.88$ GeV^{-2}) in all cases for the heavy-quark states.

Naturally, we may also calculate the spectrum of F's with $m_s=0.29$ GeV using the general form (2.1). The predicted masses of D's and F's, etc. are given in Table V with two values for the

TABLE IV. The K^* trajectory with a least-squares fit to a straight line (I) and then (II) fits to the mass formula (2.1) with one massive quark (m_S) and one massless quark. In the latter case the "slope parameter" α' used is that obtained from the ρ trajectory by least-squares fit. The "defect" J_0 and mass m_S are obtained by fitting the $K^*(0.892)$ and $K^*(1.434)$. To see the sensitivity to the quark mass we also fit the $K^*(0.892)$ and $K^*(1.784)$ and compare with the intermediate case where m_S is given by the average of these fitted values. All quantities are in GeV units.

Observed masses	$K^*(0.892)$ (±0.004)	$K^*(1.434)$ (±0.005)	$K^*(1.784)$ (±0.010)	
Case I Linear fit				Intercept
$\alpha'=0.837$	0.904	1.418	1.791	$\alpha_0=0.316$
Case II				Defect
$\alpha'=0.886$				
$m=0.250$	fit	1.416	fit	$J_0=0.506$
$m=0.330$	fit	fit	1.805	$=0.595$
$m=0.290$ (average)	fit	1.425	1.794	$=0.550$

TABLE V. Predicted masses in GeV of rotationally excited D's and F's based upon (2.1). Predictions for two different heavy-quark masses are shown: m_c = 1.51 GeV allows one to parameterize semiclassically the observed $c\bar{c}$ spectrum, and m_c =1.65 GeV is obtained by a detailed study of the phenomenology of the charmonium spectrum (Ref. 8). The universal slope parameter α' =0.886 GeV^{-2} obtained from the ρ spectrum is used. The "defects" J_0 are obtained by fitting the l =0 states to the observed masses of the D's and F's (Refs. 9 and 10).

	fit $l=0$	$l=1$	$l=2$	$l=3$	
F m_c =1.65	2.030	2.620	2.960	3.233	
m_c =1.51	2.030	2.539	2.869	3.139	
F^* m_c =1.65	2.140	2.660	2.990	3.259	(max J)
m_c =1.51	2.140	2.596	2.914	3.177	
m_s =0.29 obtained from K^* spectrum					
D m_c =1.65	1.865	2.492	2.845	3.126	
1.51	1.865	2.409	2.752	3.029	
D^* 1.65	2.009	2.546	2.885	3.160	(max J)
1.51	2.009	2.482	2.808	3.077	
J/ψ m_c=1.51	3.097 fit	3.554 fit	3.840 (predicted) maximum J state ($J=1$, $l=2$ 3.77)		

charmed-quark mass. In one case we take the m_c which allows a fit of the charmonium spectrum using the semiclassical formula. In another case we used the charmed quark mass which best fits the detailed calculation of charmonium including some of the effects of short-range interactions which are important in $l=0$ states.

IV. CONCLUSIONS

We have shown in the case of the light-quark mesons that the observed flavor variations in the Chew-Frautschi plots can be accounted for by quark mass differences in a semiclassical relativistic model based upon a universal, flavor-independent linear confining interaction.[6] We have also indicated how this can be tested when the rotationally excited states of D's and F's are discovered.

ACKNOWLEDGMENT

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[1]See Table I.

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