Spin forces in charmonium spectroscopy

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The observed $\psi'(3.68)$-$\psi''(3.77)$ structure in e^+e^- annihilation implies the existence of a tensor-force effect in charmonium spectroscopy. We show that the magnitude of the effect is consistent with an analysis of ^3P_J levels using a phenomenological Hamiltonian linear in both $\vec{L}\cdot\vec{S}$ and S_{12}, the tensor force. Adding a linear $\vec{\sigma}_1\cdot\vec{\sigma}_2$ term one can account for all known charmonium levels below 4 GeV and predict the location of the rest of the spectrum, in particular the $^1P_1$ state at 3.27 GeV.

In this paper we explore some of the consequences of a two-body effective-Hamiltonian description of the observed charmonium resonances in e^+e^- annihilation. We parametrize the Hamiltonian in the form

$$H = H_0(L) + a\vec{\sigma}_1\cdot\vec{\sigma}_2 + b\,\vec{L}\cdot\vec{S} + c\,S_{12}\,, \qquad (1)$$

where the coefficients a, b, and c are small compared to the energy splittings within $H_0(L)$. These coefficients may vary with principal and angular momentum quantum numbers. Nevertheless, we find experimental indications that the variations in a and c are small. This leads us to explore the consequences of using constant coefficients in Eq. (1). Our approach and results differ from conventional potential models. In particular, we predict a low-lying $^1P_1$ level at 3.27 GeV, which can be produced by the hadronic decay of the $\psi'(3.684)$.

The recently observed resonance[1] $\psi''(3.772)$ has a substantial decay width into e^+e^- pairs, $\Gamma_{ee}(3.77) = 0.37 \pm 0.10$ keV. This observation is not compatible with its assignment as a pure $^3D_1$ system of charmonium; some $^3S_1$ component is required. This is the first example of configuration mixing within a given value of J in the sequence of charmonium states. Such a mixing can arise through the tensor force term of the Hamiltonian, Eq. (1). Identifying the $\psi'(3.684)$ as the $^3S_1 - {}^3D_1$ state orthogonal to the ψ'', one[1] concludes from the observed $\Gamma_{ee}(3.77)$ that the mixing angle is $(23\pm3)^0$.

If we assume that the imaginary part of the effective Hamiltonian $\underline{H}$ in the $^3S_1$-$^3D_1$ system is small compared to its real part,[2] we can determine $\underline{H}$ in terms of the physical mass and the mixing angle. In the $^3S_1$-$^3D_1$ basis, this system can be described by

$$H = \begin{pmatrix} 3.70 & \pm 0.03 \\ \pm 0.03 & 3.76 \end{pmatrix} \text{ GeV}\,. \qquad (2)$$

The off-diagonal matrix element in Eq. (2) can only come from the S_{12} term in Eq. (1). Using the standard definition,

$$S_{12} = 3\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2\,. \qquad (3)$$

It has the matrix elements

$$S_{12} = \begin{pmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & -2 \end{pmatrix}\,. \qquad (4)$$

From Eqs. (2) and (4) we find

$$c \simeq \pm 11 \text{ MeV}\,, \qquad (5)$$

where the sign ambiguity is due to the ambiguity in the off-diagonal element in Eq. (2).

Let us compare the value of c found in this way with the value of c found in the analysis of the two observed splittings of the $^3P_0(3.413)$, $^3P_1(3.508)$, and $^3P_2(3.552)$ charmonium states.[3,4] These splittings are due only to the $\vec{L}\cdot\vec{S}$ and S_{12} terms in Eq. (1). We fit b and c to the observed splittings and find

$$b \simeq 34 \text{ MeV}, \quad c \simeq 10 \text{ MeV}\,. \qquad (6)$$

Surprisingly enough, the spectroscopic value of c in Eq. (6) is almost equal to the positive solution of Eq. (5). We use the fact that the two values of c are so close as a justification for the assumption that the b values of the P and D state levels will be the same. This allows us to determine the location of the other ^3D states in terms of the location of the $^3D_1$ state at 3.76 GeV [Eq. (2)]: we expect the $^3D_{2,3}$ states to be at

3.87, 3.94 GeV. The $^3D_2$ state has $J^{PC}=2^{--}$; therefore its $D\bar{D}$ decay is forbidden by C invariance. While the $\bar{D}D^*$ or $D\bar{D}\pi$ mode is not forbidden, these channels are so close to threshold that they would be strongly suppressed. Therefore we expect $^3D_2$ to be a very narrow state, which may have an appreciable branching ratio into $\gamma+\chi$. By contrast, the $^3D_3$ is a 3^{--} state which will be broad by virtue of open $D\bar{D}$ and $D\bar{D}^*$ channels.

Until this point we have only used the $\vec{L}\cdot\vec{S}$ and S_{12} terms of Eq. (1). We summarize our results in Fig. 1. Using the values of b and c [Eq. (6)], we can find the central masses of all of the triplet ($S=1$) levels of the P, S, and D systems. As a consistency check, we note that the difference between the $1^3D(3.88)$ and $2^3S(3.70)$ spectral values is, indeed, larger than the parameters b and c. Hence a perturbative approach of the type used for the $\vec{L}\cdot\vec{S}$ and S_{12} terms in Eq. (1) seems to be justified.

The analysis of the $\vec{\sigma}_1\cdot\vec{\sigma}_2$ term in $\underline{H}$ suffers from the questionable identification of the 2.83 and 3.45 levels as η_c and η_c'. These assignments lead to $\vec{\sigma}_1\cdot\vec{\sigma}_2$ terms which are much bigger than expected in charmonium potential models; in addition, the calculation of photonic transitions to and from these levels is in complete disagreement with experiment.[5] Nonetheless, if that identification is made, the phenomenological equality

$$m_\psi - m_{\eta_c} \simeq m_{\psi'} - m_{\eta_c'}\,, \qquad (7)$$

implies that the coefficient a of Eq. (1) is also approximately constant. Its value is

$$a \simeq 66 \text{ MeV}\,, \qquad (8)$$

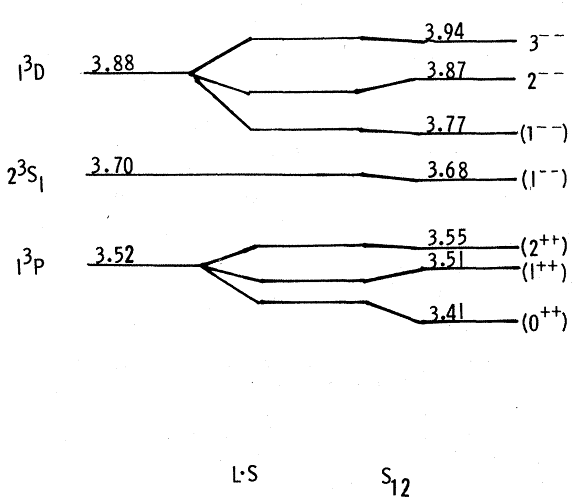

FIG. 1. Mass splittings of triplet states due to $\vec{L}\cdot\vec{S}$ and S_{12} forces as determined by the analysis in the text. The levels on the right are identified according to their J^{PC} values. Experimentally observed states are enclosed in parentheses.

which is definitely larger than the other coefficients and casts some doubt on the validity of the perturbative treatment. This leads us to a situation in which all of the coefficients in Eq. (1) are constant over the energy range studied. In particular, we shall assume that $a(1P)\simeq a(1S)$.

This drastic assumption is certainly outside the realm of potential models. We feel that the identifications $\eta_c(2.83)$ and $\eta_c'(3.45)$ have already forced us outside of the usual potential-model approach. However, we assume that the effective-Hamiltonian description is valid. Since we have previously encountered the situation where the parameter c did not change with L, we explore the consequences of assuming that a remains constant. Using the previously established value of $1^3P(3.52)$ we predict that the $^1P_1$ state lies at approximately 3.27 GeV. A state so far below the $\psi'(3.684)$ is amenable to detection through the hadronic decay of the ψ'. It can be formed[6] by

$$\psi' \to (2\pi)_{L=1} + {}^1P_1\,, \qquad (9)$$

or radiatively from the decay

$$\eta_c' \to \gamma + {}^1P_1\,. \qquad (10)$$

Analogously, it can decay into $2\pi+\eta_c$ or into $\gamma+\eta_c$. We expect $^1P_1$ to be narrow because the first decay mode is forbidden by the Okubo-

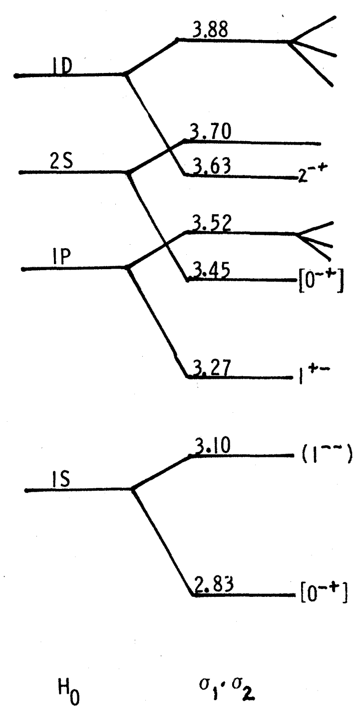

FIG. 2. Mass splittings due to $\vec{\sigma}_1\cdot\vec{\sigma}_2$ force. The 0^{-+} states, which lead to the big splittings that dominate this figure, are enclosed in square brackets because of uncertainty in their identification.

Zweig-Iizuka rule and the second is electromagnetic. The effect of the $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ term is displayed in Fig. 2 where we have also included a ${}^1D_2$ state at 3.63 GeV obtained through the further assumption that $a(1D) \simeq a(2S)$.

Our results disagree with previous approaches to charmonium spectroscopy which start from an ansatz for H_0—generally incorporating linearly rising vector potentials—and derive a, b, and c from $O((v/c)^2)$ relativistic corrections.[7] These models obtain the right signs for the coefficients but have some problems in predicting their size and configuration dependence. Characteristically their values decrease with increasing mass; as we have seen above, at least part of this behavior requires modification. For example, Schnitzer[8] introduced an anomalous quark magnetic moment to account for the size and behavior of the spin forces in the potential model.

Our approach is phenomenological, using the observed trends of the data as a guide to help sort out the qualitative features of the observed spectrum. The relation of the Hamiltonian (1) to the charmonium picture lies in the two-body language we use in its explicit construction. However, the effects incorporated in the coefficients of Eq. (1) may well go beyond potential models. In particular, off-shell contributions of closed charmed channels[9] can have strong effects on the terms that we discussed. Moreover, nonperturbative QCD effects,[10] may cause considerable deviation from simple potential models which were fashioned after QED predecessors.

A big challenge in charmonium spectroscopy above 3.8 GeV is to find out where the naive $c\bar{c}$ picture breaks down and how to amend it. Every peak in the observed R spectrum in this region represents a resonance which has a component of a ${}^3S_1$, $c\bar{c}$ state. An interesting question is at what mass will the number and spacing of the resonances necessitate the inclusion of states beyond those of simple charmonium. The use of a phenomenological Hamiltonian like Eq. (1) may help in deciding this issue.

Since this paper was completed, there has been another measurement[11] of the leptonic decay width of the $\psi''(3.772)$: $\Gamma_{ee}(3.77) = 0.18 \pm 0.06$ keV. If we use this width, the mixing angle is modified from 23° to a new value of 16°; however the off-diagonal matrix elements vary[2] as $\tan^2\alpha$, and hence c determined from Eqs. (2) and (4) is reduced in magnitude only to 8.5 MeV. This is nearly as good a fit to the c of Eq. (6) as the c of Eq. (5), based on the first measurement of $\Gamma_{ee}(3.77)$. Our hypotheses and conclusions are therefore little changed by this more recent measurement.

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[2]For an extensive discussion of this point see D. Horn and D. E. Novoseller (unpublished).

[3]The same analysis for the χ states was carried out by J. D. Jackson in SLAC Report No. 198, 1976 (unpublished), p. 147.

[4]See, e.g., the review paper by G. J. Feldman and M. L. Perl [Phys. Rep. 33C, 285 (1977)] for a summary of the relevant experimental data, theoretical interpretation, and list of references.

[5]For a recent discussion and summary see J. D. Jackson, in *Proceedings of the* 1977 *European Conference on Particle Physics*, *Budapest*, edited by L. Jenik and I. Montray (CRIP, Budapest, 1978); K. Gottfried, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies*, *Hamburg*, *1977*, edited by F. Gutbrod (DESY, Hamburg, 1977).

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